MECH 6323 - HW 1

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1 Problem 1

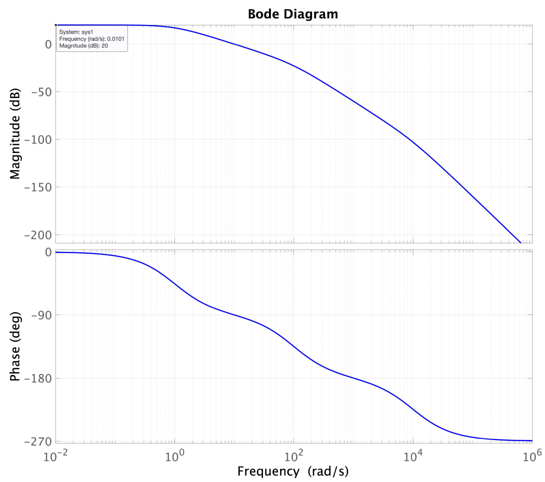
2 Problem 3

Problem: For each of the bode plots:

1. Determine the breakpoints and the transfer function.

2. Determine the gain cross-over frequency ω_c and the phase cross-over frequency ω_{180}.

2.1 Bode Plot 1:



2.1.1 Gain, Poles, and Zeros:

1. **Gain:** 20 db = 10

2. **Poles:**

   (a) $10^0 = 1$ rad/s

   (b) $10^2 = 100$ rad/s

   (c) $10^4 = 10,000$ rad/s

3. Zeros: (NA)

Transfer Function:

$$H(s) = \frac{10}{\left(1 + \frac{s}{1}\right)\left(1 + \frac{s}{100}\right)\left(1 + \frac{s}{10000}\right)}$$

2.1.2 Cross-over Frequency:

1. $\omega_c = 10^1 = 10$ rad/s

2. $\omega_{180} = 10^3 = 100$ rad/s

2.2 Bode Plot 2:

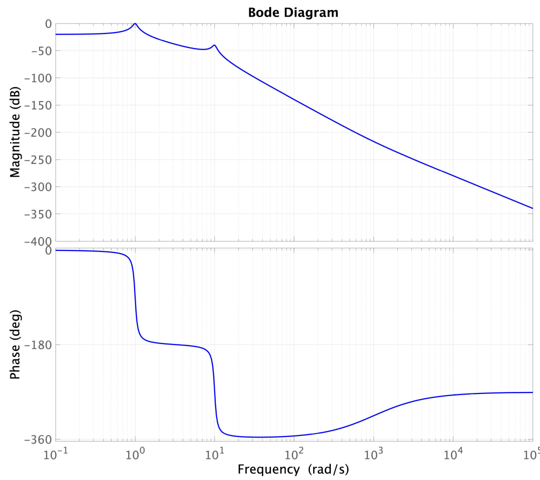


Figure 1: Bode Plot 2

2.2.1 Gain, Poles, and Zeros:

1. **Gain:** -20 db $= \frac{1}{10}$
2. **Poles:**
   (a) $10^0 = 1$ rad/s (complex)
   (b) $10^1 = 10$ rad/s (complex)
3. Zeros:
   (a) $10^3 = 1,000$ rad/s

2.2.2 Transfer Function:

$$H(s) = \frac{(1+\frac{s}{1000})}{10\big(\frac{1}{1}\big(s^2+2(\frac{1}{10})(1)s+(1)^2\big)\big)\big(\frac{1}{10}\big(s^2+2(\frac{1}{10})(10)s+(10)^2\big)\big)} = \frac{(s+1000)}{(s^2+0.2s+1)(s^2+2s+100)}$$

Assuming a Q-factor of around 10 to get the complex response.

2.2.3 Cross-over Frequency:

1. $\omega_c = 10^0 = 1$ rad/s
2. $\omega_{180} = 10^3 = 100$ rad/s

3 Problem 4

Consider the interconection of Problem 1 with the PI controller

$$C(s) = \frac{10(s+3)}{s}$$

and plant

$$P(s) = \frac{-0.5(s^2-2000)}{(s-3)(s^2+50s+1000)}$$

3.1 Is the feedback system stable? Why?

$$\begin{aligned}\frac{C(s)P(s)}{1+C(s)P(s)} &= \frac{\frac{10(s+3)}{s}\frac{-0.5(s^2-2000)}{(s-3)(s^2+50s+1000)}}{1+\frac{10(s+3)}{s}\frac{-0.5(s^2-2000)}{(s-3)(s^2+50s+1000)}} \\ &\approx \frac{-5s(s+3)(s-3)(s^2-2000)(s^2+50s+1000)}{s(s-3)(s^2+11.73s+73.9)(s^2+30.27s+406)(s^2+50s+1000)}\end{aligned}$$

Yes and No. Internally it is not fully stable since it has a pole/zero pair at $s = 3$; however, if we only care about TF after cancellations, then it is stable.

3.2 Find phase and cross-over frequencies.

Problem: Use the Bode plot of the open loop transfer function $L(s) = C(s)P(s)$ to find the phase cross-over frequencies ω_0 such that $L(j\omega_0) = 180\,\text{deg}$. Use this information to compute the gain margin(s) of the feedback system. Check your answers using the *allmargin* command in MATLAB.

Solution: As marked in the Bode Plot seen in Figure 3.2, the gain cross-over frequency is $\omega_c = 10$ resulting in a phase margin around 25 deg. Similarly, the phase cross-over occurs around $\omega_{180} = 4$ or $\omega_{180} = 25$, resulting in gain margins of around ± 10 dB or around $g_0 = 0.3$ and $g_0 = 3$ respectively.

Verification with *allmargin* resulted in similar and likely more precise and accurate results:

1. Gain Margin(s)
   (a) $g_0 = 0.3585$ at $\omega_{180} = 3.5966$
   (b) $g_0 = 2.6490$ at $\omega_{180} = 26.3797$
2. Phase Margin
   (a) $PM = 27.5718$ at $\omega_c = 10.2049$

3.3 Gain Margin Closed-loop poles

Problem: For each gain margin g_0 obtained in the previous part, construct the closed-loop using the perturbed loop transfer function $g_0L(s)$ and verify that the closed-loop has poles at $\pm\omega_0$.

Solution:

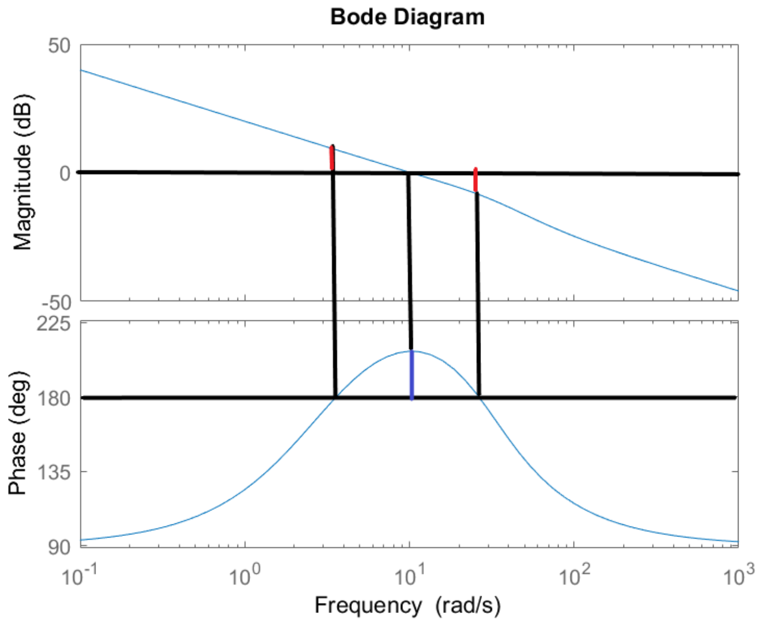


Figure 2: Open Loop Bode Plot of $L(S) = C(s)P(s)$

3.3.1 $g_0 = 0.3585$

Let $g_0 = 0.3585$,

$$g_0 L(s) = \frac{-1.7927(s+3)(s+44.72)(s-44.72)}{s(s-3)(s^2+50s+1000)}$$

and the closed-loop becomes

$$H = \frac{-1.7927(s-44.72)(s+44.72)(s+3)}{(s^2+0.0008164s+12.93)(s^2+45.21s+831.7)}$$

This has complex poles located at $-0.0004 \pm j3.596$, which is essentially roots at $\pm j\omega_{180}$.

3.3.2 $g_0 = 2.6490$

Let $g_0 = 2.6490$,

$$g_0 L(s) = \frac{-13.245(s+3)(s+44.72)(s-44.72)}{s(s-3)(s^2+50s+1000)}$$

and the closed-loop becomes

$$H = \frac{-13.245(s+3)(s+44.72)(s-44.72)}{(s+29.94)(s+3.814)(s^2+0.003402s+696)}$$

This has complex poles located at $-0.0017 \pm j26.3818$, which is essentially roots at $\pm j\omega_{180}$.

3.4 $\|S-T\|_\infty$

Problem: Compute $\|S-T\|_\infty$ and the corresponding frequency ω_p where the peak gain of $S-T$ is achieved.

Solution: