Problem01

Saturday, April 2, 2022 05:58

1. [15 points] Consider the feedback system shown in Figure 1 where $G(s)$ is the nominal plant model and $\Delta(s)$ is stable. Assume that $G(s)$, $K(s)$, and $\Delta(s)$ are all SISO.

   (a) The dashed box represents an uncertain model $\hat{G}(s)$ that depends on both $G(s)$ and $\Delta(s)$. What is the set of models $\mathcal{A}$ corresponding to this block diagram?

   (b) What can you conclude about the classical gain margins if the feedback system is stable for all $\|\Delta\|_\infty < 0.5$?

   (c) Find a necessary and sufficient condition for $K(s)$ to stabilize all $\hat{G}(s) \in \mathcal{A}$. Briefly describe a proof that your condition is sufficient for $K(s)$ to achieve robust stability. You do not need to prove necessity.

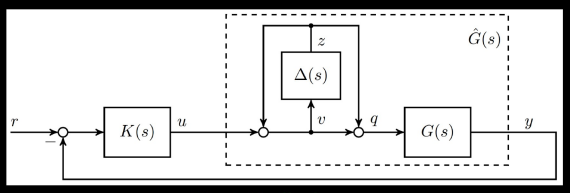

Fig. 1: Feedback system

a) $y = G(s)q$

$q = v + z$

$v = u + z$

$z = \Delta(s)v$

$y = G(s)(v+z)$

$z = \Delta(s)(u+z)$

$(1-\Delta(s))z = u$

$z = (1-\Delta(s))^{-1}u$

$(v+z) = u+z+z$

$(v+z) = u+2z$

$(v+z) = u+2(1-\Delta(s))^{-1}u$

$(v+z) = \left(1+2(1-\Delta(s))^{-1}\right)u$

$y = G(s)\left(1+2(1-\Delta(s))^{-1}\right)u$

$\hat{G}(s) = \left(1+2(1-\Delta(s))^{-1}\right)G(s)$

MIMO

$$\mathcal{A} := \left\{ \hat{G}(s) = \left(1+2(1-\Delta(s))^{-1}\right)G(s) \;:\; \Delta(s) \text{ stable} \right\}$$

since we assume SISO,

$$\hat{G}(s) = \frac{(1-\Delta(s))+2}{1-\Delta(s)}\,G(s)$$

$$\hat{G}(s) = \frac{3 - \Delta(s)}{1 - \Delta(s)} G(s)$$

$$\mathcal{A} := \left\{ \hat{G}(s) = \frac{3 - \Delta(s)}{1 - \Delta(s)} G(s) \;:\; \forall s : s - \Delta(s) = 0 \Rightarrow \mathrm{Re}\{s\} < 0 \right\}$$

---

b) $\|\Delta(s)\| < 0.5$

$$y = \left(\frac{3 - \Delta(s)}{1 - \Delta(s)} \right) G(s) K(s) (y - r)$$

$$\frac{\left((3 - \Delta(s)) G(s) K(s) - (1 - \Delta(s)) \right)}{1 - \Delta(s)} y = \frac{(3 - \Delta(s)) G(s) K(s)}{1 - \Delta(s)} r(s)$$

$$y = \frac{3GK - GK\Delta}{(3GK - 1) + (1 - GK)\Delta} r$$

$$\left(3GK - 1 - \frac{1 + GK}{2}, \; 3GK - 1 + \frac{1 - GK}{2} \right)$$

$$\left(\frac{5GK - 3}{2}, \; \frac{5GK + 3}{2} \right)$$

c)