MECH 6327 - Homework 2

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1 Problem Set 1: Convex Sets

1.1 Problem 2.5

Problem:
What is the distance between two parallel hyperplanes: $\{x \in \Re^n | a^T x = b_1\}$ and $\{x \in \Re^n | a^T x = b_2\}$?

Solution:
Under the assumption that $a \in \Re^n$ and $b_1, b_2 \in \Re$, the quantity $a^T x_0$ represents the component of x_0 in the normal direction. Similarly, the quantities b_1 and b_2 represent the euclidean distance of the hyperplane from the origin (in the normal direction). Since the hyperplanes are parrellel, the distance between them is the difference between their offsets:

$$\text{Distance between hyperplanes: } b_1 - b_2 \tag{1}$$

1.2 Problem 2.7

Problem:
Voronoi description of halfspace. Let a and b be distinct points in $\Re^n$. Show that the set of all points that are closer to a than b via the euclidean norm is a halfspace. Describe it explicitly as an inequality and draw a picture.

Solution:
The set of all points closer to a then b can be defined as:

$$\{x \in \Re^n \mid \|x - a\|_2 \leq \|x - b\|_2\} \tag{2}$$

The boundary defining this halfspace will be a plane defined by the normal vector c representing the distance between a and b, and the offset coefficient d describing intersection of the plane through the halfway point between a and b. The quantities c and d can therefore be defined by:

$$\begin{aligned} c &= b - a \\ d &= \frac{c^T a + c^T b}{2} \\ &= \frac{1}{2} c^T (a + b) \end{aligned} \tag{3}$$

The halfspace, that is equivenlent to x, can be described by the following:

$$\{x \in \Re^n \mid c^T x \leq d\} \tag{4}$$

1.3 Problem 2.12

Problem:
Multiple choice problem from book....
Solution:

1.4 Problem 2.28

Problem:

Solution:

1.5 Problem 2.33

Problem:

Solution:

2 Problem Set 2: Convex Functions

2.1 Problem 3.6

Problem:

Solution:

2.2 Problem 3.16

Problem:

Solution:

2.3 Problem 3.18a

Problem:

Solution:

2.4 Problem 3.22

Problem:

Solution: