Problem02

Saturday, April 2, 2022 06:00

2. [20 points] Consider a string of two vehicles as shown in Figure 2. The leading (first) vehicle is tracking a reference r_1 and the following (second) vehicle is tracking a reference $r_2 = x_1 - \delta$ to maintain a distance of δ with the first vehicle. The second vehicle uses a radar device to measure the distance to the first vehicle and compute the error $e_2 = r_2 - x_2$. Let each vehicle be modeled by the transfer function $G(s)$ and assume that both vehicles use the same control law $K(s)$. Figure 3 shows the feedback diagram for the two vehicle string.

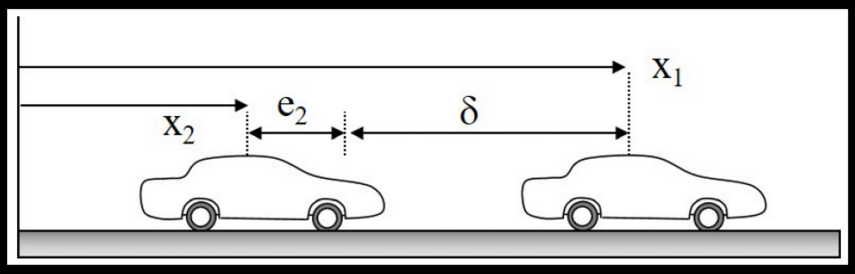

Fig. 2: String of two vehicles

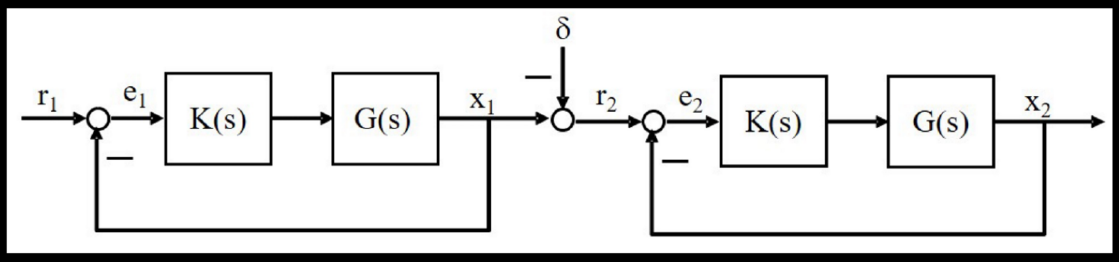

Fig. 3: Feedback diagram for two-vehicle system

(a) Compute the transfer function from: (i) r_1 to e_1, (ii) r_1 to x_1, (iii) r_2 to e_2, and (iv) r_2 to x_2. Your answers should be expressed in terms of the sensitivity $S(s) = \frac{1}{1+G(s)K(s)}$ and complementary sensitivity $T(s) = \frac{G(s)K(s)}{1+G(s)K(s)}$.

$e_1 = r_1 - x_1$

$x_1 = KG\,e_1$

$e_1 = r_1 - KG\,e_1$

$r_1 = (1+KG)\,e_1 \Rightarrow$ i)

$$\frac{e_1}{r_1} = \frac{1}{1+KG} = S(s)$$

$r_1 = (1 + KG) e_1 \Rightarrow$ i) $\boxed{\frac{e_1}{r_1} = \frac{1}{1+KG} = S(s)}$

$x_1 = KG(r_1 - x_1)$ ii)

$(1 + KG) x_1 = KG r_1 \Rightarrow \boxed{\frac{x_1}{r_1} = \frac{KG}{1+KG} = T(s)}$

iii)

$e_2 = r_2 - x_2$

$r_2 = (1 + KG) e_1$

$\boxed{\frac{e_2}{r_2} = \frac{1}{1+KG} = S(s)}$

$x_2 = KG e_2$

$x_2 = KG\left(\frac{1}{1+KG}\right) r_2 \Rightarrow \boxed{\frac{x_2}{r_2} = \frac{KG}{1+KG} = T(s)}$

(b) What is the transfer function from the first vehicle's reference r_1 to the tracking error for the second vehicle e_2? Note: By linearity you may assume $\delta = 0$ in this calculation.

$r_1 \to e_2$

$\frac{x_1}{r_1} = T(s) = \frac{r_2}{r_1} \Rightarrow \boxed{\frac{e_2}{r_1} = S(s) T(s)}$

(c) The goal is for the first vehicle to track the reference command r_1. In addition, the second vehicle should achieve a much smaller tracking error than the first vehicle, i.e., we would like $|e_2|$ to be much smaller than $|e_1|$. Use your results from the previous parts to express these objectives in terms of $S(s)$ and $T(s)$. You may assume that the reference command r_1 mainly consists of low frequency content.

(d) Is it possible to achieve the two goals in part (c)? If yes, describe how you would design $K(s)$ to achieve these goals. If no, then describe a constraint that prevents you from achieving both goals.