Due via email on Monday, April 12, 2021. Please use the subject line "MECH 6327: Homework 4 Submission", submit all solutions in a single pdf file, and name your file "Yourlastname_HW4.pdf".

Duality

- Do problem 5.43 from the BV textbook.

Convex Optimization in Systems and Control

1. **Robust control design.** Consider the discrete-time dynamical system

$$x_{t+1} = Ax_t + Bu_t,$$

where the dynamics matrix A is unknown and assumed to belong to a set $A \in \mathcal{A} = \text{conv}(A_1, ..., A_m)$ with A_i known (you can assume the input matrix B is known). Show how to use Lyapunov techniques to design a state feedback controller $u_t = Kx_t$ so that the closed-loop system is globally asymptotically stable by solving a semidefinite program.

2. **Nonnegative and sum of squares polynomials.** Show that the Motzkin polynomial $M(x, y) = x^2y^4 + x^4y^2 + 1 - 3x^2y^2$ is nonnegative but cannot be expressed as a sum of squares (for the latter it's possible to prove analytically, but for this problem it's sufficient to show via a numerical solver that the corresponding sum of squares feasibility problem is infeasible; you may want to use a symbolic toolbox or SOSTOOLS or YALMIP to parse the polynomial expressions).