4) $x_{t+1} = Ax_t + Bu_t$

$A \in \mathcal{A} = \operatorname{conv}(A_1, \dots, A_m)$

$u_t = Kx_t$

$\hat{A}_i = (A_i + BK)$ } Must Be Stable $\forall A_i \in \mathcal{A}$

$\hat{A}_i^T P \hat{A}_i \prec P$

$(A_i + BK)^T P (A_i + BK) - P \prec 0$

$\cancel{(A_i^T P + K^T B^T P)(A_i + BK) - P \prec 0}$

$\cancel{A_i^T P A_i + A_i^T P B K + K^T B^T P A_i + K^T B^T P B K - P \prec 0}$

Schur's compliment

$$\begin{bmatrix} P & \hat{A}_i^T \\ \hat{A}_i & P^{-1} \end{bmatrix} \succ 0 \quad \forall i$$

→ Summing All w/ α_i to find ANY Point

$$\begin{bmatrix} P & (A_i + BK)^T \\ A_i + BK & P^{-1} \end{bmatrix} \succ 0$$

... Stable ~~∀~~ A_i in $\mathcal{A}$

4) cont.

$$\cancel{\begin{bmatrix} P & 0 \\ 0 & P^{-1} \end{bmatrix} + \begin{bmatrix} 0 & A_i^T \\ A_i & 0 \end{bmatrix} + \begin{bmatrix} 0 & K^T B^T \\ BK & 0 \end{bmatrix} \succ 0} \;?$$

~~Solve w/ LMI:~~ $\cancel{\begin{bmatrix} P & \hat{A}_i^T \\ \hat{A}_i & P^{-1} \end{bmatrix} \succ 0}$

$\cancel{K = B^{-1}(\hat{A}_i - A_i)}$

$\cancel{A_i^T P A_i}$

solve LMI: $\begin{bmatrix} P & \hat{A}_i^T \\ \hat{A}_i & P^{-1} \end{bmatrix} \succ 0 \quad \forall i = 1, \dots, m$

$$B^{-1}(\hat{A}_i - A_i) - K = 0$$

$$K = B^{-1}(\hat{A}_i - A_i)$$

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2) $M(x,y) = x^2y^4 + x^4y^2 + 1 - 3x^2y^2$

Nonnegativity: AM-GM $n=3$

$\sqrt[n]{\prod x_i} \leq \frac{1}{n}\sum_{i=1}^{n} x_i$

$\sqrt[3]{(x^4y^2)(y^2x^4)(1)} \leq \frac{1}{3}(x^4y^2 + y^2x^4 + 1)$

$3x^2y^2 \leq x^4y^2 + y^2x^4 + 1$

$0 \leq x^4y^2 + y^2x^4 - 3x^2y^2 + 1$

SOS Infeasability: using Yalmip

Problem 5.43:

$$\text{minimize} \quad f^T x$$
$$\text{subject to} \quad \|A_i x + b_i\|_2 \le c_i^T x + d_i, \quad i = 1, \dots, m$$

Let $y_i = A_i x + b_i$
$t_i = c_i^T x + d_i$

Lagrange: $L(x, \lambda, \nu) = f(x) + \sum \lambda g(x)$

$$L(x, \lambda_i, \nu_i) = f^T x + \sum_{i=1}^m \lambda_i (\|y_i\|_2 - t_i) + \sum_{i=1}^m \nu_i (A_i x + b - y_i) + \sum_{i=1}^m \nu_{i+m}^T (c_i x + d_i - t_i)$$

$$= -\sum_i \nu_i d_i + \sum_i \nu_{i+m} b_i$$

$$L(x, \lambda_i, \nu_i) = f^T x + \sum_i \lambda_i \|y_i\| + \sum_i \nu_i^T (y_i - A_i x + b_i) - \lambda_i (c_i x + d_i)$$

~~Let $\nu_i = \|y_i\|$~~

$$= \sum_i \nu_i^T b_i - \nu_i^T d_i$$
$$+ f^T x - \nu_i A x - \lambda_i c x$$
$$- \lambda_i \|y_i\| - \lambda$$

5.43: cont.

$$\begin{aligned} \text{Minimize} \quad & f^T x \\ \text{Subject to} \quad & \|y_i\|_2 \leq t_i \\ & y_i = A_i x + b_i \\ & t_i = c_i x + d_i \end{aligned}$$

$$L(x, y, t, \lambda_i, \nu_i, \mu_i) = \sim$$

$$= f^T x + \sum_{i=1}^m \lambda_i (\|y_i\|_2 - t_i) + \sum_{i=1}^m \nu_i^T (y_i - A_i x - b_i) + \sum_{i=1}^m \mu_i^T (t_i - C_i x + d_i)$$

$$= \left[f^T + \sum_{i=1}^m (-\mu_i^T C_i^T + \nu_i^T A_i)\right] x + \sum_{i=1}^m \lambda_i \|y_i\|_2 + \nu_i^T y_i + \sum_{i=1}^m (-\lambda_i + \mu_i) t_i$$

$$- \left(\sum_{i=1}^m b^T \nu_i - d_i \mu_i\right)$$

Minimize by x: $f^T = \left(\sum_{i=1}^m C_i \mu_i - A_i^T \nu_i\right)^T$

Minimize over y_i: $\inf \sim \begin{cases} 0, & \lambda_i \|y_i\|_2 \geq \|\nu_i\|_2 \cancel{y_i} \\ -\infty, & \text{else} \end{cases}$

Minimize over t_i: $\mu_i = \lambda_i$

minimum @ Maximum of $\sum_{i=1}^m b^T \nu_i - d_i \mu_i$

5.43: a cont.

$$\inf_{x, \gamma_i, t_i} \begin{cases} -\sum_{i=1}^{m} b^T \gamma_i - d\mu_i , & \|\gamma_i\| \leq \lambda_i \\ & \mu_i = \lambda_i \\ -\infty , \text{else} & f + \sum_{i=1}^{m} A_i^T \gamma_i - c_i \mu_i \end{cases}$$

Dual Problem:

Maximize $g(\lambda_i, \gamma_i, \mu_i) = \sum_{i=1}^{m} b^T \gamma_i - d\mu_i$

Subject to $f + \sum_{i=1}^{m} A_i^T \gamma_i - c_i \mu_i$

$\|\gamma_i\| \leq \mu_i$

Correct... but μ and γ backwards

Fix when typing...

5.43:b)

SOCP is given as

$$\text{minimize} \quad f^T x$$
$$\text{subject to} \quad \|A_i x + b_i\|_2 \leq c_i^T x + d_i \quad \forall i = 1, \ldots, m$$

The Conic approach: ← Ref SOCP Duality lecture...

~~Known fact: $\max_{(u,s): \|u\|_2 \leq \lambda} u^T y - t\lambda = \max_{\lambda \geq 0} \lambda(\|y\|_2 - t)$~~

$$\min_x \ \langle c, x \rangle$$
$$Ax = b$$
$$x \in K$$

From Ref 2

Dual $\Updownarrow$

K^* = dual cone of K

$$\max_{y,s} \ \langle b, y \rangle$$
$$Ay + s = c$$
$$s \in K^*$$

Notes... conic Def method...

5.43: b)

$\langle f, x \rangle$

minimize $f^T x$

subject to $\|A_i x + b_i\| \leq c_i^T x + d_i$, $i = 1, \dots, m$

Conic Dual

$Ax = b$

$x \in K \leftarrow \|y\| \leq t$

Maximize $\langle b, y \rangle$

Subject to $A y + s = c$

$s \in K^* \leftarrow$ self dual $\|y\| \leq t$

$\|u\| \leq v$

$$\underset{u, v}{\text{Maximize}} \quad \sum b_i^T u_i - d_i v_i$$

$$\text{subject to} \quad \sum \left(A_i^T u_i - c_i v_i\right) + f$$

$$\|u_i\| \leq v_i$$