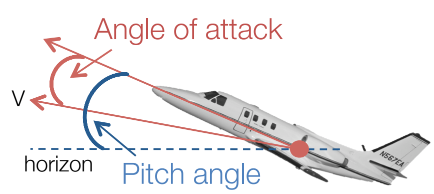
Due via email on Monday, May 3, 2021. Please use the subject line "MECH 6327: Homework 5 Submission", submit all solutions in a single pdf file, and name your file "Yourlastname_HW5.pdf".

Control Design via Semidefinite Programming

Consider the linearized model of a Cessna Citation Aircraft (at 5000m, at speed 128.2 m/s)

$$\dot{x} = \begin{bmatrix} -1.2822 & 0 & 0.98 & 0 \\ 0 & 0 & 1 & 0 \\ -5.4293 & 0 & -1.8366 & 0 \\ -128.2 & 128.2 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} -0.3 \\ 0 \\ -17 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} w, \quad y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

with control input u = elevator angle, states x_1 = angle of attack, x_2 = pitch angle, x_4 = pitch rate, and x_4 = altitude (all relative to nominal), performance output y, and pitch rate disturbance input w. Use the semidefinite programming formulations discussed in class to compute the optimal state feedback controller to minimize



- the system $\mathcal{H}_2$ norm from the disturbance input w to the performance output y;
- the system $\mathcal{H}_\infty$ norm from the disturbance input w to the performance output y.

Also provide the optimal value of the respective closed-loop system norms.

Model Predictive Control (MPC)

Suppose the elevator angle input is limited to ± 0.262rad (± 15 degrees), the elevator angle *rate* is limited to ± 0.524rad/s (± 30 degrees/s), and we would like to limit the pitch angle to ± 0.349rad (± 20 degrees). Consider the time discretized system with sampling period $dt = 0.25s$, and discrete-time dynamics matrices $A = e^{A_c dt}$, $B = dtB_c + \frac{1}{2}dtAcBc + \frac{1}{6}dt^2 A_c^2 B_c$, where A_c and B_c are the above continuous time state space matrices. Complete the following steps:

- Compute the optimal infinite-horizon LQR controller by solving the discrete-time algebraic Riccati equation with $Q = I$ and $R = 10$ (i.e., in Matlab compute $[P, E, K] = dare(A, B, Q, R)$). Simulate the closed-loop system from initial state $x_0 = [0, 0, 0, 10]^T$ but saturate the LQR controller to the input (magnitude and rate) constraints (i.e., if the LQR controller computes an input above or below 15 degrees, saturate it to 15 degrees, and if it computes an input more than $60dt$ degrees different from the previous step, saturate the difference to $60dt$). Comment on the results.

- Implement a model predictive controller with horizon $T_h = 10$ sample periods and constant quadratic cost parameters $Q = I$ and $R = 10$ that explicitly accounts for the elevator angle and pitch constraints (the input rate constraint can by enforced in discrete time by using $|u(t+1) - u(t)| \leq 0.524dt$, and be sure to limit the actual applied input based on the previous applied input as well). Simulate the closed-loop system from initial state $x_0 = [0, 0, 0, 10]^T$ and compare to the LQR controller. Determine using your implementation how short the MPC planning horizon can be reduced before stability problems arise.