Path Planning for Autonomous Vehicles based on Nonlinear MPC with using a Kinematic Bicycle Model

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MECH 6V29 - MPC
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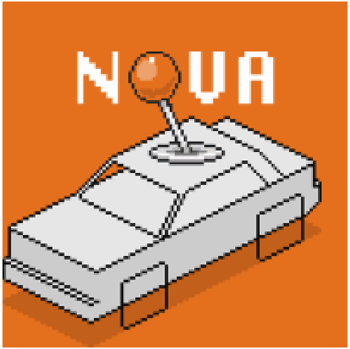
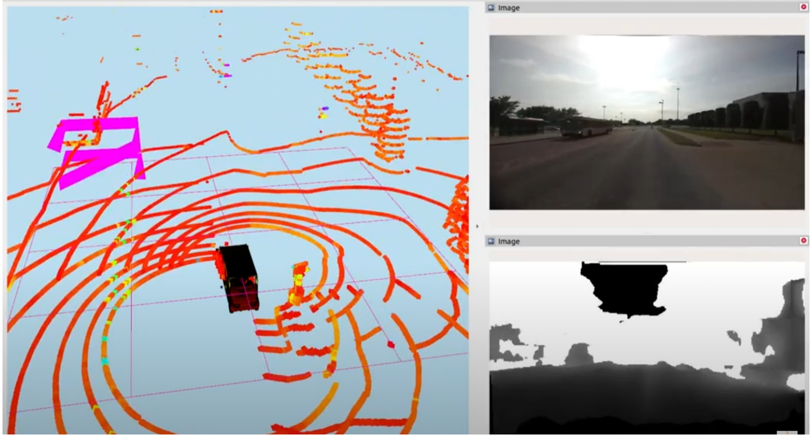
Outline

Outline

NOVA: Hail Bopp [1]

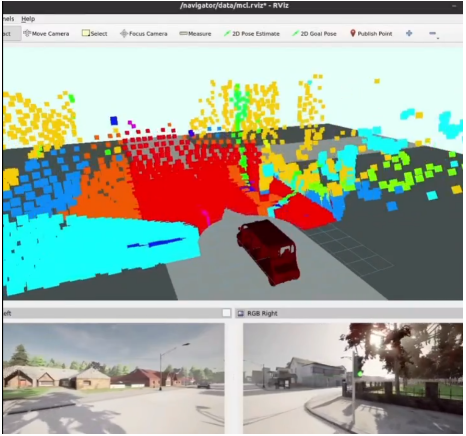


NOVA: Perception [1]



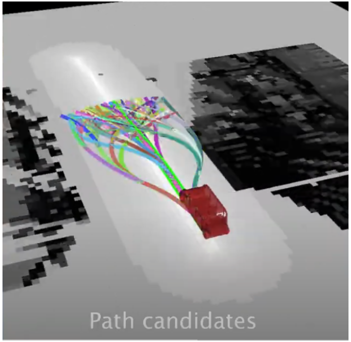
NOVA: Navigator [1]

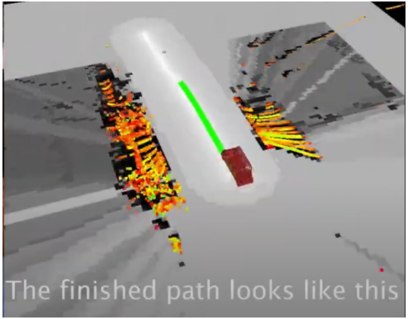
- Navigator is the self-driving software stack being developed by NOVA.
- Simulations done in then deployed to Hail Bopp.



Path Planning Objective [1]

Current Approach (random path generation and ranking)





Objective: Develop NL-MPC based path planning that is better than the current aproach. (Spoiler: NL-MPC is too complicated for this portion of Navigator's stack at this stage and many more efficient techniques exist. MPC will become more benificial when robustness/security gaurentees are required/desired).

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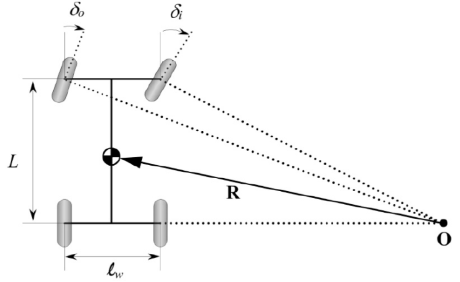
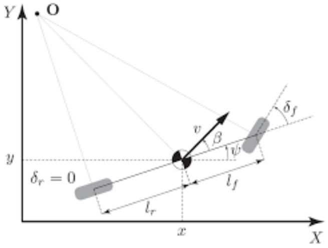
Vehicle Kinematic Models [2, 3]

Akerman Steering Model



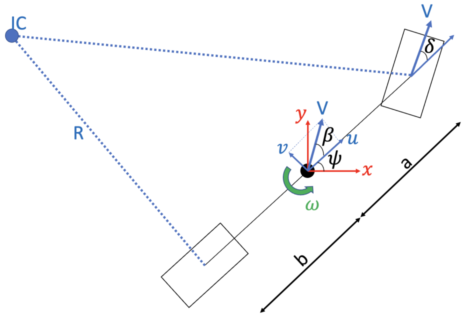
Bicycle Approximation



Akerman steering can be approximated as a bicycle model in most cases.

$$\delta \approx \frac{\delta_o + \delta_i}{2}$$

Kinematic Bicycle Model [4]



Simple nonlinear kinematics model equations:

$$\begin{cases} \dot{x} = V\cos(\psi + \beta) \\ \dot{y} = V\sin(\psi + \beta) \\ \dot{\psi} = \dfrac{V\cos(\beta)}{l_f + l_r}(\tan(\delta_f) - \tan(\delta_r)) \\ \dot{\theta} = \psi \end{cases} \tag{1}$$

where

$$\beta = \tan^{-1}\left(\frac{l_f\tan(\delta_r) + l_r\tan(\delta_f)}{l_f + l_r}\right) \tag{2}$$

$\delta_r = 0$, $l_f = a = 0.7[m]$, and $l_r = b = 0.7[m]$.

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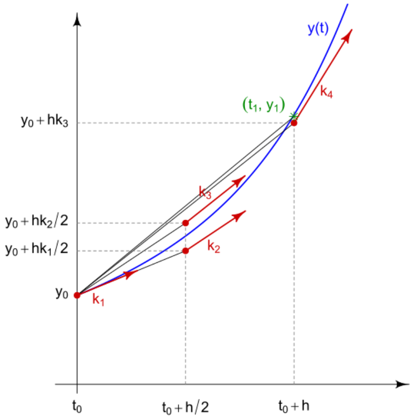
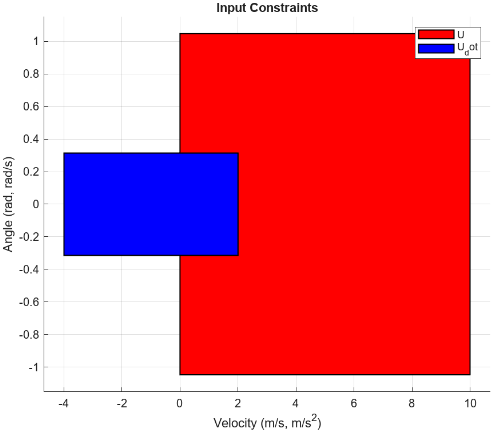
Model Discretization[5]

Discretized with $\Delta t = 1$ [s] using **RK4 method** (euler method doesn't work without very small Δt)



Input Constraints

$V \in [0, 10]$ [m/s]
$\theta \in \pm\pi/3$ [rad]
$\dot{V} \in [-4, 2]$ [m/s$\hat{2}$]
$\dot{\theta} \in \pm\pi/10$ [rad/s]



MPC Optimization Problem

Parameters:

- $N = 15$
- $\Delta t = 1$ [s]
- Multiple $q(x_k, u_k)$ and $p(x_n)$ tested
- $h(x_k, u_k) \mid u_k \in \mathbf{U} \wedge \dot{u}_k \in \dot{\mathbf{U}}$
- (for closed-loop sim: $T = 10$ [s])

$$J_0^*(x_0) = \min_{U_0} \sum_{k=0}^{N-1} q(x_k, u_k) + p(x_N)$$

$$\text{s.t.}$$

$$x_{k+1} = f(x_k, u_k),\ k \in \{0, 1, \ldots, N-1\}$$

$$h(x_k, u_k) \leq 0, \quad k \in \{0, 1, \ldots, N-1\}$$

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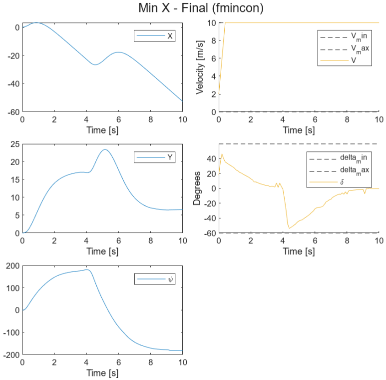
fmincon vs ipopt (Computation Time)

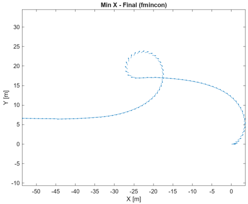
The results and computation time are GREATLY affected by the selection of the nonlinear optimization solver used.
The following is a simple comparison of computation time (just for demonstration... not a controlled case study)

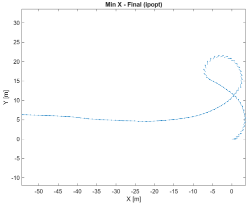
	FMINCON	**IPOPT**
Min Y - Unstructured	243.79	164.75
Min Y - Final	235.91	155.6
Min Y - Ref X and theta	438.67	71.79
Min X - Final	**1077.6**	**164.58**
Min X with +180	395.71	191.69
Min X with special ref	**344.52**	**165.16**
Max Y - Final	**331.71**	**140.49**
Max Y - Ref X theta	**665.82**	**85.492**

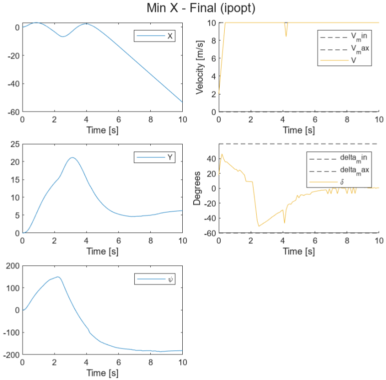
fmincon vs ipopt (Min X w/ Final - Both Fail)

$$J = x_{N+1}$$



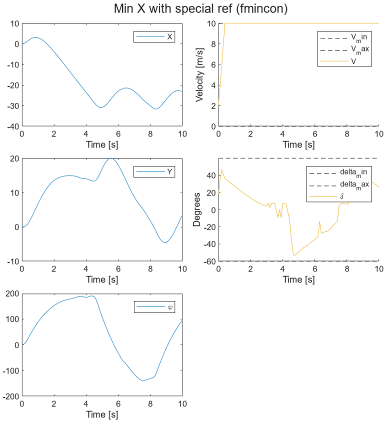


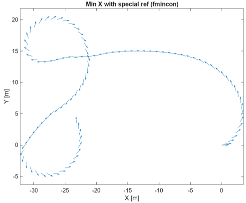


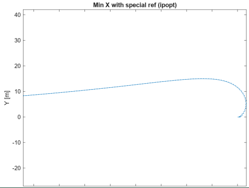


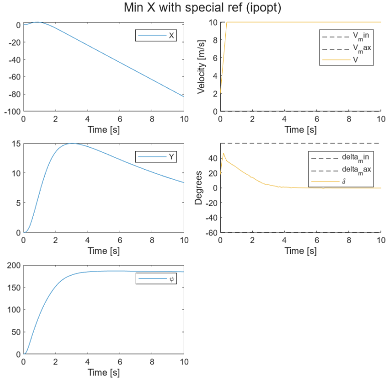
fmincon vs ipopt (Min X w/ special ref - fmincon fails)

$$J_k = x_k + (y_k - 0)^2/100 + (\theta_k - \pi)^2$$



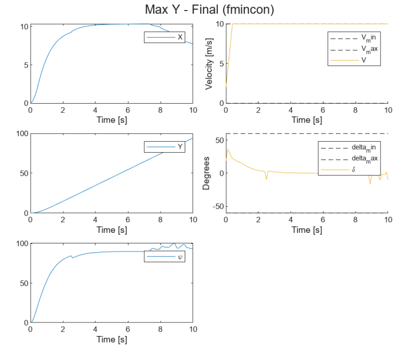


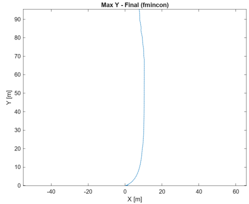


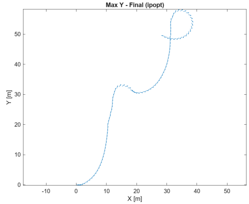


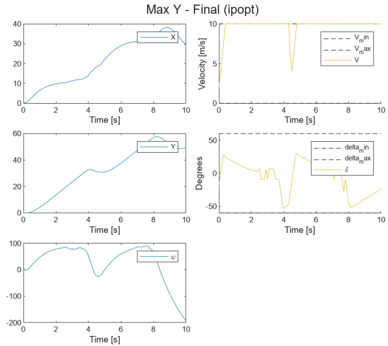
fmincon vs ipopt (Max Y w/ Final - ipopt fails)

$$J = -y_{N+1}$$



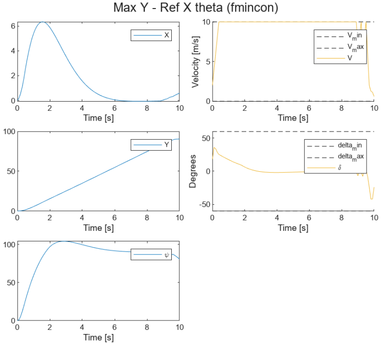


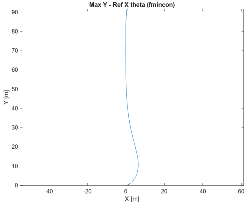


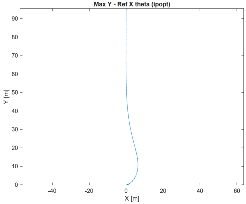


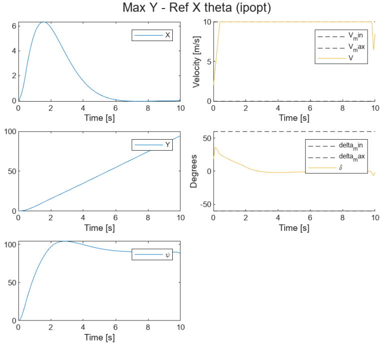
fmincon vs ipopt (Max Y with refs)

$$J_k = -y_k + (x_k - 0)^2 + (\theta_k - \pi/2)^2$$









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