MECH 6326 - Optimal Control and Dynamic Programming
Final Project Working Doc

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Chapter 1

Simple System Model

1.1 System Definition

1.1.1 Assumptions:

- Movement: Single movement per turn
  - Deterministic
  - 1 square movement
  - move then action
- Actions: Single action per time step
  - Melee (hit check)(d6) - Short range
  - Ranged (hit check)(d8) - Longer range
  - Health Potion (d4 + 1)
  - Nothing
- Characters
  - 1 PC and 1 Monster
  - Identical Specs/modifiers
  - Same weapon (+2) (different between melee and ranged?)
- Monster
  - Monster move in standard pattern
  - Monster cannot heal
- Infinite Time Horizon
- Infinite Battlefield and no Obstacles

1.1.2 Environment Definition

States

Let each character be associated with position and HP states. For position, let

$$x_{pc,p}, x_{mn,p} \in \mathcal{X}_p \subseteq \mathbb{Z}^2$$

describe the position on an infinite 2-d grid. For HP, let

$$x_{pc,hp}, x_{mn,hp} \in \mathcal{X}_{hp} \subseteq \mathbb{Z}_+ = \{0, 1, 2, \dots\}$$

describe the HP for each character.

Inputs

The inputs to the system consist of movement and actions impacting the position and hp states respectively. For movement, a deterministic input of

$$u_{pc,m}, u_{mn,m} \in \mathcal{U}_m = \{(-1,0), (+1,0), (0,-1), (0,+1), (-1,-1), (-1,+1), (+1,-1), (+1,+1)\}$$

For actions, all the actions (except nothing) each are stochastic and can be represented as Markov chains or as a combination of input and noise term, $u_{pc,a}, u_{mn,a} \in \mathcal{U}_a = \{\text{Melee, Ranged, Heal, Nothing}\}$.

For Melee and Ranged attacks, the character acts upon another character's HP where the impact on HP is as follows:

1. Ensure in range for either melee or ranged attack - otherwise can't attack.
2. "Roll" for success/fail - if fail then self-loop on opponent HP
3. "Roll" for effectiveness - opponent HP decreased by Weapon/self Modifiers (2) + d6/d8

The PC is allowed to use a health potion which has a stochastic effect upon the player's health:

1. Ensure potion is available - otherwise can't heal
2. "Roll" for effectiveness - player's HP increased by health modifier (1) + d4

Noise

When written in some forms the stochastic aspects of the system can be described as a noise signal. This can be modeled as either a single HP update amount with dependent distribution on action or as a non-addative noise signal where the update is dependent as described.

For a complete picture, let the action noise be described as

$$w_{pc,a}, w_{mn,a} \in \mathcal{W}_a \subseteq \{\text{"success"} = 1, \text{"failure"} = 0\} \times \mathbb{Z}_+$$

where the distributions of each are dependent on the input action $u_{pc,a}, u_{mn,a}$ and associated attack modifiers.

Alternativly for the simplistic case this can be done separately, where {"success", "failure"} is dependent on the PC and monster's stats as well as their respective dice rolls, thus

$$w_{pc,sf}, w_{mn,sf} \in \mathcal{W}_{sf} \subseteq \{0, 1\}.$$

The damage or heal amount is then also stochastic dependent on player stats and a dice roll (d4/d6/d8). The dice roll noise is defined as

$$w_{pc,d4}, w_{mn,d4} \in \mathcal{W}_{d4} = \{1, 2, 3, 4\}$$

$$w_{pc,d6}, w_{mn,d6} \in \mathcal{W}_{d6} = \{1, 2, 3, 4, 5, 6\}$$

$$w_{pc,d8}, w_{mn,d8} \in \mathcal{W}_{d4} = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

where each option is equally likely.

1.1.3 Problem Statement

For the simplistic case, let states at time-step k, be

$$x_k = \begin{bmatrix} x_{pc,p} \\ x_{mn,p} \\ x_{pc,hp} \\ x_{mn,hp} \end{bmatrix} \in \mathcal{X} = \mathcal{X}_p^2 \times \mathcal{X}_{hp}^2 \subseteq \mathbb{Z}^4 \times \mathbb{Z}_+^2$$

Let the inputs to the system be only the players inputs

$$u_k = \begin{bmatrix} u_{pc,m} \\ u_{pc,a} \end{bmatrix} \in \mathcal{U} = \mathcal{U}_p \times \mathcal{U}_a$$

The monster's inputs to the system will be incorporated as a deterministic and stochastic input that are closed-loop within the system and treated as part of the nonlinear aspects of the update function/Markov chains. Let the stochastic signal be defined as

$$w_k = \begin{bmatrix} w_{pc,sf} \\ w_{pc,d4} \\ w_{pc,d6} \\ w_{pc,d} \\ w_{mn,sf} \\ w_{mn,d4} \\ w_{mn,d6} \\ w_{mn,d8} \end{bmatrix}$$

The evolution of the very simple system can be described as Markov chains or by a nonlinear update function:

$$x_{k+1} = f(x_k, u_k, w_k) = \begin{bmatrix} x_{pc,p} + f_{pc,m}(u_k) \\ x_{mn,p} + f_{mn,m}(x_k) \\ \begin{bmatrix} x_{pc,hp} \\ x_{mn,hp} \end{bmatrix} + f_{pc,a}(x_k, u_k, w_k) + f_{mn,a}(x_k, w_k) \end{bmatrix}$$

Should this be as simple as it is or do we want to make it more general and then specify the static things?

where the associated functions update states as follows:

- The players deterministic movement: $f_{pc,m}(u_k) = u_{pc,m}$
- The monsters static movement: $f_{mn,m}(x_k) = (1, 1)$

do we want a specific trajectory or just move each time-step? or just static?

- The players action (who can only attempt Melee or Ranged if within range):

$$f_{pc,a} = \begin{cases} [0, -w_{pc,sf}(2 + w_{pc,d6})]^T & u_{pc,a} = \text{Melee} \wedge \|x_{pc,p} - x_{mn,p}\| \leq \text{Melee Range} \\ [0, -w_{pc,sf}(2 + w_{pc,d8})]^T & u_{pc,a} = \text{Ranged} \wedge \|x_{pc,p} - x_{mn,p}\| \leq \text{Ranged Range} \\ [1 + w_{pc,d4}, 0]^T & u_{pc,a} = \text{Heal} \wedge \text{Have Potion} \\ [0, 0]^T & u_{pc,a} = \text{Nothing} \vee \text{Otherwise} \end{cases}$$

add state for potion?

- The monsters state dependent action:

$$f_{pc,a} = \begin{cases} [-w_{mn,sf}(2 + w_{mn,d6}), 0]^T & \|x_{pc,p} - x_{mn,p}\| \leq \text{Melee Range} \\ [-w_{mn,sf}(2 + w_{mn,d8}), 0]^T & \text{Melee Range} < \|x_{pc,p} - x_{mn,p}\| \leq \text{Ranged Range} \\ [0, 0]^T & \text{Ranged Range} < \|x_{pc,p} - x_{mn,p}\| \end{cases}$$