MECH 6v29 - Model Predictive Control
Homework 3

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Contents

Problem 1

Problem Data:

1a)

Problem Data:

$$\begin{aligned} \mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} \quad & \mathcal{W} = \{\mathbf{w} = \mathbf{B}z \;:\; |z| \le 0.3\} \\ \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad & \mathcal{Y} = \{\mathbf{y} \in \mathbb{R}^3 \;:\; \|\mathbf{y}\|_\infty \le 1\} \end{aligned} \tag{1}$$

Prediction Horizon: $N = 10$
Initial Condition: $\mathbf{x}_0 = 0$

1b) Nilpotent candidate controller

$\Lambda(\mathbf{A} + \mathbf{BK}) = 0$
Result w/ Acker: $\mathbf{K} = \begin{bmatrix} -1 & -1.5 \end{bmatrix}$
Same as in [?].

1c) Output constraint tightening

From reference (using different notation):

$$\begin{aligned} \mathcal{Y}_0 &= \mathcal{Y} \\ \mathcal{Y}_{j+1} = \mathcal{Y}_j \ominus (\mathbf{C} + \mathbf{DK})\mathbf{L}_j \mathcal{W}, \quad & \forall_{j \in \{0,\ldots,N-1\}} \end{aligned} \tag{2}$$

where $\mathbf{L}_j = (\mathbf{A} + \mathbf{BK})^j$.[1]
Or equivalently, using the time-invarience of K and some version of the Cayley-Hamilton theorem,

$$\mathcal{Y}_j = \mathcal{Y} \ominus \bigoplus_{i=1,\ldots,n} (\mathbf{C} + \mathbf{DK})(\mathbf{A} + \mathbf{BK})^{i-1} \tag{3}$$

(eliminating if the power is negative...)
TODO: double check this... (pretty sure this falls under some distributed property...)
For this system, $(\mathbf{C} + \mathbf{DK}) = \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{K} \end{bmatrix}$

$$\begin{aligned} \mathcal{Y}_0 &= \{\mathbf{y} \in \mathbb{R}^3 \;:\; \|\mathbf{y}\|_\infty \le 1\} \\ &= \{\mathbf{y} \in \mathbb{R}^3 \;:\; |y_1| \le 1, |y_2| \le 1, |y_3| \le 1\} \\ \mathcal{Y}_1 &= \mathcal{Y}_0 \ominus (\mathbf{C} + \mathbf{DK})\mathcal{W} \\ &= \mathcal{Y}_0 \ominus \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{K} \end{bmatrix} \{\mathbf{B}w \in \mathbb{R} \;:\; |w| \le 0.3\} \\ &= \{\mathbf{y} \in \mathbb{R}^3 \;:\; |y_1| \le 0.85, |y_2| \le 0.7, |y_3| \le 0.4\} \\ \mathcal{Y}_1 &= \mathcal{Y}_1 \ominus (\mathbf{C} + \mathbf{DK})(\mathbf{A} + \mathbf{BK})\mathcal{W} \\ &= \mathcal{Y}_1 \ominus \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{K} \end{bmatrix} \{\mathbf{B}w \in \mathbb{R} \;:\; |w| \le 0.3\} \\ &= \{\mathbf{y} \in \mathbb{R}^3 \;:\; |y_1| \le 0.7, |y_2| \le 0.4, |y_3| \le 0.1\} \end{aligned} \tag{4}$$

which is the same for the remaining since $(\mathbf{A} + \mathbf{BK})^2 = \mathbf{0}$.

[1] not explicitly, but eliminating time-variance that's what it is...

A Code

1a) Github

See my github repo for all my course related materials: `https://github.com/jonaswagner2826/MECH6v29_MPC`

1b) Matlab results

MATLAB code and results are attached.