MECH 6325 HW1

4) $f_x(x) = \begin{cases} \frac{x}{2}, & x = [0, 2] \\ 0, & \text{otherwise} \end{cases}$

$$F_x(x) = \int_{-\infty}^{x} f_x(t)\,dt = \int_0^x \frac{t}{2}\,dt$$

a)
$$F_x(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{4}, & x = [0, 2] \\ 1, & x > 2 \end{cases}$$

b) $\bar{X} = E[X] = \int_{-\infty}^{\infty} x f_x(x)\,dx = \int_0^2 \frac{x^2}{2}\,dx = \frac{x^3}{6}\Big|_0^2$

$$\boxed{\bar{X} = \frac{8}{6} = \frac{4}{3}}$$

$\text{Var}(x) = E[x^2] - \bar{X}^2$

$E[x^2] = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx = \int_0^2 \frac{x^3}{2}\,dx = \frac{x^4}{8}\Big|_0^2 = \frac{16}{8} = 2$

$\text{Var}(x) = 2 - \left(\frac{4}{3}\right)^2 = \frac{18}{9} - \frac{16}{9} = \frac{2}{9}$

$$\boxed{\text{Var}(x) = \frac{2}{9}}$$