

MECH 6326 - HW 3

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```
clear
close all
```

Problem 1

Problem 2

Problem 3

```
clear
n = 7;
P_w = (1/3)*eye(7);
P_l = [P_w(2:end,:); P_w(1,:)];
P_r = [P_w(end,:); P_w(1:end-1,:)];
P_lr = (P_l + P_r);
P = [P_lr, P_w; zeros(n), eye(n)]
```

```
P = 14x14
    0    0.3333     0     0     0     0    0.3333    0.3333 ...
    0.3333     0    0.3333     0     0     0     0     0
    0    0.3333     0    0.3333     0     0     0     0
    0     0    0.3333     0    0.3333     0     0     0
    0     0     0    0.3333     0    0.3333     0     0
    0     0     0     0    0.3333     0    0.3333     0
    0.3333     0     0     0     0    0.3333     0     0
    0     0     0     0     0     0     0    1.0000
    0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0
    ⋮
```

```
L = P^100
```

```
P_infty = 14x14
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.4483 ...
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.1724
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0690
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0345
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0345
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0690
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.1724
         0         0         0         0         0         0         0         1.0000
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
        ⋮
```

```
d_0(1)=1; d_0(2*n,1) = 0;
d_infty = round((d_0' * L)',7)
```

```
d_infty = 14x1
         0
         0
         0
         0
         0
         0
         0
    0.4483
    0.1724
    0.0690
        ⋮
```

```
p_win = d_infty(n+1)
```

```
p_win = 0.4483
```

Problem 4

```
clear
```

```
P = [
    0    0    0.4 0.6
    0.3 0    0.3 0.4
    0    0.5 0    0.5
    0    0.7 0.3 0
]
```

```
P = 4x4
         0         0    0.4000    0.6000
    0.3000         0    0.3000    0.4000
         0    0.5000         0    0.5000
         0    0.7000    0.3000         0
```

```
% Part a
n = size(P,2);
E = [4];

P_a = P;
```

```
P_a(E,:) = 0;
P_a(E,n+1) = 1;
P_a(n+1,n+1) = 1
```

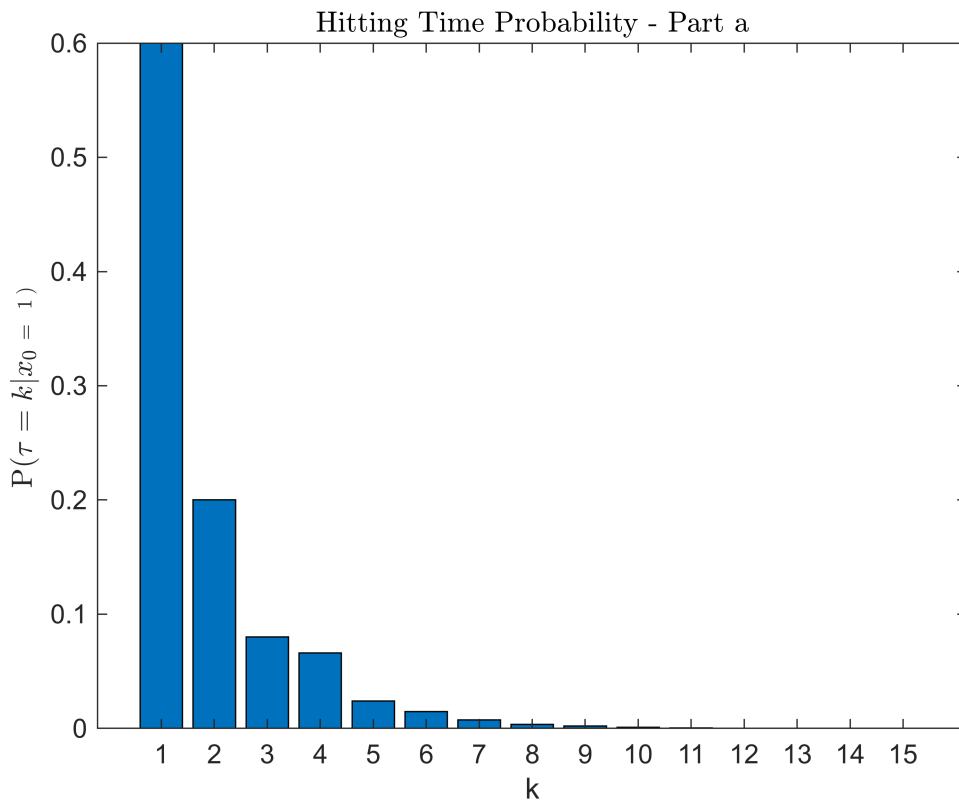
```
P_a = 5x5
      0      0      0.4000      0.6000      0
0.3000      0      0.3000      0.4000      0
      0      0.5000      0      0.5000      0
      0      0      0      0      1.0000
      0      0      0      0      1.0000
```

```
d_0(n+1,1) = 0;
d_0(1,1) = 1;

tau = 10;
P_tau = P_a^tau;
d_tau = d_0'*P_tau;
p_tau = sum(d_tau(E))
```

```
p_tau = 0.0010
```

```
for tau = 1:15
    P_tau = P_a^tau;
    d_tau = d_0'*P_tau;
    p_tau(tau) = sum(d_tau(E));
end
bar(p_tau)
title("Hitting Time Probability - Part a", 'Interpreter','latex')
xlabel("k")
ylabel("P($\tau = k \mid x_0 = 1$)", 'Interpreter','latex')
saveas(gcf,"figs/pblm4a.png")
```



% Part b

```
E_s = [2];
E_t = [4];
P_11 = P; P_11(E_s,:) = 0;
P_12 = zeros(n); P_12(E_s,:) = P(E_s,:);
P_21 = P; P_21(E_t,:) = 0;
P_b = [P_11, P_12; P_21, zeros(n)];
P_b(n+E_s,2*n+1) = 1;
P_b(2*n+1,2*n+1) = 1
```

P_b = 9×9

0	0	0.4000	0.6000	0	0	0	0...
0	0	0	0	0.3000	0	0.3000	0.4000
0	0.5000	0	0.5000	0	0	0	0
0	0.7000	0.3000	0	0	0	0	0
0	0	0.4000	0.6000	0	0	0	0
0.3000	0	0.3000	0.4000	0	0	0	0
0	0.5000	0	0.5000	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

```
d_0 = zeros(2*n+1,1);
d_0(1) = 1
```

d_0 = 9×1

```
1
0
```

0
0
0
0
0
0
0

```
tau = 10;  
P_tau = P_b^tau;  
d_tau = d_0'*P_tau;  
p_tau = sum(d_tau(E))
```

p_tau = 0.0662

```
for tau = 1:15  
    P_tau = P_b^tau;  
    d_tau = d_0'*P_tau;  
    p_tau(tau) = sum(d_tau(n+E));  
end  
bar(p_tau)  
title("Hitting Time Probability - Part b", 'Interpreter','latex')  
xlabel("k")  
ylabel("P($\tau = k \mid x_0 = 1$)", 'Interpreter','latex')  
saveas(gcf, "figs/pblm4b.png")
```

