Assignment 4

(SYSM 6v80.001 / MECH 6v29.001 – Multiagent Robotic Systems)

- The submission deadline is **06 April 2022 (Wednesday)** 5:00 PM (CT).

Problem 1 — 10 points

State a summary of ***Notes 13–15***, (which include the topics of persistence, combinatorial coverage and graph grammars) preferably by creating a concept map diagram (flow diagram). The whole purpose is to make sure that we are clear about the ***bigger picture***, and reiterate why are we doing and discussing the specific topics in the class. Do not merely write the topics, instead create connections between topics to clarify the flow of information.

Problem 2 — 10 points

We studied a way to assign edge directions in a minimally rigid graph to make them constraint consistent, and hence, minimally persistent. Assign orientations to edges in the following graphs to make them *persistent*. Show all the steps of the Henneberg construction.

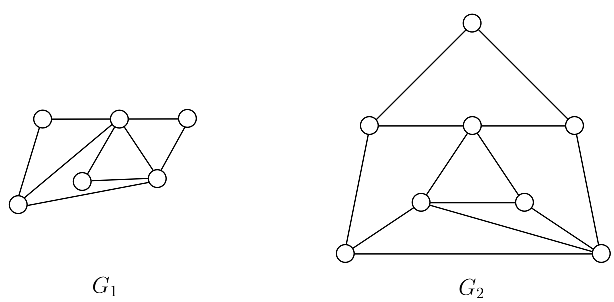

Figure 1: Minimally rigid graphs for Problem 2.

Problem 3 — 10 points

We have discussed that for a directed graph G_d to be *minimally persistent*, it must satisfy the following two conditions: (a) The corresponding undirected graph G must be *minimally rigid*, and (b) the direction of edges must make G_d *constraint consistent*. Moreover, We have seen how to achieve minimally rigid (undirected) graphs using *Henneberg construction rules* and then assign them directions to make them constraint consistent. Now, your job is to solve the following problem (for the $d = 2$-dimensional case).

For any odd integer N, provide an algorithm/method to construct directed graphs that are

1. *minimally persistent, and*
2. *balanced at the same time.*

Please also illustrate your method through example(s). (Recall that balanced graph is the one where in-degree = out-degree for every node.)

Problem 4 — 10 points

Draw a Gabriel graph induced by the following set of 14 points. The coordinates of points are:

$$P = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 9 & 7 & 14 & 9 & 12 & 6 & 6 & 12 & 1 & 1 & 2 & 5 & 12 & 12 \end{bmatrix}$$

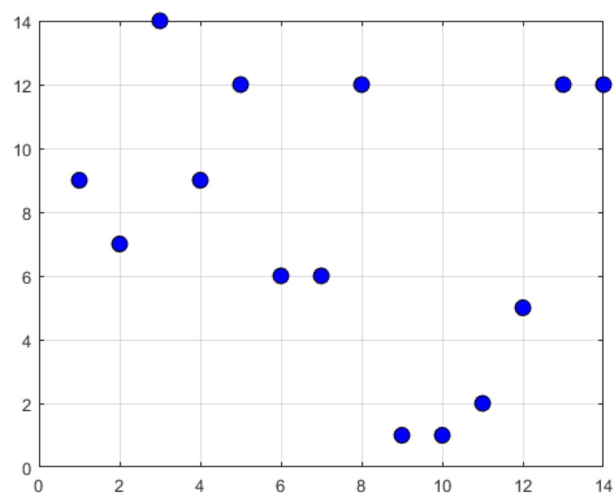

Figure 2:

Problem 5 — 15 points

Part a: Show that Gabriel graphs are planar.

Part b: Show that Gabriel graphs are connected.

Problem 6 — 10 points

Graph grammars are a good tool for generating networks with some specific structure. Assume that we have a total of 2^n nodes for some positive integer n. Consider the following rules:

$$R_0: \quad a \qquad a \longrightarrow \ell_1 \_\_ c$$

$$R_1: \quad \ell_i \qquad \ell_i \longrightarrow \ell_{i+1} \_\_ c\,; \qquad 1 \leq i \leq n-1$$

What structure do we get at the end? (Later in the course, we will see that the graph obtained by such graph grammars are completely controllable with only a single leader. So, we can also use graph grammars to generate controllable networks).

Problem 7 — 15 points

Consider the three graph grammars in Figure 3. Assume that initially all agents are labelled α. What is the final product expected after applying these graph grammars.

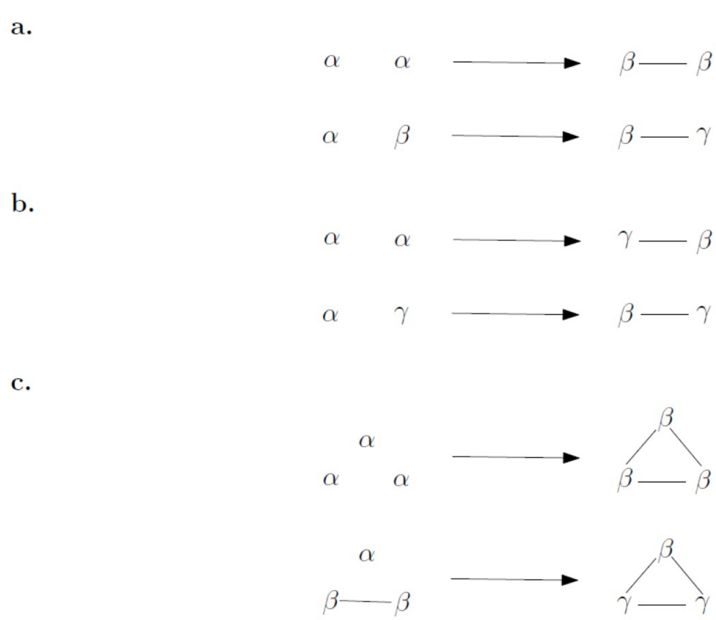

Figure 3: Graph grammars for Problem 7.