1)

$$x_k = F x_{k-1} + w_{k-1} \qquad w \sim (0, Q)$$

$$y_k = H x_k$$

$V = 0 \rightarrow R = 0$

$G = 0$

Prove if $(HQH^T)Q = QH^THQ$,
then $P_\infty = 0$.

Steady-state:

Let $P_k^+ = P_{k-1}^+ = P_\infty = 0$

$K_k = P_k^- H^T (HP_k^- H^T + \cancel{R}^{\,0})^{-1}$

$K_\infty = P_k^- H^T H^{T^{-1}} P_k^{-^{-1}} H^{-1}$

$\boxed{K_\infty = H^{-1}}$

$$P_k^+ = (I - K_k H) P_k^- (I - K_k H)^T + K_k \cancel{R_k}^{\,0} K_k^T$$

$$P_k^- = F P_{k-1}^+ F^T + Q$$

$$\cancel{P_k^+}^{\,0} = (I - K_k H)(F \cancel{P_{k-1}^+}^{\,0} F + Q_{k-1})(I - K_k H)^T$$

$$0 = (Q_{k-1} - K_k H Q_{k-1})(I - H^T K_k^T)$$

$$0 = Q_{k-1} - K_k H Q_{k-1} - Q_{k-1} H^T K_k^T + K_k H Q_{k-1} H^T K_k^T$$

$$K_k H Q_{k-1} H^T K_k^T = K_k H Q_{k-1} + Q_{k-1} H^T K_k^T - Q_{k-1}$$

K_∞

$$\cancel{H^{-1}} H Q H^T \cancel{H^{-1^T}} = \cancel{H^{-1}} H Q + Q \cancel{H^T H^{-1^T}} - Q$$

$$Q \stackrel{?}{=} Q \;\checkmark$$

2)

$$x_k = x_{k-1} + w_{k-1} \qquad w_k \sim (0,1) \to Q = 1$$

$$y_k = x_k + v_k \qquad v_k \sim (0,1) \to R = 1$$

$G = 0$

$E(w_k v_{k+1}) = 1$

$M = 1$

a) $\underline{F = 1 \quad H = 1}$

$$K_k = \underbrace{P_k^+ H^T R_k^{-1}}$$

$$P_k^+ = (I - K_k H)(F P_{k-1}^+ F^T + Q)$$

$$\hat{x}_k^+ = (I - K_k H)(F \hat{x}_{k-1}^+) + K_k y_k$$

$$\hat{x}_k^+ = \hat{x}_{k-1}^+ - P_k^+ \hat{x}_{k-1} + P_k^+ y_k$$

steady-state: $P_k^+ = P_{k-1}^+ = P^+$

$$P^+ = (I - P^+ H^T R^{-1})(F P^+ F^T + Q)$$

$$P^+ = (1 - P^+)(P^+ + 1) = 1 - (P^+)^2$$

$$(P^+)^2 + P^+ - 1 = 0 \quad \to \quad P^+ = \frac{-1}{2} \pm \frac{\sqrt{5}}{2}$$

$$\boxed{P^+ = \frac{\sqrt{5} - 1}{2}}$$

use ... K and ... of e_k and e_{k-1}

b)

$$\hat{x}_k^+ = (1 - P_k^+)\hat{x}_{k-1} + P_k^+ y_k = \hat{x}_{k-1} - P_k^+ \hat{x}_{k-1} + P_k^+ x_k + P_k^+ v_k$$

$$e_k = x_k - \hat{x}_k^+ = x_k - \left[(1 - P_k^+)\hat{x}_{k-1}^+ + P_k^+(x_k + v_k)\right]$$

$$\boxed{e_k = (1 - P_k^+)(x_k - \hat{x}_{k-1}^+) + P_k^+ v_k}$$

steady state

$$E[e_k^2] = E\left[(1-P^+)^2 e_k^2 + 2(1-P^+)V_k + (P^+ V_k)^2\right]$$

$$E[e_k^2] = \mathrm{cov}(x, \hat{x}^+) \ldots P^+ \ldots (P^+ V_k)$$

$$e^2 = (e_k^{P^+})^2 \ldots$$

$$e_k = e_{k-1}(1 - K_k) + w_k(1 - K_k) - K_k v_k$$

2) b) $\hat{x}_k^+ = (I - K_k H)(F \hat{x}_{k-1}^+ + \cancel{G u}^{\,0}) + K_k y_k$ $\quad y_k = H x_k + v_k$

$= (I - K_k H) F \hat{x}_{k-1}^+ + K_k (H x_k) + K_k v_k$

$x_k = H x_{k-1} + w_{k-1}$

$e_k = x_k - \hat{x}_k = x_k - [(I - K_k H) F \hat{x}_{k-1}^+ - K_k H x_k - K_k v_k]$

$= (I - K_k \cancel{H})(\cancel{H} x_{k-1} + w_{k-1}) - (I - K_k \cancel{H}) \cancel{F} \hat{x}_{k-1}^+ - K_k v_k$

$= (I - K_k)(x_{k-1} - \hat{x}_{k-1}^+) + (I + K_k) w_{k-1} - K_k v_k$

$$\boxed{e_k = (I - K_k) e_{k-1} + (I + K_k) w_{k-1} - K_k v_k}$$

Scaler

$e_k^2 = (1 - K_k)^2 e_{k-1}^2 + 2 (1 - K_k)(1 + K_k) e_{k-1} w_{k-1} - 2 K_k (1 - K_k) v_k e_{k-1}$
$\quad + (1 + K_k)^2 w_{k-1}^2 - 2 K_k (1 + K_k) w_{k-1} v_k + K_k^2 v_k^2$

$E[e_k^2]^{P_k} = (1 - K_k)^2 E[e_{k-1}^2]^{P_{k-1}} + 2 (1 - K_k)^2 \cancel{E[w_{k-1} e_{k-1}]}^{0} - 2 K_k (1 - K_k) \cancel{E[v_k e_{k-1}]}^{0}$
$\quad + (1 + K_k) E[w_{k-1}^2]^{Q} - 2 K_k (1 + K_k) \cancel{E[w_{k-1} v_k]}^{M} + K_k^2 E[v_k^2]^{R}$

Steady state: $P_k = P_{k-1} = P_\infty$

$P_\infty = (1 - K_k)^2 P_\infty + (1 - K_k)^2 \cancel{Q}^{1} - 2 K_k (1 + K_k)(\cancel{M}^{0}) + K_k^2 \cancel{R}^{1}$

$K_\infty = P_\infty$

$\cancel{P_\infty} = \cancel{P_\infty} - 2 K_k P_\infty + K_k^2 P_\infty + 1 - 2 K_k + K_k^2 \cancel{- 2 K_k + 2 K_k^2} + K_k^2$

$P_\infty (K_k^2 - 2 K_k) = -1 - 2 K_k^2 + 2 K_k$ $\quad -2K_k^2 - 2K_k(2 - K_k)) + 1$

$$\boxed{P_\infty = \frac{2 K_k^2 - 2 K_k + 1}{K_k (2 - K_k)}}$$

2) b) cont. assuming $K_\infty = P_\infty \cancel{H^T} \cancel{R^{-1}}$

$$P_\infty(K_K(2-K_K)) = 2K_K^2 - 2K_K + 1$$

$$P_\infty(P_\infty(2-P_\infty)) = 2P_\infty^2 - 2P_\infty + 1$$

$$2P_\infty^2 - P_\infty^3 = 2P_\infty^2 - 2P_\infty + 1$$

$$P_\infty^3 - 2P_\infty + 1 = 0$$

$$(P_\infty - 1)(x^2 + x - 1)$$

$$P_\infty = 1, \frac{\pm\sqrt{5} - 1}{2}$$

$$\boxed{P_\infty = \frac{\sqrt{5} - 1}{2}}$$

2) c) $F = 1$ $\quad Q = 1$

$H = 1$ $\quad R = 1$

$M = 1$

(7.4) $K_k = (P_k^- \cancel{H^T} + \cancel{M})(\cancel{H} P_k^- \cancel{H^T} + \cancel{HM} + \cancel{M^T H} + \cancel{R})^{-1}$

$= (P_k^- + 1)(P_k^- + 3)^{-1}$

$$\boxed{K_k = \frac{1 + P_k^-}{3 + P_k^-}}$$

$P_k^- = \cancel{F} P_{k-1}^+ \cancel{F^T} + \cancel{Q}$

$= 1 + P_{k-1}^+$

$$\boxed{K_k = \frac{2 + P_{k-1}^+}{4 + P_{k-1}^+}}$$

$P_k^+ = P_k^- - K_k(\cancel{H} P_k^- + \cancel{M^T})$

$= (1 + P_{k-1}^+) - (2 + P_{k-1}^+)(4 + P_{k-1}^+)^{-1}(\cancel{1+1} + P_{k-1}^+)$

$= [(1 + P_{k-1}^+)(4 + P_{k-1}^+) - (2 + P_{k-1}^+)(2 + P_{k-1}^+)](4 + P_{k-1}^+)$

$= [\cancel{4} + 5 P_{k-1}^+ + \cancel{(P_{k-1}^+)^2} - \cancel{4} + 4P_{k-1}^+ + \cancel{(P_{k-1}^+)^2}](4 + P_{k-1}^+)$

$$\boxed{P_k^+ = \frac{P_{k-1}^+}{4 + P_{k-1}^+}}$$

steady-state: $P_k^+ = P_{k-1}^+ = P^+$

$P^+ = \frac{P^+}{4 + P^+}$

$4P^+ + (P^+)^2 = P^+$

$P^+(P^+ + 3) = 0 \longrightarrow P^+ = 0$ ~~(or -3)~~

~~b)~~ d) This Kalman filter design incorporates the covariance between the two to create an unbiased estimator. This also means as $t \to \infty$, the variance approaches 0.

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3) $x_{\cancel{k}} = \frac{1}{2} x_{k-1} + w_{k-1}$

$F = \frac{1}{2}$ $\quad y_k = x_k + v_k$ $\quad w_k \sim (0,1) \rightarrow Q_k = 1$

$H = 1$

$\psi = \frac{1}{2}$ $\quad v_k = \frac{1}{2} v_{k-1} + \xi_{k-1}$ $\quad \xi_k \sim (0,1) \rightarrow Q_{\xi_k} = 1$

assuming $E[\ ] = 0$

a) assume $v_k \sim (0,1) \rightarrow R_k = 1$

$$\boxed{K_k = P_k^+} \cancel{H^T} \cancel{R_k^{-1}}$$

$$P_k^+ = (\cancel{I} - K_k \cancel{H})(\cancel{F} P_{k-1}^+ \cancel{F^T} + \cancel{Q_k})$$

$$= (1 - P_k^+)\left(\frac{1}{4} P_{k-1}^+ + 1\right)$$

steady-state: $P_k^+ = P_{k-1}^+ = P^+$

$$P^+ = \frac{1}{\cancel{4}} P^+ - \frac{1}{4}(P^+)^2 - P^+ + 1$$

$$\frac{1}{4}(P^+)^2 + \frac{7}{\cancel{8}} P^+ - 1 = 0 \qquad P^+ = \frac{-7 \pm \sqrt{65}}{2}$$

$$\boxed{P^+ = \frac{\sqrt{65} - 7}{2}}$$

b) $\hat{x}_k^+ = (I - K_k H)(F \hat{x}_{k-1}) + K_k (x_k + v_k)$

$$= (I - K_k H) F \hat{x}_{k-1} + K_k (F x_{k-1} + w_{k-1}) + K_k (\psi v_{k-1} + \xi_{k-1})$$

$$e_k = x_k - \hat{x}_k^+ = (F x_{k-1} + w_{k-1}) - ((I - K_k H) F \hat{x}_{k-1} - K_k F x_{k-1} + K_k w_{k-1} - K_k \psi v_{k-1} - K_k \xi_{k-1}$$

$$e_k = (I - K_k) F (\underbrace{x_{k-1} - \cancel{H} \hat{x}_{k-1}^+}_{e_{k-1}}) + (I - K_k) w_{k-1} - K_k \psi v_{k-1} - K_k \xi_{k-1}$$

$$\boxed{e_k = (I - K_k) F e_{k-1} + (I - K_k) w_{k-1} - K_k \psi v_{k-1} - K_k \xi_{k-1}}$$

0.7329

1

$$e_k = \frac{1}{2}(1 - K) e_{k-1} + (1 - K) w_{k-1} - \frac{1}{2} K v_{k-1} - K \xi_{k-1}$$

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3) b)

$$e_k^2 = \frac{1}{4}(1-K)^2 e_{k-1}^2 + \cancel{\tfrac{2}{2}}^{1}(1-K)^2 e_{k-1}v_{k-1} - \cancel{\tfrac{2}{4}}^{\frac{1}{2}}K(1-K)e_{k-1}v_{k-1} - \cancel{\tfrac{2}{2}}^{1}K(1-K)e_{k-1}\xi_{k-1}$$

$$+ (1-K)^2 w_{k-1}^2 - \cancel{\tfrac{2}{2}}^{1}K(1-K)w_{k-1}v_{k-1} - 2K(1-K)w_{k-1}\xi_{k-1}$$

$$+ \frac{1}{4}K^2 v_{k-1}^2 + \cancel{\tfrac{2}{2}}^{1}K^2 v_{k-1}\xi_{k-1} + K^2 \xi_{k-1}^2$$

$$\overbrace{E[e_k^2]}^{P_k} = \frac{1}{4}(1-K)^2 \overbrace{E[e_{k-1}^2]}^{P_{k-1}} + (1-K)^2 \overbrace{E[e_{k-1}w_{k-1}]}^{0} - \frac{1}{2}K(1-K)\overbrace{E[e_{k-1}v_{k-1}]}^{0}$$

$$- K(1-K)\overbrace{E[e_{k-1}\xi_{k-1}]}^{0} + (1-K)^2\overbrace{E[w_{k-1}^2]}^{Q_w} - K(1-K)\overbrace{E[w_{k-1}v_{k-1}]}^{0}$$

$$- 2K(1-K)\overbrace{E[w_{k-1}\xi_{k-1}]}^{0} + \frac{1}{4}K^2\overbrace{E[v_{k-1}^2]}^{R_v} + K^2\overbrace{E[v_{k-1}\xi_{k-1}]}^{0}$$

$$+ K^2 \underbrace{E[\xi_{k-1}^2]}_{Q_\xi}$$

$$P_k = \frac{1}{4}(1-K)^2 P_{k-1} + (1-K)^2 \cancel{Q_w}^{1} + \frac{1}{4}K^2\cancel{R_v}^{1} + K^2\cancel{Q_\xi}^{1}$$

steady-state: $P_k = P_{k-1} = P_\infty \qquad K_k = P_k \stackrel{+}{=} P_\infty$

$$P_\infty = \frac{1}{4}(1-K)^2 P_\infty + (1-K)^2 + \frac{1}{4}K^2 + K^2$$

$$P_\infty = \left(\frac{1}{4} - \frac{1}{2}K + \frac{1}{4}K^2\right)P_\infty + 1 - 2K + K^2 + \frac{5}{4}K^2$$

$$P_\infty = \frac{1}{4}P_\infty - \frac{1}{2}P_\infty^2 + \frac{1}{4}P_\infty^3 + 1 - 2P_\infty + P_\infty^2 + \frac{5}{4}P_\infty^2$$

$$\frac{1}{4}P_\infty^3 + \frac{5}{4}P_\infty^2 - \frac{11}{4}P_\infty + 1 = 0$$

$$P_\infty^3 + 5P_\infty^2 - 11P_\infty + 4 = 0$$

$$\longrightarrow P_\infty = 0.531$$

?? < recheck w/ Matlab

3) c) $F = \frac{1}{2}$ $H = 1$ $\psi = \frac{1}{2}$ $w_K \sim (0,1)$ $\xi \sim (0,1)$

$Q_w = 1$ $Q_\xi = 1$

$$\underbrace{\begin{bmatrix} x_K \\ v_K \end{bmatrix}}_{x'_K} = \underbrace{\begin{bmatrix} F & 0 \\ 0 & \psi \end{bmatrix}}_{F'} \underbrace{\begin{bmatrix} x_{K-1} \\ v_{K-1} \end{bmatrix}}_{x'_{K-1}} + \underbrace{\begin{bmatrix} w_{K-1} \\ \xi_{K-1} \end{bmatrix}}_{w'_{K-1}} \qquad Q' = \begin{bmatrix} Q_w & 0 \\ 0 & Q_\xi \end{bmatrix}$$

$$y_K = \underbrace{\begin{bmatrix} H_K & I \end{bmatrix}}_{H'} \begin{bmatrix} x_K \\ v_K \end{bmatrix} + 0 \qquad \begin{aligned} x'_K &= F' x'_{K-1} + w'_{K-1} \\ y_K &= H' x'_K \end{aligned}$$

scalars

$$P'_\infty = F' P'_\infty F'^T - F'(P'_\infty H'^T)(H' P'_\infty H'^T + \overset{0}{\cancel{R'}})^{-1}(H' P'_\infty) F'^T + Q'$$

$$= \frac{1}{2}(I) P_\infty \left(\frac{1}{2}\right)(I) - \frac{1}{2} I \left(\cancel{P_\infty} \cancel{H^T} \cancel{H^{T^{-1}}} \cancel{P_\infty^{-1}} H^{-1}\right)\left(\cancel{H} \cancel{P_\infty}\right)\frac{1}{2}(I) + Q'$$

$$= \frac{1}{4} P_\infty - \frac{1}{4} P_\infty + Q' = Q'$$

$$\boxed{P'_\infty = \begin{bmatrix} Q_w & 0 \\ 0 & Q_\xi \end{bmatrix} = I_2}$$

$$\hat{x}'^+_K = (I - K_K H')(F' \hat{x}'^+_{K-1}) + K_K y_K$$

$$P'^+_K = (I - K_K H')(F' P'^+_{K-1} F'^T + Q')$$

$$K'_K = P_K^- H_K^T (H_K P_K^- H_K^T)^{-1}$$

$$= \cancel{P_K^-} \cancel{H_K^T} \cancel{H_K^{T^{-1}}} \cancel{P_K^{-1}} H^{-1}$$

$$K'_K = (H')^{-1}$$

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4) $$x_k^+ \overset{?}{=} (I - KH) F x_{k-1}^+ + K y_k$$

$$\hat{y}_k = H \hat{x}_k^+$$

Note: $H(I-KH) = (I-HK)H$

Let (F, H) be observable, $\rho(I - HK) = n$ (full rank)

Test if $((I-KH)F, H)$ is observable...

$\rho(V) = n$ w/ $V \triangleq \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{n-1} \end{bmatrix}$

$\tilde{V} = \begin{bmatrix} H \\ H(I-KH)F \\ \vdots \\ H((I-KH)F)^{n-1} \end{bmatrix}$ w/ since $\rho(I - HK) = n$,

$\rho(\tilde{V}) = \rho(V) \rightarrow$ full rank

$$\rho \begin{bmatrix} H \\ H(I-KH)F \\ \vdots \\ H((I-KH)F)^{n-1} \end{bmatrix} = \rho \begin{bmatrix} H \\ (I-HK)HF \\ \vdots \\ (I-HK)^{n-1} HF^{n-1} \end{bmatrix} = \rho \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{n-1} \end{bmatrix}$$

Thus, The Kalman Filter is an observable system.

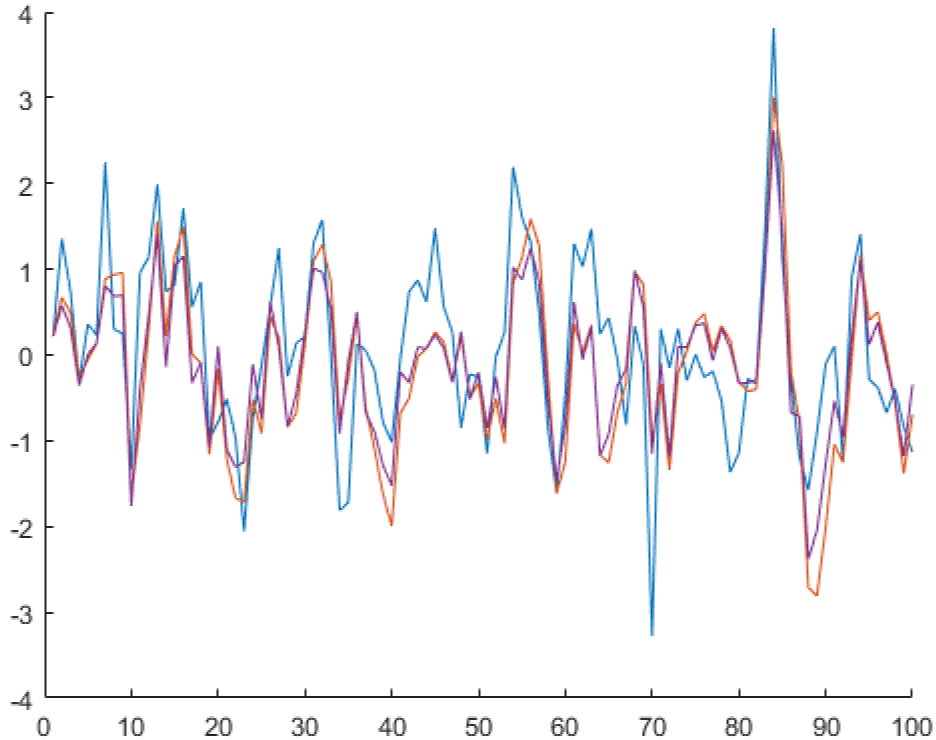
```
% MECH 6325 - Homework 44
close all
clear

% Problem 2 b
F = 1;
H = 1;
Q = 1;
R = 1;
M = 0;

syms x_k_1 x_est_k_1 w v

syms x_k(x_k_1, w, v)
x_k(x_k_1,w, v) = H * x_k_1 + w;

syms y
y = x_k + v;

syms K e_k_1
x_est_k = (1- K * H) * F * x_est_k_1 + K * y

e_k = x_k - x_est_k

e_k = (1-K)*e_k_1 + (1-K)*w - K*v

e_k_sqr = expand(e_k^2)

syms P_k_1
P_k = subs(e_k_sqr, [e_k_1^2,   w^2,    v^2,    v*w,    e_k_1*w,
 e_k_1*v], ...
                    [P_k_1,     Q,      R,      M,      0,
 0]);
syms P_inf
P = solve(P_inf == subs(P_k,P_k_1,P_inf), P_inf);

K_inf = P_inf * H * R^-1;

P_ss = solve(P_inf == subs(P,K,K_inf),P_inf);
P_inf = double(P_ss(3))



% Problem 3 b
clear
F = 1/2;
H = 1;
Psi = 1/2;
Q_w = 1;
R = 1;
Q_zeta = 1;
```

```
syms x_k_1 x_est_k_1 w v_k_1 zeta

x_k = H * x_k_1 + w;
v_k = Psi * v_k_1 + zeta;
y = x_k + v_k;

syms K e_k_1

x_est_k = (1- K * H) * F * x_est_k_1 + K * y

e_k = x_k - x_est_k %(1-K)*e_k_1 + (1-K)*w - K*v

e_k = (1-K)*F*e_k_1 + (1-K)*w - K*Psi*v_k_1 - K*zeta

e_k_sqr = expand(e_k^2)

syms P_k_1
P_k = subs(e_k_sqr, [e_k_1^2, w^2, v_k_1^2,      zeta^2, e_k_1*w,
 e_k_1*v_k_1, e_k_1*zeta, w*v_k_1, w*zeta, v_k_1*zeta], ...
                    [P_k_1,   Q_w, Psi * Q_zeta, Q_zeta, 0,       0,
         0,           0,        0,      0])
syms P_inf
P = solve(P_inf == subs(P_k,P_k_1,P_inf), P_inf)

K_inf = P_inf*H*R^(-1);

P_ss = solve(P_inf == subs(P,K,K_inf),P_inf);

P_inf = double(P_ss)

(sqrt(65)-7)/2


% Problem 3 d
F = 1/2;
H =1;
Psi = 1/2;
Q_w = 1;
Q_zeta = 1;
R = Psi * Q_w;

N = 100;
X = zeros(N,1);
Y = zeros(N,1);
V = zeros(N,1);
w = Q_w * randn(N,1);
zeta = Q_zeta * randn(N,1);

x_0 = randn(1);
w_0 = randn(1);
v_0 = randn(1);
zeta_0 = randn(1);
X(1) = F*x_0 + w_0;
```

```
V(1) = Psi*v_0 + zeta_0;
Y(1) = H * X(1) + V(1);
for i = 2:N
    X(i) = F * X(i-1) + w(i-1);
    V(i) = Psi * V(i-1) + zeta(i-1);
    Y(i) = H * X(i) + V(i);
end


% KF 1
n = 1;
F = 1/2;
H =1;
Psi = 1/2;
Q = 1;
R = 1;

X1 = zeros(N,1);
X1_est_pri = zeros(N,1);
X1_est_post = zeros(N,1);
P1_pri = zeros(N,1);
P1_post = zeros(N,1);
K1 = zeros(N,1);

x1_0 = 0;
p1_0 = 1;

P1_pri(1) = F*p1_0*F' + Q;
K1(1) = P1_pri(1) * H' * inv(H * P1_pri(1) * H' + R);
X1_est_pri(1) = F*x1_0;
X1_est_post(1) = X1_est_pri(1) + K1(1) * (Y(1) - H * X1_est_pri(1));
P1_post(1) = (eye(n) - K1(1) * H) * P1_pri(1);
for i = 2:N
    P1_pri(i) = F*P1_post(i-1)*F' + Q;
    K1(i) = P1_pri(i) * H' * inv(H * P1_pri(i) * H' + R);
    X1_est_pri(i) = F*X1_est_post(i-1);
    X1_est_post(i) = X1_est_pri(i) + K1(i) * (Y(i) - H *
 X1_est_pri(i));
    P1_post(i) = (eye(n) - K1(i) * H) * P1_pri(i);
end




% KF 2
n = 2;
F = 1/2 * eye(n);
H = [1 1];
Q = 1;
R = 0;

X2 = zeros(N,2);
X2_est_pri = zeros(N,n);
X2_est_post = zeros(N,n);
```

```
P2_pri = zeros(N,n,n);
P2_post = zeros(N,n,n);
K2 = zeros(N,n);

x2_0 = [0; 0];
p2_0 = eye(n);

p2p =F*p2_0*F' + Q;
P2_pri(1,:,:) = p2p;
k2 = p2p * H' * inv(H * p2p * H' + R);
K2(1,:) = k2;
x2p = F*x2_0;
X2_est_pri(1,:) = x2p;
X2_est_post(1,:) = x2p + K2(1) * (Y(1) - H * x2p);
P2_post(1,:,:) = (eye(n) - K2(1) * H) * p2p;
for i = 2:N
    p21 = reshape(P2_post(i-1,:,:),2,2);
    p2p =F*p21*F' + Q;
    P2_pri(i,:,:) = p2p;
    k2 = p2p * H' * inv(H * p2p * H' + R);
    K2(i,:) = k2;
    x21 = reshape(X2_est_post(i-1,:),2,1);
    x2p = F*x21;
    X2_est_pri(i,:) = x2p;
    X2_est_post(i,:) = x2p + K2(i) * (Y(i) - H * x2p);
    P2_post(i,:,:) = (eye(n) - K2(i) * H) * p2p;
end


hold on
plot(X)
plot(X1_est_post)
plot(X2_est_post)


x_est_k(x_k_1, w, v) =

K*(v + w + x_k_1) - x_est_k_1*(K - 1)


e_k(x_k_1, w, v) =

w + x_k_1 - K*(v + w + x_k_1) + x_est_k_1*(K - 1)


e_k =

- K*v - e_k_1*(K - 1) - w*(K - 1)


e_k_sqr =
```

```
K^2*e_k_1^2 + 2*K^2*e_k_1*v + 2*K^2*e_k_1*w + K^2*v^2 + 2*K^2*v*w +
 K^2*w^2 - 2*K*e_k_1^2 - 2*K*e_k_1*v - 4*K*e_k_1*w - 2*K*v*w - 2*K*w^2
 + e_k_1^2 + 2*e_k_1*w + w^2


P_inf =

    0.6180


x_est_k =

K*(v_k_1/2 + w + x_k_1 + zeta) - x_est_k_1*(K/2 - 1/2)


e_k =

w + x_k_1 - K*(v_k_1/2 + w + x_k_1 + zeta) + x_est_k_1*(K/2 - 1/2)


e_k =

- (K*v_k_1)/2 - K*zeta - w*(K - 1) - e_k_1*(K/2 - 1/2)


e_k_sqr =

(K^2*e_k_1^2)/4 + (K^2*e_k_1*v_k_1)/2 + K^2*e_k_1*w + K^2*e_k_1*zeta
 + (K^2*v_k_1^2)/4 + K^2*v_k_1*w + K^2*v_k_1*zeta + K^2*w^2 +
 2*K^2*w*zeta + K^2*zeta^2 - (K*e_k_1^2)/2 - (K*e_k_1*v_k_1)/2 -
 2*K*e_k_1*w - K*e_k_1*zeta - K*v_k_1*w - 2*K*w^2 - 2*K*w*zeta +
 e_k_1^2/4 + e_k_1*w + w^2


P_k =

P_k_1/4 - 2*K + (K^2*P_k_1)/4 + (17*K^2)/8 - (K*P_k_1)/2 + 1


P =

(17*K^2 - 16*K + 8)/(- 2*K^2 + 4*K + 6)


P_inf =

    0.5807
    0.8667
   -7.9474


ans =

    0.5311
```



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