Due Sunday 04/10/22 (10:00pm)

1. Let M and N be matrices of suitable dimension and let Δ be a structured uncertainty. Prove or disprove the follow:

   (a) $\mu_\Delta(M) = 0 \implies M = 0$.

   (b) $\mu_\Delta(M_1 + M_2) \le \mu_\Delta(M_1) + \mu_\Delta(M_2)$.

   (c) $\mu_\Delta(\alpha M) = |\alpha|\, \mu_\Delta(M)$.

   (d) $\mu_\Delta(I) = 1$.

   (e) $\mu_\Delta(MN) \le \bar{\sigma}(M)\, \mu_\Delta(N)$.

   (f) $\mu_\Delta(MN) \le \bar{\sigma}(N)\, \mu_\Delta(M)$.

2. Let $\Delta = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix}$, where Δ_i are structured uncertainties. Show that

$$\mu_\Delta\left(\begin{bmatrix} M_{11} & M_{12} \\ 0 & M_{22} \end{bmatrix}\right) = \max\{\, \mu_{\Delta_1}(M_{11}),\, \mu_{\Delta_2}(M_{22})\,\}.$$

3. Work through the exercise on *Computing Complex* μ. The attached zip file includes relevant m-files along with the pdf for this exercise.

4. Consider a simple model for a car:

$$m\,\dot{v} = -b\,v + F$$

   where v is the velocity, m is the mass, b is the wind drag coefficient, and F is the force generated due to the engine. Assume that F is proportional to the engine throttle angle: $F = cu$, where u is the engine throttle and c is the force constant. The vehicle model can be written as

$$m\,\dot{v} = -b\,v + c\,u.$$

   Moreover, assume the throttle actuator dynamics from u_{cmd} to u can be modeled as a first-order lag, $\frac{1}{\tau s+1}$. Thus, the nominal vehicle model from u_{cmd} to v is given by $P(s) = \frac{c}{ms+b}\frac{1}{\tau s+1}$. The parameter values for the nominal model are given by $m = 2150$ kg, $b = 20$ N s/m, $c = 150$ N/deg, and $\tau = 0.1$ sec.

   (a) A cruise control algorithm is designed to control the vehicle velocity v to track a desired speed v_{des} set by the driver. Let this tracking objective be specified as the requirement $|S(j\omega)| \le |B(j\omega)|$ for all ω with the performance bound

$$B(s) = \frac{s + 0.02}{0.5\,s + 1}.$$

   Provide a brief interpretation for the performance objective specified by the bound $B(s)$.

(b) Design a simple proportional control law that achieves nominal performance. What is the time constant of the closed-loop system? Submit a single Bode magnitude plot with $S(j\omega)$ and $|B(j\omega)|$.

(c) Next we consider the effect of model uncertainty. The vehicle mass will vary depending on the number of passengers, etc. Assume $m = 2150 \pm 150$ kg. The other parameters will also have some uncertainty. Assume b is 20% uncertain and c is 10% uncertain. Finally, assume the actuator time constant is in the range $\tau \in [0.05, 0.2]$. Denote the set of all models that arise over these parameter ranges by $\mathcal{A}$. For simplicity, we will "cover" the uncertainty with a multiplicative model. Specifically, we will choose an uncertainty weight W_u such that the multiplicative uncertainty set $\mathcal{M}$ described by W_u contains all models in $\mathcal{A}$, i.e., $\mathcal{A} \subset \mathcal{M}$. Generate 20 samples $\hat{P}_i \in \mathcal{A}$ by randomly sampling the uncertain parameter values. The relative error between these samples and the nominal model is given by $R_i := \frac{|P - \hat{P}_i|}{|P|}$. Plot the relative error vs frequency for all 20 samples. Choose first order uncertainty weight $W_u(s) = \frac{a_1 s + a_2}{s + a_3}$ such that $|W_u(j\omega)| \geq \max_i R_i(\omega)$ for all ω. This will ensure that $\mathcal{M}$ contains all samples and hence we approximately have $\mathcal{A} \subset \mathcal{M}$. Plot the magnitude of your weight $|W_u(j\omega)|$ on the same plot as the relative errors.

(d) Does your proportional control law robustly stabilize all plants in $\mathcal{M}$ for the weight W_u designed in the previous part?

(e) Construct the closed-loop sensitivity function $\hat{S}_i$ for each of the 20 samples $\hat{P}_i$ generated in part (c). Hand in a single Bode magnitude plot with $\hat{S}(j\omega)$ (for $i = 1, \ldots, 20$) and $|B(j\omega)|$. Does your proportional control law achieve robust performance on these samples of the plant dynamics, i.e., does K achieve the performance objective for all plants $\{\hat{P}_i\}_{i=1}^{20} \subset \mathcal{M}$?

Comment: the steps covered in the last problem can be automated using MATLAB commands ureal and ucover. While we will be using these commands later on in the course, for now, you can complete this problem without them.