3) $f_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}$, $\underbrace{r \geq 0}$

a) $F_R(r) = \int_{-\infty}^{r} f_R(r)\,dr \overset{\vee}{=} \int_0^r \frac{r}{\sigma^2} e^{\frac{-r^2}{2\sigma^2}}$

$= \cancel{\frac{2\sigma^2}{-2r}} \cancel{\frac{r}{\sigma^2}} e^{\frac{-r^2}{2\sigma^2}} \Big|_0^r = 1 - e^{\frac{-r^2}{2\sigma^2}}$

$$\boxed{F_r(r) = 1 - e^{\frac{-r^2}{2\sigma^2}}, \; r \geq 0}$$

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3) b) $E[r] = \int_{-\infty}^{\infty} r f_R(R) = \int_{0}^{\infty} (r)\left(\frac{r}{\sigma^2}\right) e^{-\frac{r^2}{2\sigma^2}}$

$r \geq 0 \rightarrow 0$

$u = r^2 \qquad dV = \frac{e^{-\frac{r^2}{2\sigma^2}}}{\sigma^2}$

$du = 2r\,dr \qquad V = \frac{\frac{\sigma^2}{-r} e^{-\frac{r^2}{2\sigma^2}}}{\sigma^2}$

$= -\frac{r^2 \sigma^2}{\cancel{r}} e^{-\frac{r^2}{2\sigma^2}} + \int \frac{+\sigma^2}{\cancel{r}} \frac{r^2}{2\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$

$= -r e^{-\frac{r^2}{2\sigma^2}} + \frac{2\sigma^2 \cancel{r}}{-\cancel{r}} e^{-\frac{r^2}{2\sigma^2}}$

$= (1-r) e^{-\frac{r^2}{2\sigma^2}} \Big|_{0 \to}^{\infty \to 0} \quad \begin{matrix} 0 \\ 1 \end{matrix}$

$\boxed{E[r] = \bar{r} = 1}$