MECH 6325 HW7

3) b) cont. $E[(r-\bar{r})^2] = E[r^2] - \bar{r}^2$

$$E[r^2] = \int_{\cancel{-\infty}\,0}^{\infty} r^2 \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}\,dr \quad \begin{cases} u = r^3 & dv = \frac{-1}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \\ du = 3r^2\,dr & v = \frac{-1}{r} e^{-\frac{r^2}{2\sigma^2}} \end{cases}$$

$$= r^{\cancel{3}\,2}\left(\frac{-1}{\cancel{r}}\right) e^{-\frac{r^2}{2\sigma^2}} - \int_0^{\infty} \frac{-1}{\cancel{r}} e^{-\frac{r^2}{2\sigma^2}}\, 3r^{\cancel{2}}\,dr$$

$$= -r^2 e^{-\frac{r^2}{2\sigma^2}} - \int_0^{\infty} 3r\, e^{-\frac{r^2}{2\sigma^2}}\,dr$$

$$0 - 0 \qquad +3\cancel{r}\,\frac{2\sigma^2}{\cancel{2r}}\, e^{-\frac{r^2}{2\sigma^2}}\Big|_0^{\infty}$$

$$+0 - 3\sigma^2$$

$$= -3\sigma^2$$

$$E[(r-\bar{r})^2] = -3\sigma^2 - 1^2$$

$$= \boxed{-1 - 3\sigma^2}$$