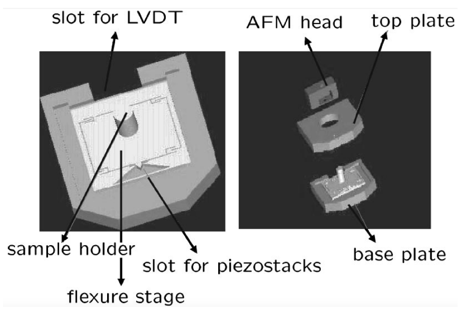
Due: Sunday 05/08/22 at 10:00pm

Do not send me m-files. Please upload your solutions (as a single pdf-file) to eLearning in addition to sending them to me via email.

1. In this problem, you will design a control law for a nano-positioning stage. These devices can achieve very high precision positioning which is important in applications such as atomic force microscopes (AFMs). The right side of Figure 1 shows a feedback diagram of a nanopositioning device. The system consists of piezo-electric actuation, a flexure stage, and a detection system. As illustrated in the feedback diagram, the flexure stage interacts with the head of an AFM. The left side of Figure 1 shows a diagram of the flexure stage for a nanopositioning device. Typical design requirements for the control law include high bandwidth, high resolution and good robustness.



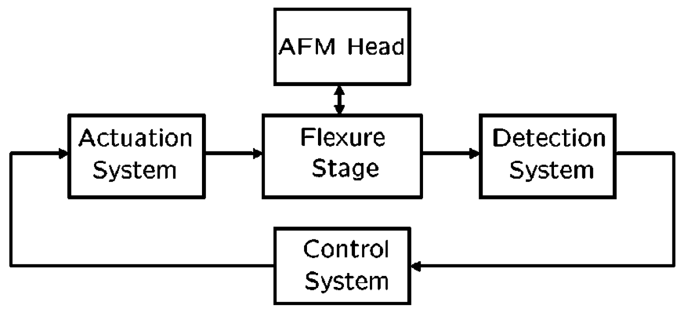




Fig. 1: Nanopositioning flexure stage (left) and feedback diagram (right); figures adapted from Salapaka et. al, *Rev. Sci. Instrum.* 2002.

(a) Frequency responses were obtained for the nano-positioning stage. The file **npresp.mat** contains the frequency response data as an **frd** object. Develop a transfer function model for the stage whose frequency response approximates this experimental response. You can use the **fitfrd** function to fit the data. Denote the transfer function model by $G(s)$. You should be able to obtain a good fit with $G(s)$ of order less than 10.

(b) As a baseline, first design a PI control law that stabilizes the system $G(s)$. To ensure good robustness your PI controller should guarantee a gain margin $\geq 1.5$ and a phase margin $75^o$. What is the maximum closed-loop bandwidth achievable by a PI control law which satisfies these margin requirements? In this problem, the closed loop bandwidth is defined as the frequency at which $|S(j\omega)|$ crosses the $-3$dB point from below. We will denote your PI control design as $C_{PI}$.

(c) Now design a control law using the mixed sensitivity design approach. You may use the **mixsyn** command to compute your control law. Choose the appropriate weighting functions and design a linear controller $C_\infty$ such that:

- The bandwidth (see definition above) is approximately 250Hz.
- The upper bound on $|S(j\omega)|$, which is a measure of robustness, should be 1.5.
- Below the bandwidth, $|S(j\omega)|$ should have a slope of approximately 20dB per decade.
- The DC gain of $S$, which is a measure of the steady state tracking error for a step input, should be lower than $-80$dB.

- $|T(j\omega)|$ should be less than $-3$dB at 500Hz. This is to reduce the effect of noise.
- The upper bound on $|T(j\omega)|$, which is a measure of robustness, should be 1.5.
- $|T(j\omega)|$ should be less than $-40$dB as $\omega \to \infty$.
- To limit the control effort, $|C_\infty S(j\omega)| \leq 10$ for all $\omega$.

(d) What are the gain and phase margins achieved by $C_\infty$? Plot both $C_{PI}$ and $C_\infty$ on the same Bode plot. In addition, create a second plot that has both loop transfer functions, $GC_{PI}$ and $GC_\infty$.

2. Before attempting this problem, go through the example MATLAB codes on the use of **sysic** in the *Matlab Codes* folder on eLearning.

The Generic Transport Model (GTM) is a turbine powered subscale model of a civilian transport aircraft, which was developed by NASA Langley as a platform to validate control laws. The model has a wing span of 7 ft, and weighs around 55 lbs. Under normal operation, the aircraft flies at an altitude of 700 to 1100 ft, and with an airspeed of 70 and 85 knots. In this problem you will use the signal-weighted $H_\infty$ method to design a control law for the GTM.

A nonlinear simulation model of the GTM has been developed from extensive wind tunnel and flight tests. The model can be linearized at a particular flight condition, yielding a linear model of the aircraft dynamics. The nominal fight condition for control design is level flight at 800 ft and 80 knots. The longitudinal short-period dynamics, denoted $G$, are described by the following state equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \alpha \\ q \end{bmatrix} = \begin{bmatrix} -2.4714 & 0.9514 \\ -43.9070 & -3.4738 \end{bmatrix}\begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -0.2501 \\ -44.9478 \end{bmatrix}\delta_{elev}$$

where the state vector corresponds to angle-of-attack $\alpha$ [rad] and pitch rate $q$ [rad/s]. The control surface input is elevator deflection $\delta_{elev}$ [rad]. The elevator actuator is modeled as a 5 Hz (= 31.42 rad/sec) first-order filter with DC gain equal to 1. This actuator saturates at 0.349 rads, i.e., it is physically constrained to $\delta_{elev} \leq 0.349$ rads. A rate gyro sensor measures pitch rate with noise that has standard deviation of 0.0067 rad/sec.

The main control objective is to design a Stability Augmentation System (SAS) to increase damping in the aircraft's oscillatory modes. A model-matching $H_\infty$ control problem is formulated to achieve the desired robustness and performance characteristics. The longitudinal control system interconnection is shown in Figure 2. The inputs to the controller are pilot longitudinal stick command and pitch rate feedback. The actual input commands are assumed to have magnitude $\leq 0.2618$ rads. The ideal model for matching is denoted as "Model" in the diagram. It will be chosen to mimic the open-loop aircraft behavior at low and high frequency but with improved damping in the oscillatory modes.

(a) Choose the model for the ideal response as $G_{damp}(s)A(s)$ where $A(s)$ is the same as the actuator dynamics and $G_{damp}(s)$ is the same as the aircraft short-period dynamics $G$ but with the damping ratio of the poles increased to $\zeta = 0.8$.

(b) Choose all weighting functions in the design interconnection (Figure 2) to be constants based on the information provided above. Specifically, the weights should roughly normalize all input/output signals. In addition, you may assume that the maximum disturbance is 15% of the maximum actuator level. The maximum error between the closed-loop pitch rate and the ideal response should be roughly less than or equal to 0.01 rad/sec.

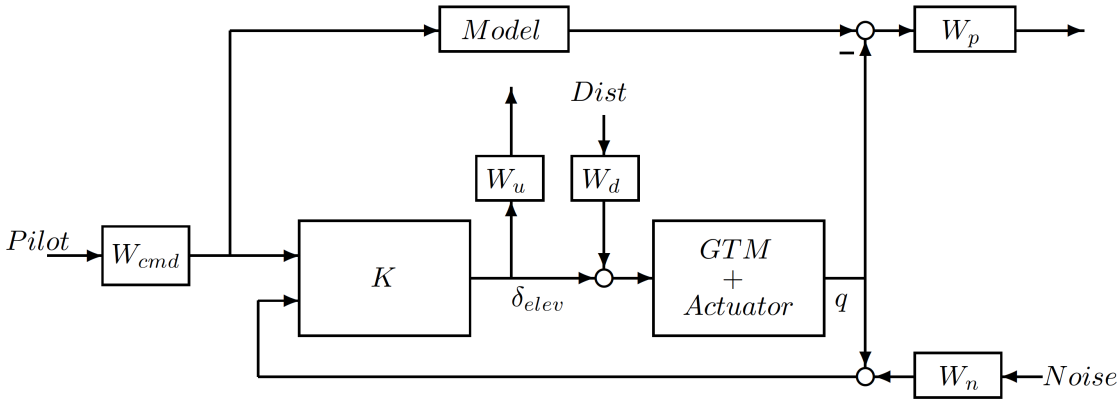



Fig. 2: Longitudinal system interconnection.

Note that in Figure 2, Pilot, Dist, and Noise are normalized signals. The actual signals in the feedback loop are the outputs of the corresponding weights.

(c) Use **sysic** to create the generalized plant $P$ with inputs [Pilot, Dist, Noise, $\delta_{elev}]^T$ and outputs $[W_p,\ W_u,\ W_{cmd},\ q + W_n]^T$. If you prefer, you may use **connect** to build this generalized plant instead of **sysic**.

(d) Use **hinfsyn** to perform the $H_\infty$ controller design for your generalized plant. What value of $\gamma$ was achieved in the design? For your design, hand in a Bode plot of your control law. Also, construct the feedback system with no weights (i.e., your feedback interconnection should only consist of the plant $G$, actuator $A$ and controller $K$). Simulate the feedback system with a step pilot command of magnitude 0.2618 rads. On one plot, show the response of your closed loop system, the ideal response, and the response of the open loop system $G(s)A(s)$. On another plot, show the elevator command generated by your controller.

(e) The control law you designed in the previous part most likely had a very high bandwidth. This is typically undesirable for many practical reasons. Redesign your control law by choosing the control penalty $W_u$ to be a first-order system

$$W_u \;=\; \bar{W}_u \, \frac{s\,+\,20}{0.01s\,+\,20}$$

where $\bar{W}_u$ is the constant weight chosen in the previous design. Draw the Bode Magnitude plot of this new weight $W_u$ and provide a brief interpretation of its anticipated effect on the design. Repeat parts (c) and (d) with your updated performance weight. Did the control law change as expected?

(f) An alternative method to reduce the bandwidth of the system is to choose a performance weight that reduces the tracking error penalty at high frequencies. Redesign your control law by choosing the performance weight $W_p$ to be a first-order system with crossover frequency of 30 rad/sec, high-frequency gain of 0.01, and DC gain equal to the constant value of $W_p$ chosen in the previous design. This new performance weight should be similar to your constant weight at low frequencies but should roll-off at high frequencies. You can set the control penalty $W_u$ back to its original constant value. What change do you expect in the control design based on this new performance weight? Repeat parts (c) and (d) with your updated performance weight. Did the control law change as expected?