MECH 6323 - HW 07

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Problem 1

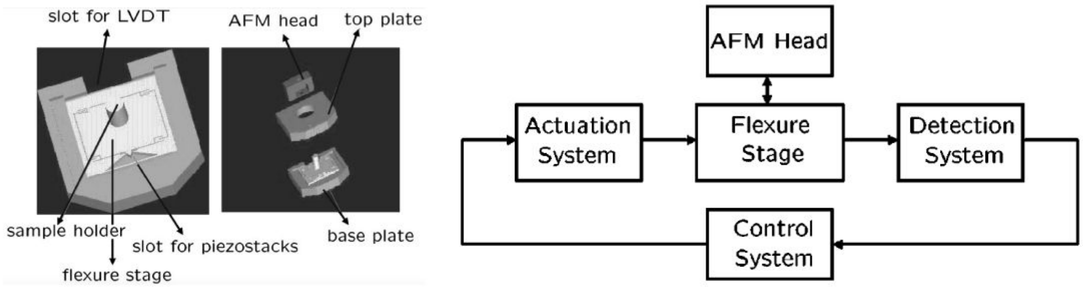


Fig. 1: Nanopositioning flexure stage (left) and feedback diagram (right); figures adapted from Salapaka et. al, *Rev. Sci. Instrum.* 2002.

```
clear
close all
```

Part a

Load data: G, w, nano_sp.Gfr

```
% load('C:\Users\Jonas\OneDrive - The University of Texas at Dallas\2022_Spring\MECH6323\Homewo
nano_rsp = load('npresp.mat')
```

```
nano_rsp = struct with fields:
    Gfr: [1×1 frd]
      w: [1748×1 double]
      G: [1×1×1748 double]
```

```
omega_min = min(nano_rsp.w);
omega_max = max(nano_rsp.w);
```

Estimate system transfer function

```
tf_order = 6;
G_sys = fitfrd(nano_rsp.Gfr, tf_order)
```

```
G_sys =

  A = 
          x1     x2     x3     x4     x5     x6
   x1  -1468   8540  -8540   8540  -8540   4270
   x2  -4681   6559  -3757   3757  -3757   1878
   x3  -2174   4348  -7150   9952  -9952   4976
   x4  -4695   9390  -9390   6588  -3786   1893
   x5  -1687   3374  -3374   3374  -6176   4489
   x6  -3428   6856  -6856   6856  -6856  625.7
```

```
  B = 
          u1
   x1  5.156
   x2  11.36
   x3   16.7
   x4  18.15
   x5  11.59
   x6  11.18

  C = 
           x1      x2      x3      x4      x5      x6
   y1   114.1  -228.1   228.1  -228.1   228.1  -114.1

  D = 
           u1
   y1  0.08876

Continuous-time state-space model.
```

```
G_sys_tf = tf(G_sys)
```

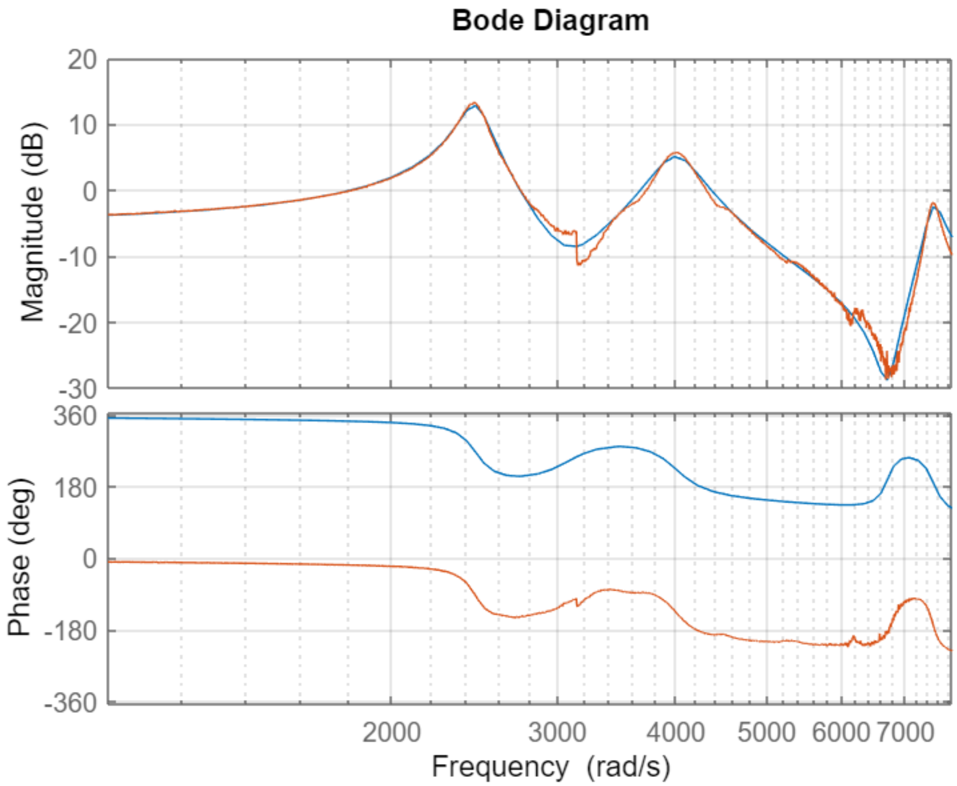
```
G_sys_tf =
 
  0.08876 s^6 - 876.1 s^5 + 1.136e07 s^4 - 4.345e10 s^3 + 4.097e14 s^2 - 2.095e17 s + 3.082e21
  ---------------------------------------------------------------------------------------------
       s^6 + 1021 s^5 + 7.856e07 s^4 + 5.129e10 s^3 + 1.342e15 s^2 + 3.65e17 s + 5.421e21
 
Continuous-time transfer function.
```

```
G_sys_zpk = zpk(G_sys)
```

```
G_sys_zpk =
 
  0.088762 (s^2 + 526.7s + 9.417e06) (s^2 + 276.6s + 4.494e07) (s^2 - 1.067e04s + 8.207e07)
  ------------------------------------------------------------------------------------------
         (s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)
 
Continuous-time zero/pole/gain model.
```

Bode Diagram Data

```
figure()
bode(G_sys)
hold on
bode(nano_rsp.Gfr)
grid on
xlim([omega_min, omega_max])
```



Clearly this plot is a good estimation for the frequency response of the system, noting that the phase of the system is offset by a 360 degree phase shift (which implies a need for another set of integrators)

Part b

PI - Implimentation

```
PM_min = 75;
opt = pidtuneOptions( ...
    'PhaseMargin', PM_min, ...
    'DesignFocus', 'disturbance-rejection' ...
    )
```

```
opt =
  pidtune with properties:

        PhaseMargin: 75
    NumUnstablePoles: 0
        DesignFocus: 'disturbance-rejection'
```

```
[C_pi, info] = pidtune(G_sys, 'pi', opt)
```

```
C_pi =

        1
  Ki * ---
        s

  with Ki = 240
```

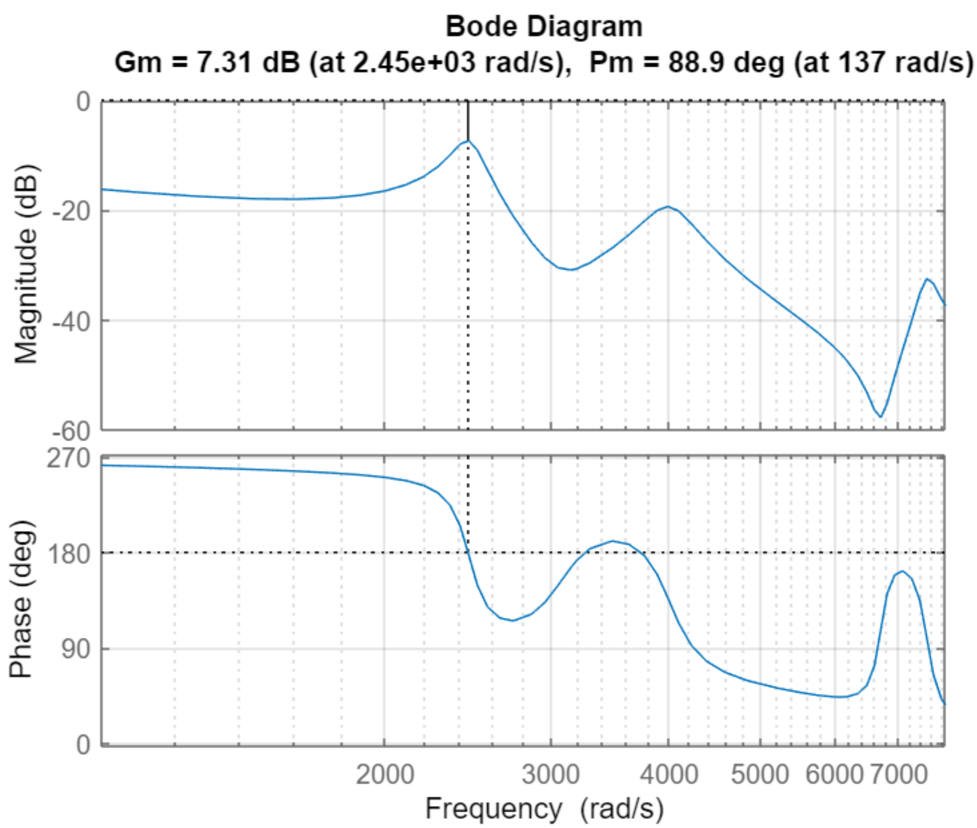
```
Continuous-time I-only controller.
info = struct with fields:
                Stable: 1
    CrossoverFrequency: 136.5946
           PhaseMargin: 88.9408
```

```
allmargin(C_pi * G_sys)
```

```
ans = struct with fields:
     GainMargin: [2.3213 31.8215 13.3190]
    GMFrequency: [2.4472e+03 3.2556e+03 3.7346e+03]
    PhaseMargin: 88.9408
    PMFrequency: 136.5946
    DelayMargin: 0.0114
    DMFrequency: 136.5946
         Stable: 1
```

Bode Diagram of Margin

```
figure
margin(C_pi * G_sys_tf)
xlim([omega_min, omega_max])
grid on
```



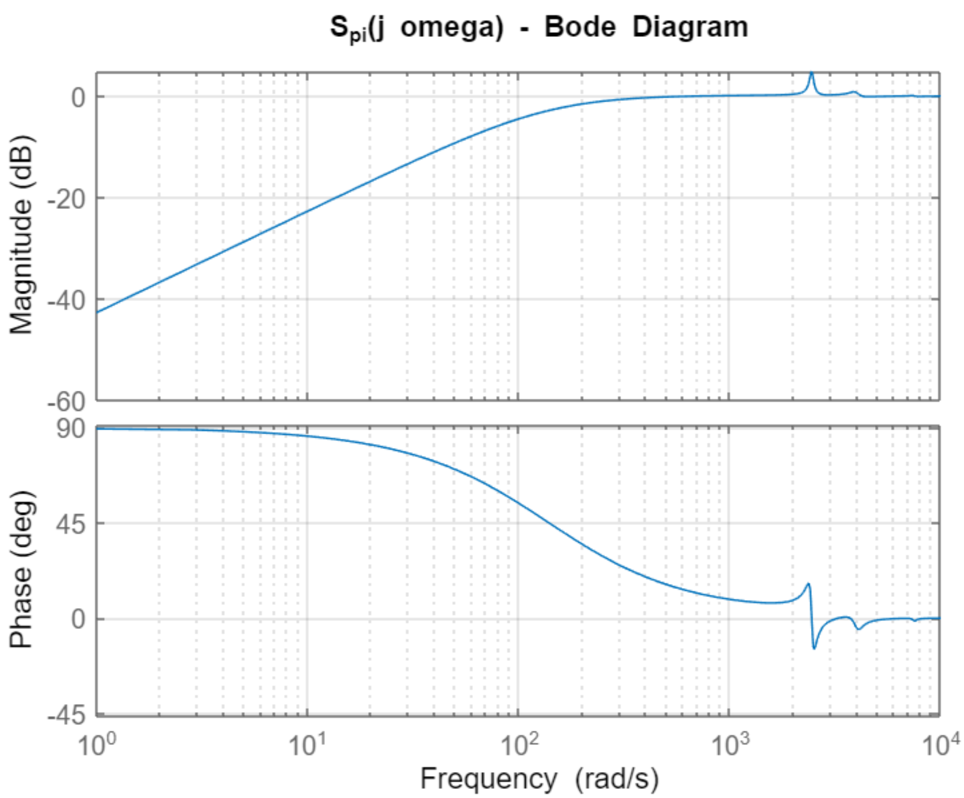
As a result, the single only integral controller is a pretty weird result. However, looking at the results it does make sense.

Sensivity Transfer Function

```
S_pi = zpk(1/(1+C_pi*G_sys))
```

```
S_pi =

       s (s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)
  ---------------------------------------------------------------------------------
  (s+138.6) (s^2 + 108.1s + 5.995e06) (s^2 + 446.2s + 1.584e07) (s^2 + 349.8s + 5.615e07)

Continuous-time zero/pole/gain model.
```

```
figure
bode(S_pi)
title('S_{pi}(j omega) - Bode Diagram')
grid on
```

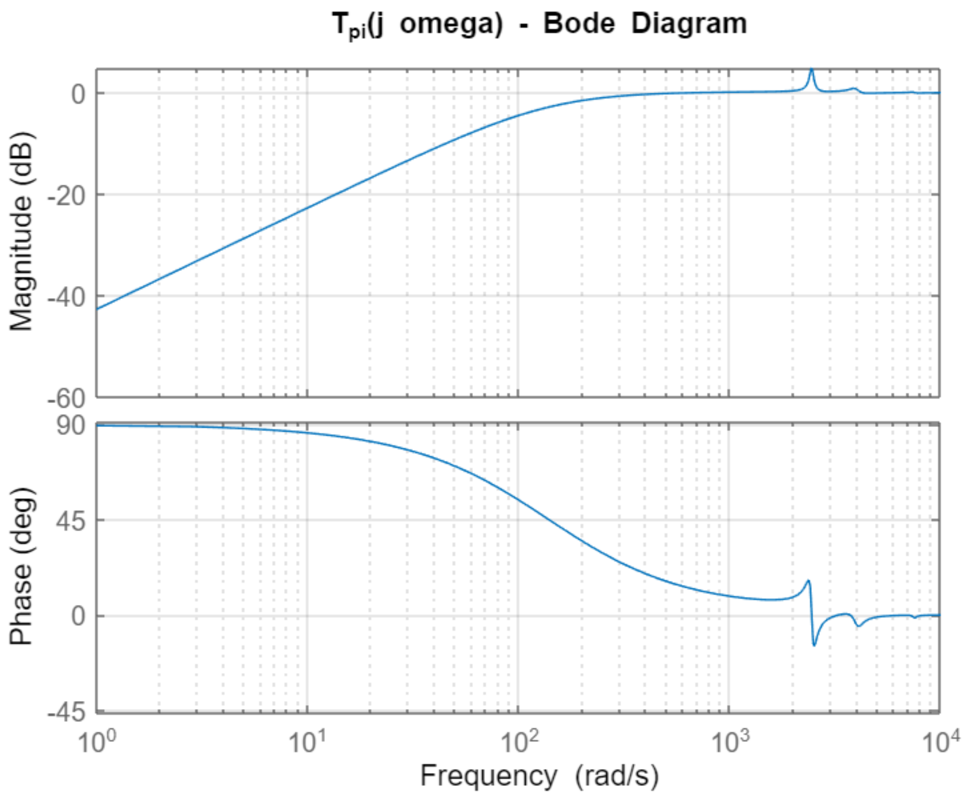


Complimentary Transfer Function

```
T_pi = zpk(1/(1+C_pi*G_sys))
```

```
T_pi =

       s (s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)
  ---------------------------------------------------------------------------------
  (s+138.6) (s^2 + 108.1s + 5.995e06) (s^2 + 446.2s + 1.584e07) (s^2 + 349.8s + 5.615e07)

Continuous-time zero/pole/gain model.
```

```
figure
bode(T_pi)
title('T_{pi}(j omega) - Bode Diagram')
grid on
```



Bandwith Calculation

```
bw_threshold = -3; %db
bw = getGainCrossover(S_pi, db2mag(bw_threshold))
```

```
bw = 134.4361
```

Double Integrator implimenation

Instead we can also include an additional integrator into the controller, i.e.

```
G_int = tf(1,[1, 0]);
[C_pi_int, info] = pidtune(G_int * G_sys, 'pi', opt)
```

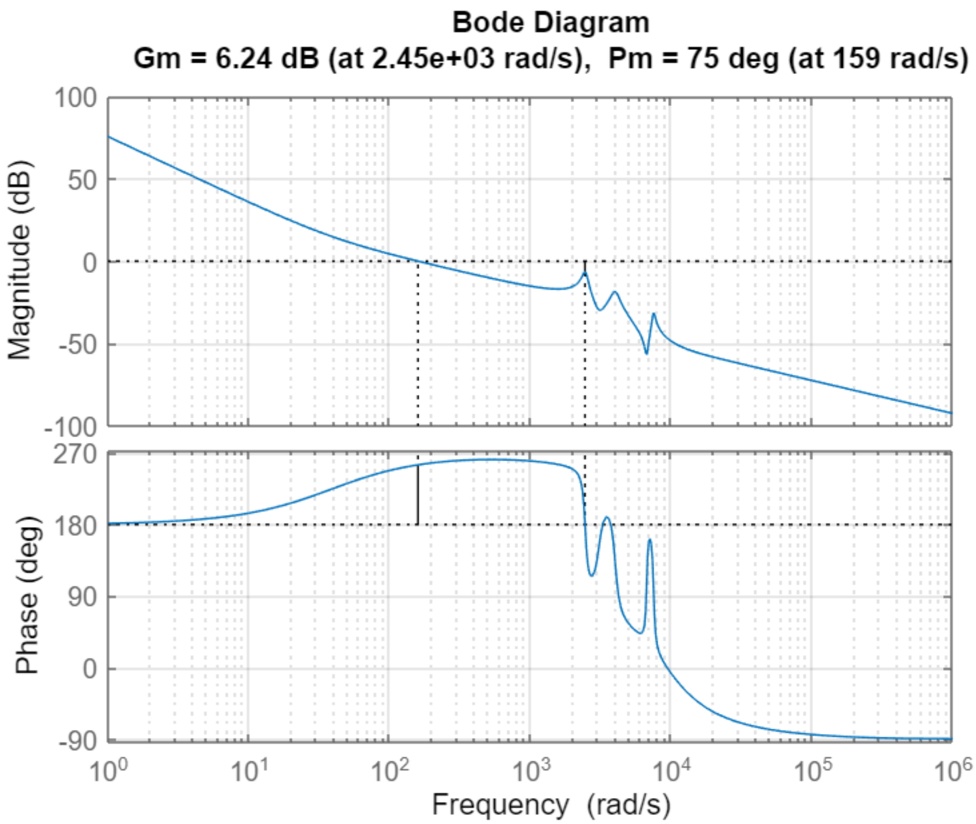
```
C_pi_int =

             1
  Kp + Ki * ---
             s

  with Kp = 271, Ki = 1.06e+04

Continuous-time PI controller in parallel form.
info = struct with fields:
                Stable: 1
     CrossoverFrequency: 159.0729
           PhaseMargin: 75.0000
```

```
allmargin(C_pi_int * G_int * G_sys)
```

```
ans = struct with fields:
```

```
     GainMargin: [0 2.0518 27.8933 11.9425]
    GMFrequency: [0 2.4457e+03 3.2626e+03 3.7282e+03]
    PhaseMargin: 75.0002
    PMFrequency: 159.0759
    DelayMargin: 0.0082
    DMFrequency: 159.0759
         Stable: 1
```

```
figure
margin(C_pi_int * G_int * G_sys)
grid on
```

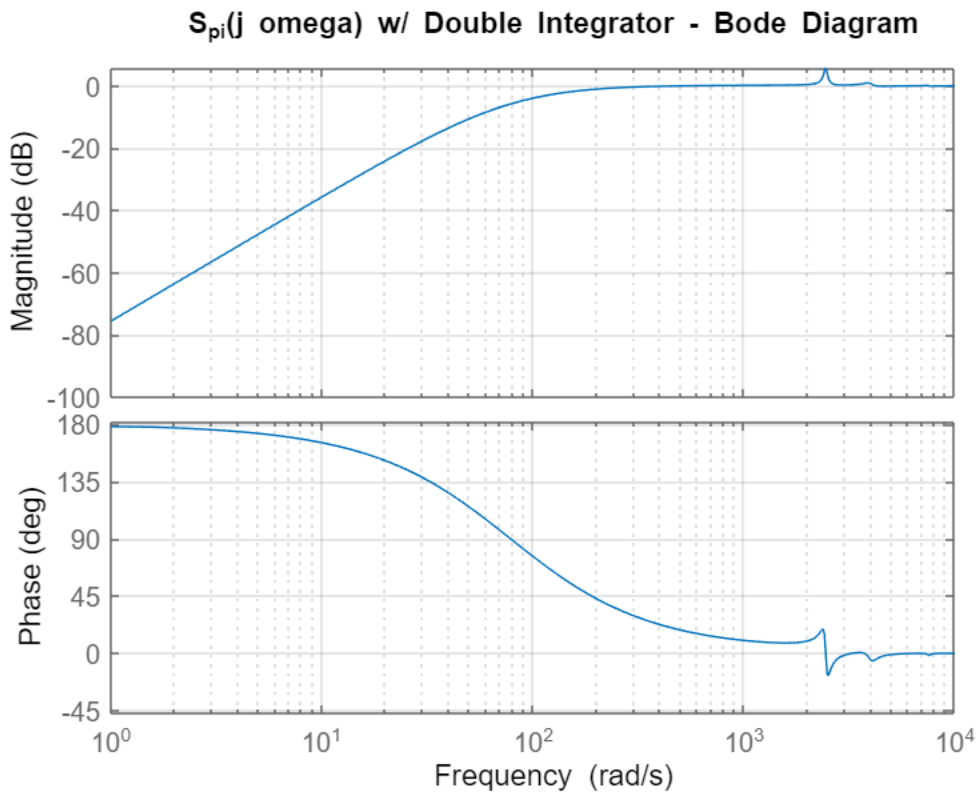


Sensivity Transfer Function

```
S_pi_int = zpk(1/(1+C_pi_int*G_int*G_sys))
```

```
S_pi_int =

            s^2 (s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)
  ----------------------------------------------------------------------------------------------------
  (s^2 + 156.3s + 6120) (s^2 + 97.81s + 5.989e06) (s^2 + 441.9s + 1.582e07) (s^2 + 349.5s + 5.614e07)

Continuous-time zero/pole/gain model.
```

```
figure
bode(S_pi_int)
title('S_{pi}(j omega) w/ Double Integrator - Bode Diagram')
grid on
```

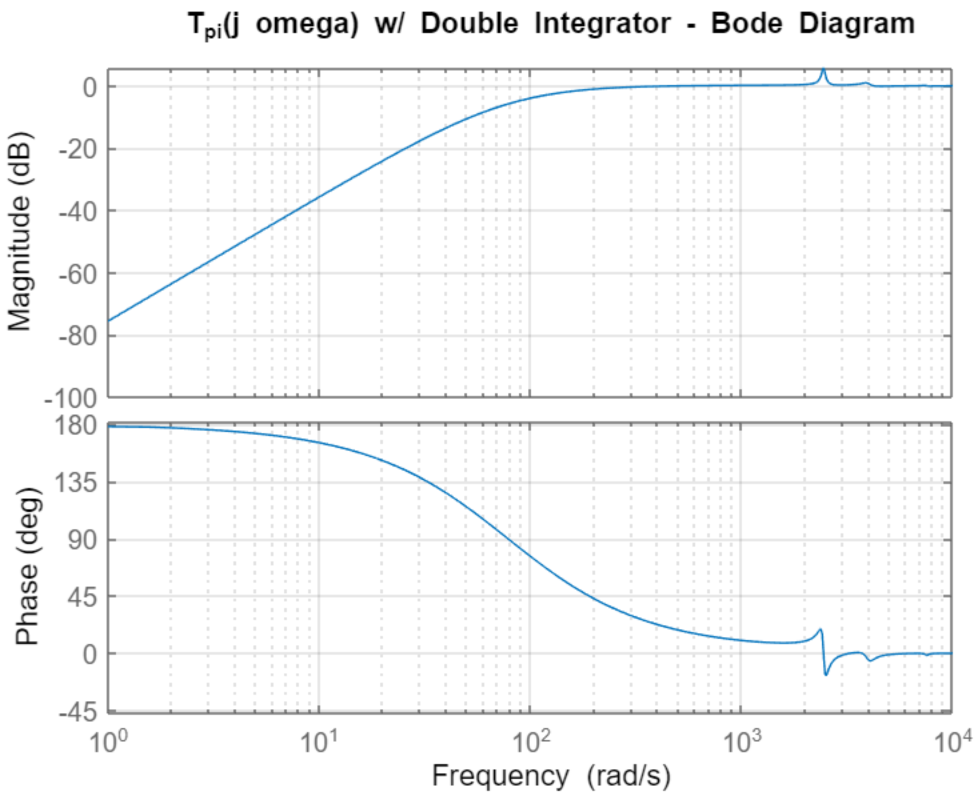


Complimentary Transfer Function

```
T_pi_int = zpk(1/(1+C_pi_int*G_int*G_sys))
```

```
T_pi_int =

            s^2 (s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)
  ----------------------------------------------------------------------------------------------------
  (s^2 + 156.3s + 6120) (s^2 + 97.81s + 5.989e06) (s^2 + 441.9s + 1.582e07) (s^2 + 349.5s + 5.614e07)

Continuous-time zero/pole/gain model.
```

```
figure
bode(T_pi_int)
title('T_{pi}(j omega) w/ Double Integrator - Bode Diagram')
grid on
```



Bandwith Calculation

```
bw_threshold = -3; %db
bw = getGainCrossover(S_pi_int, db2mag(bw_threshold))
```

```
bw = 117.7945
```

The bandwidth of this method is not as great as the original PI implimenation. I think the performance is better in certain situation though and may be worth implimenting (even if it isn't really a PI controller)

Part c

Specs:

1. Bandwidth ($|S(j\omega)| = -3$ dB) is around 250 Hz
2. $|S(j\omega)| \leq 1.5 \ \ \forall_\omega$
3. Slope below bandwidth = 20 dB/decade
4. DC gain of $S \leq -80$dB
5. $|T(j\omega)| < -3$dB @ 500 Hz
6. $|T(j\omega)| \leq 1.5 \ \ \forall_\omega$
7. $|T(j\omega)| < -40$dB as $\omega \to \infty$
8. $|C_\infty S(j\omega)| \leq 10 \ \ \forall_\omega$

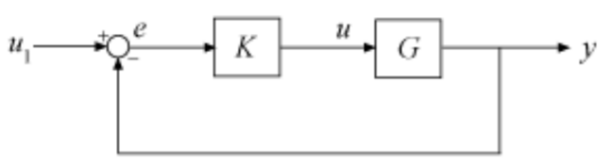
Design Weights

In order design using the mixed sensitivity design approach, we shape $S_{pi}(s)$ and $T_{pi}(s)$ to achieve the desired performance and robustness specs using weighting functions that are inverse of thoose desired shapes.

`[K,CL,gamma,info] = mixsyn(G,W1,W2,W3)` computes a controller that minimizes the H_∞ norm of the weighted closed-loop transfer function

$$M(s) = \begin{bmatrix} W_1 S \\ W_2 K S \\ W_3 T \end{bmatrix},$$

where $S = (I + GK)^{-1}$ and $T = (I - S)$ is the complementary sensitivity of the following control system.



`mixsyn` computes the controller `K` that yields the minimum $||M(s)||_\infty$, which is returned as `gamma`. For the returned controller K,

$$\begin{aligned} ||S||_\infty &\le \gamma |W_1^{-1}| \\ ||KS||_\infty &\le \gamma |W_2^{-1}| \\ ||T||_\infty &\le \gamma |W_3^{-1}|. \end{aligned}$$

Description

`makeweight` is a convenient way to specify loop shapes, target gain profiles, or weighting functions for applications such as controller synthesis and control system tuning.

`W = makeweight(dcgain,[freq,mag],hfgain)` creates a first-order, continuous-time weight $W(s)$ satisfying these constraints: example

$$\begin{aligned} W(0) &= \text{dcgain} \\ W(\text{Inf}) &= \text{hfgain} \\ |W(j \cdot \text{freq})| &= \text{mag}. \end{aligned}$$

In other words, the gain of `W` passes through `mag` at the finite frequency `freq`.

W_1 - Shaping S: $||S||_\infty \le \gamma |W_1^{-1}|$

```
dcgain_1 = db2mag(-80);% Spec 4
hfgain_1 = 1.5; % Spec 2
bw_1 = 250; % Spec 1
W_1 = makeweight(dcgain_1, [2*pi*bw_1, db2mag(-3)], hfgain_1)
```

```
W_1 =

  A = 
           x1
   x1   -2934
```

```
  B =
       u1
   x1  64

  C =
           x1
   y1  -68.77

  D =
        u1
   y1  1.5

Continuous-time state-space model.
```

W_2 - Shaping KS: $\quad ||KS||_\infty \leq \gamma |W_2^{-1}|$

```
u_max = 10;
W_2 = tf(1/u_max)
```

```
W_2 =

  0.1

Static gain.
```

W_3 - Shaping T: $\quad ||T||_\infty \leq \gamma |W_3^{-1}|$

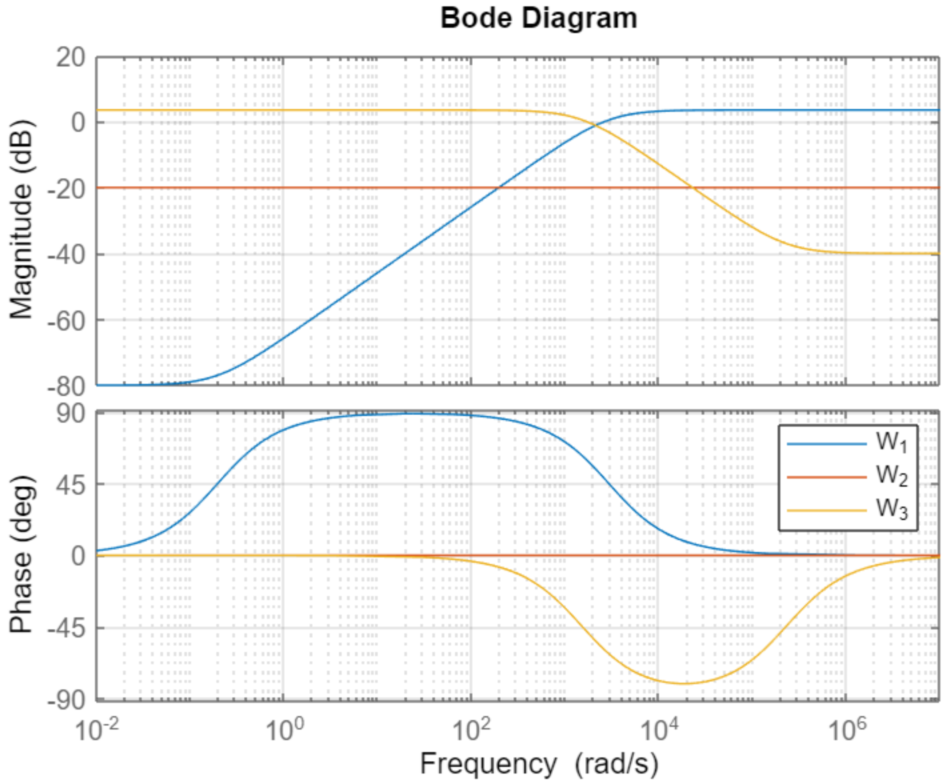
```
dcgain_3 = 1.5; % Spec 6 (max KS should be less then 1.5)
hfgain_3 = db2mag(-40); % Spec 7
bw_3 = 450; %should be less then 500 to ensure below -3dB @ 500 Hz
W_3 = makeweight(dcgain_3, [2*pi*bw_3, db2mag(-3)], hfgain_3)
```

```
W_3 =

  A =
          x1
   x1  -1513

  B =
       u1
   x1  64

  C =
          x1
   y1  35.24

  D =
         u1
   y1  0.01

Continuous-time state-space model.
```

Plotting Weighting functions

```
figure
hold on
bode(W_1)
bode(W_2)
```

```
bode(W_3)
legend('W_1','W_2','W_3')
grid on
```



H_∞ controller Calculation

```
[C_Hinf,CL,gamma,info] = mixsyn(G_sys,W_1,W_2,W_3)
```

```
C_Hinf =

  A =
              x1          x2          x3          x4          x5          x6          x7          x8
   x1       -2934   -1.51e-10   2.547e-11   4.657e-10  -9.313e-10   6.985e-10  -1.164e-10   8.731e-11
   x2   9.353e+04  -1.527e+04  -5.738e+04  -3.866e+06   4.894e+06  -4.182e+06   2.764e+06  -6.636e+05
   x3   8.488e+04  -1.249e+04  -6.017e+04  -3.487e+06    4.42e+06  -3.774e+06   2.486e+06  -5.914e+05
   x4   1.871e+05  -2.752e+04   -1.34e+05  -7.697e+06   9.756e+06  -8.331e+06   5.494e+06  -1.311e+06
   x5    2.75e+05  -4.046e+04  -1.924e+05  -1.132e+07   1.434e+07  -1.224e+07   8.074e+06  -1.925e+06
   x6   2.989e+05  -4.397e+04  -2.114e+05   -1.23e+07   1.558e+07  -1.331e+07   8.781e+06  -2.095e+06
   x7   1.908e+05  -2.807e+04  -1.336e+05  -7.853e+06    9.95e+06  -8.497e+06   5.601e+06  -1.334e+06
   x8   1.841e+05  -2.709e+04  -1.308e+05  -7.576e+06     9.6e+06  -8.198e+06   5.405e+06  -1.291e+06

  B =
          u1
   x1     64
   x2  113.4
   x3    103
   x4  226.9
   x5  333.6
   x6  362.5
   x7  231.4
   x8  223.3
```

```
  C = 
             x1          x2          x3          x4          x5          x6          x7          x8
   y1   1.646e+04       -2422  -1.139e+04   -6.78e+05     8.59e+05  -7.336e+05   4.839e+05  -1.155e+05
 
  D = 
         u1
   y1  19.97
 
Continuous-time state-space model.


CL =
 
  A = 
               x1          x2          x3          x4          x5          x6          x7          x8          x9
   x1       -2934           0       -2633        5266       -5266        5266       -5266        2633  -3.373e+04
   x2           0       -1513        2633       -5266        5266       -5266        5266       -2633   3.373e+04
   x3           0           0       -5704   1.701e+04  -1.701e+04   1.701e+04  -1.701e+04        8506   3.062e+04
   x4           0           0  -1.402e+04   2.523e+04  -2.243e+04   2.243e+04  -2.243e+04   1.121e+04   6.747e+04
   x5           0           0   -1.59e+04    3.18e+04   -3.46e+04    3.74e+04   -3.74e+04    1.87e+04    9.92e+04
   x6           0           0  -1.961e+04   3.922e+04  -3.922e+04   3.642e+04  -3.362e+04   1.681e+04   1.078e+05
   x7           0           0  -1.121e+04   2.241e+04  -2.241e+04   2.241e+04  -2.522e+04   1.401e+04    6.88e+04
   x8           0           0  -1.262e+04   2.523e+04  -2.523e+04   2.523e+04  -2.523e+04        9815   6.641e+04
   x9           0           0       -2633        5266       -5266        5266       -5266        2633  -3.667e+04
   x10          0           0       -4667        9335       -9335        9335       -9335        4667   3.373e+04
   x11          0           0       -4236        8472       -8472        8472       -8472        4236   3.062e+04
   x12          0           0       -9335   1.867e+04  -1.867e+04   1.867e+04  -1.867e+04        9335   6.747e+04
   x13          0           0  -1.372e+04   2.745e+04  -2.745e+04   2.745e+04  -2.745e+04   1.372e+04    9.92e+04
   x14          0           0  -1.492e+04   2.983e+04  -2.983e+04   2.983e+04  -2.983e+04   1.492e+04   1.078e+05
   x15          0           0       -9520   1.904e+04  -1.904e+04   1.904e+04  -1.904e+04        9520    6.88e+04
   x16          0           0       -9189   1.838e+04  -1.838e+04   1.838e+04  -1.838e+04        9189   6.641e+04
 
              x16
   x1    2.367e+05
   x2   -2.367e+05
   x3   -2.148e+05
   x4   -4.734e+05
   x5    -6.96e+05
   x6   -7.564e+05
   x7   -4.828e+05
   x8    -4.66e+05
   x9    2.367e+05
   x10   -2.44e+05
   x11  -2.106e+05
   x12  -4.715e+05
   x13  -6.911e+05
   x14  -7.545e+05
   x15  -4.783e+05
   x16  -4.654e+05
 
  B = 
           u1
   x1   23.08
   x2   40.92
   x3   37.13
   x4   81.83
   x5   120.3
   x6   130.8
   x7   83.45
   x8   80.55
   x9   23.08
   x10  40.92
   x11  37.13
   x12  81.83
```

```
    x13  120.3
    x14  130.8
    x15  83.45
    x16  80.55

  C = 
             x1        x2        x3        x4        x5        x6        x7        x8        x9
   y1    -68.77         0    -61.72     123.4    -123.4     123.4    -123.4     61.72    -790.6
   y2         0         0    -82.16     164.3    -164.3     164.3    -164.3     82.16     593.8
   y3         0     35.24    0.4114   -0.8229    0.8229   -0.8229    0.8229   -0.4114     5.271

            x16
   y1      5548
   y2     -4167
   y3    -36.99

  D = 
           u1
   y1   0.541
   y2  0.7203
   y3  0.006393

Input groups:
    Name    Channels
     U1        1

Output groups:
    Name    Channels
     Y1      1,2,3

Continuous-time state-space model.
gamma = 1.4549
info = 
  hinfINFO with properties:

    gamma: 1.4549
        X: [8×8 double]
        Y: [8×8 double]
       Ku: [-325.4596 42.8192 1.8687e+03 9.4406e+03 -1.4372e+04 1.3922e+04 -1.0845e+04 3.5761e+03]
       Kw: [-869.6446 127.2423 943.6919 3.5037e+04 -4.4783e+04 3.8441e+04 -2.5510e+04 6.1675e+03]
       Lx: [8×1 double]
       Lu: 7.2026
     Preg: [5×2 ss]
       AS: [2×2 ss]
```

Part d

H_∞-controller Margin Calculations

```
allmargin(C_Hinf*G_sys)
```

```
ans = struct with fields:
     GainMargin: 6.5542
    GMFrequency: 4.2065e+03
    PhaseMargin: [53.6057 -125.7859]
    PMFrequency: [1.2243e+03 1.6588e+06]
    DelayMargin: [7.6422e-04 2.4644e-06 0]
    DMFrequency: [1.2243e+03 1.6588e+06 Inf]
         Stable: 1
```

```
figure
```

```
margin(C_Hinf * G_sys)
grid on
```

PI - vs H_∞ - controller Bode Comparrision

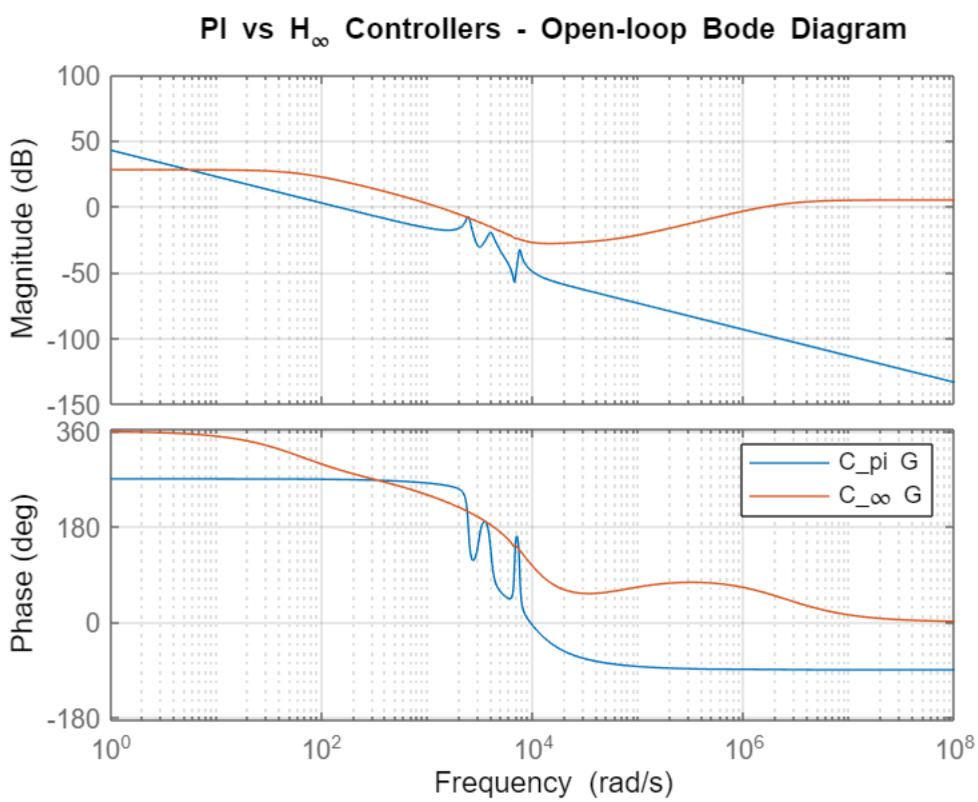
```
figure
hold on
bode(C_pi)
bode(C_Hinf)
legend('C_{pi}', 'C_{H_\infty}')
grid on
```

Loop Transfer Functions

```
sys_pi = feedback(C_pi * G_sys, 1);
sys_Hinf = feedback(C_Hinf * G_sys, 1);
```

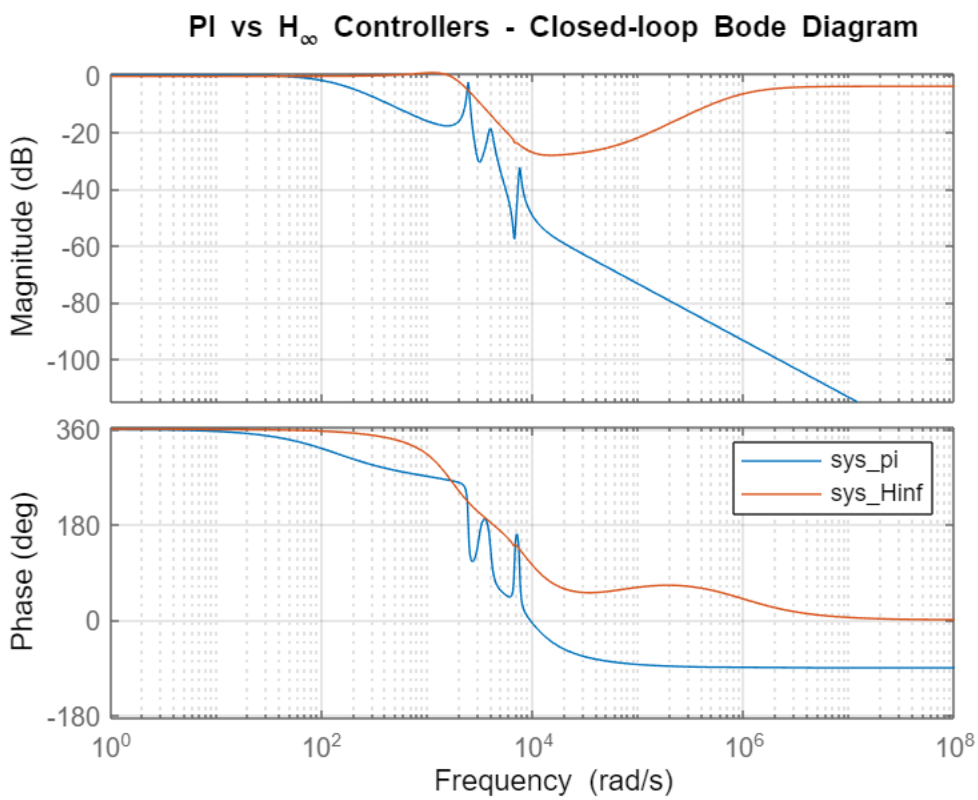
Bode Open Loop

```
figure
hold on
bode(C_pi * G_sys, C_Hinf * G_sys)
legend('C_{pi} G', 'C_\infty G')
title('PI vs H_\infty Controllers - Open-loop Bode Diagram')
grid on
```



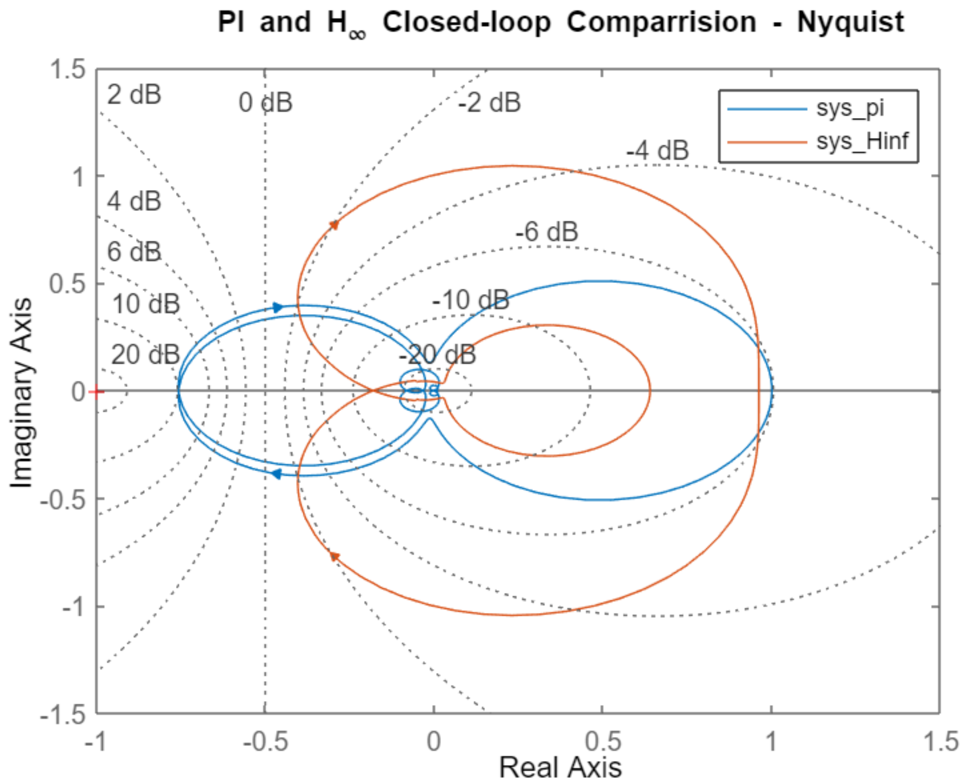
Bode Closed Loop

```
figure
hold on
bode(sys_pi,sys_Hinf)
legend
title('PI vs H_\infty Controllers - Closed-loop Bode Diagram')
grid on
```



Nyquist Loops

```
figure
nyquist(sys_pi, sys_Hinf)
legend
title('PI and H_\infty Closed-loop Comparrision - Nyquist')
grid on
```



Problem 2

The Generic Transport Model (GTM) is a turbine powered subscale model of a civilian transport aircraft, which was developed by NASA Langley as a platform to validate control laws. The model has a wing span of 7 ft, and weighs around 55 lbs. Under normal operation, the aircraft flies at an altitude of 700 to 1100 ft, and with an airspeed of 70 and 85 knots. In this problem you will use the signal-weighted H_∞ method to design a control law for the GTM.

A nonlinear simulation model of the GTM has been developed from extensive wind tunnel and flight tests. The model can be linearized at a particular flight condition, yielding a linear model of the aircraft dynamics. The nominal fight condition for control design is level flight at 800 ft and 80 knots. The longitudinal short-period dynamics, denoted G, are described by the following state equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \alpha \\ q \end{bmatrix} = \begin{bmatrix} -2.4714 & 0.9514 \\ -43.9070 & -3.4738 \end{bmatrix}\begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -0.2501 \\ -44.9478 \end{bmatrix}\delta_{elev}$$

where the state vector corresponds to angle-of-attack α [rad] and pitch rate q [rad/s]. The control surface input is elevator deflection δ_{elev} [rad]. The elevator actuator is modeled as a 5 Hz (= 31.42 rad/sec) first-order filter with DC gain equal to 1. This actuator saturates at 0.349 rads, i.e., it is physically constrained to $\delta_{elev} \leq 0.349$ rads. A rate gyro sensor measures pitch rate with noise that has standard deviation of 0.0067 rad/sec.

Final Parts

```
fname = matlab.desktop.editor.getActiveFilename;
export(fname, 'MECH6323_HW07.pdf')
```

```
ans =
'C:\Users\Jonas\OneDrive - The University of Texas at Dallas\2022_Spring\MECH6323\Homework\HW07\MECH6323_HW07.pdf'
```