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MPC HW 1 - Problem 1

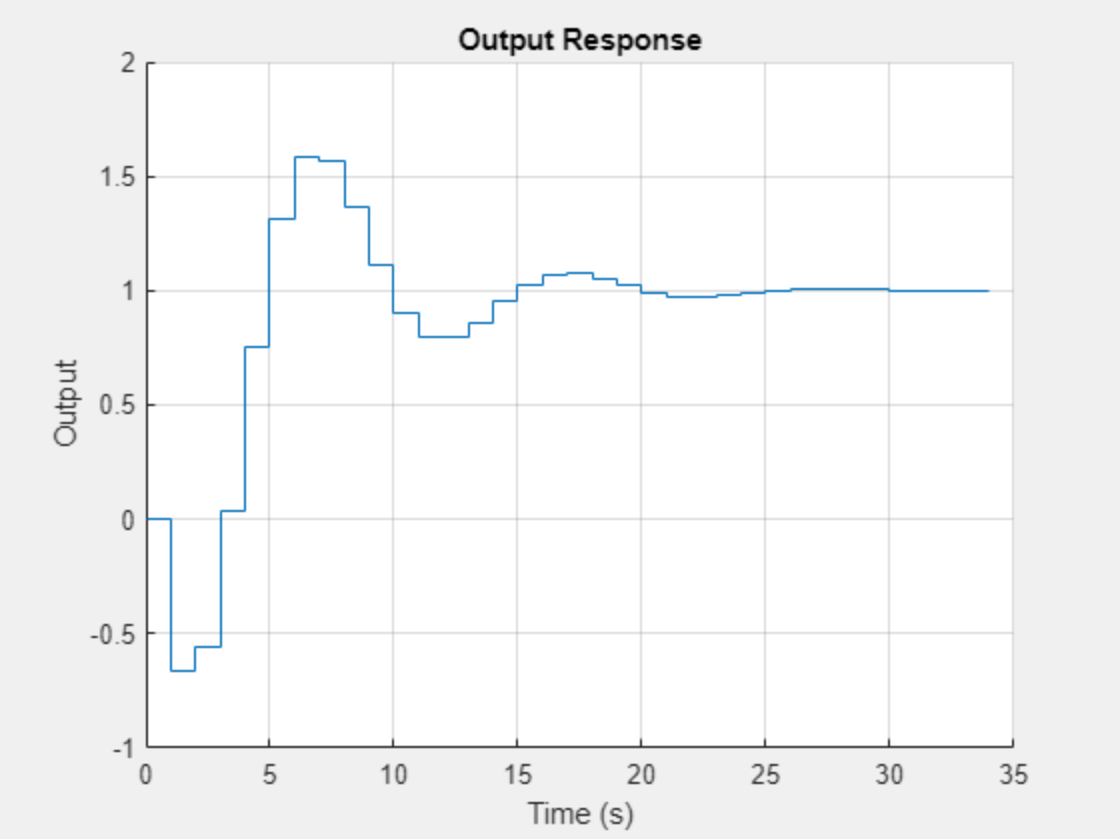
```
clear
close all
subfolder = fileparts(mfilename('fullpath'));
if ~isfolder('figs'); mkdir('figs'); end

% Problem Information
A = [4/3, -2/3; 1, 0];
B = [1; 0];
C = [-2/3, 1];
D = 0;
dt = 1;
sys = ss(A,B,C,D,dt);

% Size parameters
nx = size(A,1);
nu = size(B,2);
```

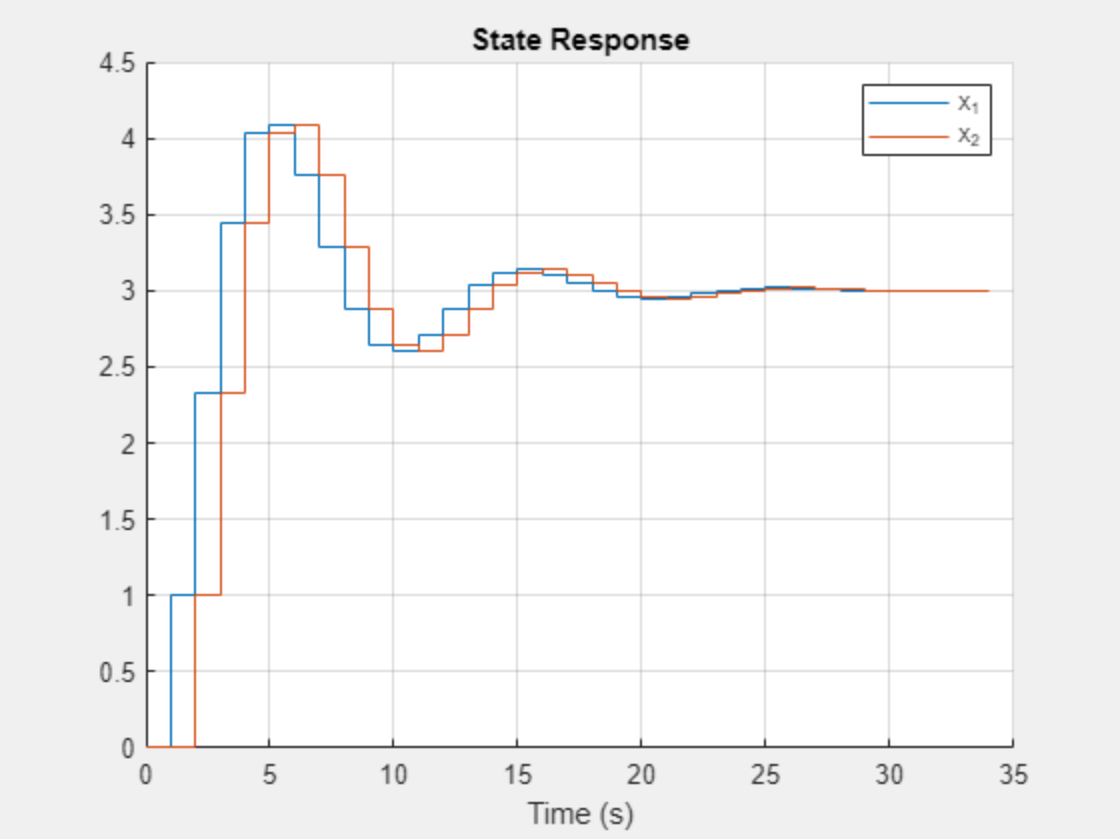
Part 1a

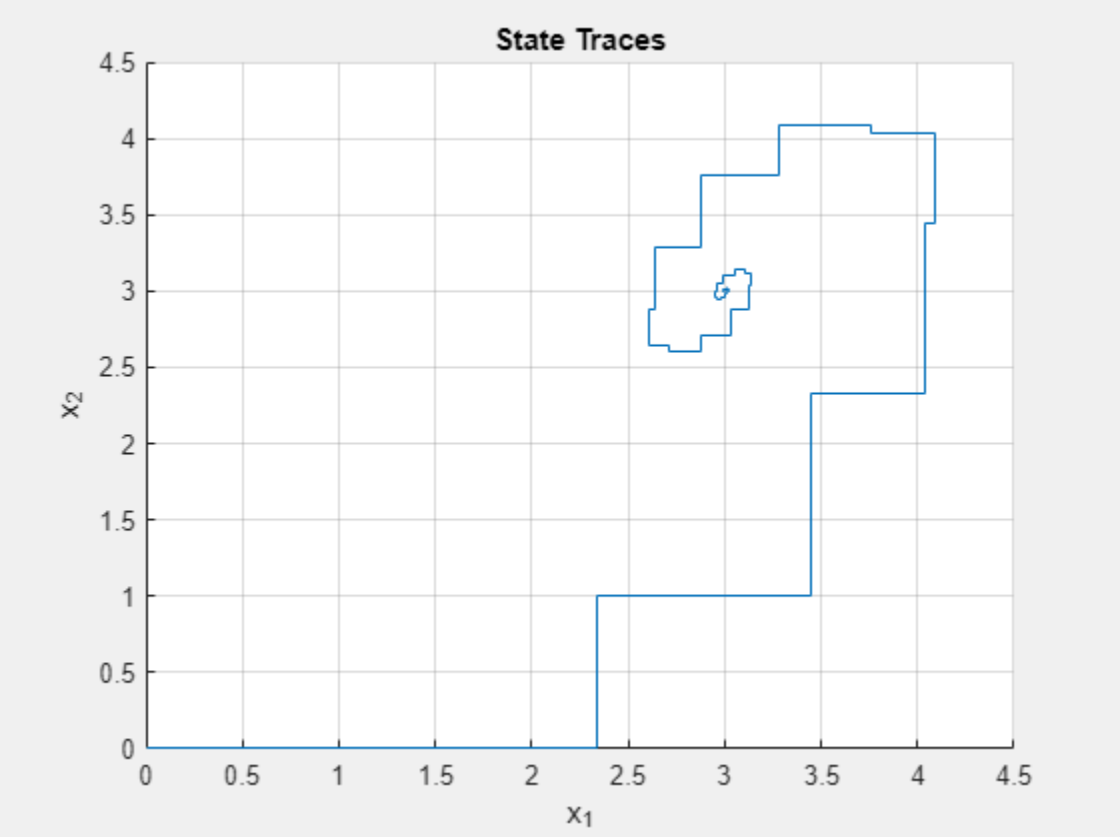
Step Response

```
[y,t,x] = step(sys);

% Output Response
figName = 'pblm1a_fig1';
fig = figure(WindowStyle="normal");
hold on; grid on;
stairs(t,y,"DisplayName",'Output');
title('Output Response')
xlabel('Time (s)')
ylabel('Output')
saveas(fig,[subfolder,filesep,'figs',filesep,figName],'png')

% State Response
figName = 'pblm1a_fig2';
fig = figure(WindowStyle="normal");
```

```
hold on; grid on;
stairs(t,x(:,1),"DisplayName",'x_1');
stairs(t,x(:,2),"DisplayName",'x_2');
legend
title('State Response')
xlabel('Time (s)')
ylabel('')
saveas(fig,[subfolder,filesep,'figs',filesep,figName],'png')

% State Response
figName = 'pblm1a_fig3';
fig = figure(WindowStyle="normal");
hold on; grid on;
stairs(x(:,1),x(:,2))
title('State Traces')
xlabel('x_1')
ylabel('x_2')
saveas(fig,[subfolder,filesep,'figs',filesep,figName],'png')
```




State Response
x1
x2
Time (s)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
5
10
15
20
25
30
35


State Traces
x2
x1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

Part 1b

MPC Parameters (standard optimization)

```
Q = C'*C + 1e-3*eye(nx);
R = 1e-3;
P = Q;
N = 5; % prediction horizon
```

Batch method

Construct S_x,and S_u

```
for i = 1:N+1
    S_x_{i} = A^(i-1);
    for j = 1:nu
        S_u__{j} = eye(nu)*A^(i-2+j-1)*B;
    end
    S_u_{i} = horzcat(S_u__{:});
end
S_x = vertcat(S_x_{:});
S_u = tril(vertcat(S_u_{:}),-1);

% Construct Qbar and Rbar
Qbar = blkdiag(kron(Q,eye(N)),P);
Rbar = kron(R,eye(N));

% H, F, Y
H = S_u'*Qbar*S_u + R;
F = S_x'*Qbar*S_u;
Y = S_x'*Qbar*S_x;

% Results
Ustar = @(x_0) -H\F'*x_0;
ustar = @(x_0) [eye(nx),zeros(nx,nx*(N-1))]*-H\F'*x_0;
K_0_batch = -H\F';

disp('Batch Method:')
disp('K = '); disp(K_0_batch);
```

```
Batch Method:
K =
   -0.2753    0.6666
```

Dynamic Programing Method

```
F_fun = @(P_k) -(B'*P_k*B+R)\B'*P_k*A;
P_fun = @(P_kp1) A'*P_kp1*A + Q - A'*P_kp1*B*inv(B'*P_kp1*B+R)*B'*P_kp1*A;

P_{N} = Q;
```

```
for k = N-1:-1:1
    P_{k} = P_fun(P_{k+1});
    F_{k} = F_fun(P_{k});
end
P_0 = P_fun(P_{1});
F_0 = F_fun(P_0);

ustar = @(x) F_0*x;
K_0_dynprog = F_0;

disp('Dynamic Programing Method:')
disp('K = '); disp(K_0_dynprog);

% results
disp('They are not the same, or at least not with a time-horrizon on $N=5$')
```

```
Dynamic Programing Method:
K =
   -0.1739    0.6655

They are not the same, or at least not with a time-horrizon on $N=5$
```

Part c

```
disp('Batch version')
A_K_batch = A+B*K_0_batch
eig_batch = eig(A_K_batch)

disp('The system is not closed-loop stable acording to this as there is an
 eigen value >= 1')

disp('Dynamic Programing Method')
A_K_dynprog = A+B*K_0_dynprog
eigh_dynprog = eig(A_K_dynprog)

disp('The system is not closed-loop stable acording to this as there is an
 eigen value >= 1')
```

```
Batch version

A_K_batch =

    1.0580   -0.0001
    1.0000         0


eig_batch =

    1.0579
    0.0001

The system is not closed-loop stable acording to this as there is an eigen
 value >= 1
```

```
Dynamic Programing Method

A_K_dynprog =

    1.1594   -0.0012
    1.0000         0


eigh_dynprog =

    1.1584
    0.0010

The system is not closed-loop stable acording to this as there is an eigen
 value >= 1
```

Part d

```
K_lqr = -dlqr(A,B,Q,R)
A_K_lqr = A+B*K_lqr
eig_lqr = eig(A_K_lqr)

disp('LQR is stable')
```

```
K_lqr =

   -0.6683    0.6660


A_K_lqr =

    0.6650   -0.0007
    1.0000         0


eig_lqr =

    0.6640
    0.0010

LQR is stable
```

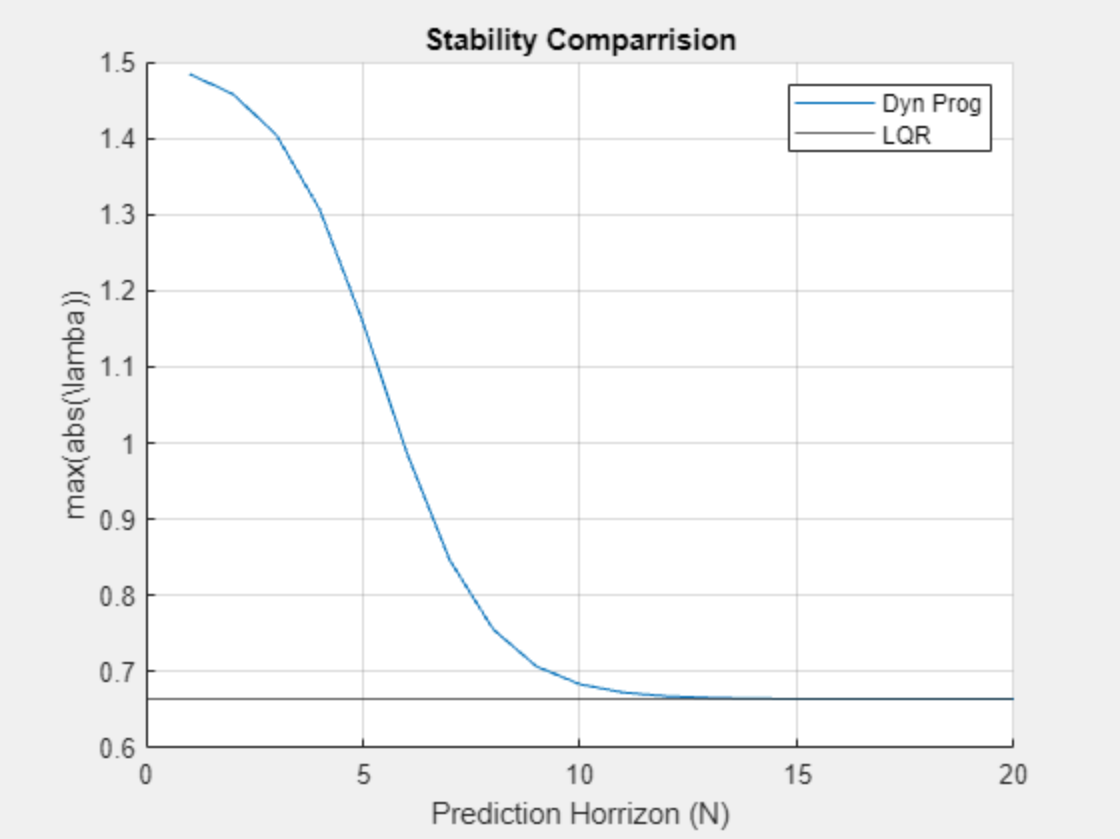
Part e

```
for N = 1:20
    F_fun = @(P_k) -(B'*P_k*B+R)\B'*P_k*A;
    P_fun = @(P_kp1) A'*P_kp1*A + Q - A'*P_kp1*B*...
        inv(B'*P_kp1*B+R)*B'*P_kp1*A;

    P_{N} = Q;
    for k = N-1:-1:1
```

```
        P_{k} = P_fun(P_{k+1});
        F_{k} = F_fun(P_{k});
    end
    P_0 = P_fun(P_{1});
    F_0 = F_fun(P_0);

    K_0_dynprog_{N} = F_0;
    A_K_dynprog_{N} = A+B*K_0_dynprog_{N};
    eig_dynprog_{N} = eig(A+B*K_0_dynprog_{N});
    max_eig_{N} = max(abs(eig_dynprog_{N}));
end

max_eig = [max_eig_{:}];

figName = 'pblm1e';
fig = figure(WindowStyle="normal");
hold on; grid on;
plot(1:N,max_eig)
yline(max(eig_lqr))
legend({'Dyn Prog', 'LQR'})
title('Stability Comparrision')
xlabel('Prediction Horrizon (N)')
ylabel('max(abs(\lamba))')
saveas(fig,[subfolder,filesep,'figs',filesep,figName],'png')
```

Warning: Error updating Text.

String scalar or character vector must have valid interpreter syntax:
max(abs(\lamba))



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