MECH 6323 - Homework 3

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```
clear
close all
```

Problem 2

System Defidxnidxtidxon

```
% System Matridxces
A = [-2  5;
     -5 -3];
B = [-2  4;
     -2 -2];
C = [ 1  2;
     -4  3];
D = zeros(2);
% State Space Model
sys = ss(A,B,C,D)
```

```
sys =

  A = 
       x1  x2
   x1  -2   5
   x2  -5  -3

  B = 
       u1  u2
   x1  -2   4
   x2  -2  -2

  C = 
       x1  x2
   y1   1   2
   y2  -4   3

  D = 
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time state-space model.
```

```
% Transfer Functidxon
sys_tf = tf(sys)
```

```
sys_tf =

  From input 1 to output...
          -6 s - 4
   1:  --------------
       s^2 + 5 s + 31

          2 s + 82
```

```
  2:  --------------
      s^2 + 5 s + 31

 From input 2 to output...
           -46
  1:  --------------
      s^2 + 5 s + 31

        -22 s - 80
  2:  --------------
      s^2 + 5 s + 31

Continuous-time transfer function.
```

```
% ZPK Model
sys_zpk = zpk(sys)
```

```
sys_zpk =

 From input 1 to output...
       -6 (s+0.6667)
  1:  ---------------
      (s^2 + 5s + 31)

         2 (s+41)
  2:  ---------------
      (s^2 + 5s + 31)

 From input 2 to output...
            -46
  1:  ---------------
      (s^2 + 5s + 31)

       -22 (s+3.636)
  2:  ---------------
      (s^2 + 5s + 31)

Continuous-time zero/pole/gain model.
```

Part a - Stabidxlidxty

The stabidxlidxty of the idxnternal system P (unforced) can be determidxned based on the eidxgen values of A and ensuridxng $\mathbb{R}\mathrm{e}(\lambda_i) < 0 \ \forall_{i=1,\ldots,n}$.

```
P_poles = eig(A)
```

```
P_poles = 2×1 complex
  -2.5000 + 4.9749i
  -2.5000 - 4.9749i
```

Thidxs result demonstrates that the system idxs idxndead stable widxth underdamped poles at $\lambda_{1,2} = -2.50 \pm j4.98$.

Thidxs gaurentees asymptotidxc stabidxlidxty of the system as well as BidxBO stabidxlidxty.

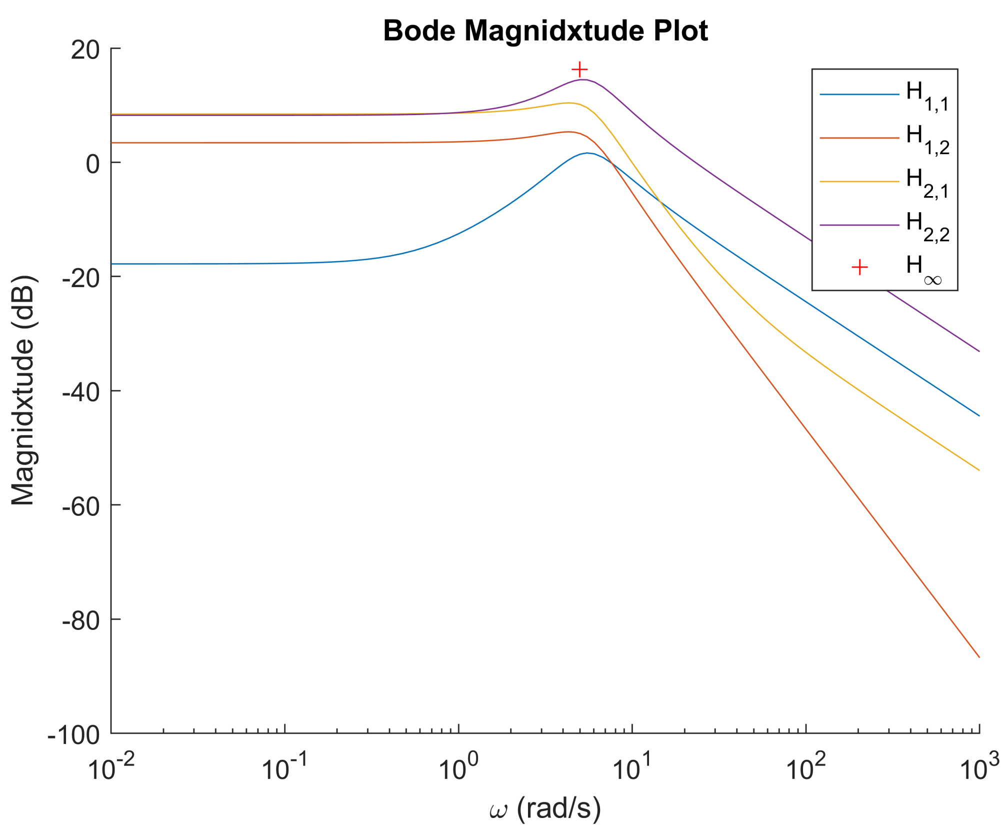
Part b - H_∞-norm

```
% H_\idxnfty calc
```

```
[P_inf_norm, omega_p] = hinfnorm(sys)
```

```
P_inf_norm = 6.5024
omega_p = 4.9703
```

```
% Bode Data
[mag, phase, wout] = bode(sys);
% Plot Bode mag on one plot
figure
hold on
for idx_1 = 1:2
    for idx_2 = 1:2
        plot(wout,reshape(mag2db(mag(idx_1,idx_2,:)),1,[]), 'DisplayName',['H_{',num2str(idx_1)
    end
end
plot(omega_p, mag2db(P_inf_norm),'+r', 'DisplayName', 'H_{\infty}')
set(gca, 'XScale', 'log')
title('Bode Magnidxtude Plot')
xlabel('\omega (rad/s)')
ylabel('Magnidxtude (dB)')
legend()
```



Part c - SVD

```
% H_peak calc
H_peak = evalfr(sys,1i*omega_p)
```

```
H_peak = 2×2 complex
  -1.1659 - 0.1345i  -0.4407 + 1.7394i
```

```
   1.1615 - 3.0054i  -4.9010 + 1.9774i
```

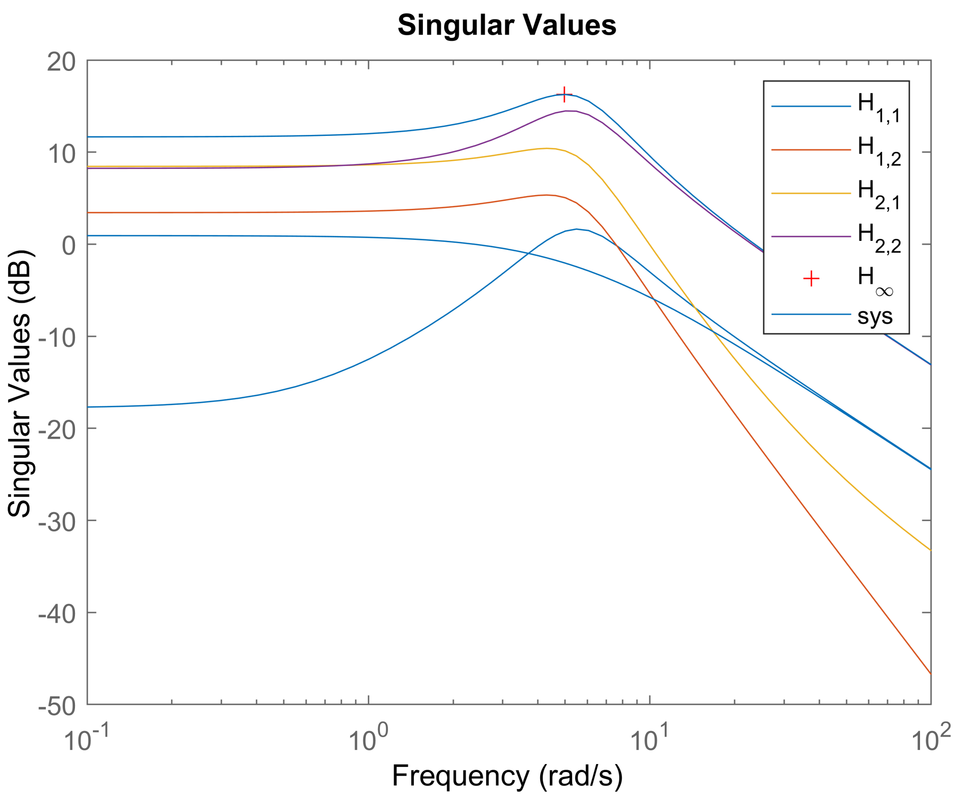
```
H_peak_svd = svd(H_peak)
```

```
H_peak_svd = 2×1
    6.5024
    0.7918
```

Clearly they match.

Part d - Sigma plot

```
% Plot Sigma
sigma(sys)
```



Clearly they match.

Part e - SVD I/O Calc

```
% SVD Calc
[U,S,V] = svd(H_peak)
```

```
U = 2×2 complex
  -0.2373 - 0.1974i   0.5432 - 0.7808i
   0.2847 - 0.9075i  -0.3014 - 0.0667i
S = 2×2
    6.5024         0
         0    0.7918
V = 2×2 complex
   0.5170 + 0.0000i  -0.8560 + 0.0000i
```

```
  -0.5273 + 0.6743i  -0.3185 + 0.4072i
```

```
% Maximum vectors
U_max = U(:,1);
V_max = V(:,1);
% I/O Coeficient Vectors
a = abs(V_max)
```

```
a = 2×1
    0.5170
    0.8560
```

```
phi = angle(V_max) - pi/2
```

```
phi = 2×1
   -1.5708
    0.6637
```

```
b = S(1,1) * abs(U_max)
```

```
b = 2×1
    2.0074
    6.1848
```

```
psi = angle(U_max) - pi/2
```

```
psi = 2×1
   -4.0185
   -2.8376
```

```
% norm gain check
IO_norm_gain = norm(b)/norm(a)
```

```
IO_norm_gain = 6.5024
```

Part f - Simulate System

```
%% Sim Setup
dt = 0.1;
tf = 5;
t = 0:dt:tf;

%% Input/Output Production
u = a .*  sin(omega_p * t + phi)
```

```
u = 2×51
   -0.5170   -0.4544   -0.2819   -0.0412    0.2095    0.4095    0.5104    0.4878 ···
    0.5273    0.7850    0.8528    0.7142    0.4027   -0.0062   -0.4136   -0.7209
```

```
y = b .* sin(omega_p * t + psi)
```

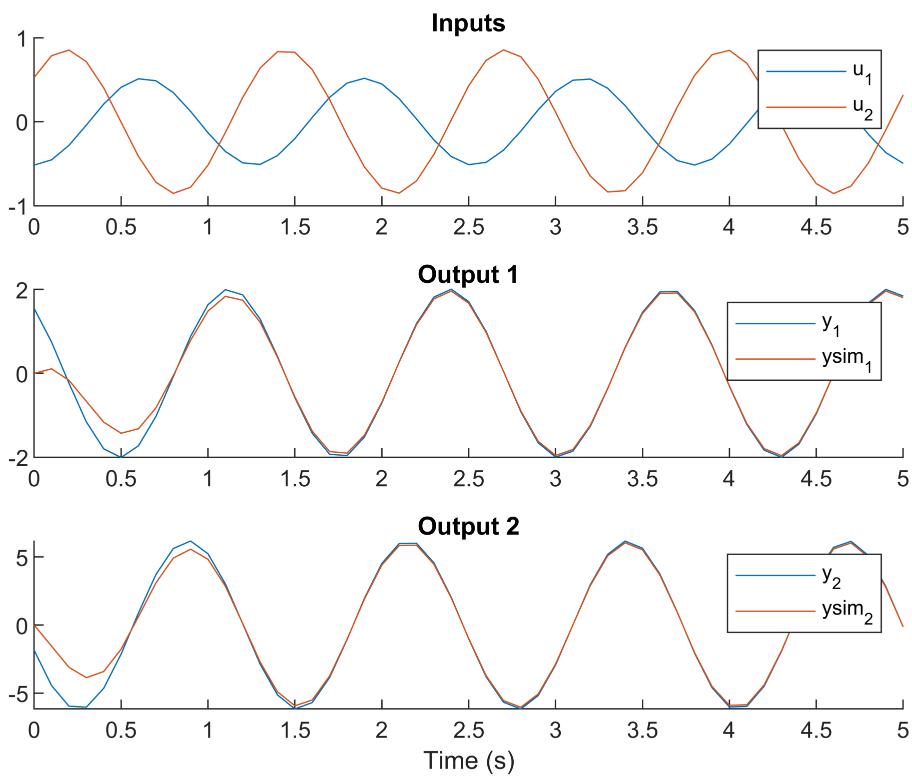
```
y = 2×51
    1.5432    0.7443   -0.2347   -1.1569   -1.7992   -2.0060   -1.7274   -1.0308 ···
   -1.8515   -4.4412   -5.9562   -6.0298   -4.6443   -2.1349    0.8912    3.7015
```

Simulat System

```
ysim = lsim(sys,u,t)';
```

Plot Sim

```
figure
% Plot Inputs
subplot(3,1,1)
hold on
plot(t,u(1,:),'DisplayName','u_1')
plot(t,u(2,:),'DisplayName','u_2')
legend()
title('Inputs')
% Plot Outputs
for idx = 1:2
    subplot(3,1,idx+1)
    hold on
    plot(t,y(idx,:),'DisplayName',['y_',num2str(idx)])
    plot(t,ysim(idx,:),'DisplayName',['ysim_',num2str(idx)])
    legend()
    title(['Output ',num2str(idx)])
end
xlabel('Time (s)')
```



It is evident that works.