Assignment 3

(SYSM 6v80.001 / MECH 6v29.001 – Multiagent Robotic Systems)

Problem 2:

Checking the rank of the rigidity matrix gives that

$$\text{rank}(R(q)) = 8 < 2N - 3 = 9$$

and hence graph **a** is not rigid.

Simiarly, in both the case **b** and **c**

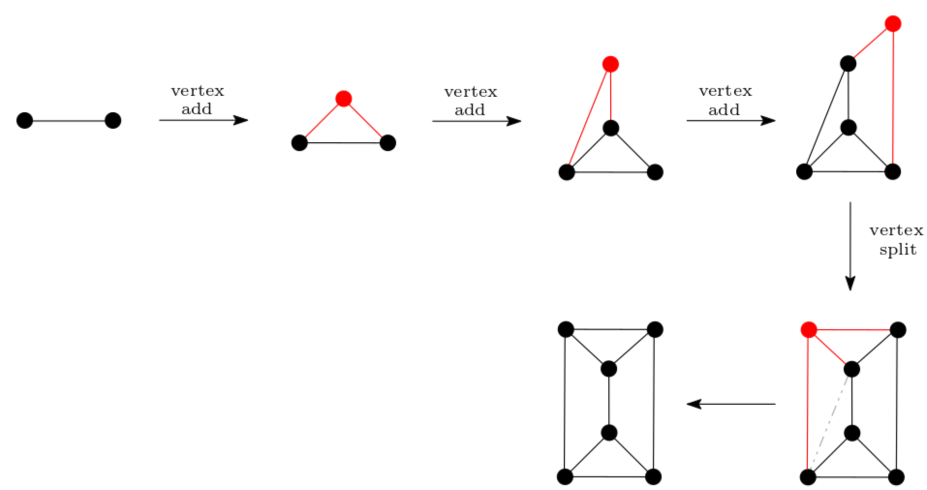
$$\text{rank}(R(q)) = 9 = 2N - 3$$

and hence both of those graphs are rigid.

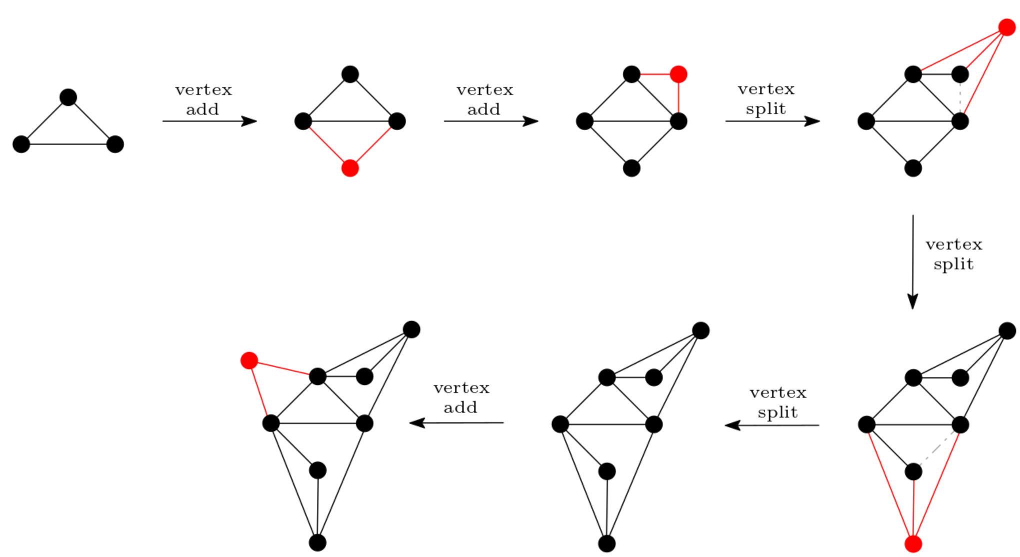
Problem 3:

(A) See notes 12_3 (vertex addition and vertex split).

(B) See the sequence below.



(C) See the sequence below.



Problem 4:

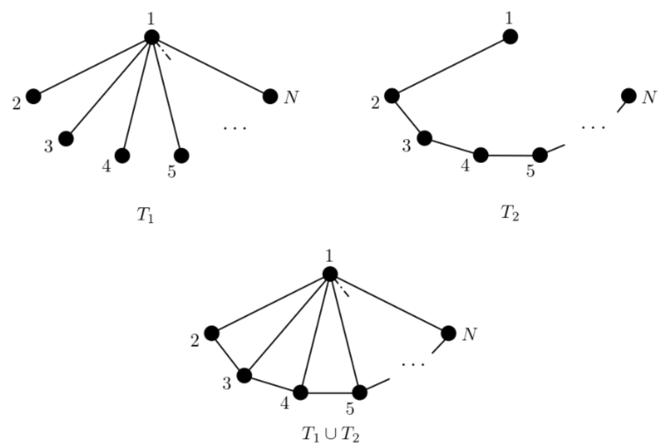
A tree with N nodes has N-1 edges. So, if we take a union of trees that have the same vertex set, the number of edges in that union can be at most 2N – 2. Note that for a minimally rigid graph, we need 2N-3 edges. Thus, we satisfy at least “enough edges” requirement.

Now, consider the following construction.

T_1 is a star graph with node 1 as the central node and (N-1) nodes as leaves.

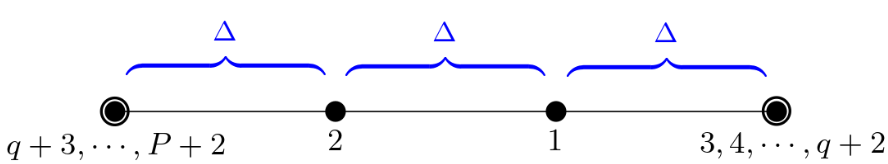
T_2 is a path graph with node 1 as one of the end nodes.

If we take their union, as illustrated in the below figure, we get a triangulated graph with exactly 2N-3 edges. It can be verified that such a graph is indeed minimally rigid.



Problem 5:

(A) Assuming (without loss of generality) that the agents stay on a line, we can place two agents Δ apart, i.e., such that $x_2 - x_1 = \Delta$. Now, place the remaining agents on the line. It is clear that they should pull x_2 away from x_1 as much as possible. As such, place q of the remaining P agents Δ away from x1 and the remaining (P – q) agents Δ away from x_2.



$$\dot{x_1}(t) = -\big(x_1(t) - x_2(t)\big) - \sum_{i=3}^{q+2} (x_1(t) - x_i(t))$$

$$= -\Delta - (-q\Delta) = -\Delta + q\Delta$$

Similarly,

$$\dot{x_2}(t) = -\big(x_2(t) - x_1(t)\big) - \sum_{i=q+3}^{P+2} (x_2(t) - x_i(t))$$

$$= \Delta - (P-q)\Delta.$$

Now, we will compute $x_1(t+\delta t)$,

$$x_1(t+\delta t) \approx x_1(t) + \delta t\, \dot{x}_1(t)$$

$$= x_1(t) + (-\Delta + q\Delta)\delta t.$$

And $x_2(t+\delta t)$.

$$x_2(t+\delta t) \approx x_2(t) + \delta t\, \dot{x_2}(t)$$

$$= x_2(t) + (\Delta - (P-q)\Delta)\delta t.$$

Now,

$$x_1(t+\delta t) - x_2(t+\delta t) = x_1(t) - x_2(t) + (-1 + q - 1 + (P-q))\Delta\delta t$$

$$= x_1 - x_2(t) + (P-2)\Delta\delta t = \Delta + (P-2)\Delta\delta t$$

Hence, it does not matter where we place the remaining agents (q can be anything between 0 and P) as long as they are Δ away from x_1 or x_2. Hence, the configuration is indeed optimal albeit not a unique optimum.

(B) Since the established worst-case scenario does not render $\dot{d}$ positive if P ≤ 2, the network does not break in this case. As such, we have shown that if the network has at most 4 agents, if it starts connected, it stays connected.

Problem 6:

(A) Use weights! For example, our old friend …

$$\dot{x}_i = -\sum_{j\in N(i)} \frac{2\Delta - \|x_i - x_j\|}{(\Delta - \|x_i - x_j\|)^2}(x_i - x_j)$$

will do the trick.

(B) Here we need similar weights that a) go to infinity when the distance approaches Δ and that goes to negative infinity (or at least something large) as the distance goes to zero.