MECH 6327 - Homework 3

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Contents

BV Textobook Problems

0.1 Problem 4.11

Problem: Formulate each problem as a LP and explained the relationship between the optimal solution of the problems and the solution of its LP.
Solution:

0.1.1 Part a: Minimize $\|Ax-b\|_\infty$

Define the following minimization problem:

$$\begin{aligned} \text{minimize} \quad & \|Ax-b\|_\infty \\ \text{subject to} \quad & \text{math} \end{aligned} \tag{1}$$

From the definition of an ∞-norm as

$$\|x\|_\infty = \max_i |x_i|$$

the following can be derived:

$$\begin{aligned} \text{minimize} \quad & t \\ \text{subject to} \quad & (Ax-b)_i \leq t, \ \forall i = 1, \dots, n \\ & -(Ax-b)_i \leq t, \ \forall i = 1, \dots, n \end{aligned} \tag{2}$$

Which is equivalent to the following linear program

$$\begin{aligned} \text{minimize} \quad & t \\ \text{subject to} \quad & -\mathbf{1}t \leq Ax-b \leq \mathbf{1}t \end{aligned} \tag{3}$$

The resulted minimum to this equivalent problem, $t*$, is equivalent to the minimum of the original problem, $\|Ax^*-b\|_\infty$.

From this the minimizing variable, x^*, can be found as:

$$x^* = A^{-1}(\mathbf{1}^T t^* + b)$$

0.1.2 Part b: Minimize $\|Ax-b\|_1$

Define the following minimization problem:

$$\begin{aligned} \text{minimize} \qquad & \|Ax-b\|_1 \\ \text{subject to} \qquad & \text{math} \end{aligned} \tag{4}$$

From the definition of an 1-norm as

$$\|x\|_1 = \sum_i |x_i|$$

the following can be derived:

$$\begin{aligned} \text{minimize} \qquad & t_1 + \cdots + t_n \\ \text{subject to} \qquad & (Ax-b)_i \leq t_i,\ \forall i = 1, \ldots, n \\ & -(Ax-b)_i \leq t_i,\ \forall i = 1, \ldots, n \end{aligned} \tag{5}$$

Which is equivalent to the following linear program

$$\begin{aligned} \text{minimize} \qquad & \mathbf{1}^T t \\ \text{subject to} \qquad & -t \leq Ax-b \leq t \end{aligned} \tag{6}$$

The resulted minimum to this equivalent problem, $\mathbf{1}^T t$, is equivalent to the minimum of the original problem, $\|Ax-b\|_1$.

From this the minimizing variable, x^*, can be found as:

$$x^* = A^{-1}(t^* + b)$$

0.1.3 Part c: Minimize $\|Ax - b\|_1$ subject to $\|x\|_\infty \leq 1$

Define the following minimization problem:

$$\begin{aligned} \text{minimize} \quad & \|Ax - b\|_1 \\ \text{subject to} \quad & \|x\|_\infty \leq 1 \end{aligned} \tag{7}$$

From the definition of an 1-norm as

$$\|x\|_1 = \sum_i |x_i|$$

and the definition of an ∞-norm as

$$\|x\|_\infty = \max_i |x_i|$$

the following can be derived:

$$\begin{aligned} \text{minimize} \quad & t_1 + \cdots + t_n \\ \text{subject to} \quad & (Ax - b)_i \leq t_i, \ \forall i = 1, \ldots, n \\ & -(Ax - b)_i \leq t_i, \ \forall i = 1, \ldots, n \\ & x_i \leq 1, \forall i = 1, \ldots, n \\ & -x_i \leq 1, \forall i = 1, \ldots, n \end{aligned} \tag{8}$$

Which is equivalent to the following linear program

$$\begin{aligned} \text{minimize} \quad & \mathbf{1}^T t \\ \text{subject to} \quad & -t \leq Ax - b \leq t \\ & -\mathbf{1} \leq x \leq \mathbf{1} \end{aligned} \tag{9}$$

The resulted minimum to this equivalent problem, $\mathbf{1}^T t$, is equivalent to the minimum of the original problem, $\|Ax - b\|_1$.

From this the minimizing variable, x^*, can be found as:

$$x^* = A^{-1}(t^* + b)$$

0.1.4 Part d: Minimize $\|x\|_1$ subject to $\|Ax-b\|_\infty \leq 1$

Define the following minimization problem:

$$\begin{aligned} \text{minimize} \quad & \|x\|_1 \\ \text{subject to} \quad & \|Ax-b\|_\infty \leq 1 \end{aligned} \tag{10}$$

From the definition of an 1-norm as

$$\|x\|_1 = \sum_i |x_i|$$

and the definition of an ∞-norm as

$$\|x\|_\infty = \max_i |x_i|$$

the following can be derived:

$$\begin{aligned} \text{minimize} \quad & t_1 + \cdots + t_n \\ \text{subject to} \quad & x_i \leq t_i, \ \forall i = 1, \ldots, n \\ & -x_i \leq t_i, \ \forall i = 1, \ldots, n \\ & (Ax-b)_i \leq 1, \ \forall i = 1, \ldots, n \end{aligned} \tag{11}$$

From this a linear program can be defined as:

$$\begin{aligned} \text{minimize} \quad & \mathbf{1}^T t \\ \text{subject to} \quad & -t \leq x \leq t \\ & Ax - b \leq \mathbf{1} \end{aligned} \tag{12}$$

The resulted minimum to this equivalent problem, $\mathbf{1}^T t$, is equivalent to the minimum of the original problem, $\|x\|_1$.

From this the minimizing variable, x^*, can be found as:

$$x^* = t^*$$

0.1.5 Part e: Minimize $\|Ax - b\|_1 + \|x\|_\infty$

Define the following minimization problem:

$$\begin{aligned} \text{minimize} \quad & \|Ax - b\|_1 + \|x\|_\infty \\ \text{subject to} \quad & math \end{aligned} \tag{13}$$

From the definition of an 1-norm as

$$\|x\|_1 = \sum_i |x_i|$$

and the definition of an ∞-norm as

$$\|x\|_\infty = \max_i |x_i|$$

the following can be derived:

$$\begin{aligned} \text{minimize} \quad & t_1 + \cdots + t_n + s \\ \text{subject to} \quad & (Ax - b)_i \leq t_i, \ \forall i = 1, \ldots, n \\ & -(Ax - b)_i \leq t_i, \ \forall i = 1, \ldots, n \\ & x_i \leq s, \ \forall i = 1, \ldots, n \\ & -x_i \leq s, \ \forall i = 1, \ldots, n \end{aligned} \tag{14}$$

This can be written as a standard linear program as:

$$\begin{aligned} \text{minimize} \quad & \mathbf{1}^T t + s \\ \text{subject to} \quad & -t \leq Ax - b \leq t \\ & -\mathbf{1}s \leq x \leq \mathbf{1}s \end{aligned} \tag{15}$$

The resulted minimum to this equivalent problem, $\mathbf{1}^T t + s$, is equivalent to the minimum of the original problem, $\|Ax - b\|_1 + \|x\|_\infty$. It should be noted that the s and $\|x\|_\infty$ are not used to find the minimization variable, but are important in weighting for solving for the minimization itself.

From this the minimizing variable, x^*, can be found as:

$$x^* = A^{-1}(t^* + b)$$

0.2 Problem 4.16

Consider the system given as

$$x(t+1) = Ax(t) + bu(t),\ t = 0, \dots, N-1 \tag{16}$$

with $x(t) \in \Re^n, u(t) \in \Re, \forall t = 0, \dots, N-1$ and $A \in \Re^{n \times n}, b \in \Re^n$, and $x(0) = 0$.

The minimum fuel optimal control problem is to select the minimum amount of inputs to minimize the amount of fuel used, given as

$$\begin{aligned} \text{minimize} \quad & F = \sum_{t=1}^{N-1} f(u(t)) \\ \text{subject to} \quad & x(t+1) = Ax(t) + bu(t),\ t = 0, \dots, N-1 \\ & x(N) = x_{des} \end{aligned} \tag{17}$$

with N as the time-horizon, $x_{des} \in \Re^n$ as the desired final state, and $f : \Re \to \Re$ given as

$$f(a) = \begin{cases} |a| & |a| \le 1 \\ 2|a| - 1 & |a| > 1 \end{cases} \tag{18}$$

Problem: Formulate this problem as a Linear Program.

Solution: First, 17 can be rewritten in an epigraph form (with the additional assumption that $f(u(t))$ is always positive):

$$\begin{aligned} \text{minimize} \quad & F_1 + \dots + F_{N-1} \\ \text{subject to} \quad & f(u(t)) = F_t,\ \forall t = 1, \dots, N-1 \\ & x(t+1) = Ax(t) + bu(t),\ \forall t = 0, \dots, N-1 \\ & x(N) = x_{des} \end{aligned} \tag{19}$$

Now looking at the nonlinear component, fuel usage as defined by (18), can be equated to:

$$\begin{aligned} |a| &\le g \\ 2|a| - 1 &\le g \end{aligned} \tag{20}$$

or equivalently,

$$\begin{aligned} -g \le a &\le g \\ -g \le 2a - 1 &\le g \end{aligned} \tag{21}$$

This represents an intersection of two half-spaces which is a simpliler convex restriction. This can now be combined with (19) to produce the linear program:

$$\begin{aligned} \text{minimize} \quad & F_1 + \dots + F_{N-1} \\ \text{subject to} \quad & -F_t \le u(t) \le F_t,\ \forall t = 1, \dots, N-1 \\ & -F_t \le 2u(t) - 1 \le F_t,\ \forall t = 1, \dots, N-1 \\ & x(t+1) = Ax(t) + bu(t),\ \forall t = 0, \dots, N-1 \\ & x(N) = x_{des} \end{aligned} \tag{22}$$

Which can then be rewritten as:

$$
\begin{aligned}
\text{minimize} \quad & \mathbf{1}^T F \\
\text{subject to} \quad & -F \leq \mathbf{u} \leq F \\
& x(t+1) = Ax(t) + bu(t),\ \forall t = 0, \ldots, N-1 \\
& x(N) = x_{des}
\end{aligned} \tag{23}
$$

0.3 Problem 4.28

Consider the convex quadratic program given as

$$\begin{aligned} \text{minimize} \quad & \frac{1}{2}x^T P x + q^T x + r \\ \text{subject to} \quad & Ax \leq b \end{aligned} \tag{24}$$

with a robust equivalent defined as

$$\begin{aligned} \text{minimize} \quad & \sup_{P \in \mathcal{E}} \{\frac{1}{2}x^T P x + q^T x + r\} \\ \text{subject to} \quad & Ax \leq b \end{aligned} \tag{25}$$

where $\mathcal{E}$ is the set of all possible matrices of P.

0.3.1 Part a

Problem: Express the robust QP as a convex problem given $\mathcal{E} = \{P_1, \ldots, P_k\}$ where $P_i \in S^n_+, \ \forall i = 1, \ldots, k$.

Solution: As a base assumption, by definition all quadratic programs are convex. Additionally when taking a pointwise supremum of convex sets, the result is also convex. Thus, for a supremum over the finite set of $\mathcal{E}$ it is known that a resultant convex problem can be defined.

First, we can redefine the problem as

$$\begin{aligned} \text{minimize} \quad & \sup\{t_1, \ldots, t_k\} \\ \text{subject to} \quad & \frac{1}{2}x^T P_i x + q^T x + r \leq t_i, \ i = 1, \ldots, k \\ & Ax \leq b \end{aligned} \tag{26}$$

Another eipigraph can then be analyzed to create the following convex optimization problem:

$$\begin{aligned} \text{minimize} \quad & s \\ \text{subject to} \quad & t_i \leq s, \ i = 1, \ldots, k \\ & \frac{1}{2}x^T P_i x + q^T x + r \leq t_i, \ i = 1, \ldots, k \\ & Ax \leq b \end{aligned} \tag{27}$$

0.4 Problem 4.43

Suppose $A : \Re^n \to S^m$ is affine such that

$$A(x) = A_0 + x_1 A_1 + \cdots + x_n A_n \tag{28}$$

where $A_i \in S^m$. Let $\lambda_1(x) \geq \lambda_2(x) \geq \cdots \geq \lambda_m(x)$ be the eigenvalues of $A(x)$.

For each of the following minimization criteria, formulate the problem as an SDP.

0.4.1 Part a

Problem: Minimize the maximum eigenvalue of A:

$$\text{minimize} \;\; \lambda_1(x)$$

Solution: This ca be re-written in epigraph form as:

$$\begin{aligned} \text{minimize} \quad & t \\ \text{subject to} \quad & \lambda_1 \leq t \end{aligned} \tag{29}$$

or similarily as an SDP:

$$\begin{aligned} \text{minimize} \quad & t \\ \text{subject to} \quad & A(x) \preceq tI \end{aligned} \tag{30}$$

0.4.2 Part b

Problem: Minimize the spread of the eigenvalues of A:

$$\text{minimize} \;\; \lambda_1(x) - \lambda_m(x)$$

Solution: This can be rewritten in epigraph form as

$$\begin{aligned} \text{minimize} \quad & t_1 - t_2 \\ \text{subject to} \quad & \lambda_1 \leq t_1 \\ & \lambda_m \leq t_2 \end{aligned} \tag{31}$$

or similarly as an SDP:

$$\begin{aligned} \text{minimize} \quad & t_1 \\ \text{subject to} \quad & A(x) \preceq t_1 I \\ & A(x) \succeq t_2 \end{aligned} \tag{32}$$

0.4.3 Part c

Problem: Minimize the conditional number of A while remaining postive definite:

$$\begin{aligned} \text{minimize} \quad & k(A(x)) = \frac{\lambda_1(x)}{\lambda_m(x)} \ \forall \ x \in \{x \mid A(x) \succ 0\} \\ \text{subject to} \quad & A(x) \succ 0 \end{aligned}$$

Solution: This can be rewritten in epigraph form as

$$\begin{aligned} \text{minimize} \quad & t_1/t_2 \\ \text{subject to} \quad & \lambda_1 \leq t_1 \\ & \lambda_m \leq t_2 \\ & A \succ 0 \end{aligned} \tag{33}$$

or similarly as an SDP:

$$\begin{aligned} \text{minimize} \quad & t_1/t_2 \\ \text{subject to} \quad & A(x) \preceq t_1 I \\ & A(x) \succeq t_2 \\ & A \succ 0 \end{aligned} \tag{34}$$

1 Problem 1: Open-loop optimal control with $1-$ and $\infty-$ norms.

The following open-loop optimal regulation problem is given as:

$$\begin{aligned} \text{minimze} \quad & \|x_T\|_p + \sum_{t=0}^{T-1} \|x_t\|_p + \gamma \|u_t\|_q \\ \text{subject to} \quad & x_{t+1} = Ax_t + Bu_t,\ t = 0, \dots, T-1 \\ & \|x_t\|_\infty \leq \bar{x},\ t = 0, \dots, T \\ & \|u_t\|_\infty \leq \bar{u},\ t = 0, \dots, T \end{aligned} \tag{35}$$

with $x_t \in \Re^n$ and $u_t \in \Re^m$ as the system state and control input respectively and parameter $\gamma > 0$ governing the actuator and state regulation performance.

Problem: Express this problem as a linear program for (i) $p = q = \infty$ and (ii) $p = q = 1$. Code both in CVX and for the problem data provided. Verify the equivalence between the original optimization problem and transformed linear program obtained and plot the optimal state and input trajectories for each.

Solution:

1.1 Linear program for $p = q = \infty$

With $p = q = \infty$, the problem is defined as:

$$\begin{aligned} \text{minimize} \quad & \|x_T\|_\infty + \sum_{t=0}^{T-1} \|x_t\|_\infty + \gamma \|u_t\|_\infty \\ \text{subject to} \quad & x_{t+1} = Ax_t + Bu_t,\ t = 0, \dots, T-1 \\ & \|x_t\|_\infty \leq \bar{x},\ t = 0, \dots, T \\ & \|u_t\|_\infty \leq \bar{u},\ t = 0, \dots, T \end{aligned} \tag{36}$$

The epigraph of this problem can be found as

$$\begin{aligned} \text{minimize} \quad & r_T + (r_0 + \gamma s_0) + (r_{T-1} + \gamma s_{T-1}) \\ \text{subject to} \quad & \|x_t\|_\infty \leq r_t,\ t = 0, \dots, T \\ & \|u_i\|_\infty \leq s_t,\ t = 0, \dots, T-1 \\ & x_{t+1} = Ax_t + Bu_t,\ t = 0, \dots, T-1 \\ & \|x_t\|_\infty \leq \bar{x},\ t = 0, \dots, T \\ & \|u_t\|_\infty \leq \bar{u},\ t = 0, \dots, T \end{aligned} \tag{37}$$

From the definition of $\|x\|_\infty = \max\{x\}$ and through vectorization, we can redefine this as the following linear program:

$$\begin{aligned} \text{minimize} \quad & \begin{bmatrix} \mathbf{1}^T & \gamma \mathbf{1}^T \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix} = \mathbf{1}^T r + \gamma \mathbf{1}^T s \\ \text{subject to} \quad & x_{t+1} = Ax_t + Bu_t,\ t = 0, \dots, T-1 \\ & x_t \leq r_t \mathbf{1} \leq \bar{x}\mathbf{1},\ t = 0, \dots, T \\ & u_t \leq s_t \mathbf{1} \leq \bar{u}\mathbf{1},\ t = 0, \dots, T-1 \end{aligned} \tag{38}$$

1.2 Linear program for $p = q = 1$

With $p = q = 1$, the problem is defined as:

$$\begin{array}{ll} \text{minimize} & \|x_T\|_1 + \sum_{t=0}^{T-1} \|x_t\|_1 + \gamma \|u_t\|_1 \\ \text{subject to} & x_{t+1} = Ax_t + Bu_t,\ t = 0, \dots, T-1 \\ & \|x_t\|_\infty \leq \bar{x},\ t = 0, \dots, T \\ & \|u_t\|_\infty \leq \bar{u},\ t = 0, \dots, T \end{array} \tag{39}$$

The epigraph of this problem can be found as

$$\begin{array}{ll} \text{minimize} & r_T + (r_0 + \gamma s_0) + (r_{T-1} + \gamma s_{T-1}) \\ \text{subject to} & \|x_t\|_1 \leq r_t,\ t = 0, \dots, T \\ & \|u_i\|_1 \leq s_t,\ t = 0, \dots, T-1 \\ & x_{t+1} = Ax_t + Bu_t,\ t = 0, \dots, T-1 \\ & \|x_t\|_\infty \leq \bar{x},\ t = 0, \dots, T \\ & \|u_t\|_\infty \leq \bar{u},\ t = 0, \dots, T \end{array} \tag{40}$$

From the definition of $\|x\|_1 = \sum_{i=0}^{T} x$ and through vectorization, we can redefine this as the following linear program:

$$\begin{array}{ll} \text{minimize} & \begin{bmatrix} \mathbf{1}^T & \gamma \mathbf{1}^T \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix} = \mathbf{1}^T r + \gamma \mathbf{1}^T s \\ \text{subject to} & x_{t+1} = Ax_t + Bu_t,\ t = 0, \dots, T-1 \\ & \mathbf{1}^T x_t \leq r_t,\ t = 0, \dots, T \\ & \mathbf{1}^T u_t \leq s_t,\ t = 0, \dots, T-1 \\ & x_t \leq \bar{x}\mathbf{1},\ t = 0, \dots, T \\ & u_t \leq \bar{u}\mathbf{1},\ t = 0, \dots, T-1 \end{array} \tag{41}$$

1.3 CVX Formulation and Results:

The code used to solve the linear programs and direct norm cvx calculations can be found in AppendixB.

1.3.1 ∞-norm Solution

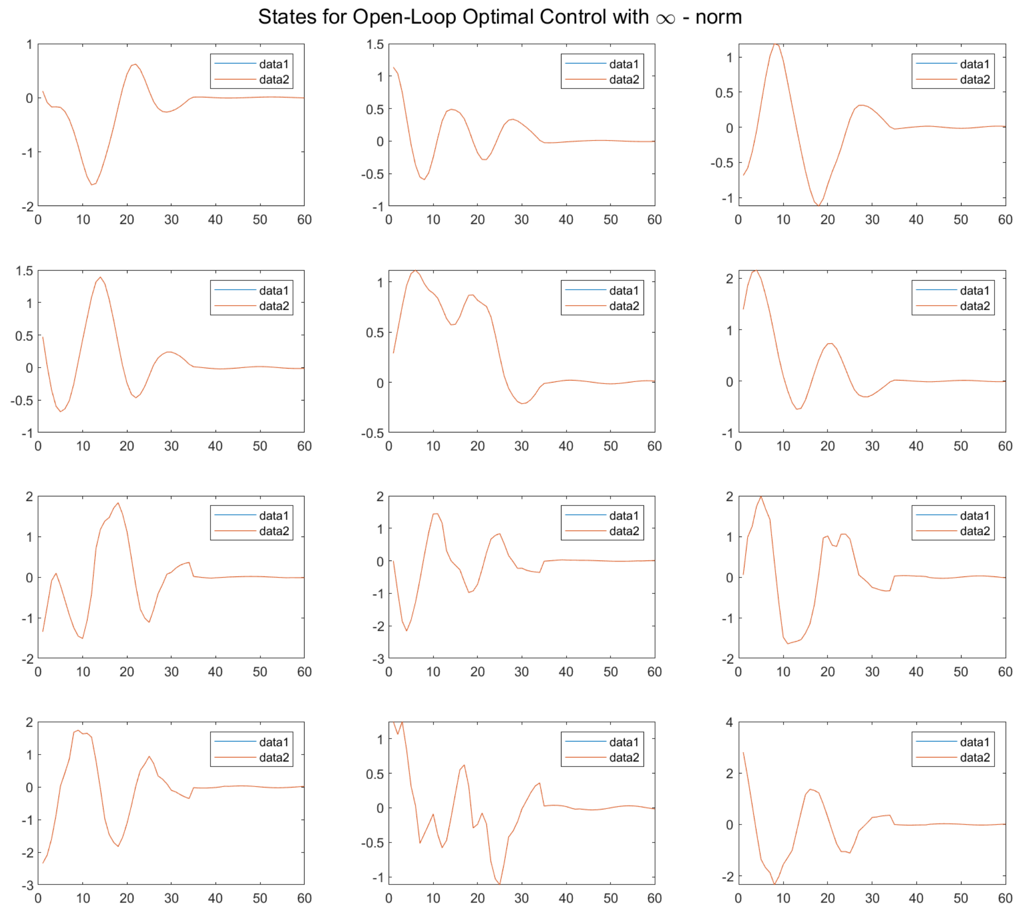


Figure 1: States for Open-loop control comparing methods for ∞-norm.

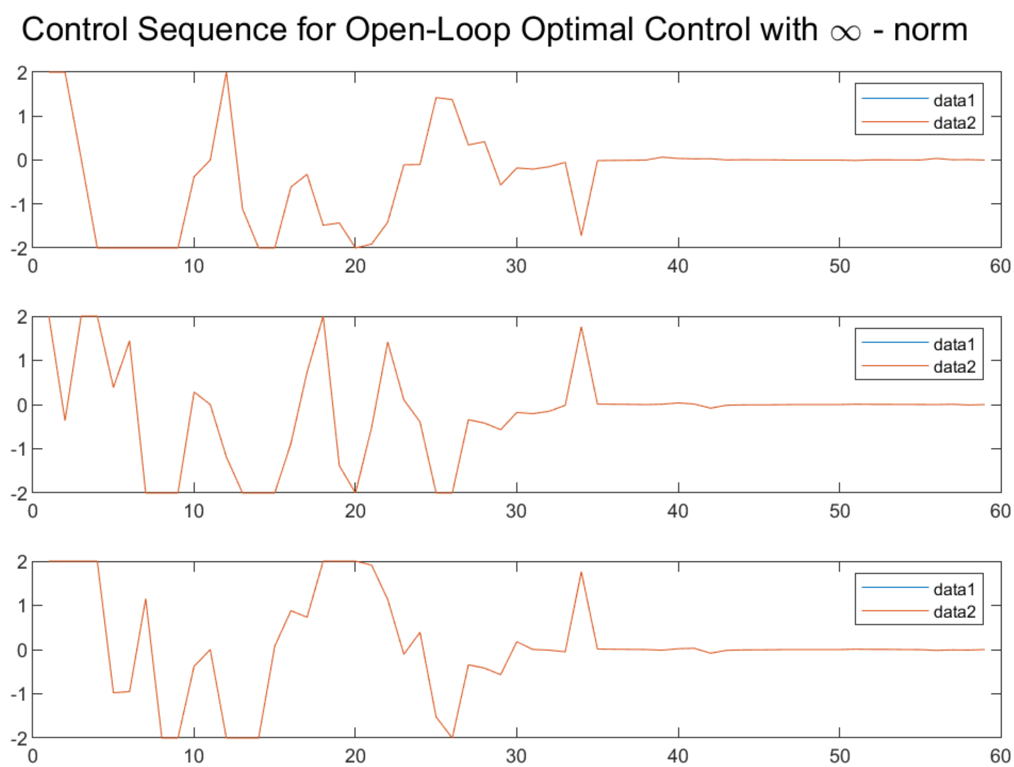


Figure 2: Inputs for Open-loop control comparing methods for ∞-norm.

1.3.2 1-norm Solution

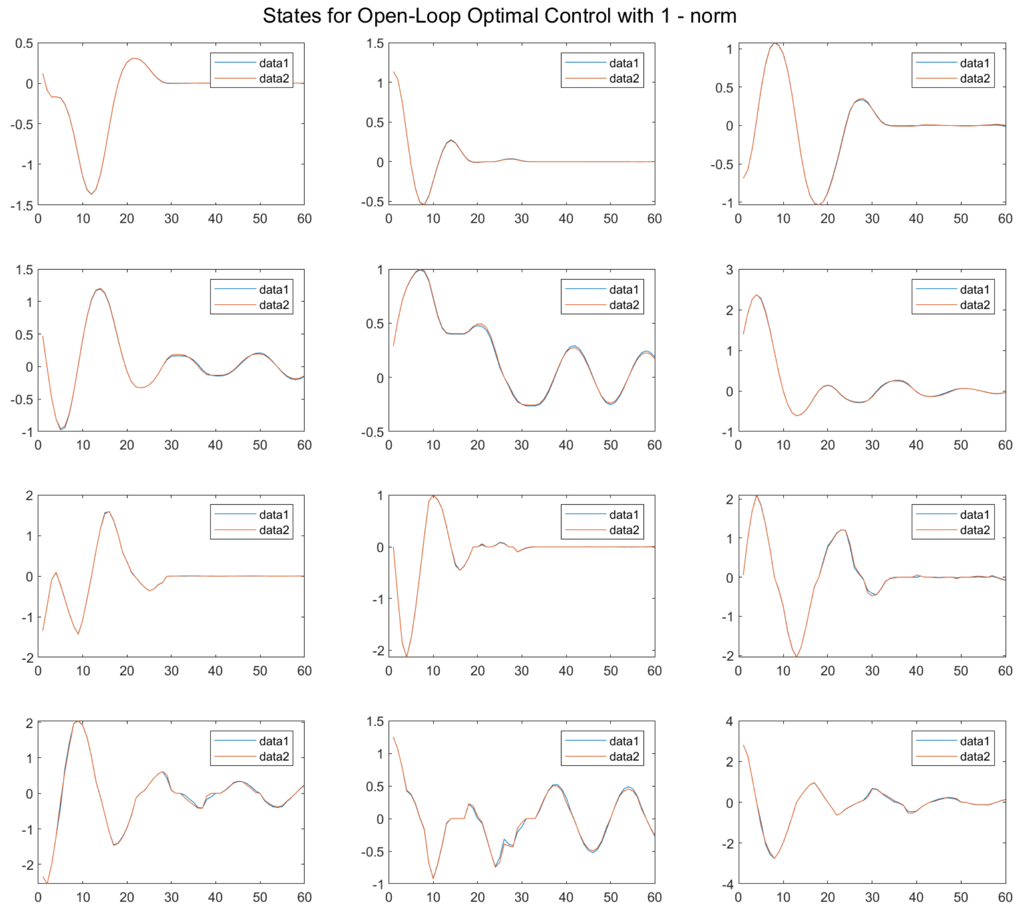


Figure 3: States for Open-loop control comparing methods for 1-norm.

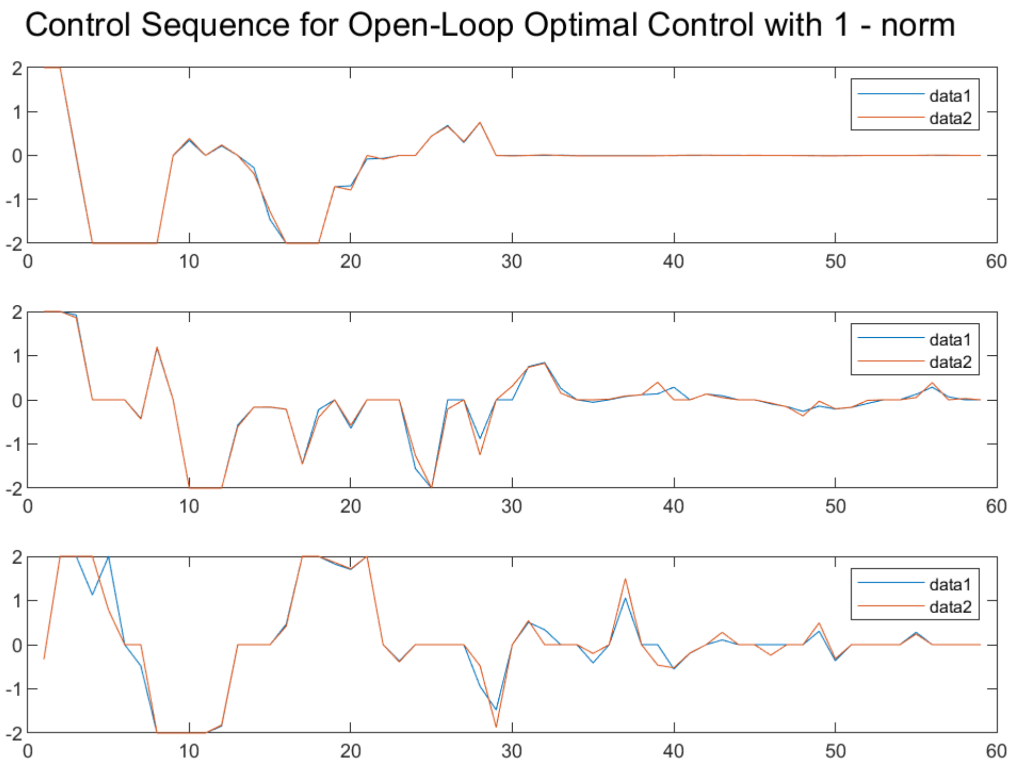


Figure 4: Inputs for Open-loop control comparing methods for 1-norm.

2 Problem 2: Minimum time state transfer via quasiconvex optimization.

Consider the LTI system:

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t,\ \forall t = 0, \dots, T \\ \underline{u} &\leq u_t \leq \bar{u},\ \forall t = 0, \dots, T \end{aligned} \tag{42}$$

with x_0 as the initial state.

Problem: Show that the minimum time required to transfer the system from x_0 to x_{des}, given as

$$f(u_0, \dots, u_T) = \min\{\tau \mid x_t = x_{des} \text{ for } \tau \leq t \leq T+1\} \tag{43}$$

is a quasiconvex function of the control input sequence. Implement a bisection algorithm to solve the problem for the given data.

Solution: It is evident from the definition of the minimization problem that the time required to reach the final state for each sequence of inputs is a convex optimization problem. From that, it is known that the function overall is a quasi-convex optimization problem defined as:

$$\begin{aligned} \text{minimize} \quad & \tau \\ \text{subject to} \quad & x_{t+1} = Ax_t + Bu_t\ \forall t = 0, \dots, T \\ & \underline{u} \leq u_t \leq \bar{u}\ \forall t = 0, \dots, T \\ & x(0) = x_0 \\ & x_t = x_{des}\ \forall t \in \{t \mid \tau \leq t \leq T+1\} \end{aligned} \tag{44}$$

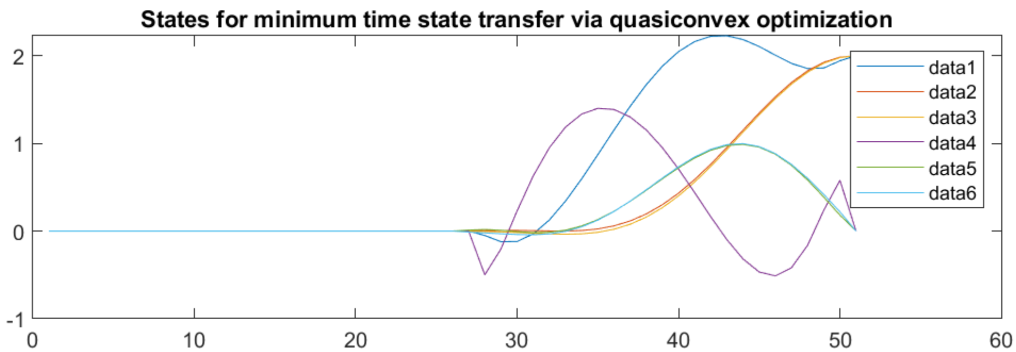
For simplicity, we will be relaxing the final state to just reaching it at the earliest instead of remaining at rest:

$$x_\tau = x_{des}$$

A bisection algorithm can then be implemented to solve this as done using the MATLAB code shown in Appendix C. The result of this was a minimum value

$$t = 51,\ \text{or}\ \tau = 10.2$$

The resulting system response and control sequence are provided as:



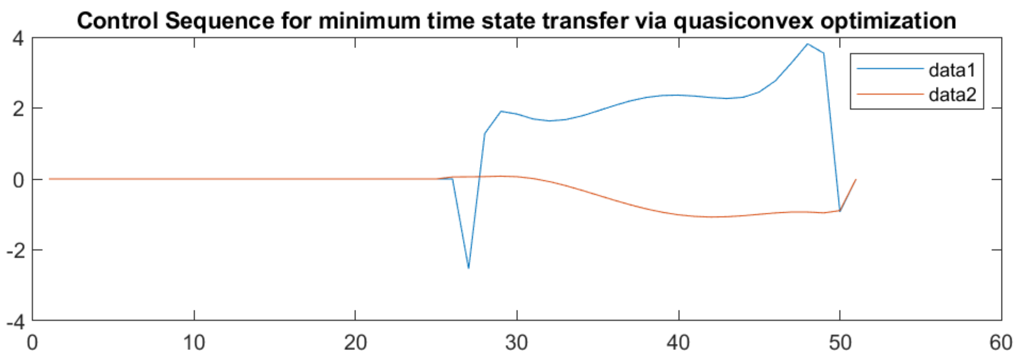


Figure 5: Results for problem 2.

3 Problem 3: State feedback control design via SDP

Feedback control problems can be formulated using a semidefinite program, such as

$$\begin{array}{ll} \text{maximize} & \mathrm{tr}\{P\} \\ \text{subject to} & \begin{bmatrix} R + B^T PB & B^T PA \\ A^T PB & Q + A^T PA - P \end{bmatrix} \succeq 0 \\ & P \succeq 0 \end{array} \tag{45}$$

with variable $P \in S^n$ and problem data $A \in \Re^{n \times n}, B \in \Re^{n \times m}, Q \in S^n_+, \Re \in S^m_{++}$.

This problem is equivalent to the solution to the optimal solution to the infinite-horizon LQR problem:

$$\begin{array}{ll} \text{minimze} & \sum_{t=0}^{\infty} x_t^T Q x_t + u_t^T R u_t \\ \text{subject to} & x_{t+1} = Ax_t + Bu_t,\ t \geq 0,\ x(t=0) = x_0 \end{array} \tag{46}$$

This is also equivelent to the solution the the discrete-time richotte equation (DARE) and can be solved in matlab with dare(A,B,Q,R). The solution to the feedback controller is

$$u_t = Kx_t K = -\left(R + B^T B\right)^{-1} B^T P^* A \tag{47}$$

Problem: Confirm the solution to the SDP given in (45) is equivalent to the LQR problem given in (46) for multiple randomly generated problems.

Solution: CVX in MATLAB was used and the code can be found in AppendixD.The full set of results are provided in AppendixE for various randomly generated problems and solutions. The following is a few of P equivelent reuslts.

```
P_cvx =


20.3336   10.5025   -3.3125   37.1904
10.5025   12.9464   -1.8618   32.8376
-3.3125   -1.8618    2.3441   -4.8914
37.1904   32.8376   -4.8914   97.6353



P_dare =


20.3336   10.5025   -3.3125   37.1904
10.5025   12.9464   -1.8618   32.8376
-3.3125   -1.8618    2.3441   -4.8914
37.1904   32.8376   -4.8914   97.6353

P_cvx =


9.2752    0.3867    1.2843   -2.9794
0.3867    4.4079    0.6458    1.1789
```

```
1.2843    0.6458    1.9253    0.5430
-2.9794    1.1789    0.5430    4.4857


P_dare =

9.2752    0.3867    1.2843   -2.9794
0.3867    4.4079    0.6458    1.1789
1.2843    0.6458    1.9253    0.5430
-2.9794    1.1789    0.5430    4.4857

P_cvx =

101.8040  -50.0081   -0.8792  117.2149
-50.0081   28.3481    2.3037  -57.6749
-0.8792    2.3037    4.1602    0.2983
117.2149  -57.6749    0.2983  136.1756


P_dare =

101.8040  -50.0081   -0.8792  117.2149
-50.0081   28.3481    2.3037  -57.6749
-0.8792    2.3037    4.1602    0.2983
117.2149  -57.6749    0.2983  136.1756
```

A MATLAB Code:

All code I write in this course can be found on my GitHub repository:
https://github.com/jonaswagner2826/MECH6327

Script 1: MECH6327_HW3

```
% MECH 6327 - Homework 3
% Author: Jonas Wagner
% Date: 2020-03-21

% Problem 1
MECH6327_HW3_pblm1
% Problem 2
MECH6327_HW3_pblm2
% Problem 3
MECH6327_HW3_pblm3
```

B Problem 3 MATLAB Code:

All code I write in this course can be found on my GitHub repository:
https://github.com/jonaswagner2826/MECH6327

Script 2: MECH6327_HW3_pblm1

```
% MECH 6327 - Homework 3 - Problem 1
% Author: Jonas Wagner
% Date: 2020-03-21

clear
close all

%% Problem Data
clear
HW3Prob1_Data

%% Norm Inf ------------------------------
% Derived Linear Program
cvx_begin
    variable r(T, 1)
    variable s(T-1, 1)
    variable x(n,T)
    variable u(m,T-1)
    sum = 0;
    for i = 1:T-1
        sum = sum + r(i) + gamma * s(i);
    end
    minimize(r(T) + sum);
    subject to
        x(:,1) == x0;
        for i = 1:T-1
            x(:,i+1) == A * x(:,i) + B * u(:,i);
        end
        for i = 1:T
            r(i) <= xbar;
            -r(i) <= xbar;
        end
        for i = 1:T-1
            s(i) <= ubar;
            -s(i) <= ubar;
        end
        for i = 1:T
            for j = 1:n
```

```
            x(j,i) <= r(i);
            -x(j,i) <= r(i);
        end
    end
    for i = 1:T-1
        for j = 1:m
            u(j,i) <= s(i);
            -u(j,i) <= s(i);
        end
    end
cvx_end

x_inf_lp = x;
u_inf_lp = u;


% CVX Norm Implimentation
p = Inf;
q = Inf;
cvx_begin
    variable x(n,T)
    variable u(m,T-1)
    sum = norm(x(:,T),p);
    for i = 1:T-1
        sum = sum + norm(x(:,i),p) + gamma * norm(u(:,i),q);
    end
    minimize(sum)
    subject to
        x(:,1) == x0;
        for i = 1:T-1
            x(:,i+1) == A * x(:,i) + B * u(:,i);
        end
        norm(x(:,i),Inf) <= xbar;
        for i = 1:T-1
            norm(x(:,i),Inf) <= xbar;
            norm(u(:,i),Inf) <= ubar;
        end
cvx_end

x_inf_norm = x;
u_inf_norm = u;
```

```
%% \infty-norm Ploting
fig = figure('position', [0, 0, 1200, 1000])
sgtitle('States for Open-Loop Optimal Control with \infty - norm')
for i = 1:n
    subplot(ceil(n/3),3,i)
    plot(x_inf_lp(i,:))
    hold on
    plot(x_inf_norm(i,:))
    legend
end
saveas(fig,fullfile([pwd '\\' 'Homework' '\\' 'HW3' '\\' 'fig'],'pblm1_inftyn_x.png'))

fig = figure('position', [0, 0, 750, 500])
sgtitle('Control Sequence for Open-Loop Optimal Control with \infty - norm')
for i = 1:m
    subplot(3,1,i)
    plot(u_inf_lp(i,:))
    hold on
    plot(u_inf_norm(i,:))
    legend
end
saveas(fig,fullfile([pwd '\\' 'Homework' '\\' 'HW3' '\\' 'fig'],'pblm1_inftyn_u.png'))


%% Norm 1 ------------------------------
% Derived Linear Program
cvx_begin
    variable r(n,T)
    variable s(m,T-1)
    variable x(n,T)
    variable u(m,T-1)
    sum = 0;
    for i = 1:T-1
        for j = 1:n
            sum = sum + r(j,i);
        end
        for j = 1:m
            sum = sum + gamma * s(j,i);
        end
    end
    minimize(r(T) + sum);
    subject to
        x(:,1) == x0;
```

```
        for i = 1:T-1
            x(:,i+1) == A * x(:,i) + B * u(:,i);
        end
        -xbar <= x(:,:) <= xbar;
        -ubar <= u(:,:) <= ubar;
        -r <= x <= r;
        -s <= u <= s;
cvx_end

x_1_lp = x;
u_1_lp = u;


% CVX Norm Implimentation
p = 1;
q = 1;
cvx_begin
    variable x(n,T)
    variable u(m,T-1)
    sum = norm(x(:,T),p);
    for i = 1:T-1
        sum = sum + norm(x(:,i),p) + gamma * norm(u(:,i),q);
    end
    minimize(sum)
    subject to
        x(:,1) == x0;
        for i = 1:T-1
            x(:,i+1) == A * x(:,i) + B * u(:,i);
        end
        norm(x(:,i),Inf) <= xbar;
        for i = 1:T-1
            norm(x(:,i),Inf) <= xbar;
            norm(u(:,i),Inf) <= ubar;
        end
cvx_end

x_1_norm = x;
u_1_norm = u;




%% 1-norm Ploting
```

```
fig = figure('position', [0, 0, 1200, 1000])
sgtitle('States for Open-Loop Optimal Control with 1 - norm')
for i = 1:n
    subplot(ceil(n/3),3,i)
    plot(x_1_lp(i,:))
    hold on
    plot(x_1_norm(i,:))
    legend
end
saveas(fig,fullfile([pwd '\\' 'Homework' '\\' 'HW3' '\\' 'fig'],'pblm1_1n_x.png'))

fig = figure('position', [0, 0, 750, 500])
sgtitle('Control Sequence for Open-Loop Optimal Control with 1 - norm')
for i = 1:m
    subplot(3,1,i)
    plot(u_1_lp(i,:))
    hold on
    plot(u_1_norm(i,:))
    legend
end
saveas(fig,fullfile([pwd '\\' 'Homework' '\\' 'HW3' '\\' 'fig'],'pblm1_1n_u.png'))
```

C Problem 3 MATLAB Code:

All code I write in this course can be found on my GitHub repository:
https://github.com/jonaswagner2826/MECH6327

Script 3: MECH6327_HW3_pblm2

```
% MECH 6327 - Homework 3 - Problem 2
% Author: Jonas Wagner
% Date: 2020-03-21

%% Problem Data
clear
close all
HW3Prob2_Data

%% Bisection Attempt
lower = 0;
upper = 100;
error = 1;

umax = [umax1; umax2];

t_all = [];

while (upper - lower) > error
    T = upper;
    t = floor(1/2 * (lower + upper));
    cvx_begin
        variable x(n,T)
        variable u(m,T)
        x(:,1) == x0;
        for i = 1:T-1
            x(:,i+1) == A * x(:,i) + B * u(:,i);
        end
        for i = 1:T
            -umax <= u(i) <= umax;
        end
        x(:,t) == xdes;
    cvx_end
    if abs(cvx_optval) <= 1
        upper = t;
    else
        lower = t;
    end
```

```
    t_all = [t_all,t];
end

if abs(cvx_optval) <= 1
    tau = t * ts;
    T = t;
else
    tau = (t-1) * ts;
    T = t-1;
end

cvx_begin
    variable x(n,T)
    variable u(m,T)
    x(:,1) == x0;
    for i = 1:T-1
        x(:,i+1) == A * x(:,i) + B * u(:,i);
    end
    for i = 1:T
        -umax <= u(i) <= umax;
    end
    x(:,t) == xdes;
cvx_end

t_all

tau

%% Ploting
t = ts * T;

fig = figure('position',[0,0,750,500]);
subplot(2,1,1)
for i = 1:n
    plot(x(i,:))
    hold on
end
title('States for minimum time state transfer via quasiconvex optimization')
legend
% saveas(fig,fullfile([pwd '\\' 'Homework' '\\' 'HW3' '\\' 'fig'],'pblm2_x.png'))
%
% fig = figure
subplot(2,1,2)
```

```
for i = 1:m
    plot(u(i,:))
    hold on
end
title('Control Sequence for minimum time state transfer via quasiconvex optimization')
legend
saveas(fig,fullfile([pwd '\\' 'Homework' '\\' 'HW3' '\\' 'fig'],'pblm2.png'))
```

D Problem 3 MATLAB Code:

All code I write in this course can be found on my GitHub repository:
https://github.com/jonaswagner2826/MECH6327

Script 4: MECH6327_HW3_pblm3

```
% MECH 6327 - Homework 3 - Problem 3
% Author: Jonas Wagner
% Date: 2020-03-21

clear

%% Random Problem Generation
n = 4;
m = 2;
A = randn(n,n)
B = randn(n,m)
Q = randPDMatrix(n)
R = randPDMatrix(m)

%% Optimiztation SDP Solution
cvx_begin sdp
    variable P(n,n) symmetric
    minimize(-trace(P))
    subject to
        M = [R + B' * P * B, B' * P * A;
         A' * P * B, Q + A' * P * A - P] >= 0;
        P >= 0;
cvx_end

P_cvx = P
K_cvx = - inv(R + B'*P*B)*B'*P*A
%% DARE Function

[P_dare,K_dare] = idare(A,B,Q,R)

P_cvx
P_dare




%% Random Matrix Generation
% Random Symetric Matrix
```

```
function S = randSMatrix(n)
    A = randn(n,n);
    S = (A + A')/2;
end
% Random Semi-ositive Definite Matrix
function SPD = randSPDMatrix(n)
    A = randn(n-1,n);
    SPD = A' * A;
end
% Random Positive Definite Matrix
function PD = randPDMatrix(n)
    A = rand(n,n);
    PD = A' * A;
% S = randSMatrix(n);
% PD = S + n * eye(n);
end
```

E Problem 3 MATLAB Results:

```
>> MECH6327_HW3_pblm3

A =

1.1002   -1.1372    1.1077    0.2641
0.1751    0.6430    0.8205    3.1585
1.0036   -0.0128   -0.8176    1.2266
1.5110    0.9143   -0.1265    2.3206


B =

0.4145    1.2416
0.2118   -0.1576
0.6132   -1.3736
-0.5278    0.8708


Q =

0.4766    0.4525    0.2565    0.4911
0.4525    0.5808    0.3777    0.7950
0.2565    0.3777    0.4440    0.4396
0.4911    0.7950    0.4396    1.2997


R =

1.1591    0.8154
0.8154    0.7716


Calling SDPT3 4.0: 31 variables, 10 equality constraints
For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------

num. of constraints = 10
dim. of sdp    var  = 10,   num. of sdp  blk  =  2
*******************************************************************
SDPT3: Infeasible path-following algorithms
*******************************************************************
version  predcorr  gam  expon  scale_data
```

```
HKM      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
0|0.000|0.000|8.4e+01|1.2e+01|1.5e+03| 4.731862e+01  0.000000e+00| 0:0:00| chol  1  1
1|0.894|0.821|8.9e+00|2.3e+00|2.2e+02| 2.141617e+01  1.453677e+01| 0:0:00| chol  1  1
2|0.833|0.841|1.5e+00|3.7e-01|5.1e+01| 2.431086e+01  1.330905e+01| 0:0:00| chol  1  1
3|0.515|0.846|7.2e-01|5.7e-02|2.1e+01| 2.140755e+01  2.132680e+01| 0:0:00| chol  1  1
4|0.187|0.237|5.9e-01|4.4e-02|1.9e+01| 2.434592e+01  7.419460e+01| 0:0:00| chol  1  1
5|0.061|0.042|5.5e-01|4.2e-02|2.7e+01| 3.321322e+01  1.216786e+02| 0:0:00| chol  1  1
6|0.104|0.026|4.9e-01|4.1e-02|5.4e+01| 6.400240e+01  6.740367e+01| 0:0:00| chol  1  1
7|0.129|0.433|4.3e-01|2.3e-02|5.0e+01| 7.772307e+01  1.153429e+02| 0:0:00| chol  1  1
8|1.000|0.825|1.4e-08|4.0e-03|4.5e+01| 1.616373e+02  1.181693e+02| 0:0:00| chol  1  1
9|0.962|0.980|2.6e-07|7.9e-05|1.8e+00| 1.344089e+02  1.326294e+02| 0:0:00| chol  1  1
10|0.965|0.977|9.4e-09|1.8e-06|5.8e-02| 1.333014e+02  1.332444e+02| 0:0:00| chol  1  1
11|0.958|1.000|3.9e-10|1.9e-09|3.6e-03| 1.332623e+02  1.332587e+02| 0:0:00| chol  1  1
12|0.987|1.000|7.9e-12|7.9e-11|2.8e-04| 1.332596e+02  1.332594e+02| 0:0:00| chol  1  1
13|0.952|0.987|1.2e-11|2.6e-12|1.3e-05| 1.332595e+02  1.332595e+02| 0:0:00| chol  1  1
14|1.000|1.000|5.2e-12|2.3e-12|1.2e-06| 1.332595e+02  1.332595e+02| 0:0:00|
stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
number of iterations   = 14
primal objective value =  1.33259457e+02
dual   objective value =  1.33259456e+02
gap := trace(XZ)       = 1.15e-06
relative gap           = 4.31e-09
actual relative gap    = 4.31e-09
rel. primal infeas (scaled problem)   = 5.20e-12
rel. dual     "        "       "      = 2.32e-12
rel. primal infeas (unscaled problem) = 0.00e+00
rel. dual     "        "       "      = 0.00e+00
norm(X), norm(y), norm(Z) = 1.7e+02, 1.1e+02, 1.7e+03
norm(A), norm(b), norm(C) = 3.5e+01, 3.0e+00, 3.9e+00
Total CPU time (secs)  = 0.47
CPU time per iteration = 0.03
termination code       =  0
DIMACS: 7.8e-12  0.0e+00  4.0e-12  0.0e+00  4.3e-09  4.3e-09
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): -133.259
```

```
P_cvx =

20.3336   10.5025   -3.3125   37.1904
10.5025   12.9464   -1.8618   32.8376
-3.3125   -1.8618    2.3441   -4.8914
37.1904   32.8376   -4.8914   97.6353


K_cvx =

1.3793    2.0277   -0.1335    4.7523
-1.0233    0.0284   -0.4975   -1.2125


P_dare =

20.3336   10.5025   -3.3125   37.1904
10.5025   12.9464   -1.8618   32.8376
-3.3125   -1.8618    2.3441   -4.8914
37.1904   32.8376   -4.8914   97.6353


K_dare =

-1.3793   -2.0277    0.1335   -4.7523
1.0233   -0.0284    0.4975    1.2125


P_cvx =

20.3336   10.5025   -3.3125   37.1904
10.5025   12.9464   -1.8618   32.8376
-3.3125   -1.8618    2.3441   -4.8914
37.1904   32.8376   -4.8914   97.6353


P_dare =

20.3336   10.5025   -3.3125   37.1904
10.5025   12.9464   -1.8618   32.8376
-3.3125   -1.8618    2.3441   -4.8914
37.1904   32.8376   -4.8914   97.6353
```

```
>> MECH6327_HW3_pblm3

A =

-0.8568    1.3798   -0.6563    1.1284
0.0484    0.0951   -0.1250    0.7425
-0.6649   -0.4271   -0.5305    1.1436
1.4527    0.5108    0.1056   -0.9147


B =

0.1798    1.2963
-0.9833    1.0992
0.3848    0.6532
0.3257   -0.5051


Q =

0.3838    0.3714    0.4771    0.6771
0.3714    1.2344    1.2453    1.0731
0.4771    1.2453    1.4076    1.4208
0.6771    1.0731    1.4208    2.1397


R =

0.5081    0.7407
0.7407    1.1909


Calling SDPT3 4.0: 31 variables, 10 equality constraints
For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------

num. of constraints = 10
dim. of sdp    var  = 10,   num. of sdp  blk  =  2
*******************************************************************
SDPT3: Infeasible path-following algorithms
*******************************************************************
version  predcorr  gam  expon  scale_data
HKM      1      0.000   1        0
```

```
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
0|0.000|0.000|4.4e+01|5.2e+00|1.0e+03| 6.864540e+01  0.000000e+00| 0:0:00| chol  1  1
1|0.886|0.852|5.0e+00|8.2e-01|1.7e+02| 5.707109e+01  1.265425e+01| 0:0:00| chol  1  1
2|0.782|1.000|1.1e+00|5.5e-03|5.2e+01| 5.048488e+01  1.175262e+01| 0:0:00| chol  1  1
3|0.752|1.000|2.7e-01|5.5e-04|1.6e+01| 2.602445e+01  1.535157e+01| 0:0:00| chol  1  1
4|1.000|0.756|1.8e-07|1.8e-04|7.3e+00| 2.578672e+01  1.846254e+01| 0:0:00| chol  1  1
5|0.933|1.000|1.8e-08|5.6e-06|5.8e-01| 2.048293e+01  1.990168e+01| 0:0:00| chol  1  1
6|0.964|0.970|2.0e-09|7.1e-07|2.9e-02| 2.011379e+01  2.008495e+01| 0:0:00| chol  1  1
7|0.965|1.000|6.3e-10|5.6e-08|1.7e-03| 2.009536e+01  2.009368e+01| 0:0:00| chol  1  1
8|0.994|1.000|1.8e-10|1.3e-10|1.2e-04| 2.009415e+01  2.009404e+01| 0:0:00| chol  1  1
9|0.953|0.987|6.7e-11|3.7e-11|5.3e-06| 2.009409e+01  2.009408e+01| 0:0:00| chol  1  1
10|1.000|1.000|6.7e-15|1.3e-11|1.2e-06| 2.009408e+01  2.009408e+01| 0:0:00| chol  1  1
11|1.000|1.000|1.7e-14|1.0e-12|1.3e-08| 2.009408e+01  2.009408e+01| 0:0:00|
stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
number of iterations   = 11
primal objective value =  2.00940824e+01
dual   objective value =  2.00940824e+01
gap := trace(XZ)       = 1.33e-08
relative gap           = 3.22e-10
actual relative gap    = 3.22e-10
rel. primal infeas (scaled problem)   = 1.70e-14
rel. dual     "        "       "      = 1.00e-12
rel. primal infeas (unscaled problem) = 0.00e+00
rel. dual     "        "       "      = 0.00e+00
norm(X), norm(y), norm(Z) = 8.9e+00, 1.2e+01, 8.9e+01
norm(A), norm(b), norm(C) = 1.8e+01, 3.0e+00, 5.7e+00
Total CPU time (secs)  = 0.42
CPU time per iteration = 0.04
termination code       =  0
DIMACS: 2.6e-14  0.0e+00  1.8e-12  0.0e+00  3.2e-10  3.2e-10
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): -20.0941


P_cvx =

9.2752    0.3867    1.2843   -2.9794
0.3867    4.4079    0.6458    1.1789
```

```
1.2843    0.6458    1.9253    0.5430
-2.9794    1.1789    0.5430    4.4857


K_cvx =

0.5888   -0.3683    0.2601   -0.1295
0.7294   -0.5856    0.4291   -0.9410


P_dare =

9.2752    0.3867    1.2843   -2.9794
0.3867    4.4079    0.6458    1.1789
1.2843    0.6458    1.9253    0.5430
-2.9794    1.1789    0.5430    4.4857


K_dare =

-0.5888    0.3683   -0.2601    0.1295
-0.7294    0.5856   -0.4291    0.9410


P_cvx =

9.2752    0.3867    1.2843   -2.9794
0.3867    4.4079    0.6458    1.1789
1.2843    0.6458    1.9253    0.5430
-2.9794    1.1789    0.5430    4.4857


P_dare =

9.2752    0.3867    1.2843   -2.9794
0.3867    4.4079    0.6458    1.1789
1.2843    0.6458    1.9253    0.5430
-2.9794    1.1789    0.5430    4.4857

>> MECH6327_HW3_pblm3

A =
```

```
-1.5312    0.7880    1.6345   -0.9443
0.5046    0.2982   -0.6235   -0.6712
-0.8642   -0.1637   -1.3501    0.5767
-0.3766    0.6067   -1.1622   -2.0858


B =

0.2360    0.0076
-0.7784   -0.9376
1.0996   -0.6816
-0.8556   -0.2601


Q =

1.5071    1.3058    1.2427    1.5227
1.3058    2.0018    1.1626    1.6402
1.2427    1.1626    1.9380    2.0005
1.5227    1.6402    2.0005    2.2226


R =

0.1582    0.2334
0.2334    0.4393


Calling SDPT3 4.0: 31 variables, 10 equality constraints
For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------

num. of constraints = 10
dim. of sdp    var  = 10,   num. of sdp  blk  =  2
*******************************************************************
SDPT3: Infeasible path-following algorithms
*******************************************************************
version  predcorr  gam  expon  scale_data
HKM      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
0|0.000|0.000|4.4e+01|3.9e+00|1.0e+03| 8.266937e+01  0.000000e+00| 0:0:00| chol  1  1
1|0.818|0.880|8.1e+00|5.1e-01|1.7e+02| 3.173665e+01  9.183288e+00| 0:0:00| chol  1  1
```

```
2|0.761|0.758|1.9e+00|1.3e-01|6.0e+01| 2.745402e+01  1.125760e+01| 0:0:00| chol  1  1
3|0.553|0.897|8.6e-01|1.3e-02|2.5e+01| 2.195296e+01  1.974743e+01| 0:0:00| chol  1  1
4|0.169|0.214|7.2e-01|1.1e-02|2.1e+01| 2.472518e+01  1.029151e+02| 0:0:00| chol  1  1
5|0.017|0.022|7.1e-01|1.0e-02|3.3e+01| 3.109723e+01  1.695860e+02| 0:0:00| chol  1  1
6|0.061|0.035|6.6e-01|1.0e-02|7.8e+01| 8.019566e+01  2.054055e+02| 0:0:00| chol  1  1
7|0.423|0.391|3.8e-01|6.1e-03|1.4e+02| 2.228571e+02  2.222143e+02| 0:0:00| chol  1  1
8|1.000|0.531|8.3e-06|2.8e-03|1.1e+02| 3.445733e+02  2.448139e+02| 0:0:00| chol  2  1
9|0.952|1.000|1.7e-06|1.7e-06|1.6e+01| 2.823759e+02  2.660703e+02| 0:0:00| chol  1  1
10|0.950|0.991|8.3e-08|3.5e-07|1.1e+00| 2.712737e+02  2.701719e+02| 0:0:00| chol  1  1
11|1.000|1.000|2.1e-10|1.7e-08|1.1e-01| 2.705570e+02  2.704431e+02| 0:0:00| chol  1  1
12|0.959|0.980|1.7e-10|4.8e-10|4.0e-03| 2.704909e+02  2.704869e+02| 0:0:00| chol  1  1
13|0.993|1.000|1.0e-10|3.4e-11|2.4e-04| 2.704880e+02  2.704878e+02| 0:0:00| chol  1  1
14|0.954|0.989|8.0e-11|2.1e-11|1.1e-05| 2.704879e+02  2.704879e+02| 0:0:00| chol  1  1
15|1.000|1.000|1.7e-10|1.6e-11|1.0e-06| 2.704879e+02  2.704879e+02| 0:0:00|
stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
number of iterations   = 15
primal objective value =  2.70487886e+02
dual   objective value =  2.70487885e+02
gap := trace(XZ)       = 1.04e-06
relative gap           = 1.92e-09
actual relative gap    = 1.87e-09
rel. primal infeas (scaled problem)   = 1.72e-10
rel. dual     "        "       "      = 1.59e-11
rel. primal infeas (unscaled problem) = 0.00e+00
rel. dual     "        "       "      = 0.00e+00
norm(X), norm(y), norm(Z) = 8.9e+02, 2.2e+02, 1.4e+03
norm(A), norm(b), norm(C) = 2.2e+01, 3.0e+00, 7.5e+00
Total CPU time (secs)  = 0.45
CPU time per iteration = 0.03
termination code       =  0
DIMACS: 2.6e-10  0.0e+00  3.7e-11  0.0e+00  1.9e-09  1.9e-09
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): -270.488


P_cvx =

101.8040  -50.0081   -0.8792  117.2149
-50.0081   28.3481    2.3037  -57.6749
```

```
-0.8792    2.3037    4.1602    0.2983
117.2149  -57.6749    0.2983  136.1756


K_cvx =

-4.7337    3.0098    1.0219   -6.7728
0.2527   -0.0799   -1.2428    0.2020


P_dare =

101.8040  -50.0081   -0.8792  117.2149
-50.0081   28.3481    2.3037  -57.6749
-0.8792    2.3037    4.1602    0.2983
117.2149  -57.6749    0.2983  136.1756


K_dare =

4.7337   -3.0098   -1.0219    6.7728
-0.2527    0.0799    1.2428   -0.2020


P_cvx =

101.8040  -50.0081   -0.8792  117.2149
-50.0081   28.3481    2.3037  -57.6749
-0.8792    2.3037    4.1602    0.2983
117.2149  -57.6749    0.2983  136.1756


P_dare =

101.8040  -50.0081   -0.8792  117.2149
-50.0081   28.3481    2.3037  -57.6749
-0.8792    2.3037    4.1602    0.2983
117.2149  -57.6749    0.2983  136.1756
```