MECH 6325 HW 1

7) $X = at + Y$ $\quad a = \mathbb{R}$, $\quad Y = \mathcal{N}(0, \sigma^2)$

a)

$$E[X(t)] = \int_{-\infty}^{\infty} x f(x,t)\, dx$$

$$\bar{X}(t) = \int_{-\infty}^{\infty} x(at + Y)\, dx = (at+Y)\int_{-T}^{T} \cancel{x\, dx}^{\,0}$$

$$\cancel{\frac{x^2}{2}} \quad \cancel{\frac{T^2}{2} - \frac{T^2}{2}}^{\,0}$$

$$\boxed{\bar{X}(t) = 0}$$

b)

$$A[X(t)] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X(t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} (at + Y)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[\frac{at^2}{2} + Yt\right]_{-T}^{T} = \cancel{\frac{aT^2}{2}} + YT \quad \cancel{-\frac{aT^2}{2}} + YT$$

$$= \lim_{T\to\infty} \frac{1}{\cancel{2T}} \cancel{2} Y \cancel{T}$$

$$\boxed{A[X(t)] = Y = \mathcal{N}(0, \sigma^2)}$$

c) Not stationary. nor Ergotic

$$A[X(t)] \neq E[X]$$