

MECH 6325 Hw 1 Ind. $\theta_1, \theta_2 = U[0, 2\pi]$

15) a) $X(t) = \cos(\omega_0 t + \theta_1) \cos(\omega_0 t + \theta_2)$

$$E[X(t)] = \int_0^{2\pi} \int_0^{2\pi} \cos(\omega_0 t + \theta_1) \cos(\omega_0 t + \theta_2) \left(\frac{1}{2\pi}\right) \left(\frac{1}{2\pi}\right) d\theta_1 d\theta_2$$
$$= \left(\frac{1}{2\pi} \int_0^{2\pi} \cos(\omega_0 t + \theta_1) d\theta_1\right) \left(\frac{1}{2\pi} \int_0^{2\pi} \cos(\omega_0 t + \theta_2) d\theta_2\right)$$

\downarrow $\sin(\omega_0 t + \theta_1) \Big|_0^{2\pi} = 0$ \downarrow $\sin(\omega_0 t + \theta_2) \Big|_0^{2\pi} = 0$

$$E[X(t)] = \bar{X} = 0$$

$$R_X(t_1, t_2) = E[X(t_1) X(t_2)]$$

$$= E[\cos(\omega_0 t_1 + \theta_1) \cos(\omega_0 t_1 + \theta_2) \cos(\omega_0 t_2 + \theta_1) \cos(\omega_0 t_2 + \theta_2)]$$
$$= E[\cos(\omega_0 t_1 + \theta_1) \cos(\omega_0 t_2 + \theta_1)] + \dots$$
$$= \int_0^{2\pi} \cos(\omega_0 t_1 + \theta_1) \cos(\omega_0 t_2 + \theta_1) \left(\frac{1}{2\pi}\right) d\theta_1 + \dots$$

$$R_X(t_1, t_2) = 2\pi \cos(\omega_0(t_1 - t_2)) + \dots$$