# MECH 6323 - Final Project

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```
clc
clear
close all
```

## Plant Definition

```
a = 1.228;
b = 1.5618;
m = 1500;
I = 12;
% alpha_f = 0.1;
% alpha_r = 0.1;
```

$x_1 = \dot{\theta}$ yaw rate

$x_2 = V_\xi$ lateral velocity

$x_3 = V_\eta$ longitudinal velocity

$x_4 = x$ longitudinal position with respect to fixed reference

$x_5 = y$ lateral position with respece to fixed reference

$x_6 = \theta$ yaw angle

$$\dot{x}_1 = \frac{\left(aP_f\delta + bF_{\xi f} - bF_{\xi r}\right)}{I}$$

$$\dot{x}_2 = \frac{\left(P_f\delta + F_{\xi f} + F_{\xi r}\right)}{m} - x_3x_1$$

$$\dot{x}_3 = \frac{\left(P_f + P_r - F_{\xi f}\delta\right)}{m} - x_2x_1$$

$$\dot{x}_4 = -x_2 \sin x_6 + x_3 \cos x_6$$

$$\dot{x}_5 = x_2 \cos x_6 + x_3 \cos x_6$$

$$\dot{x}_6 = x_1$$

```
x_dot = @(x, u) [
    (a*u(2)*u(1) + b*u(4) - b*u(5))/I;
    (u(2)*u(1) + u(4) + u(5))/m - x(3)*x(1);
    (u(2) + u(3) - u(5)*u(1))/m - x(2)*x(1);
    -x(2)*sin(x(6)) + x(3)*cos(x(6));
    x(2)*cos(x(6)) + x(3)*cos(x(6));
    x(1);
];
```

$x = [\dot{\theta}, V_\zeta, V_\eta, x, y, \theta]^T$

$u = [\delta, P_f, P_r, F_{\zeta f}, F_{\zeta r}]^T$

```
x_sym = [
    sym('theta_dot')
    sym('V_zeta')
    sym('V_eta')
    sym('x')
    sym('y')
    sym('theta')
]
```

x_sym =

$$\begin{pmatrix} \dot{\theta} \\ V_\zeta \\ V_\eta \\ x \\ y \\ \theta \end{pmatrix}$$

```
u_sym = [
    sym('delta')
    sym('P_f')
    sym('P_r')
    sym('F_zeta_f')
    sym('F_zeta_r')
]
```

u_sym =

$$\begin{pmatrix} \delta \\ P_f \\ P_r \\ F_{\zeta,f} \\ F_{\zeta,r} \end{pmatrix}$$

```
x_dot_sym = x_dot(x_sym, u_sym)
```

x_dot_sym =

$$\begin{pmatrix} \frac{2603\, F_{\zeta,f}}{20000} - \frac{2603\, F_{\zeta,r}}{20000} + \frac{307\, P_f\, \delta}{3000} \\ \frac{F_{\zeta,f}}{1500} + \frac{F_{\zeta,r}}{1500} + \frac{P_f\, \delta}{1500} - V_\eta\, \dot{\theta} \\ \frac{P_f}{1500} + \frac{P_r}{1500} - \frac{F_{\zeta,r}\, \delta}{1500} - V_\zeta\, \dot{\theta} \\ V_\eta \cos(\theta) - V_\zeta \sin(\theta) \\ V_\eta \cos(\theta) + V_\zeta \cos(\theta) \\ \dot{\theta} \end{pmatrix}$$

## Linearize Model

This is an unrealistic thing to do outside of single time-steps

```
A_sym = jacobian(x_dot(x_sym,u_sym),x_sym)
```

A_sym =

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -V_\eta & 0 & -\dot{\theta} & 0 & 0 & 0 \\ -V_\zeta & -\dot{\theta} & 0 & 0 & 0 & 0 \\ 0 & -\sin(\theta) & \cos(\theta) & 0 & 0 & -V_\zeta \cos(\theta) - V_\eta \sin(\theta) \\ 0 & \cos(\theta) & \cos(\theta) & 0 & 0 & -V_\eta \sin(\theta) - V_\zeta \sin(\theta) \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
B_sym = jacobian(x_dot(x_sym,u_sym),u_sym)
```

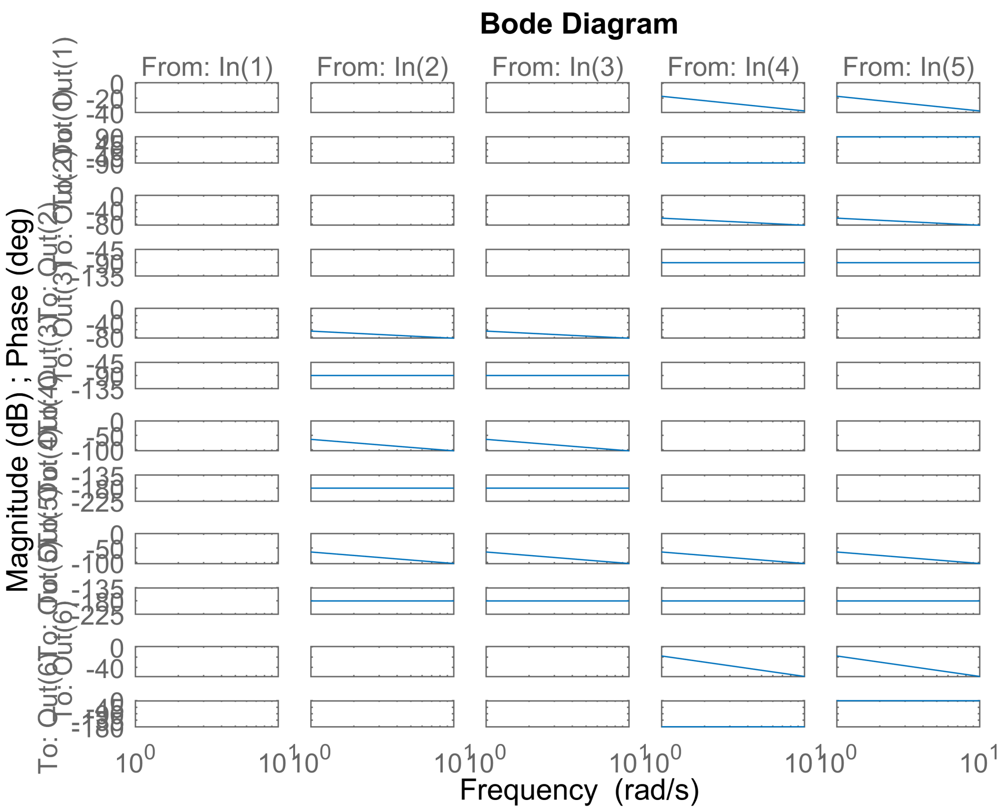
B_sym =

$$\begin{pmatrix} \frac{307\, P_f}{3000} & \frac{307\, \delta}{3000} & 0 & \frac{2603}{20000} & -\frac{2603}{20000} \\ \frac{P_f}{1500} & \frac{\delta}{1500} & 0 & \frac{1}{1500} & \frac{1}{1500} \\ -\frac{F_{\zeta,r}}{1500} & \frac{1}{1500} & \frac{1}{1500} & 0 & -\frac{\delta}{1500} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
A = @(x_0, u_0) double(subs(subs(A_sym,x_sym,x_0),u_sym,u_0));
B = @(x_0, u_0) double(subs(subs(B_sym,x_sym,x_0),u_sym,u_0));

x_0 = zeros(6,1);
u_0 = zeros(5,1);

sys_lti = ss(A(x_0,u_0), B(x_0,u_0), eye(6), 0)
```

```
sys_lti =

  A =
       x1  x2  x3  x4  x5  x6
   x1   0   0   0   0   0   0
   x2   0   0   0   0   0   0
   x3   0   0   0   0   0   0
   x4   0   0   1   0   0   0
   x5   0   1   1   0   0   0
   x6   1   0   0   0   0   0

  B =
              u1         u2         u3         u4         u5
   x1          0          0          0     0.1301    -0.1301
   x2          0          0          0  0.0006667  0.0006667
   x3          0  0.0006667  0.0006667          0          0
   x4          0          0          0          0          0
   x5          0          0          0          0          0
   x6          0          0          0          0          0

  C =
       x1  x2  x3  x4  x5  x6
   y1   1   0   0   0   0   0
   y2   0   1   0   0   0   0
   y3   0   0   1   0   0   0
   y4   0   0   0   1   0   0
   y5   0   0   0   0   1   0
   y6   0   0   0   0   0   1

  D =
       u1  u2  u3  u4  u5
   y1   0   0   0   0   0
   y2   0   0   0   0   0
   y3   0   0   0   0   0
   y4   0   0   0   0   0
   y5   0   0   0   0   0
   y6   0   0   0   0   0

Continuous-time state-space model.
```

```
bode(sys_lti)
```



Bode Diagram
From: In(1)
From: In(2)
From: In(3)
From: In(4)
From: In(5)
Magnitude (dB) ; Phase (deg)
Frequency (rad/s)