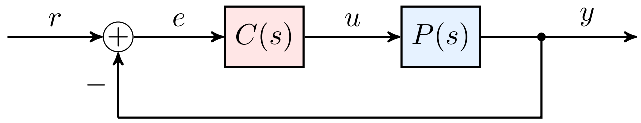
Due Thursday 02/14/2022 (at the beginning of the class)

1. Consider the negative feedback interconnection:



   (a) If possible, give an example of P and C transfer functions such that $\dfrac{1}{1+PC}$ and $\dfrac{P}{1+PC}$ are stable, but $\dfrac{C}{1+PC}$ is not.

   (b) If possible, give an example of P and C transfer functions such that $\dfrac{C}{1+PC}$ and $\dfrac{P}{1+PC}$ are stable, but $\dfrac{1}{1+PC}$ is not.

   (c) If possible, give an example of P and C transfer functions such that $\dfrac{1}{1+PC}$ and $\dfrac{C}{1+PC}$ are stable, but $\dfrac{P}{1+PC}$ is not.

2. For the following transfer functions, obtain the asymptotes of the Bode plots and compare them with the Bode plots generated in MATLAB. Include the results from MATLAB.

   (a) $\dfrac{100s+100}{s^2+110s+1000}$

   (b) $\dfrac{10s}{s^2+3s}$

   (c) $\dfrac{-100s}{(s+1)^2(s+10)}$

   (d) $30\left(\dfrac{s+10}{s^2+3s+50}\right)$

   (e) $4\left(\dfrac{s^2+s+25}{s^3+100s^2}\right)$

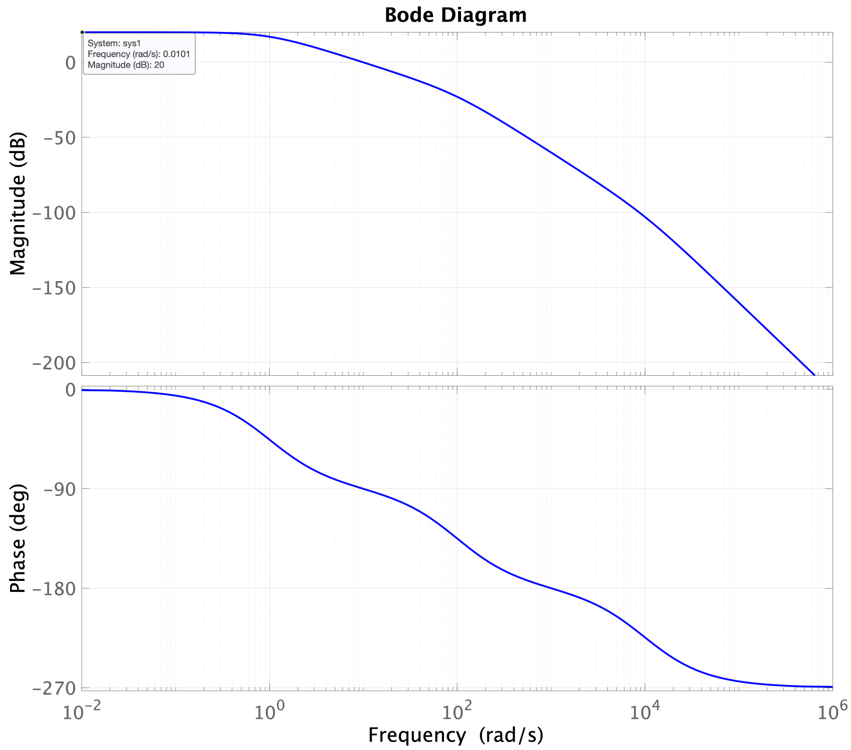
   (f) $\dfrac{10}{s^2(1+0.2s)(1+0.5s)}$

3. For the following two Bode plots,

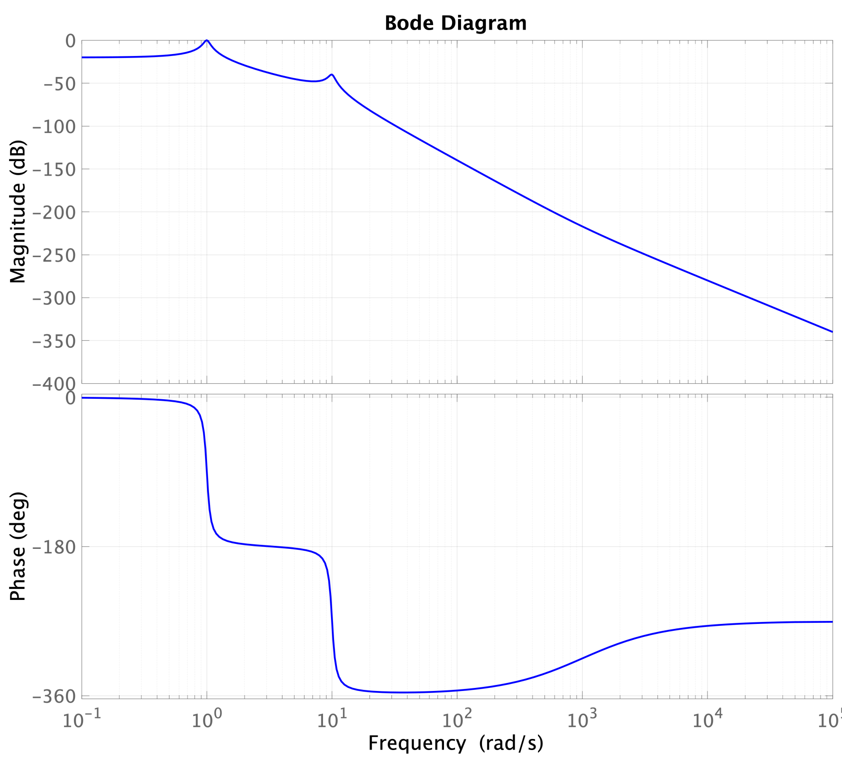
   (a) Determine the breakpoints and the transfer function.

   (b) Determine the gain cross-over frequency ω_c and the phase cross-over frequency ω_{180}.

Bode plot 1:



Bode plot 2:



4. Consider the feedback interconnection of Problem 1 with the PI controller $C(s) = \dfrac{10(s+3)}{s}$ and plant $P(s) = \dfrac{-0.5(s^2-2000)}{(s-3)(s^2+50s+1000)}$.

   (a) Is the feedback system stable? Why?

   (b) Use the Bode plot of the open loop transfer function $L(s) = P(s)C(s)$ to find the phase cross-over frequencies ω_0 such that $\angle L(j\omega_0) = \pm 180^\circ$. Use this information to compute the gain margin(s) of the feedback system. Check your answers using the **allmargin** command in MATLAB.

   (c) For each gain margin g_0 obtained in the previous part, construct the closed-loop using the perturbed loop transfer function $g_0 L(s)$ and verify that the closed-loop has poles at $\pm j\omega_0$.

   (d) Compute $\|S-T\|_\infty$ and the corresponding frequency ω_p where the peak gain of $S-T$ is achieved.

   (e) What is the symmetric disk margin m for this plant and controller? Verify your answer using **dm** = **diskmargin**(P∗C). Note that the **diskmargin** command uses the convention **m** = **dm.DiskMargin**/2.

   (f) Construct an α on the boundary of Disk$\left(\dfrac{1-m}{1+m}, \dfrac{1+m}{1-m}\right)$ such that the perturbed closed-loop $S_\alpha := \dfrac{1}{1+\alpha L(s)}$ has a pole at $j\omega_p$. Verify your construction by forming S_α and demonstrating that it has a pole at $j\omega_p$.
   Hint: Assume $\|S-T\|_\infty = 1/m$ at frequency ω_p. Then there exists a complex number $S(j\omega_p) - T(j\omega_p) = 1/z$ where $|z| = m$. Algebraically show that $\alpha = \dfrac{1+z}{1-z}$ satisfies $1 + \alpha L(j\omega_p) = 0$ and this α is in the symmetric disk defined by m.