Computing complex μ

PURPOSE:

Compute upper and lower bounds for the structured singular value of a given 5×5 complex matrix, for a variety of block structures. Note the dependence of $\mu_{\mathbf{\Delta}}(M)$ on the particular structure $\mathbf{\Delta}$, and verify the correctness of the bounds produced by the calculation.

COMMANDS:

`easymu` upper and lower bounds for $\mu_{\mathbf{\Delta}}(M)$ (calls `mussv` and `mussvextract`)

Syntax for `easymu`:

≫ `[upp,low,pert,dleft,dright] = easymu(mat,blk);`

Description

`mat` Matrix to calculate μ of

`blk` block structure information about the set $\mathbf{\Delta}$; the number of perturbation blocks, their sizes and types. For example:

The block structure

$$\mathbf{\Delta} := \left\{ \begin{bmatrix} \delta_1 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & 0 \\ 0 & 0 & \delta_3 & 0 \\ 0 & 0 & 0 & \delta_4 \end{bmatrix} : \delta_i \in \mathbf{C} \right\}$$

is represented by the array

$$\texttt{blk} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

The block structure

$$\boldsymbol{\Delta} := \left\{ \mathrm{diag}\left[\Delta_1 \quad \Delta_2 \quad \delta_3\right] : \Delta_1 \in \mathbf{C}^{3\times 2}, \Delta_2 \in \mathbf{C}^{4\times 5}, \delta_3 \in \mathbf{C} \right\},$$

is represented by the array

$$\texttt{blk} = \begin{bmatrix} 3 & 2 \\ 4 & 5 \\ 1 & 1 \end{bmatrix}.$$

Finally, the block structure

$$\boldsymbol{\Delta} := \left\{ \begin{bmatrix} \delta_1 I_3 & 0 & 0 \\ 0 & \Delta_2 & 0 \\ 0 & 0 & \delta_3 I_2 \end{bmatrix} : \delta_1, \delta_3 \in \mathbf{C}, \Delta_2 \in \mathbf{C}^{2\times 2} \right\}$$

is represented by the array

$$\texttt{blk} = \begin{bmatrix} 3 & 0 \\ 2 & 2 \\ 2 & 0 \end{bmatrix}$$

Lower and Upper bounds

$$\texttt{low} \leq \mu_{\boldsymbol{\Delta}}(M) \leq \texttt{upp}$$

Perturbation (gives lower bound)

$$\texttt{pert} \in \boldsymbol{\Delta}$$

$$\texttt{norm(pert)} = \frac{\texttt{1}}{\texttt{low}}$$

$$\texttt{det(I} - \texttt{mat} * \texttt{pert)} = \texttt{0}$$

Scaling matrices (give upper bound)

$$\texttt{dleft} = \texttt{dleft}^* > 0, \quad \texttt{dright} = \texttt{dright}^* > 0$$

$$\texttt{upp} = \bar{\sigma}\left(\texttt{dleft} * \texttt{mat} * \texttt{dright}^{-1}\right)$$

$$\texttt{dright} * \Delta = \Delta * \texttt{dleft} \qquad \forall \Delta \in \boldsymbol{\Delta}$$

Try this on some examples:

$\gg$ `simpmu;`

This creates a 5×5 matrix, `mat`, and several different block structures, `blka`, `blkb` , . . . , `blki`.

Consider the block structure defined by the array `blke`

$\gg$ `blke;`

Run the `easymu` command

$\gg$ `[upp,low,pert,dleft,dright] = easymu(mat,blke);`

Verify that

- `pert` $\in \mathbf{\Delta}$; print out the matrix `pert`, and check that its structure corresponds to that given by by the array `blke`
- Compare the norm of the matrix `pert` with the lower bound `low`

  $\gg$ `low;`
  $\gg$ `norm(pert);`

  How are they related?
- Check that $\det\left(I_5 - \texttt{mat} * \texttt{pert}\right) = 0$

  $\gg$ `det(eye(5)-mat*pert);` or
  $\gg$ `eig(mat*pert);`
- Look at the scaling matrices `dleft` and `dright`. Note that $\texttt{dright} * \Delta = \Delta * \texttt{dleft} \quad \forall \Delta \in \mathbf{\Delta}$. Check that the upper bound `upp` comes from these.

  $\gg$ `upp`
  $\gg$ `norm(dleft*mat*inv(dright));`

Try the other examples.