

# MECH 6323 - HW 07

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## Problem 1

In this problem, you will design a control law for a nano-positioning stage. These devices can achieve very high precision positioning which is important in applications such as atomic force microscopes (AFMs). The right side of Figure 1 shows a feedback diagram of a nanopositioning device. The system consists of piezo-electric actuation, a flexure stage, and a detection system. As illustrated in the feedback diagram, the flexure stage interacts with the head of an AFM. The left side of Figure 1 shows a diagram of the flexure stage for a nanopositioning device. Typical design requirements for the control law include high bandwidth, high resolution and good robustness.

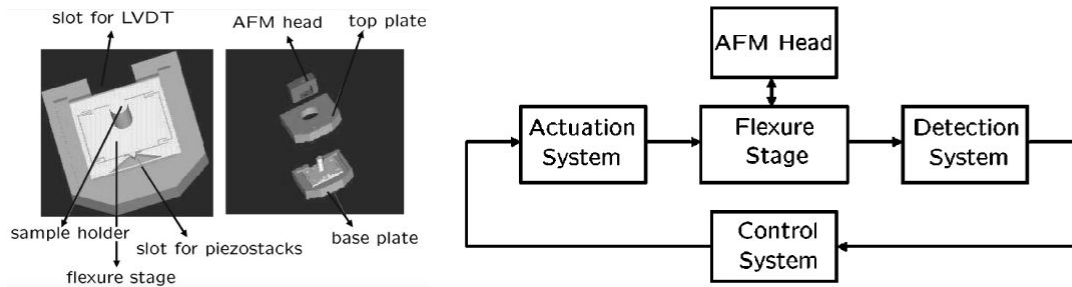


Fig. 1: Nanopositioning flexure stage (left) and feedback diagram (right); figures adapted from Salapaka et. al, *Rev. Sci. Instrum.* 2002.

```
clear
close all
```

## Part a

```
nano_rsp = load('npresp.mat')
```

```
nano_rsp = struct with fields:
  Gfr: [1x1 frd]
  w: [1748x1 double]
  G: [1x1x1748 double]
```

```
omega_min = min(nano_rsp.w);
omega_max = max(nano_rsp.w);
```

## Estimate system transfer function

```
tf_order = 6;
G_sys = fitfrd(nano_rsp.Gfr, tf_order)
```

```
G_sys =
```

```
A =
```

	x1	x2	x3	x4	x5	x6
x1	-1468	8540	-8540	8540	-8540	4270
x2	-4681	6559	-3757	3757	-3757	1878
x3	-2174	4348	-7150	9952	-9952	4976
x4	-4695	9390	-9390	6588	-3786	1893
x5	-1687	3374	-3374	3374	-6176	4489
x6	-3428	6856	-6856	6856	-6856	625.7

```
B =
      u1
x1  5.156
x2  11.36
x3  16.7
x4  18.15
x5  11.59
x6  11.18
```

```
C =
      x1      x2      x3      x4      x5      x6
y1  114.1 -228.1  228.1 -228.1  228.1 -114.1
```

```
D =
      u1
y1  0.08876
```

Continuous-time state-space model.

```
G_sys_tf = tf(G_sys)
```

```
G_sys_tf =
0.08876 s^6 - 876.1 s^5 + 1.136e07 s^4 - 4.345e10 s^3 + 4.097e14 s^2 - 2.095e17 s + 3.082e21
-----
s^6 + 1021 s^5 + 7.856e07 s^4 + 5.129e10 s^3 + 1.342e15 s^2 + 3.65e17 s + 5.421e21
```

Continuous-time transfer function.

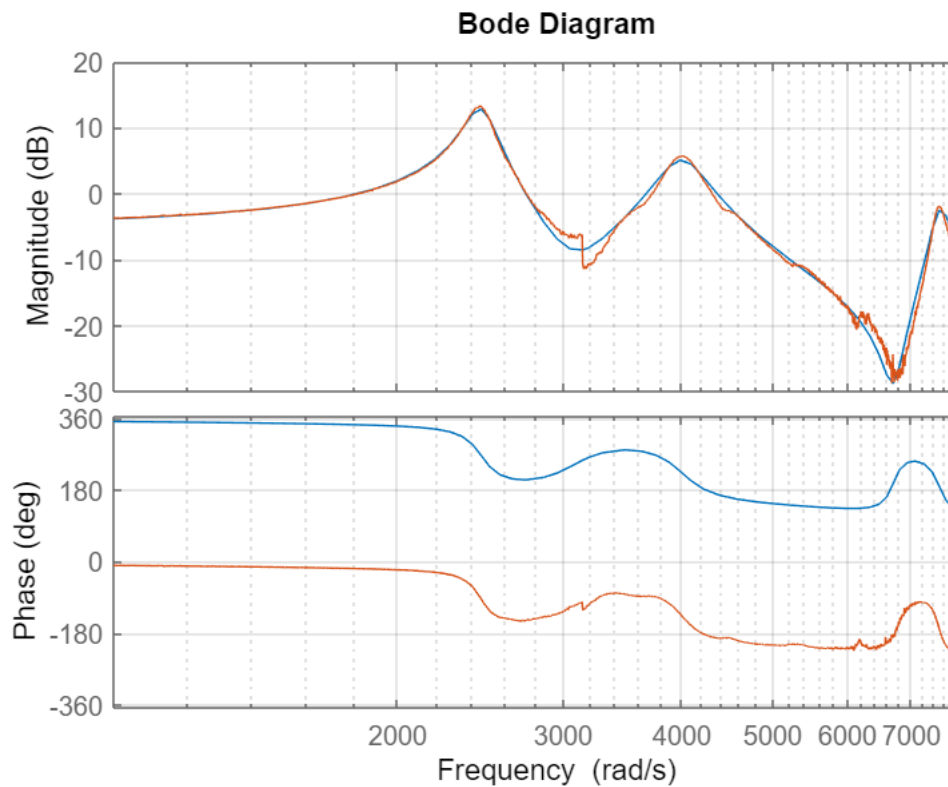
```
G_sys_zpk = zpk(G_sys)
```

```
G_sys_zpk =
0.088762 (s^2 + 526.7s + 9.417e06) (s^2 + 276.6s + 4.494e07) (s^2 - 1.067e04s + 8.207e07)
-----
(s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)
```

Continuous-time zero/pole/gain model.

## Bode Diagram Data

```
figure()
bode(G_sys)
hold on
bode(nano_rsp.Gfr)
grid on
xlim([omega_min, omega_max])
```



Clearly this plot is a good estimation for the frequency response of the system, noting that the phase of the system is offset by a 360 degree phase shift (which implies a need for another set of integrators)

## Part b

### PI - Implimentation

```
PM_min = 75;
opt = pidtuneOptions( ...
    'PhaseMargin', PM_min, ...
    'DesignFocus', 'disturbance-rejection' ...
)
```

```
opt =
    pidtune with properties:

        PhaseMargin: 75
        NumUnstablePoles: 0
        DesignFocus: 'disturbance-rejection'
```

```
[C_pi, info] = pidtune(G_sys, 'pi', opt)
```

```
C_pi =
```

```

    1
Ki * ---
    s
```

```
with Ki = 240
```

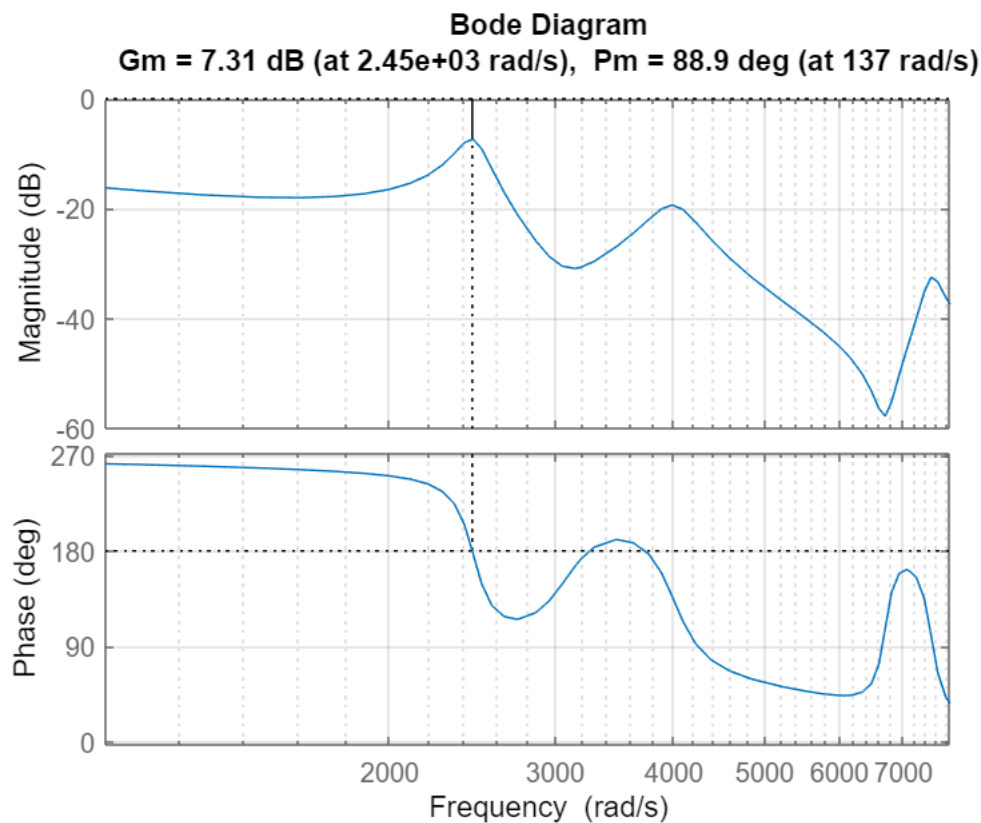
```
Continuous-time I-only controller.
info = struct with fields:
    Stable: 1
    CrossoverFrequency: 136.5946
    PhaseMargin: 88.9408
```

```
allmargin(C_pi * G_sys)
```

```
ans = struct with fields:
    GainMargin: [2.3213 31.8215 13.3190]
    GMFrequency: [2.4472e+03 3.2556e+03 3.7346e+03]
    PhaseMargin: 88.9408
    PMFrequency: 136.5946
    DelayMargin: 0.0114
    DMFrequency: 136.5946
    Stable: 1
```

## Bode Diagram of Margin

```
figure
margin(C_pi * G_sys_tf)
xlim([omega_min, omega_max])
grid on
```



As a result, the single only integral controller is a pretty weird result. However, looking at the results it does make sense.

## Sensitivity Transfer Function

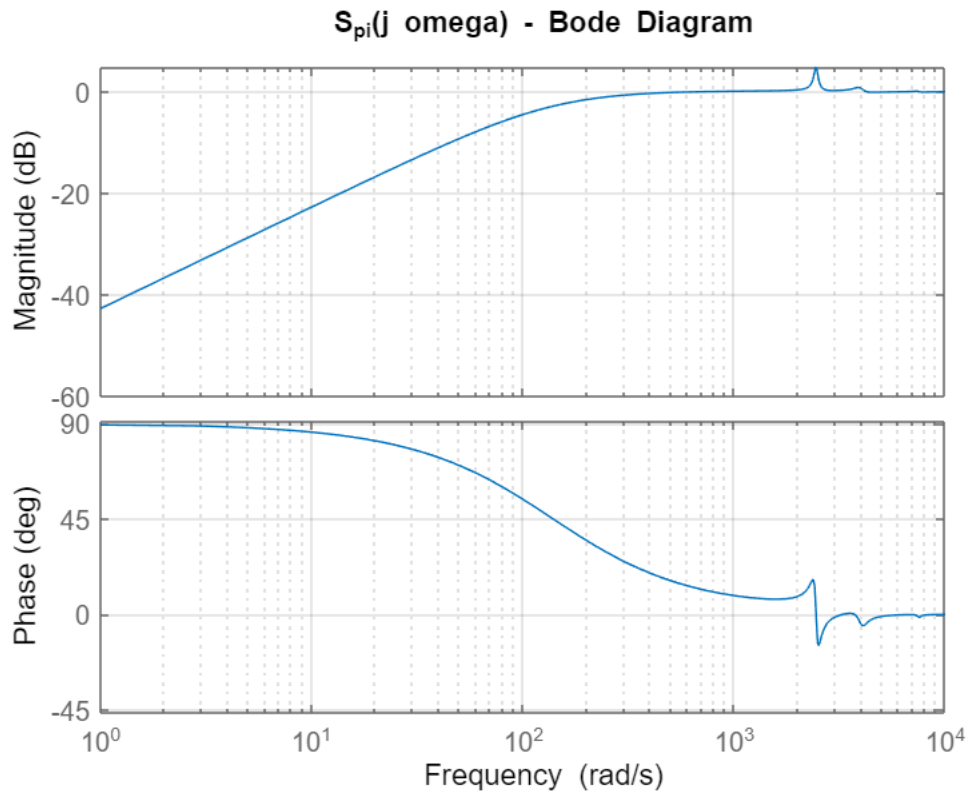
```
S_pi = zpk(1/(1+C_pi*G_sys))
```

S\_pi =

$$\frac{s (s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)}{(s+138.6) (s^2 + 108.1s + 5.995e06) (s^2 + 446.2s + 1.584e07) (s^2 + 349.8s + 5.615e07)}$$

Continuous-time zero/pole/gain model.

```
figure
bode(S_pi)
title('S_{pi}(j omega) - Bode Diagram')
grid on
```



### Complimentary Transfer Function

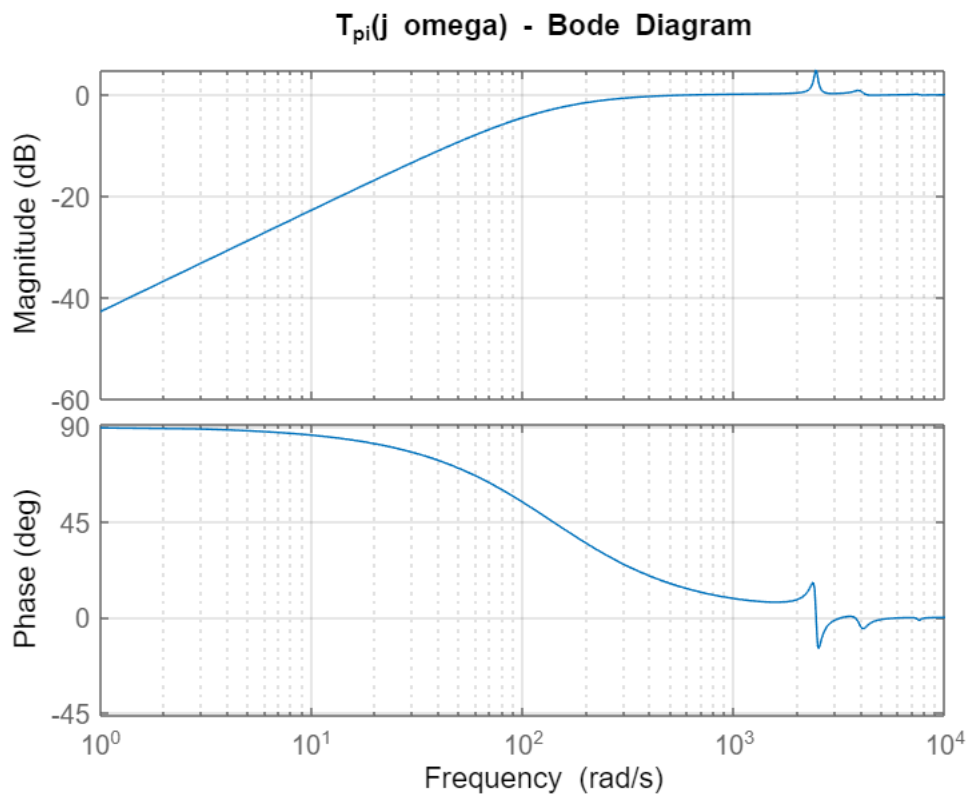
```
T_pi = zpk(1/(1+C_pi*G_sys))
```

T\_pi =

$$\frac{s (s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)}{(s+138.6) (s^2 + 108.1s + 5.995e06) (s^2 + 446.2s + 1.584e07) (s^2 + 349.8s + 5.615e07)}$$

Continuous-time zero/pole/gain model.

```
figure
bode(T_pi)
title('T_{pi}(j omega) - Bode Diagram')
grid on
```



### Bandwidth Calculation

```
bw_threshold = -3; %db
bw = getGainCrossover(S_pi, db2mag(bw_threshold))
```

```
bw = 134.4361
```

### Double Integrator implimenation

Instead we can also include an additional integrator into the controller, i.e.

```
G_int = tf(1,[1, 0]);
[C_pi_int, info] = pidtune(G_int * G_sys, 'pi', opt)
```

```
C_pi_int =
```

$$K_p + K_i * \frac{1}{s}$$

with  $K_p = 271$ ,  $K_i = 1.06e+04$

Continuous-time PI controller in parallel form.

```
info = struct with fields:
```

```
Stable: 1
```

```
CrossoverFrequency: 159.0729
```

```
PhaseMargin: 75.0000
```

```
allmargin(C_pi_int * G_int * G_sys)
```

```
ans = struct with fields:
```

```

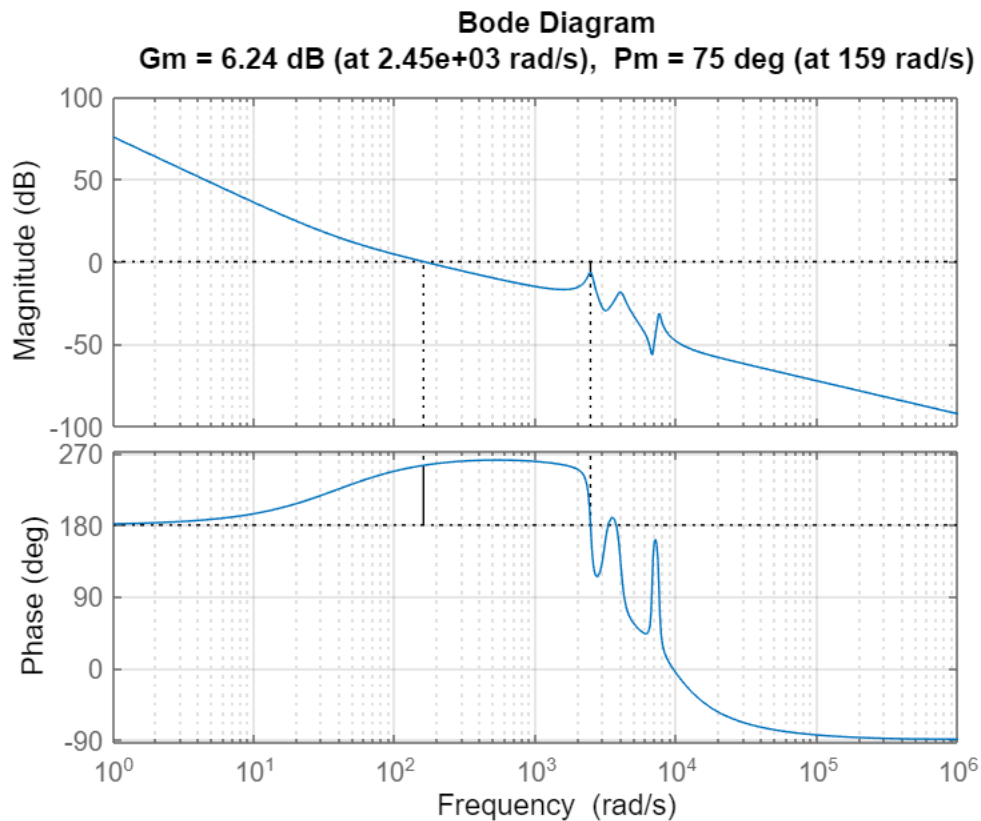
GainMargin: [0 2.0518 27.8933 11.9425]
GMFrequency: [0 2.4457e+03 3.2626e+03 3.7282e+03]
PhaseMargin: 75.0002
PMFrequency: 159.0759
DelayMargin: 0.0082
DMFrequency: 159.0759
Stable: 1

```

```

figure
margin(C_pi_int * G_int * G_sys)
grid on

```



### Sensitivity Transfer Function

```
S_pi_int = zpk(1/(1+C_pi_int*G_int*G_sys))
```

```
S_pi_int =
```

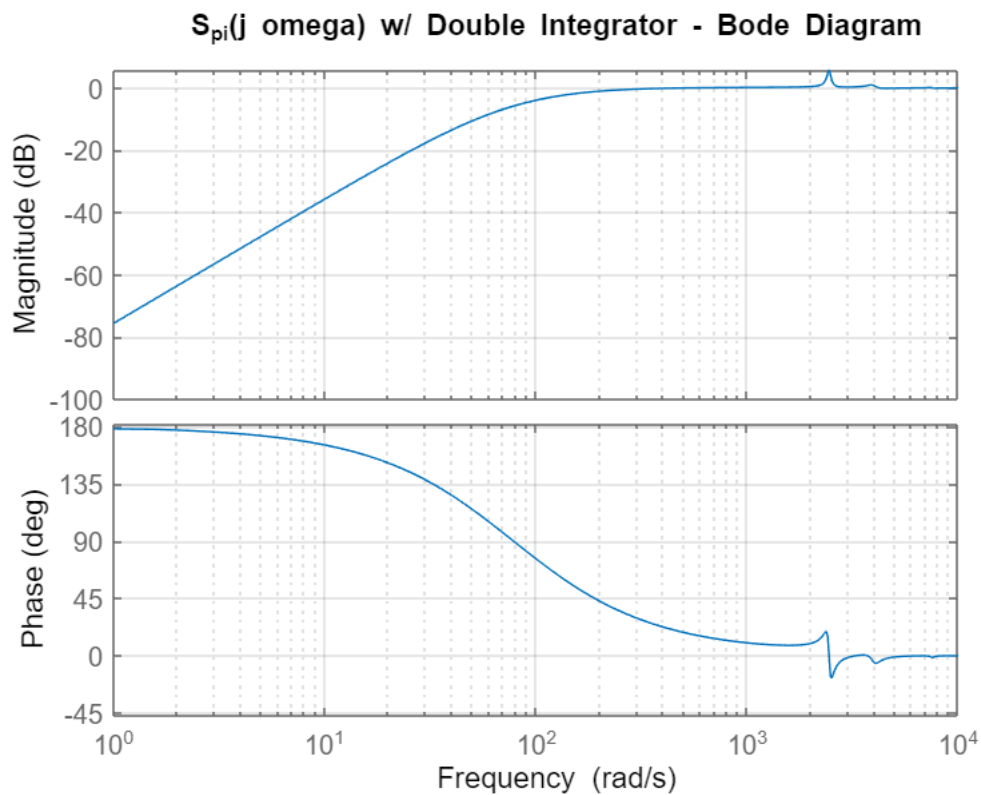
$$\frac{s^2 (s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)}{(s^2 + 156.3s + 6120) (s^2 + 97.81s + 5.989e06) (s^2 + 441.9s + 1.582e07) (s^2 + 349.5s + 5.614e07)}$$

Continuous-time zero/pole/gain model.

```

figure
bode(S_pi_int)
title('S_{pi}(j omega) w/ Double Integrator - Bode Diagram')
grid on

```



### Complimentary Transfer Function

```
T_pi_int = zpk(1/(1+C_pi_int*G_int*G_sys))
```

```
T_pi_int =
```

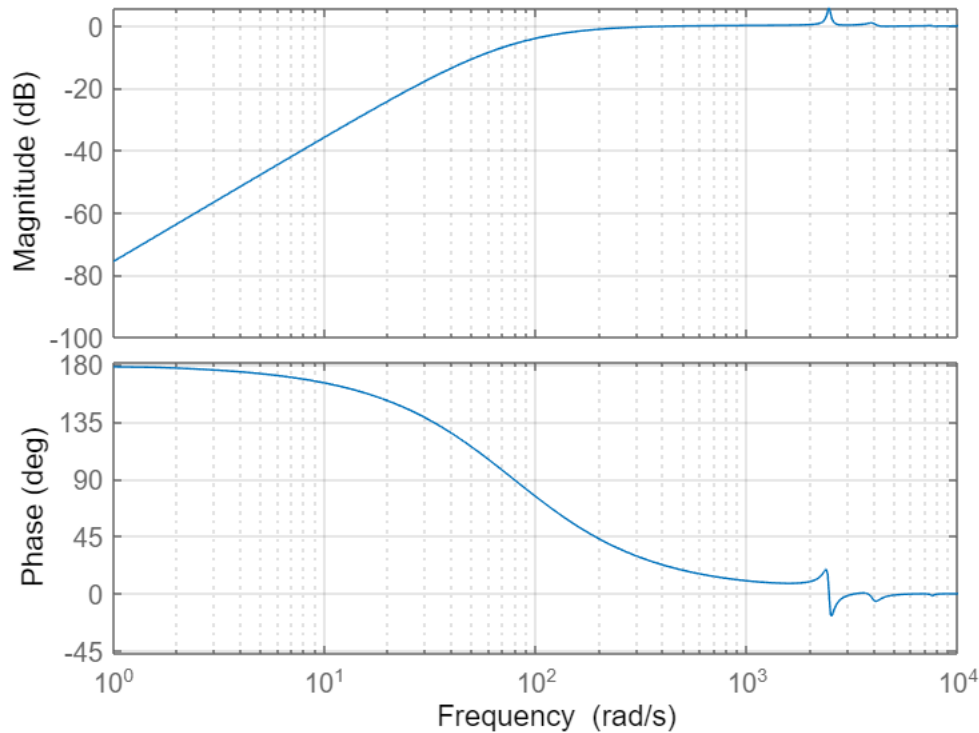
$$\frac{s^2 (s^2 + 186.2s + 6.029e06) (s^2 + 482.7s + 1.6e07) (s^2 + 352.5s + 5.621e07)}{(s^2 + 156.3s + 6120) (s^2 + 97.81s + 5.989e06) (s^2 + 441.9s + 1.582e07) (s^2 + 349.5s + 5.614e07)}$$

Continuous-time zero/pole/gain model.

```
figure
bode(T_pi_int)
title('T_{pi}(j omega) w/ Double Integrator - Bode Diagram')
grid on
```



$T_{pi}(j\omega)$  w/ Double Integrator - Bode Diagram



### Bandwidth Calculation

```
bw_threshold = -3; %db
bw = getGainCrossover(S_pi_int, db2mag(bw_threshold))
```

```
bw = 117.7945
```

The bandwidth of this method is not as great as the original PI implementation. I think the performance is better in certain situation though and may be worth implementing (even if it isn't really a PI controller)

## Part c

### Specs:

1. Bandwidth ( $|S(j\omega)| = -3$  dB) is around 250 Hz
2.  $|S(j\omega)| \leq 1.5 \quad \forall \omega$
3. Slope below bandwidth = 20 dB/decade
4. DC gain of  $S \leq -80$  dB
5.  $|T(j\omega)| < -3$  dB @ 500 Hz
6.  $|T(j\omega)| \leq 1.5 \quad \forall \omega$
7.  $|T(j\omega)| < -40$  dB as  $\omega \rightarrow \infty$
8.  $|C_\infty S(j\omega)| \leq 10 \quad \forall \omega$

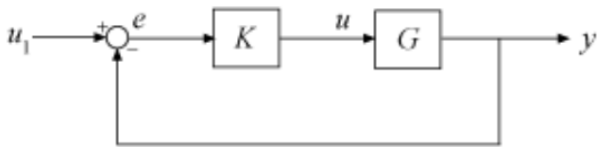
### Design Weights

In order design using the mixed sensitivity design approach, we shape  $S_{pi}(s)$  and  $T_{pi}(s)$  to achieve the desired performance and robustness specs using weighting functions that are inverse of those desired shapes.

`[K,CL,gamma,info] = mixsyn(G,W1,W2,W3)` computes a controller that minimizes the  $H_\infty$  norm of the weighted closed-loop transfer function

$$M(s) = \begin{bmatrix} W_1 S \\ W_2 K S \\ W_3 T \end{bmatrix},$$

where  $S = (I + GK)^{-1}$  and  $T = (I - S)$  is the complementary sensitivity of the following control system.



`mixsyn` computes the controller  $K$  that yields the minimum  $\|M(s)\|_\infty$ , which is returned as `gamma`. For the returned controller  $K$ ,

$$\|S\|_\infty \leq \gamma |W_1^{-1}|$$

$$\|KS\|_\infty \leq \gamma |W_2^{-1}|$$

$$\|T\|_\infty \leq \gamma |W_3^{-1}|.$$

## Description

`makeweight` is a convenient way to specify loop shapes, target gain profiles, or weighting functions for applications such as controller synthesis and control system tuning.

`W = makeweight(dcgain,[freq,mag],hfgain)` creates a first-order, continuous-time weight  $W(s)$  satisfying these constraints:

[example](#)

$$W(0) = \text{dcgain}$$

$$W(\text{Inf}) = \text{hfgain}$$

$$|W(j \cdot \text{freq})| = \text{mag}.$$

In other words, the gain of  $W$  passes through `mag` at the finite frequency `freq`.

**$W_1$  - Shaping  $S$ :**  $\|S\|_\infty \leq \gamma |W_1^{-1}|$

```
dcgain_1 = db2mag(-80); % Spec 4
hfgain_1 = 1.5; % Spec 2
bw_1 = 250; % Spec 1
W_1 = makeweight(dcgain_1, [2*pi*bw_1, db2mag(-3)], hfgain_1)
```

`W_1 =`

`A =`

`x1`

`x1 -2934`

```

B =
      u1
x1  64

C =
      x1
y1 -68.77

D =
      u1
y1  1.5

```

Continuous-time state-space model.

**$W_2$  - Shaping  $KS$ :**  $\|KS\|_\infty \leq \gamma|W_2^{-1}|$

```

u_max = 10;
W_2 = tf(1/u_max)

```

```
W_2 =
```

```
0.1
```

Static gain.

**$W_3$  - Shaping  $T$ :**  $\|T\|_\infty \leq \gamma|W_3^{-1}|$

```

dcgain_3 = 1.5; % Spec 6 (max KS should be less then 1.5)
hfgain_3 = db2mag(-40); % Spec 7
bw_3 = 450; %should be less then 500 to ensure below -3dB @ 500 Hz
W_3 = makeweight(dcgain_3, [2*pi*bw_3, db2mag(-3)], hfgain_3)

```

```
W_3 =
```

```

A =
      x1
x1 -1513

```

```

B =
      u1
x1  64

```

```

C =
      x1
y1 35.24

```

```

D =
      u1
y1 0.01

```

Continuous-time state-space model.

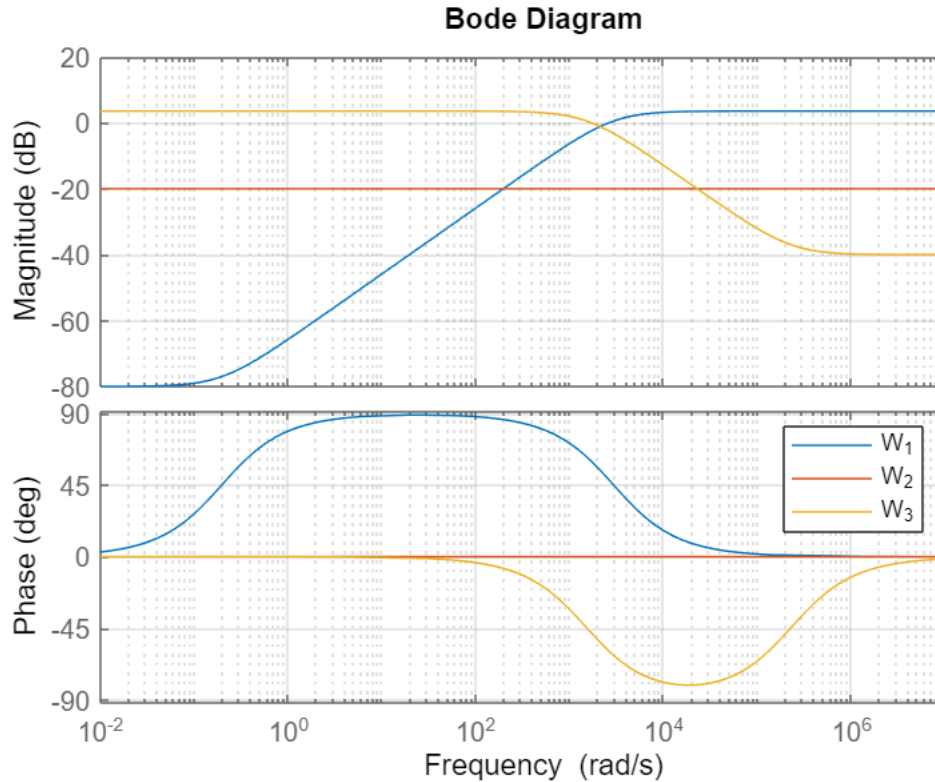
## Plotting Weighting functions

```

figure
hold on
bode(W_1)
bode(W_2)

```

```
bode(W_3)
legend('W_1','W_2','W_3')
grid on
```



## $H_\infty$ controller Calculation

```
[C_Hinf,CL,gamma,info] = mixsyn(G_sys,W_1,W_2,W_3)
```

C\_Hinf =

A =

	x1	x2	x3	x4	x5	x6	x7	x8
x1	-2934	-1.51e-10	2.547e-11	4.657e-10	-9.313e-10	6.985e-10	-1.164e-10	8.731e-11
x2	9.353e+04	-1.527e+04	-5.738e+04	-3.866e+06	4.894e+06	-4.182e+06	2.764e+06	-6.636e+05
x3	8.488e+04	-1.249e+04	-6.017e+04	-3.487e+06	4.42e+06	-3.774e+06	2.486e+06	-5.914e+05
x4	1.871e+05	-2.752e+04	-1.34e+05	-7.697e+06	9.756e+06	-8.331e+06	5.494e+06	-1.311e+06
x5	2.75e+05	-4.046e+04	-1.924e+05	-1.132e+07	1.434e+07	-1.224e+07	8.074e+06	-1.925e+06
x6	2.989e+05	-4.397e+04	-2.114e+05	-1.23e+07	1.558e+07	-1.331e+07	8.781e+06	-2.095e+06
x7	1.908e+05	-2.807e+04	-1.336e+05	-7.853e+06	9.95e+06	-8.497e+06	5.601e+06	-1.334e+06
x8	1.841e+05	-2.709e+04	-1.308e+05	-7.576e+06	9.6e+06	-8.198e+06	5.405e+06	-1.291e+06

B =

	u1
x1	64
x2	113.4
x3	103
x4	226.9
x5	333.6
x6	362.5
x7	231.4
x8	223.3

```

C =
      x1      x2      x3      x4      x5      x6      x7      x8
y1  1.646e+04 -2422 -1.139e+04 -6.78e+05  8.59e+05 -7.336e+05  4.839e+05 -1.155e+05

D =
      u1
y1  19.97

```

Continuous-time state-space model.

```

CL =

A =
      x1      x2      x3      x4      x5      x6      x7      x8      x9
x1  -2934      0    -2633     5266    -5266     5266    -5266     2633 -3.373e+04
x2      0   -1513     2633    -5266     5266    -5266     5266    -2633  3.373e+04
x3      0      0    -5704   1.701e+04 -1.701e+04  1.701e+04 -1.701e+04     8506  3.062e+04
x4      0      0  -1.402e+04  2.523e+04 -2.243e+04  2.243e+04 -2.243e+04  1.121e+04  6.747e+04
x5      0      0  -1.59e+04   3.18e+04  -3.46e+04   3.74e+04  -3.74e+04   1.87e+04   9.92e+04
x6      0      0  -1.961e+04  3.922e+04 -3.922e+04  3.642e+04 -3.362e+04  1.681e+04  1.078e+05
x7      0      0  -1.121e+04  2.241e+04 -2.241e+04  2.241e+04 -2.522e+04  1.401e+04   6.88e+04
x8      0      0  -1.262e+04  2.523e+04 -2.523e+04  2.523e+04 -2.523e+04     9815  6.641e+04
x9      0      0    -2633     5266    -5266     5266    -5266     2633 -3.667e+04
x10     0      0    -4667     9335    -9335     9335    -9335     4667  3.373e+04
x11     0      0    -4236     8472    -8472     8472    -8472     4236  3.062e+04
x12     0      0    -9335   1.867e+04 -1.867e+04  1.867e+04 -1.867e+04     9335  6.747e+04
x13     0      0  -1.372e+04  2.745e+04 -2.745e+04  2.745e+04 -2.745e+04  1.372e+04   9.92e+04
x14     0      0  -1.492e+04  2.983e+04 -2.983e+04  2.983e+04 -2.983e+04  1.492e+04  1.078e+05
x15     0      0    -9520   1.904e+04 -1.904e+04  1.904e+04 -1.904e+04     9520   6.88e+04
x16     0      0    -9189   1.838e+04 -1.838e+04  1.838e+04 -1.838e+04     9189  6.641e+04

B =
      u1
x1  23.08
x2  40.92
x3  37.13
x4  81.83
x5  120.3
x6  130.8
x7  83.45
x8  80.55
x9  23.08
x10 40.92
x11 37.13
x12 81.83
x13 120.3
x14 130.8
x15 83.45
x16 80.55

C =
      x1      x2      x3      x4      x5      x6      x7      x8      x9
y1  -68.77      0    -61.72    123.4    -123.4    123.4    -123.4     61.72 -790.6
y2      0      0    -82.16    164.3    -164.3    164.3    -164.3     82.16  593.8
y3      0    35.24     0.4114   -0.8229     0.8229   -0.8229     0.8229   -0.4114   5.271

D =
      u1
y1   0.541
y2   0.7203
y3   0.006393

```

Input groups:

Name	Channels
U1	1

Output groups:

Name	Channels
Y1	1,2,3

Continuous-time state-space model.

gamma = 1.4549

info =

hinfINFO with properties:

```

gamma: 1.4549
  X: [8x8 double]
  Y: [8x8 double]
  Ku: [-325.4596 42.8192 1.8687e+03 9.4406e+03 -1.4372e+04 1.3922e+04 -1.0845e+04 3.5761e+03]
  Kw: [-869.6446 127.2423 943.6919 3.5037e+04 -4.4783e+04 3.8441e+04 -2.5510e+04 6.1675e+03]
  Lx: [8x1 double]
  Lu: 7.2026
  Preg: [5x2 ss]
  AS: [2x2 ss]

```

## Part d

### $H_{\infty}$ -controller Margin Calculations

```
allmargin(C_Hinf*G_sys)
```

```

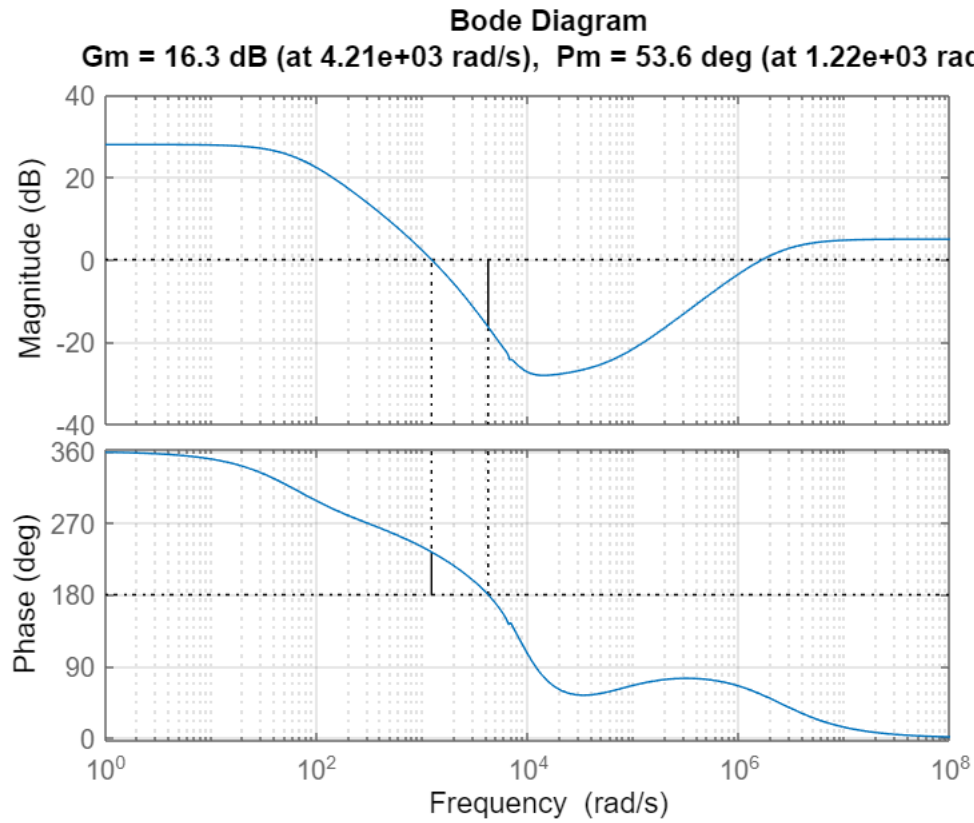
ans = struct with fields:
  GainMargin: 6.5542
  GMFrequency: 4.2065e+03
  PhaseMargin: [53.6057 -125.7859]
  PMFrequency: [1.2243e+03 1.6588e+06]
  DelayMargin: [7.6422e-04 2.4644e-06 0]
  DMFrequency: [1.2243e+03 1.6588e+06 Inf]
  Stable: 1

```

```

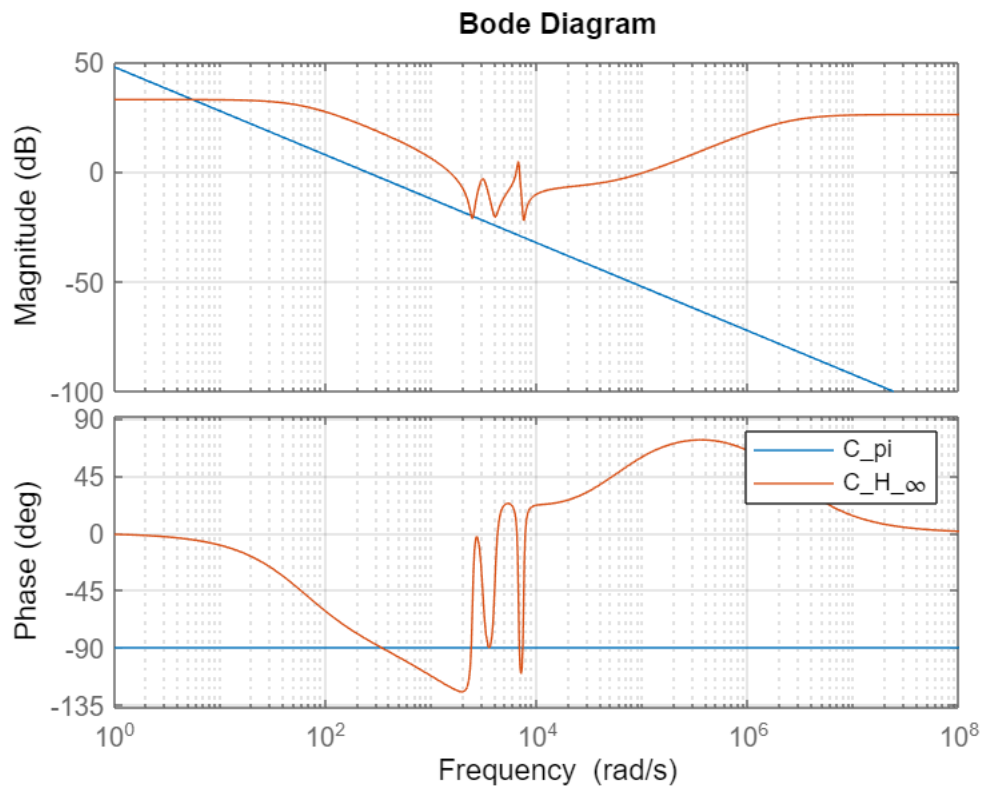
figure
margin(C_Hinf * G_sys)
grid on

```



### *PI* - vs $H_\infty$ - controller Bode Comparrison

```
figure
hold on
bode(C_pi)
bode(C_Hinf)
legend('C_{pi}', 'C_{H\infty}')
grid on
```



## Loop Transfer Functions

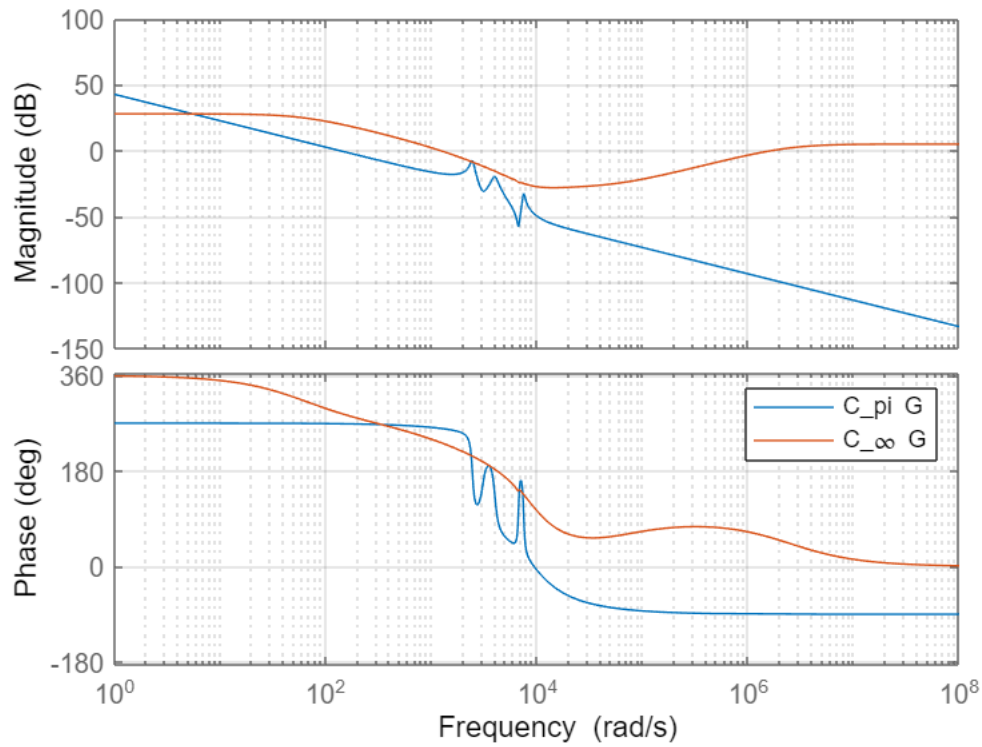
```
sys_pi = feedback(C_pi * G_sys, 1);
sys_Hinf = feedback(C_Hinf * G_sys, 1);
```

## Bode Open Loop

```
figure
hold on
bode(C_pi * G_sys, C_Hinf * G_sys)
legend('C_{pi} G', 'C_{\infty} G')
title('PI vs H_{\infty} Controllers - Open-loop Bode Diagram')
grid on
```



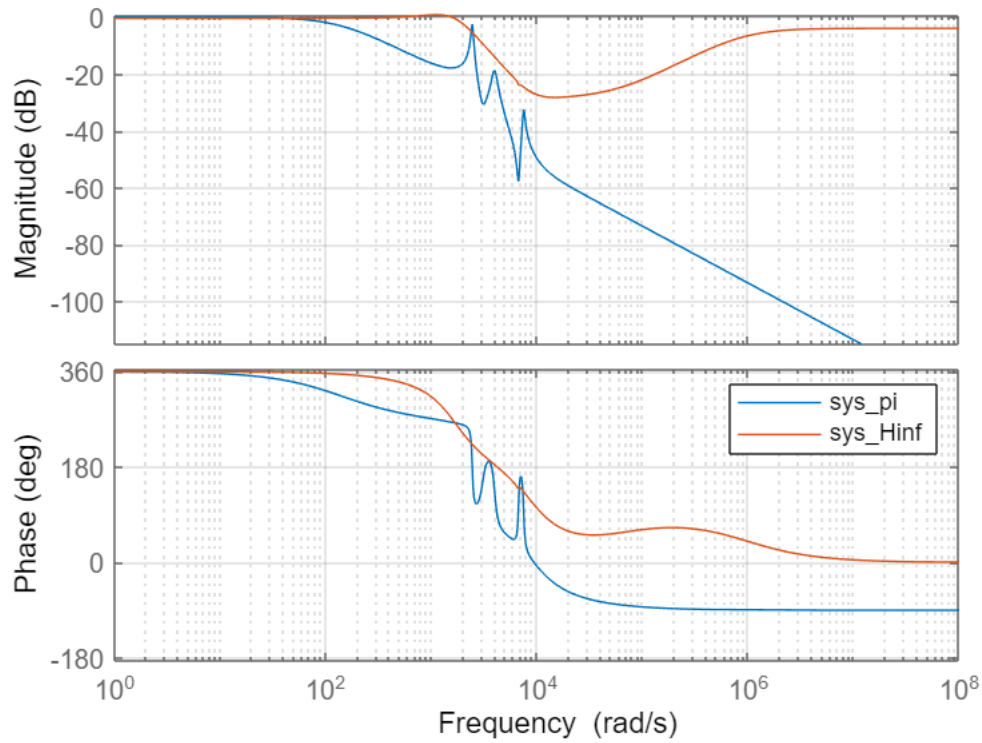
## PI vs $H_\infty$ Controllers - Open-loop Bode Diagram



## Bode Closed Loop

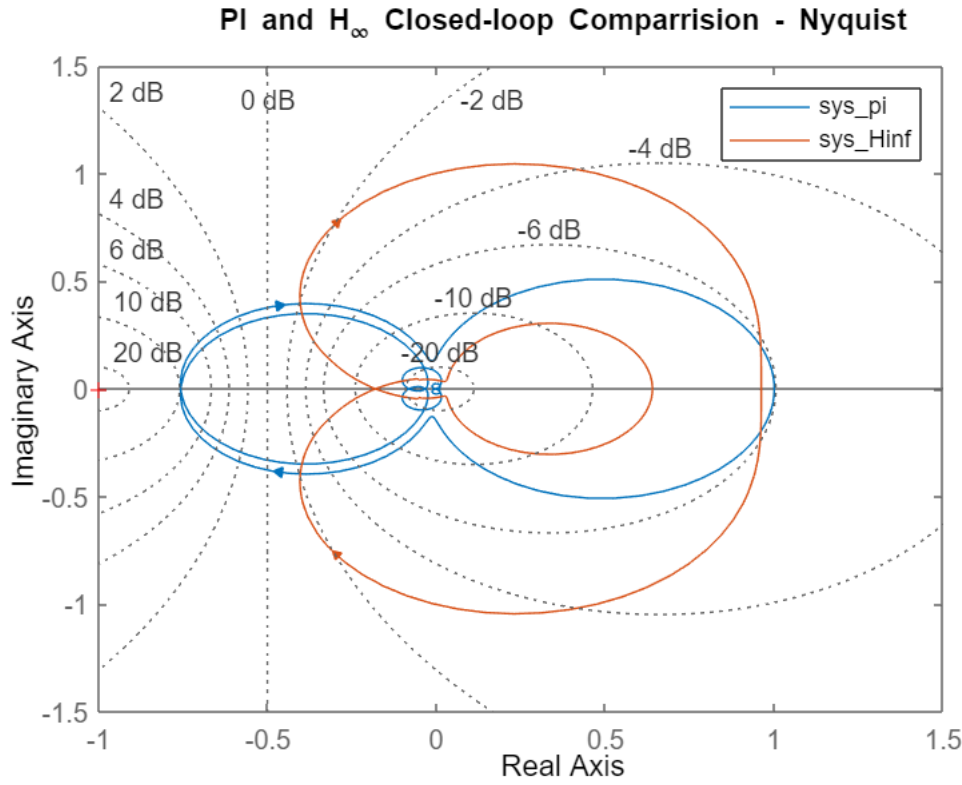
```
figure
hold on
bode(sys_pi,sys_Hinf)
legend
title('PI vs  $H_\infty$  Controllers - Closed-loop Bode Diagram')
grid on
```

## PI vs $H_\infty$ Controllers - Closed-loop Bode Diagram



## Nyquist Loops

```
figure
nyquist(sys_pi, sys_Hinf)
legend
title('PI and  $H_\infty$  Closed-loop Comparison - Nyquist')
grid on
```



## Problem 2

The Generic Transport Model (GTM) is a turbine powered subscale model of a civilian transport aircraft, which was developed by NASA Langley as a platform to validate control laws. The model has a wing span of 7 ft, and weighs around 55 lbs. Under normal operation, the aircraft flies at an altitude of 700 to 1100 ft, and with an airspeed of 70 and 85 knots. In this problem you will use the signal-weighted  $H_\infty$  method to design a control law for the GTM.

A nonlinear simulation model of the GTM has been developed from extensive wind tunnel and flight tests. The model can be linearized at a particular flight condition, yielding a linear model of the aircraft dynamics. The nominal flight condition for control design is level flight at 800 ft and 80 knots. The longitudinal short-period dynamics, denoted  $G$ , are described by the following state equation:

$$\frac{d}{dt} \begin{bmatrix} \alpha \\ q \end{bmatrix} = \begin{bmatrix} -2.4714 & 0.9514 \\ -43.9070 & -3.4738 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -0.2501 \\ -44.9478 \end{bmatrix} \delta_{elev} \quad .$$

where the state vector corresponds to angle-of-attack  $\alpha$  [rad] and pitch rate  $q$  [rad/s]. The control surface input is elevator deflection  $\delta_{elev}$  [rad]. The elevator actuator is modeled as a 5 Hz ( $= 31.42$  rad/sec) first-order filter with DC gain equal to 1. This actuator saturates at 0.349 rads, i.e., it is physically constrained to  $\delta_{elev} \leq 0.349$  rads. A rate gyro sensor measures pitch rate with noise that has standard deviation of 0.0067 rad/sec.

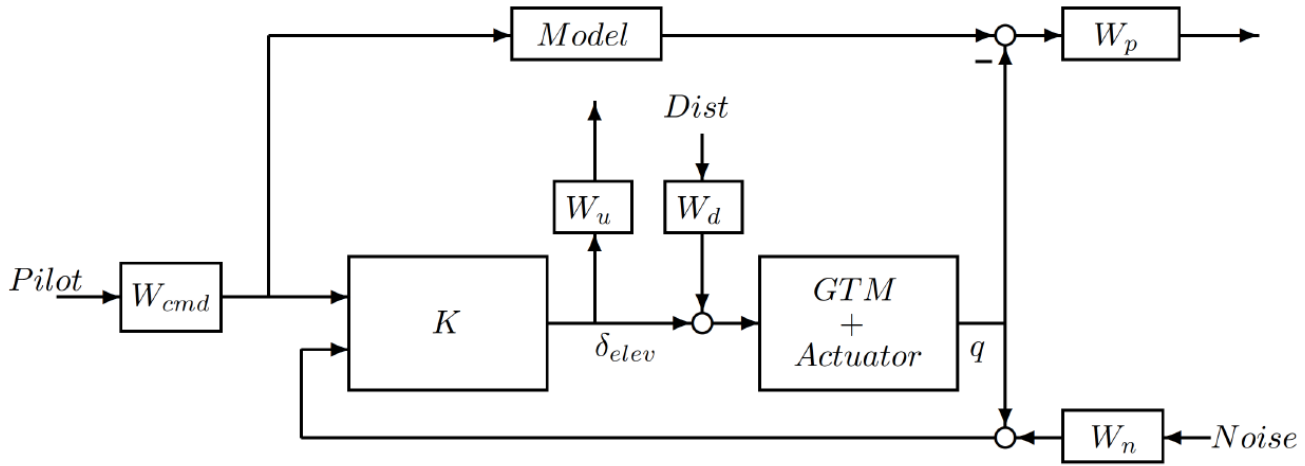


Fig. 2: Longitudinal system interconnection.

The main control objective is to design a Stability Augmentation System (SAS) to increase damping in the aircraft's oscillatory modes. A model-matching  $H_\infty$  control problem is formulated to achieve the desired robustness and performance characteristics. The longitudinal control system interconnection is shown in Figure 2. The inputs to the controller are pilot longitudinal stick command and pitch rate feedback. The actual input commands are assumed to have magnitude  $\leq 0.2618$  rads. The ideal model for matching is denoted as “Model” in the diagram. It will be chosen to mimic the open-loop aircraft behavior at low and high frequency but with improved damping in the oscillatory modes.

```
clear
close all
```

## Model Definition

### Plant Definition

```
A = [
    - 2.4714    0.9514
    -43.9070   -3.4738
];
B = [
    - 0.2501
    -44.9478
];
C = [0 1];
D = 0;
G = ss(A,B,C,D);
G.InputName = 'delta_elev_sat';
G.StateName = {'alpha', 'q'};
G.OutputName = {'q'}
```

```
G =
```

```

A =
      alpha      q
alpha -2.471  0.9514
q      -43.91  -3.474

B =
      delta_elev_s
alpha -0.2501
q      -44.95

C =
      alpha      q
q      0      1

D =
      delta_elev_s
q      0

```

Continuous-time state-space model.

## Actuator Defintion

First order Actuator filter

```

p_act = 5 * 2 * pi;
k_act = 1;
G_act = tf(k_act * p_act, [1 p_act]);
G_act.InputName = 'delta_elev_sum';
G_act.OutputName = 'delta_elev_unsat'

```

```

G_act =

From input "delta_elev_sum" to output "delta_elev_unsat":
    31.42
-----
s + 31.42

```

Continuous-time transfer function.

## Saturation?

Note: idk how to actually impliment a nonlinearity within this type of a system... so idk, do we just want another crazy filter going to zero at 0.349 somehow? idk, why aren't we using Simulink since it would be way easier (even if we did it via code...)

```

umax_act = 0.349;
G_sat = ss(1);
G_sat.InputName = 'delta_elev_unsat';
G_sat.OutputName = 'delta_elev_sat'

```

```

G_sat =

D =
      delta_elev_u
delta_elev_s      1

```

Static gain.

```

A = minreal(zpk(series(G_act, G_sat)))

```

A =

```
From input "delta_elev_sum" to output "delta_elev_sat":  
31.416  
-----  
(s+31.42)
```

Continuous-time zero/pole/gain model.

## Rate Gyro Sensor Error

Just a change in standard deviation from 1 to 0.0067...

```
std_gyro = 0.0067;  
W_n = tf(std_gyro);  
W_n.InputName = 'Noise';  
W_n.OutputName = 'q_noise'
```

W\_n =

```
From input "Noise" to output "q_noise":  
0.0067
```

Static gain.

## Part a

```
[wn_G, zeta_G, p_G] = damp(G)
```

```
wn_G = 2×1  
7.0964  
7.0964  
zeta_G = 2×1  
0.4189  
0.4189  
p_G = 2×1 complex  
-2.9726 + 6.4438i  
-2.9726 - 6.4438i
```

```
zeta_damp = 0.8;  
p_G_damp = wn_G .* (acos(zeta_damp) + 1i * asin(zeta_damp) * [1; -1]);  
G_damp = zpk(minreal(G * zpk(p_G, p_G_damp, 1)));
```

2 states removed.

```
G_damp.InputName = 'Wcmd';  
G_damp.OutputName = 'q_ideal'
```

G\_damp =

```
From input "Wcmd" to output "q_ideal":  
-44.948 (s+2.227)  
-----  
(s^2 - 9.133s + 64.15)
```

Continuous-time zero/pole/gain model.

```
[wn_G_damp, zeta_G_damp, p_G_damp] = damp(G_damp)
```

```

wn_G_damp = 2x1
    8.0097
    8.0097
zeta_G_damp = 2x1
   -0.5701
   -0.5701
p_G_damp = 2x1 complex
    4.5665 + 6.5804i
    4.5665 - 6.5804i

```

## Part b

$W_n$  - Gyro measurement error std

```
W_n % Gyro std from above
```

```
W_n =
```

```

From input "Noise" to output "q_noise":
0.0067

```

Static gain.

$W_u$  - Output to a normalized

```

W_u = tf(0.349); % delta_elev normalized from delta_elev = [-0.349,0.349] to W_cmd = [-0.2618,0.2618]
W_u.InputName = 'delta_elev';
W_u.OutputName = 'Wu'

```

```
W_u =
```

```

From input "delta_elev" to output "Wu":
0.349

```

Static gain.

$W_{cmd}$  - Pilot command input to setpoint

```

W_cmd = tf(0.2618); % Input normalized from Pilot = [-1,1] to W_cmd = [-0.2618,0.2618]
W_cmd.InputName = 'Pilot';
W_cmd.OutputName = 'Wcmd'

```

```
W_cmd =
```

```

From input "Pilot" to output "Wcmd":
0.2618

```

Static gain.

$W_p$  - Plant error output

```

error_max = 0.01;
W_p = tf(1/error_max); % Normalize from q = [-0.01,0.01] to W_p = [-1,1]
W_p.InputName = 'e';
W_p.OutputName = 'Wp'

```

```
W_p =

    From input "e" to output "Wp":
    100

Static gain.
```

$W_d$  - Disturbance Input

```
delta_limit = 0.349;
W_d_max = 0.15;
W_d = tf(delta_limit * W_d_max); % Gussing the Input normalized from u = [-1,1] to W_d = 0.15 *
W_d.InputName = 'Dist';
W_d.OutputName = 'delta_elev_dist'
```

```
W_d =

    From input "Dist" to output "delta_elev_dist":
    0.05235

Static gain.
```

## Part c

Although... I'm not sure why we are using connect (or even more outdated sysic) and all of this outdated structuring method instead of just setting this up and implimenting it within Simulink

## Connect Definitions

```
Model = G_damp;
```

```
Model =

    From input "W_cmd" to output "q_ideal":
    -44.948 (s+2.227)
    -----
    (s^2 - 9.133s + 64.15)
```

Continuous-time zero/pole/gain model.

```
% systemnames = 'G G_act Model Wn Wd Wp Wu Wcmd';
inputvar = {'Pilot', 'Dist', 'Noise', 'delta_elev'};
outputvar = {'Wp', 'Wu', 'Wcmd', 'q_sensor'};
% connect Syntax
% systemnames = 'G';
% inputvar = '[r; u]';
% outputvar = '[WS; WK; r-P]';
```

Also... what's up with  $\delta_{elev}$  as an "input"...

```
% input_to_P = '[u]';
% input_to_WS = '[r-P]';
% input_to_WK = '[u]';
% Gs = sysic;
```



```
e_sum = sumblk('e = q_ideal - q');
q_sum = sumblk('q_sensor = q + q_noise');
delta_sum = sumblk('delta_elev_sum = delta_elev + delta_elev_dist')
```

```
delta_sum =
```

```
From input "delta_elev" to output "delta_elev_sum":
```

```
1
```

```
From input "delta_elev_dist" to output "delta_elev_sum":
```

```
1
```

```
Static gain.
```

```
P = connect( ...
    G, G_act, G_sat, Model, W_n, W_d, W_p, W_u, W_cmd,...
    e_sum, q_sum, delta_sum,...
    inputvar, outputvar)
```

Warning: The following block inputs are not used: W\_cmd.

```
P =
```

```
A =
```

	alpha	q	?
alpha	-2.471	0.9514	-1.964
q	-43.91	-3.474	-353
?	0	0	-31.42

```
B =
```

	Pilot	Dist	Noise	delta_elev
alpha	0	0	0	0
q	0	0	0	0
?	0	0.2094	0	4

```
C =
```

	alpha	q	?
Wp	0	-100	0
Wu	0	0	0
Wcmd	0	0	0
q_sensor	0	1	0

```
D =
```

	Pilot	Dist	Noise	delta_elev
Wp	0	0	0	0
Wu	0	0	0	0.349
Wcmd	0.2618	0	0	0
q_sensor	0	0	0.0067	0

```
Continuous-time state-space model.
```

## Final Parts

```
fname = matlab.desktop.editor.getActiveFilename;
export(fname, 'MECH6323_HW07.pdf')
```

```
ans =
```

'C:\Users\Jonas\OneDrive - The University of Texas at Dallas\2022\_Spring\MECH6323\MECH6323\_HW07.pdf'