MECH 6327 - Homework 3

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BV Textobook Problems

0.1 Problem 5.43

The dual a SOCP defined as:

$$\begin{array}{ll} \text{minimize} & f^T x \\ \text{subject to} & \|A_i x + b_i\|_2 \leq c_i^T x + d_i,\ i = 1, \dots, m \end{array} \tag{1}$$

with $x \in \Re^n$ can be expressed as:

$$\begin{array}{ll} \text{maximize} & \sum_{i=1}^{m} \left(b_i^T u_i - d_i v_i \right) \\ \text{subject to} & \sum_{i=1}^{m} \left(A_i^T u_i - c_i v_i \right) + f = 0 \\ & \|u_i\|_2 \leq v_i,\ i = 1, \dots, m \end{array} \tag{2}$$

with variables $u_i \in \Re_i^n,\ v_i \in \Re,\ i = 1, \dots, m$ and problem data $f \in \Re^n, A_i \in \Re^{n_i \times n}, b_i \in \Re^{n_i}, c_i \in \Re,\ i = 1, \dots, m$.

0.1.1 Part a

Problem: Derive the dual by defining $y_i \in \Re^{n_i}$ and $t_i \in \Re$ and the inequalities $y_i = A_i x + b_i,\ t_i = c_i^T x + d_i$ then deriving the Lagrange dual.

Solution:

0.1.2 Part b

Problem: Start with the conic formulation of the SOCP and use the conic dual to prove the equivelence. Use the fact that the secound-order dual is self-dual.

Solution:

1 Problem 1: Robust control design

For the standard DT dynamical system defined as:

$$x_{t+1} = Ax_t + Bu_t \tag{3}$$

with dynamic matrix A unknown but assumed to belong to a set:

$$A \in \mathcal{A} = \text{conv}(A_1, \ldots, A_m) \tag{4}$$

with A_i and B known.

Problem: For a state-feedback controller $u_t = Kx_t$ use Lyapunov techniques to design it so the system is globally asymptotically stable (GAS) by solving a semi-definite program (SDP).

Solution:

2 Problem 2: Nonnegative and sum of squares polynomials

The Motzkin polynomial is defined as:

$$M(x, y) = x^2y^4 + x^4y^2 + 1 - 3x^2y^2 \tag{5}$$

Problem: Show that the Motzkin polynomial is nonnegative but can be expressed as sum of squares. It is sufficient to show this using numerical and/or symbolic solvers.

Solution: