Problem04

Saturday, April 2, 2022 06:03

4. [20 points] Consider the following set of models:

$$\mathcal{I} := \left\{ \begin{array}{c} G(s) \\ 1 + w(s)\Delta(s) \end{array} \;\middle|\; \Delta \text{ stable}, \|\Delta\|_\infty < 1 \right\}$$

$G(s)$ is SISO and $w(s)$ is a stable, proper rational transfer function.

(a) Draw a block diagram showing the structure of any $G_p \in \mathcal{I}$. Your diagram should only include blocks for G, w, and Δ.

(b) Consider the feedback diagram show in Figure 4. Assume the controller K stabilizes the nominal system G. Find a necessary and sufficient condition for K to stabilize all $G_p \in \mathcal{I}$. Provide a proof of sufficiency, i.e., prove that K achieves robust stability if your condition is satisfied. You do not need to prove necessity.

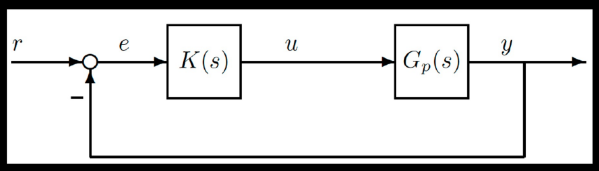
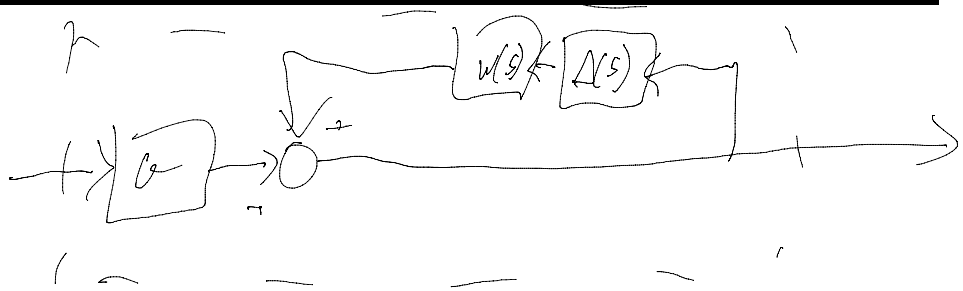


Fig. 4: Feedback loop

a)



b)

$$\frac{KG}{1 + w(Gw)}$$ stable $\forall w$

Proof:

This can be proven w/ Nyquist criteria...

(don't have time to write out specifics, but essentially that it must just remain