

MECH 6323 - Final Project

Author: Jonas Wagner

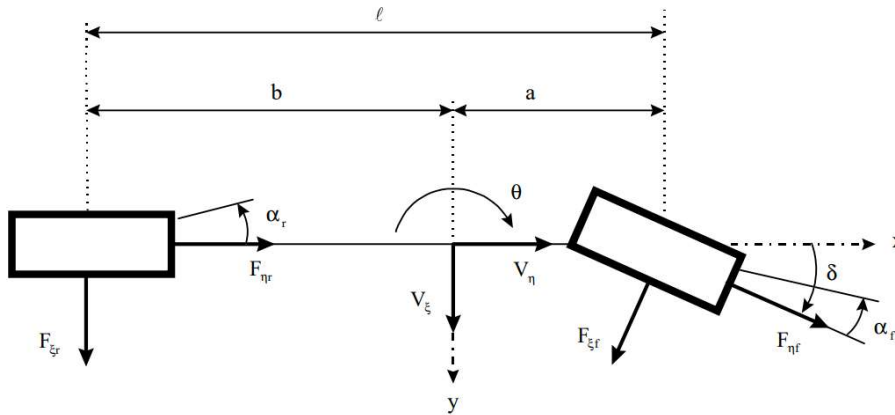
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Intro

Abstract-

In this project a model of dynamics of the Autonomous Vehicle (NOVA) will be examined. The NOVA team aims to eventually develop a more fundamental theory-based control strategy, yet there still exists many uncertain and changing parameters that determine the dynamics of the system. In this project a tool to analyse the

Nominal Plant Definition



$$\begin{aligned} I\ddot{\theta} &= aF_{\eta f}\delta + bF_{\zeta f} - bF_{\zeta r} \\ m(\dot{V}_\zeta + V_\eta\dot{\theta}) &= F_{\eta f}\delta + F_{\zeta f} + F_{\zeta r} \\ m(\dot{V}_\eta + V_\zeta\dot{\theta}) &= F_{\eta f} + F_{\eta r} + F_{\zeta f}\delta \\ \dot{x} &= -V_\zeta \sin(\theta) + V_\eta \cos(\theta) \\ \dot{y} &= V_\eta \cos(\theta) + V_\zeta \sin(\theta) \end{aligned}$$

$$\dot{x} = -V_\zeta \sin \theta + V_\eta \cos \theta$$

$$\dot{y} = V_\xi \cos \theta + V_\eta \sin \theta$$

$F_{\xi f}, F_{\xi r}$ = lateral tire force on front and rear tires

a = vehicle center of gravity location from front axle

b = vehicle center of gravity location from rear axle

m = vehicle mass

I = vehicle yaw inertia

F_f, F_r = longitudinal force on front and rear tires

V_η, V_ξ = longitudinal/lateral velocity of vehicle in body reference frame

x, y = longitudinal/lateral position of vehicle in inertial reference frame

δ = steering angle

η = longitudinal axis in body reference frame

θ = yaw angle

ξ = lateral axis in body reference frame

$$x = [\dot{\theta}, V_{\zeta}, V_{\eta}, x, y, \theta]^T$$

$$u = [\delta, F_{\eta f}, F_{\eta r}, F_{\zeta f}, F_{\zeta r}]^T$$

$$x_sym =$$

$$\begin{bmatrix} \dot{\theta} \\ V_{\zeta} \\ V_{\eta} \\ x \\ y \\ \theta \end{bmatrix}$$

$$u_sym =$$

$$\begin{bmatrix} \delta \\ F_{\zeta, f} \\ F_{\zeta, r} \\ F_{\eta, f} \\ F_{\eta, r} \end{bmatrix}$$

$$\dot{x}_1 = \frac{(aP_f \delta + bF_{\xi f} - bF_{\xi r})}{I}$$

$$\dot{x}_2 = \frac{(P_f \delta + F_{\xi f} + F_{\xi r})}{m} - x_3 x_1$$

$$\dot{x}_3 = \frac{(P_f + P_r - F_{\xi f} \delta)}{m} - x_2 x_1$$

$$\dot{x}_4 = -x_2 \sin x_6 + x_3 \cos x_6$$

$$\dot{x}_5 = x_2 \cos x_6 + x_3 \sin x_6$$

$$\dot{x}_6 = x_1$$

$$x_1 = \dot{\theta} \quad \text{yaw rate}$$

$$x_2 = V_{\xi} \quad \text{lateral velocity}$$

$$x_3 = V_{\eta} \quad \text{longitudinal velocity}$$

$$x_4 = x \quad \text{longitudinal position with respect to fixed reference}$$

$$x_5 = y \quad \text{lateral position with respect to fixed reference}$$

$$x_6 = \theta \quad \text{yaw angle}$$

Note that $F_{\eta f} = P_f$ and $F_{\eta r} = P_r$ when in an ideal plant model... effected by slippage

$$\text{bike_nonlin_dot_sym} =$$

$$\begin{bmatrix} \frac{F_{\eta,f} b - F_{\eta,r} b + F_{\zeta,f} a \delta}{I} \\ \frac{F_{\eta,f} + F_{\eta,r} + F_{\zeta,f} \delta}{m} - V_{\eta} \dot{\theta} \\ \frac{F_{\zeta,f} + F_{\zeta,r} - F_{\eta,r} \delta}{m} - V_{\zeta} \dot{\theta} \\ V_{\eta} \cos(\theta) - V_{\zeta} \sin(\theta) \\ V_{\eta} \cos(\theta) + V_{\zeta} \cos(\theta) \\ \dot{\theta} \end{bmatrix}$$

Nominal Model

$$\text{bike_nonlin_dot_nom} = \begin{bmatrix} 0.0013 F_{\eta,f} - 0.0013 F_{\eta,r} + 0.0010 F_{\zeta,f} \delta \\ 6.6667\text{e-}05 F_{\eta,f} + 6.6667\text{e-}05 F_{\eta,r} + 6.6667\text{e-}05 F_{\zeta,f} \delta - V_{\eta} \dot{\theta} \\ 6.6667\text{e-}05 F_{\zeta,f} + 6.6667\text{e-}05 F_{\zeta,r} - 6.6667\text{e-}05 F_{\eta,r} \delta - V_{\zeta} \dot{\theta} \\ V_{\eta} \cos(\theta) - V_{\zeta} \sin(\theta) \\ V_{\eta} \cos(\theta) + V_{\zeta} \cos(\theta) \\ \dot{\theta} \end{bmatrix}$$

Linearized Model

This is an unrealistic thing to do outside of single time-steps

$$\text{A_sym} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -V_{\eta} & 0 & -\dot{\theta} & 0 & 0 & 0 \\ -V_{\zeta} & -\dot{\theta} & 0 & 0 & 0 & 0 \\ 0 & -\sin(\theta) & \cos(\theta) & 0 & 0 & -V_{\zeta} \cos(\theta) - V_{\eta} \sin(\theta) \\ 0 & \cos(\theta) & \cos(\theta) & 0 & 0 & -V_{\eta} \sin(\theta) - V_{\zeta} \sin(\theta) \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{B_sym} = \begin{bmatrix} \frac{F_{\zeta,f} a}{I} & \frac{a \delta}{I} & 0 & \frac{b}{I} & -\frac{b}{I} \\ \frac{F_{\zeta,f}}{m} & \frac{\delta}{m} & 0 & \frac{1}{m} & \frac{1}{m} \\ -\frac{F_{\eta,r}}{m} & \frac{1}{m} & \frac{1}{m} & 0 & -\frac{\delta}{m} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Uncertain Plant Dynamics and Disturbances

All the calculations here are dependent on the nominal dynamics.

params = struct with fields:

```

a: 1.2280
b: 1.5618
m: 15000
I: 1200
c_f: 0.1200
c_r: 0.1600

```

Physical Disturbances

Tire Slipage

We have options to model the tire slipage better (actually what alphas are used for) but they are highly nonlinear and are also highly dependent on so it is very difficult to model accurately. Instead we will model tire with a disturbance input and have it be modeled a disturbance within the for each tire. For simplicity we will make it a parameter that is really only dependent on the unchanging parameters , , and since these are what effect this in real life.

```
W_zeta_f = 5.2418e-08
```

```
W_zeta_r = 8.4788e-08
```

Actually... since idk what I am actually talking about for vehicle dynamics, we can going to refer back to what Adi had done for the optimization and use the less specific (just that it is not derived from the parameters directly, but is just done by a single that is tuned per the vehicle operation at the specific weight/configuration/etc.

Wind Resistance

Although the NOVA vehicile will be going slow enough that wind negligence is negligible, including it seemed worth while. The effect of wind resistance itself is modeled as two components.

First, the force due to movement through air proportional to speed is added in the system directly opisite the current direction of movement. This is inaccurate in general due to car body shape but this could also be modified to be different weighting from different directions but we will instead just provide for this parameter to vary greatly.

$$F_{\eta,w} = k_{\eta,w} \cdot V_{\eta}$$

$$F_{\zeta,w} = k_{\zeta,w} \cdot V_{\zeta}$$

Secound an additive noise of general stead-state wind either fights against or assists the vehicle. Since wind direction generally changes slowly we will only be looking at lower frequencies.

$$F_{ws} = W_{ws} \cdot z_{ws} \cdot (\cos(\theta - \theta_{ws})\hat{u}_{\eta} + \sin(\theta - \theta_{ws})\hat{u}_{\zeta}), \quad z_{ws} \in [0, 1], \quad \theta_{ws} \in [0, 2\pi]$$

$$\theta_{ws} = W_{w,\theta_{ws}} z_{w,\theta_{ws}}$$

```
W_ws =
```

```

-0.1
-----
(s-0.01)

```

Continuous-time zero/pole/gain model.

```
W_ws_theta =
```

0.062832

(s-0.01)

Continuous-time zero/pole/gain model.

both a multiplicative noise on the standard are resistance of dependent on the vehicle velocity.

Currently this is not implimented in the simulation fully. Instead the following more simplistic method is used:

$$F_{\eta,r} = -k_{ws,\eta} * V_{\eta}$$

Since wind resistance is an over complication we are eliminating it for this time.

Uncertain/Nonlinear Actuators

Steering and Forward Force Delays

Both a human and autonomous driver takes a while to react, while both the steering mechanism and engine/drivetrain also contain delays. We also assume it is only a front-wheel drive vehicle, so only the front wheel will be providing force to move the vehicle.

$$\delta = W_{driver} \cdot u_{\delta}$$

W_driver =

10

(s-10)

Continuous-time zero/pole/gain model.

$$F_{eta,f} = W_{drivetrain} \cdot W_{driver} \cdot u_{gas}$$

W_drivetrain =

5

(s-0.5)

Continuous-time zero/pole/gain model.

Slip angle effects

The front slip angle is calculated as:

$$\alpha_f = \delta - \frac{a\dot{\theta} + V_{\zeta}}{V_{\eta}}$$

and the rear slip angle is

$$\alpha_r = \frac{b\dot{\theta} - V_{\zeta}}{V_{\eta}}$$

Then we have (ignoring saturation regions... remaining linear for this analysis)

$$F_{\zeta,f} = c_f \alpha_f$$

and

$$F_{\zeta,r} = c_r \alpha_r$$

where c_f and c_r are the cornering stiffnesses uncertain parameters.

Sensors and Measurement Noise

Gyroscope

The gyroscope will measure the rotational velocity of the system which will allow an observer to measure the current state with fairly high accuracy if you average it (which isn't really that useful when you would need to integrate or differentiate them) but will be noisy at higher frequencies if sampled often.

The gyro measurement output will therefore be:

$$y_{gyro} = \dot{\theta} + W_{gyro} \cdot z_{gyro}, \quad z_{gyro} \in [-1, 1]$$

$$W_{gyro} =$$

$$\frac{0.0001 \text{ (s}^{-2}\text{)}}{(s-5e05)}$$

Continuous-time zero/pole/gain model.

Spedometer

The spedometer is just a measurement directly of the Longitudinal Velocity, V_η , and will have a measurement noise that is only really apparent at higher frequencies.

The gyro measurement output will therefore be:

$$y_{sped} = V_\eta + W_{sped} \cdot z_{sped}, \quad z_{sped} \in [-1, 1]$$

$$W_{sped} =$$

$$\frac{0.1 \text{ (s}^{-10}\text{)}}{(s-1000)}$$

Continuous-time zero/pole/gain model.

GPS

The GPS system will be used to calculate the global position over a longer period of time for the system. Similar to the Gyroscope, the GPS will have error at low ends and isn't going to ever truly be accurate over-time in a real-world situation by itself without other information used by a Kalman Filter or similar in conjunction with other sensors (such as Lidar) and other localization techniques (which is a big part of what NOVA has been working on this semester). We will assume that the sensor error output from the gps will be white noise that is filtered out mostly (because of the way it is averaged and sampled so higher frequencies become less important) except for a region.

$$y_{gps,x} = x + W_{gps} \cdot z_{gps}, \quad z_{gps} \in [-1, 1]$$

$$y_{gps,y} = y + W_{gps} \cdot z_{gps}, \quad z_{gps} \in [-1, 1]$$

$$W_{gps} =$$

$$\frac{10}{\text{-----}}$$

$$s^2 + 100s + 100$$

Continuous-time transfer function.

Eyeballs (Vison & Loclaization)

The big thing that isn't included in this entire thing is the actual vision that the vehicle has. Generally this is almost a cheat catch-all of everything since although the sensors will have higher accuracy and ultimently a faster response time, it is the vision that will make things consistant. Similarly to the 'driver-delay', we will have a sensing type delay, but all of the information we are able to be awar of is available (although we will have considerably more noise.

$$y_{vision} = y + W_{vision} \cdot z_{vision}, \quad z_{vision} \in [-1, 1]$$

Parameteric Uncertainty

Parameteric Uncertainty essentially just assumes deviations from potential modeling issues. For the current analysis this is ignored in order to focus on actuator uncertainty.

Actually, I ended up deciding to do just that... but just with a more standard thing.

Plant Model and Controller Design

Nominal Plant

sys_nom =

A =

	x1	x2	x3	x4	x5	x6
x1	0	0	0	0	0	0
x2	-1	0	0.1	0	0	0
x3	-0.1	0.1	0	0	0	0
x4	0	-0.7078	0.7064	0	0	-0.7785
x5	0	0.7064	0.7064	0	0	-0.7786
x6	1	0	0	0	0	0

B =

	u1	u2	u3	u4	u5
x1	0.001023	0.0001023	0	0.001302	-0.001302
x2	6.667e-05	6.667e-06	0	6.667e-05	6.667e-05
x3	6.667e-05	6.667e-05	6.667e-05	0	6.667e-05

Reference Signal

Uncertain Plant

sys_plant_u =

Uncertain continuous-time state-space model with 6 outputs, 5 inputs, 6 states.

The model uncertainty consists of the following blocks:

- I: Uncertain real, nominal = 1.2e+03, variability = [-1,1]%, 2 occurrences
- a: Uncertain real, nominal = 1.23, variability = [-1,1]%, 1 occurrences
- b: Uncertain real, nominal = 1.56, variability = [-1,1]%, 1 occurrences
- m: Uncertain real, nominal = 1.5e+04, variability = [-1,1]%, 2 occurrences

Type "sys_plant_u.NominalValue" to see the nominal value, "get(sys_plant_u)" to see all properties

◀

The uncertain plant with all the dynamics is actually really complicated and not only is dependent on the changing parameters and nonlinearities due to actuators; but is also really complicated with conections

to the global coordinate system. We therefore can develop a local plant that incorporates less nonlinearities while being tracked separately in the global reference frame. The controller that works for this would not need to worry about that particular nonlinearity.

```
sys_plant_u_local =
```

```
Uncertain continuous-time state-space model with 4 outputs, 5 inputs, 4 states.
```

```
The model uncertainty consists of the following blocks:
```

```
I: Uncertain real, nominal = 1.2e+03, variability = [-1,1]%, 2 occurrences
```

```
a: Uncertain real, nominal = 1.23, variability = [-1,1]%, 1 occurrences
```

```
b: Uncertain real, nominal = 1.56, variability = [-1,1]%, 1 occurrences
```

```
m: Uncertain real, nominal = 1.5e+04, variability = [-1,1]%, 2 occurrences
```

```
Type "sys_plant_u_local.NominalValue" to see the nominal value, "get(sys_plant_u_local)" to see all
```

```
<
```

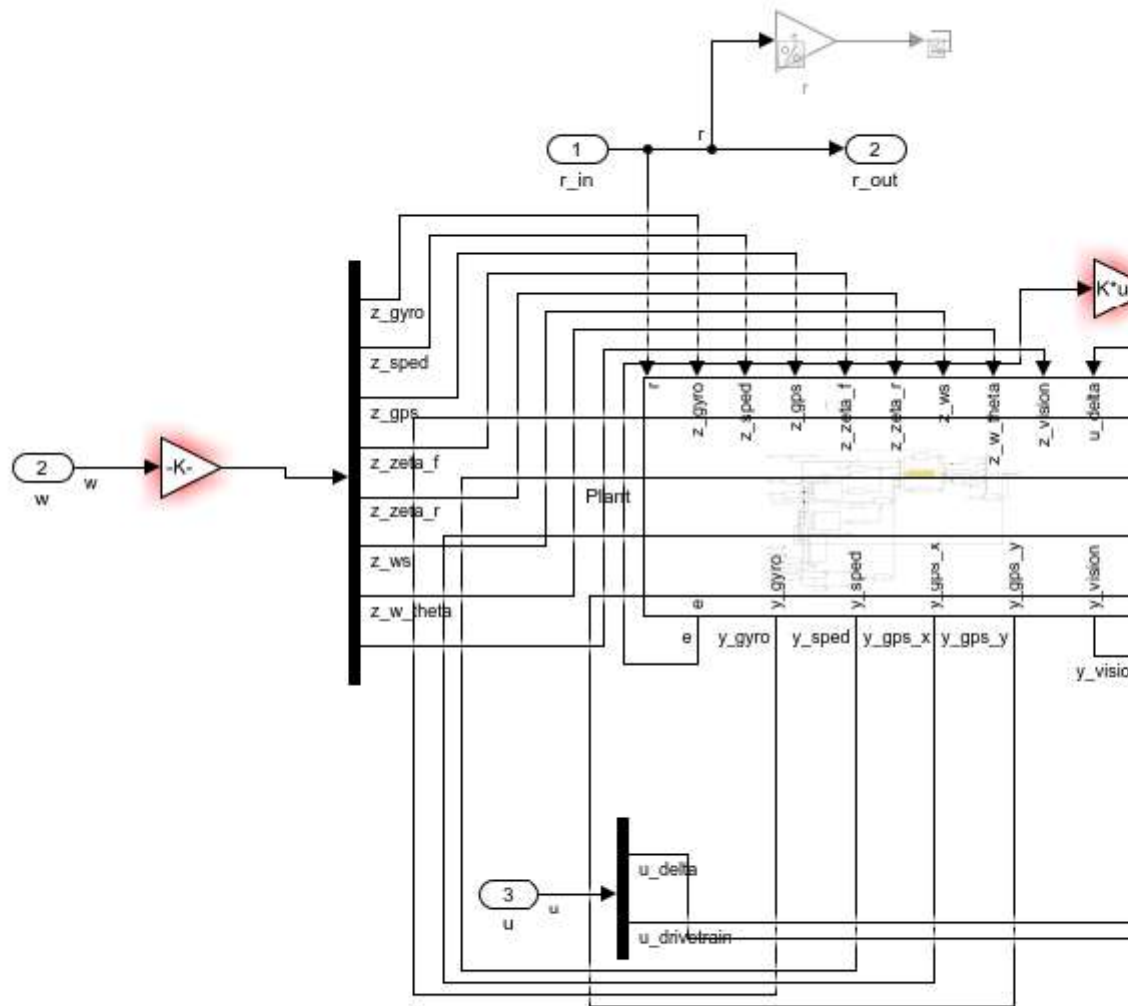
Simulink Modeling

Plant Modeling

Plant Internal Diagram

The plant internal diagram includes the original physical dynamics within a standard LTI system block and all of the weighting functions for the actuators and sensors containing the uncertainty.

plant_bike_lin.slx



fast_test.slx

Because of many issues with the complicated model, a simpler system that implemented a less restrictive actuator system (and ignores the wind disturbances which made stabilization nearly impossible) a simpler model (shown below) was created instead.

Type "sys_plant.NominalValue" to see the nominal value, "get(sys_plant)" to see all properties, an

This plant

H-infty Controller Design

The rank of our controllability is needed to ensure that the system is actually controllable.

```
ctrb_rank = 7
```

In many different equilibrium cases the rank of controllability is 1. In other words,

The rank being 1 unfortunately means that the entire project is built on false assumptions, leading to a complete failure to even have a controllable (let alone robust) system. Looking back, the assumptions made where the lateral force on the side of the wheel was solely a weighted value dependent on the

State-space model with 5 outputs, 16 inputs, and 20 states.

```
info =
```

```
  hinfINFO with properties:
```

```
  gamma: 1.0100
      X: [20x20 double]
      Y: [20x20 double]
      Ku: [5x20 double]
      Kw: [25x20 double]
      Lx: [20x16 double]
      Lu: [5x16 double]
      Preg: [27x30 ss]
      AS: [21x21 ss]
```

A controller was able to be developed, for some very specific linearizations,

Uncertain Plant Analysis

```
L =
```

Uncertain continuous-time state-space model with 5 outputs, 5 inputs, 40 states.

The model uncertainty consists of the following blocks:

```
I: Uncertain real, nominal = 1.2e+03, variability = [-1,1]%, 2 occurrences
a: Uncertain real, nominal = 1.23, variability = [-1,1]%, 1 occurrences
b: Uncertain real, nominal = 1.56, variability = [-1,1]%, 1 occurrences
```

Type "L.NominalValue" to see the nominal value, "get(L)" to see all properties, and "L.Uncertainty

It is worth noting that the controller, even in the open loop configuration, was able to eliminate dependence on both m and c_f .

```
ans = 5x1 struct
```

Fields	GainMargin	PhaseMargin	DiskMargin	LowerBound	UpperBound
1	[1,1]	[0,0]	0	0	