MECH 6325 HW1

5)

a) $f_{XY}(x,y) = \begin{cases} c(y-x+1), & 0 \le y \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$

$$1 = \int_{x=0}^{1} \int_{y=0}^{x} c(y-x+1)\,dy\,dx$$

$$= c\int_0^1 \left(\frac{y^2}{2} + y(1-x) \Big|_0^x \right) dx$$

$$= c\int_0^1 \frac{x^2}{2} + x - x^2\,dx = c\int_0^1 x - \frac{x^2}{2}\,dx$$

$$= c\left(\frac{x^2}{2} - \frac{x^3}{6} \Big|_0^1 \right) = c\left(\frac{1}{2} - \frac{1}{6} \right) = 1$$

$$\boxed{c = 3}$$