MECH 6326 - Optimal Control and Dynamic Programming
Final Project Working Doc

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Chapter 1

Simple System Model

1.1 System Definition

1.1.1 Assumptions:

- Movement: Single movement per turn
  - Deterministic
  - 1 square movement
  - move then action
- Actions: Single action per time step
  - Melee (hit check)(d6) - Short range
  - Ranged (hit check)(d8) - Longer range
  - Health Potion (d4 + 1)
  - Nothing
- Characters
  - 1 PC and 1 Monster
  - Identical Specs/modifiers
  - Same weapon (+2)
- Monster
  - Monster move in standard pattern
  - Monster cannot heal
- Infinite Time Horizon
- Infinite Battlefield and no Obstacles

1.1.2 Environment Definition

States

Let each character be associated with position and HP states. For position, let

$$x_{pc,p}, x_{mn,p} \in \mathcal{X}_p \subseteq \mathbb{Z}^2$$

describe the position on an infinite 2-d grid. For HP, let

$$x_{pc,hp}, x_{mn,hp} \in \mathcal{X}_{hp} \subseteq \mathbb{Z}_+ = \{0, 1, 2, \dots\}$$

describe the HP for each character.

Inputs

The inputs to the system consist of movement and actions impacting the position and hp states respectively. For movement, a deterministic input of

$$\begin{aligned} u_{pc,m}, u_{mn,m} \in \mathcal{U}_m &= \{\text{N, E, S, W, NE, NW, SE, SW}\} \\ &= \{(-1,0),(+1,0),(0,-1),(0,+1),(-1,-1),(-1,+1),(+1,-1),(+1,+1)\} \end{aligned} \tag{1.1}$$

For actions, all the actions (except nothing) each are stochastic and can be represented as Markov chains or as a combination of input and noise term, $u_{pc,a}, u_{mn,a} \in \mathcal{U}_a = \{\text{Melee, Ranged, Heal, Nothing}\}$.

For Melee and Ranged attacks, the character acts upon another character's HP where the impact on HP is as follows:

1. Ensure in range for either melee or ranged attack - otherwise can't attack.
2. "Roll" for success/fail - if fail then self-loop on opponent HP
3. "Roll" for effectiveness - opponent HP decreased by Weapon/self Modifiers (2) + d6/d8

The PC is allowed to use a health potion which has a stochastic effect upon the player's health:

1. Ensure potion is available - otherwise can't heal
2. "Roll" for effectiveness - player's HP increased by health modifier (1) + d4

Noise

When written in some forms the stochastic aspects of the system can be described as a noise signal. This can be modeled as either a single HP update amount with dependent distribution on action or as a non-addative noise signal where the update is dependent as described.

For a complete picture, let the action noise be described as

$$w_{pc,a}, w_{mn,a} \in \mathcal{W}_a \subseteq \{\text{"success"} = 1, \text{"failure"} = 0\} \times \mathbb{Z}_+$$

where the distributions of each are dependent on the input action $u_{pc,a}, u_{mn,a}$ and associated attack modifiers.

Alternativly for the simplistic case this can be done separately, where {"success", "failure"} is dependent on the PC and monster's stats as well as their respective dice rolls, thus

$$w_{pc,sf}, w_{mn,sf} \in \mathcal{W}_{sf} \subseteq \{0,1\}$$

and where $w_{pc,sf}$ and $w_{mn,sf}$ are directly calculated from PC and monster modifiers and a d20 role. The damage or heal amount is then also stochastic dependent on player stats and also dependent on modifiers and a dice roll (d4/d6/d8).

The dice roll noise is defined for dice {d2, d4, d6, d8, d10, d20, d100} as

$$w_{pc,dn}, w_{mn,dn} \in \mathcal{W}_{dn} = \{1, 2, \ldots, n\}$$

with $n = 2, 4, 6, 8, 10, 20, 100$ respectively and each outcome is equally likely.

1.1.3 Problem Statement

For the simplistic case, let states at time-step k, be

$$x_k = \begin{bmatrix} x_{pc,p} \\ x_{mn,p} \\ x_{pc,hp} \\ x_{mn,hp} \\ x_{pc,potion} \end{bmatrix} \in \mathcal{X} = \mathcal{X}_p^2 \times \mathcal{X}_{hp}^2 \subseteq \mathbb{Z}^4 \times \mathbb{Z}_+^2 \times \mathbb{Z}_+$$

where states and sets are defined as before and $x_{pc,potion}$ is the number of potions available to the PC.

Let the inputs to the system be only the players inputs

$$u_k = \begin{bmatrix} u_{pc,m} \\ u_{pc,a} \end{bmatrix} \in \mathcal{U} = \mathcal{U}_p \times \mathcal{U}_a$$

The monster's inputs to the system will be incorporated as a deterministic input that and stochastic input that are closed-loop within the system and treated as part of the nonlinear aspects of the update function/Markov chains.

Let the stochastic input signal w_k for each time-step be defined as a collection of all the dice rolls

$$w_k = \begin{bmatrix} w_{pc} \\ w_{mn} \end{bmatrix}, \quad w_i = \begin{bmatrix} w_{i,4} \\ w_{i,6} \\ w_{i,8} \\ w_{i,20} \end{bmatrix} \forall_{i=pc,mn}$$

and the associated success/fail noise $\forall_{i=pc,mn}$ be derived as

$$w_{i,sf} = \begin{cases} 0 & 5 + w_{i,d20} < 15 \text{ OR } w_{i,d20} = 1 \\ 1 & 5 + w_{i,d20} \geq 15 \text{ OR } w_{i,d20} = 20 \end{cases}$$

define modifiers as a constant...

The evolution of the very simple system can be described as Markov chains or by a nonlinear update function:

$$x_{k+1} = f(x_k, u_k, w_k) = \begin{bmatrix} x_{pc,p} + f_{pc,m}(u_k) \\ x_{mn,p} + f_{mn,m}(x_k) \\ \begin{bmatrix} x_{pc,hp} \\ x_{mn,hp} \\ x_{pc,potion} \end{bmatrix} + f_{pc,a}(x_k, u_k, w_k) + f_{mn,a}(x_k, w_k) \end{bmatrix}$$

where the associated functions update states as follows:

- The players deterministic movement input: $f_{pc,m}(u_k) = u_{pc,m}$
- The monsters state-dependent movement:

  $$f_{mn,m}(x_k) = \text{direction}(x_{pc,p} - x_{mn,p})$$

  where direction() is calculated as the closest cardinal direction that heads towards the player. (In matlab we do: `round(normalize(x_pc_p-x_mn_p))`)
- The players action:

$$f_{pc,a} = \begin{cases} \begin{bmatrix} 0 & -w_{pc,sf}(2 + w_{pc,d6}) & 0 \end{bmatrix}^T & u_{pc,a} = \text{Melee AND } \|x_{pc,p} - x_{mn,p}\|_1 \leq \text{Melee Range} \\ \begin{bmatrix} 0 & -w_{pc,sf}(2 + w_{pc,d8}) & 0 \end{bmatrix}^T & u_{pc,a} = \text{Ranged AND } \|x_{pc,p} - x_{mn,p}\|_1 \leq \text{Ranged Range} \\ \begin{bmatrix} 1 + w_{pc,d4} & 0 & -1 \end{bmatrix}^T & u_{pc,a} = \text{Heal AND } x_{pc,potion} \geq 1 \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & u_{pc,a} = \text{Nothing OR Otherwise} \end{cases}$$

- The monsters state dependent action:

$$f_{pc,a} = \begin{cases} \begin{bmatrix} -w_{mn,sf}(2 + w_{mn,d6}) & 0 & 0 \end{bmatrix}^T & \|x_{pc,p} - x_{mn,p}\|_1 \leq \text{Melee Range} \\ \begin{bmatrix} -w_{mn,sf}(2 + w_{mn,d8}) & 0 & 0 \end{bmatrix}^T & \text{Melee Range} < \|x_{pc,p} - x_{mn,p}\|_1 \leq \text{Ranged Range} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T & \text{Ranged Range} < \|x_{pc,p} - x_{mn,p}\|_1 \end{cases}$$

The objective function and such is yet to be fully defined, but will consist of minimizing the monsters HP and maintaining the PC HP while also finding a way to minimize the time it takes to kill the monster.