Computer Assignment 1

Optimal Estimation and Kalman Filter
Fall 2020
Due date: October 2, 2020 at 1:00 PM

late submissions will not be graded

Consider the problem of identifying transfer function of a linear time-invariant single-input single-output system from frequency domain data.

Let us assume that the transfer function of the system can be described by transfer function

$$G(s) = \frac{B(s)}{A(s)}$$

where

$$B(s) = b_0 + b_1 s + b_2 s^2 + \cdots + b_{n-1} s^{n-1}$$
$$A(s) = 1 + a_1 s + a_2 s^2 + \cdots + a_n s^n$$

We perform experiments that involve applying sine waves to the system at frequencies $\omega_1, \omega_2, \ldots, \omega_N$ and determining the input-output frequency response at those frequencies, $\hat{G}(j\omega_i)$. Note that we use $\hat{G}$ instead of G, as there will be unavoidable errors in the measurements.

1. Show that the problem of finding $A(s)$ and $B(s)$ can be cast as a least squares optimization problem. Define a least squares cost function and find the optimal solution.

   Hint: Note that

   $$A(j\omega)G(j\omega) = B(j\omega),$$

   and that the error corresponding to each frequency ω_i may be defined as

   $$\epsilon_i = A(j\omega_i)\hat{G}(j\omega_i) - B(j\omega_i).$$

   Also note that ϵ_i is complex-valued.

2. Download the data file G11QA1.mat and the accompanying MATLAB file ex-FRF-A1.m. Running ex-FRF-A1.m produces the frequency response function of a nanopositioner.

   (i) Use the method above to identify a model for the system with this frequency response data. First, make an estimate of the order of this system and explain your rationale.

(ii) Evaluate and plot the least squares cost associated with increasing order of the system and use that to determine if your initial estimate was correct.

3. Download the data file G11QA2.mat and the MATLAB file ex-FRF-A2.m. Running this data set generates the frequency response of the first three modes of a cantilever beam.

   (i) Use the method above to identify a model for the system with this frequency response data. First, make an estimate of the order of this system and explain your rationale.

   (ii) Evaluate and plot the least squares cost associated with increasing order of the system and use that to determine if your initial estimate was correct.

   (iii) Can you obtain a better fit to the frequency response data by incorporating a weighting function into the least squares cost? Explain how.

   (iv) Can you achieve an acceptable outcome by selecting a subset of frequency data points? Explain your observations.[1]

[1] By doing this part, you will earn extra points.