MECH 6323 - HW 1

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1 Problem 1

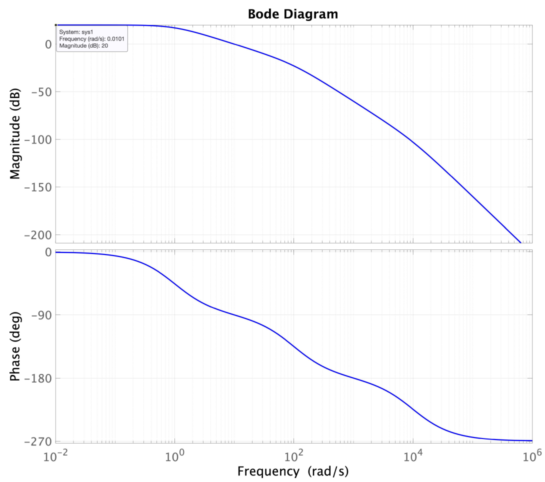
2 Problem 3

Problem: For each of the bode plots:

1. Determine the breakpoints and the transfer function.

2. Determine the gain cross-over frequency ω_c and the phase cross-over frequency ω_{180}.

2.1 Bode Plot 1:



2.1.1 Gain, Poles, and Zeros:

1. **Gain:** 20 db = 10

2. **Poles:**

   (a) $10^0 = 1$ rad/s

   (b) $10^2 = 100$ rad/s

   (c) $10^4 = 10,000$ rad/s

3. Zeros: (NA)

Transfer Function:

$$H(s) = \frac{10}{\left(1 + \frac{s}{1}\right)\left(1 + \frac{s}{100}\right)\left(1 + \frac{s}{10000}\right)}$$

2.1.2 Cross-over Frequency:

1. $\omega_c = 10^1 = 10$ rad/s

2. $\omega_{180} = 10^3 = 100$ rad/s

2.2 Bode Plot 2:

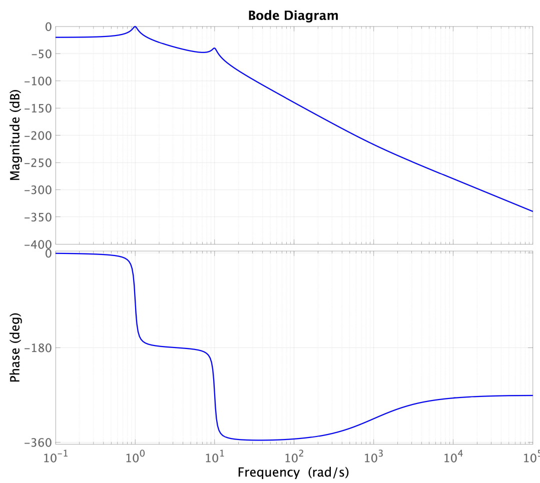


Figure 1: Bode Plot 2

2.2.1 Gain, Poles, and Zeros:

1. **Gain:** -20 db $= \frac{1}{10}$
2. **Poles:**
   (a) $10^0 = 1$ rad/s (complex)
   (b) $10^1 = 10$ rad/s (complex)
3. Zeros:
   (a) $10^3 = 1,000$ rad/s

2.2.2 Transfer Function:

$$H(s) = \frac{(1 + \frac{s}{1000})}{10\big(\frac{1}{1}\big(s^2 + 2(\frac{1}{10})(1)s + (1)^2\big)\big)\big(\frac{1}{10}\big(s^2 + 2(\frac{1}{10})(10)s + (10)^2\big)\big)} = \frac{(s + 1000)}{(s^2 + 0.2s + 1)(s^2 + 2s + 100)}$$

Assuming a Q-factor of around 10 to get the complex response.

2.2.3 Cross-over Frequency:

1. $\omega_c = 10^0 = 1$ rad/s
2. $\omega_{180} = 10^3 = 100$ rad/s

3 Problem 4

Consider the interconection of Problem 1 with the PI controller

$$C(s) = \frac{10(s+3)}{s}$$

and plant

$$P(s) = \frac{-0.5(s^2 - 2000)}{(s-3)(s^2+50s+1000)}$$

3.1 Is the feedback system stable? Why?

$$\begin{aligned}
1 + C(s)P(s) &= 1 + \frac{10(s+3)}{s}\frac{-0.5(s^2-2000)}{(s-3)(s^2+50s+1000)} \\
&= \frac{s(s-3)(s^2+50s+1000) + 10(s+3)(-0.5)(s^2-2000)}{s(s-3)(s^2+50s+1000)} \\
&\approx \frac{(s^2+11.73s+73.9)(s^2+30.27s+406)}{s(s-3)(s^2+50s+1000)}
\end{aligned}$$