MECH 6V29 - Homework 2

Author: Jonas Wagner

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Problem 2

```
N = 3;
Delta = diag(sym('degV',[N,1],'real'))
```

Delta =

$$\begin{pmatrix} \mathrm{degV}_1 & 0 & 0 \\ 0 & \mathrm{degV}_2 & 0 \\ 0 & 0 & \mathrm{degV}_3 \end{pmatrix}$$

```
A = sym('a',[N,N],'real');
A = A - eye(N).*A
```

A =

$$\begin{pmatrix} 0 & a_{1,2} & a_{1,3} \\ a_{2,1} & 0 & a_{2,3} \\ a_{3,1} & a_{3,2} & 0 \end{pmatrix}$$

```
L = Delta - A
```

L =

$$\begin{pmatrix} \mathrm{degV}_1 & -a_{1,2} & -a_{1,3} \\ -a_{2,1} & \mathrm{degV}_2 & -a_{2,3} \\ -a_{3,1} & -a_{3,2} & \mathrm{degV}_3 \end{pmatrix}$$

```
L^2
```

ans =

$$\begin{pmatrix} \mathrm{degV}_1^2 + a_{1,2}\,a_{2,1} + a_{1,3}\,a_{3,1} & a_{1,3}\,a_{3,2} - a_{1,2}\,\mathrm{degV}_1 - a_{1,2}\,\mathrm{degV}_2 & a_{1,2}\,a_{2,3} - a_{1,3}\,\mathrm{degV}_1 - a_{1,3}\,\mathrm{degV}_3 \\ a_{2,3}\,a_{3,1} - a_{2,1}\,\mathrm{degV}_1 - a_{2,1}\,\mathrm{degV}_2 & \mathrm{degV}_2^2 + a_{1,2}\,a_{2,1} + a_{2,3}\,a_{3,2} & a_{1,3}\,a_{2,1} - a_{2,3}\,\mathrm{degV}_2 - a_{2,3}\,\mathrm{degV}_3 \\ a_{2,1}\,a_{3,2} - a_{3,1}\,\mathrm{degV}_1 - a_{3,1}\,\mathrm{degV}_3 & a_{1,2}\,a_{3,1} - a_{3,2}\,\mathrm{degV}_2 - a_{3,2}\,\mathrm{degV}_3 & \mathrm{degV}_3^2 + a_{1,3}\,a_{3,1} + a_{2,3}\,a_{3,2} \end{pmatrix}$$

```
Delta*Delta
```

ans =

$$\begin{pmatrix} \mathrm{degV}_1^2 & 0 & 0 \\ 0 & \mathrm{degV}_2^2 & 0 \\ 0 & 0 & \mathrm{degV}_3^2 \end{pmatrix}$$

```
Delta*A
```

ans =

$$\begin{pmatrix} 0 & a_{1,2}\,\mathrm{degV}_1 & a_{1,3}\,\mathrm{degV}_1 \\ a_{2,1}\,\mathrm{degV}_2 & 0 & a_{2,3}\,\mathrm{degV}_2 \\ a_{3,1}\,\mathrm{degV}_3 & a_{3,2}\,\mathrm{degV}_3 & 0 \end{pmatrix}$$

```
A*Delta
```

ans =

$$\begin{pmatrix} 0 & a_{1,2}\,\mathrm{degV}_2 & a_{1,3}\,\mathrm{degV}_3 \\ a_{2,1}\,\mathrm{degV}_1 & 0 & a_{2,3}\,\mathrm{degV}_3 \\ a_{3,1}\,\mathrm{degV}_1 & a_{3,2}\,\mathrm{degV}_2 & 0 \end{pmatrix}$$

```
A*A
```

ans =

$$\begin{pmatrix} a_{1,2}\,a_{2,1}+a_{1,3}\,a_{3,1} & a_{1,3}\,a_{3,2} & a_{1,2}\,a_{2,3} \\ a_{2,3}\,a_{3,1} & a_{1,2}\,a_{2,1}+a_{2,3}\,a_{3,2} & a_{1,3}\,a_{2,1} \\ a_{2,1}\,a_{3,2} & a_{1,2}\,a_{3,1} & a_{1,3}\,a_{3,1}+a_{2,3}\,a_{3,2} \end{pmatrix}$$

```
x_sym = sym("x",[N,1])
```

x_sym =

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

```
x_dot = -L^2 * x_sym
```

x_dot =

$$\begin{pmatrix} x_3\,(a_{1,3}\,\mathrm{degV}_1 - a_{1,2}\,a_{2,3} + a_{1,3}\,\mathrm{degV}_3) - x_1\,\left(\mathrm{degV}_1{}^2 + a_{1,2}\,a_{2,1} + a_{1,3}\,a_{3,1}\right) + x_2\,(a_{1,2}\,\mathrm{degV}_1 - a_{1,3}\,a_{3,2} + a_1 \\ x_3\,(a_{2,3}\,\mathrm{degV}_2 - a_{1,3}\,a_{2,1} + a_{2,3}\,\mathrm{degV}_3) - x_2\,\left(\mathrm{degV}_2{}^2 + a_{1,2}\,a_{2,1} + a_{2,3}\,a_{3,2}\right) + x_1\,(a_{2,1}\,\mathrm{degV}_1 - a_{2,3}\,a_{3,1} + a_2 \\ x_2\,(a_{3,2}\,\mathrm{degV}_2 - a_{1,2}\,a_{3,1} + a_{3,2}\,\mathrm{degV}_3) - x_3\,\left(\mathrm{degV}_3{}^2 + a_{1,3}\,a_{3,1} + a_{2,3}\,a_{3,2}\right) + x_1\,(a_{3,1}\,\mathrm{degV}_1 - a_{2,1}\,a_{3,2} + a_3 \end{pmatrix}$$

Problem 3

```
alpha = sym('alpha',[2,1]);
A = [-2*alpha(1), alpha(1);
     alpha(2), -2*alpha(2)]
```

A =

$$\begin{pmatrix} -2\,\alpha_1 & \alpha_1 \\ \alpha_2 & -2\,\alpha_2 \end{pmatrix}$$

```
B = diag(alpha)
```

B =

$$\begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix}$$

```
u_0 = [sym('beta'); sym('gamma')]
```

```
u_0 =
```

$$\begin{pmatrix} \beta \\ \gamma \end{pmatrix}$$

```
syms s
x_infty_sym = (s *eye(2) - A) \ B * u_0
```

```
x_infty_sym =
```

$$\begin{pmatrix} \frac{\alpha_1 \alpha_2 \gamma}{\sigma_1} + \frac{\alpha_1 \beta \, (2\alpha_2 + s)}{\sigma_1} \\ \frac{\alpha_1 \alpha_2 \beta}{\sigma_1} + \frac{\alpha_2 \gamma \, (2\alpha_1 + s)}{\sigma_1} \end{pmatrix}$$

where

$$\sigma_1 = 3\,\alpha_1\,\alpha_2 + 2\,\alpha_1\,s + 2\,\alpha_2\,s + s^2$$

```
x_infty = subs(x_infty_sym,s,0)
```

```
x_infty =
```

$$\begin{pmatrix} \frac{2\beta}{3} + \frac{\gamma}{3} \\ \frac{\beta}{3} + \frac{2\gamma}{3} \end{pmatrix}$$

Problem 6

```
% G_1
D1 = [
    -1   0   1   0   0   0   0
     1  -1   0   0   0   0   0
     0   1  -1  -1   0   0   0
     0   0   0   1   1   1   0
     0   0   0   0   0  -1   1
     0   0   0   0  -1   0  -1
];
N1 = size(D1,1);
M1 = size(D1,2);

A1 = zeros(N1);
for i = 1:M1
    idx_out = find(D1(:,i)==-1,1);
    idx_in = find(D1(:,i)==1,1);
    A1(idx_out,idx_in) = 1;
end
```

```
A1
```

```
A1 = 6×6
     0     1     0     0     0     0
     0     0     1     0     0     0
     1     0     0     1     0     0
     0     0     0     0     0     0
     0     0     0     1     0     0
     0     0     0     1     1     0
```

```
DeltaIn1 = diag(sum(D1==-1,2))
```

```
DeltaIn1 = 6×6
     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     2     0     0     0
     0     0     0     0     0     0
     0     0     0     0     1     0
     0     0     0     0     0     2
```

```
DeltaOut1 = diag(sum(D1==1,2));
L1 = DeltaIn1 - A1
```

```
L1 = 6×6
     1    -1     0     0     0     0
     0     1    -1     0     0     0
    -1     0     2    -1     0     0
     0     0     0     0     0     0
     0     0     0    -1     1     0
     0     0     0    -1    -1     2
```

```
L_rank_1 = rank(L1)
```

```
L_rank_1 = 5
```

```
L_balenced_1 = sum(L1,1)
```

```
L_balenced_1 = 1×6
     0     0     1    -3     0     2
```

```
% G_2
D2 = [
    -1   0   1   0   0   0   0
     1  -1   0   0   0   0   0
     0   1  -1  -1   0   0   0
     0   0   0   1   1  -1   0
     0   0   0   0   0   1   1
     0   0   0   0  -1   0  -1
];
N2 = size(D2,1);
M2 = size(D2,2);

A2 = zeros(N2);
for i = 1:M2
    idx_out = find(D2(:,i)==-1,1);
    idx_in = find(D2(:,i)==1,1);
    A2(idx_out,idx_in) = 1;
end
A2
```

```
A2 = 6×6
     0     1     0     0     0     0
     0     0     1     0     0     0
     1     0     0     1     0     0
     0     0     0     0     1     0
     0     0     0     0     0     0
     0     0     0     1     1     0
```

```
DeltaIn2 = diag(sum(D2==-1,2))
```

```
DeltaIn2 = 6×6
     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     2     0     0     0
     0     0     0     1     0     0
     0     0     0     0     0     0
     0     0     0     0     0     2
```

```
DeltaOut2 = diag(sum(D2==1,2));
L2 = DeltaIn2 - A2
```

```
L2 = 6×6
     1    -1     0     0     0     0
     0     1    -1     0     0     0
    -1     0     2    -1     0     0
     0     0     0     1    -1     0
     0     0     0     0     0     0
     0     0     0    -1    -1     2
```

```
L_rank_2 = rank(L2)
```

```
L_rank_2 = 5
```

```
L_balenced_2 = sum(L2,1)
```

```
L_balenced_2 = 1×6
     0     0     1    -1    -2     2
```

```
% G_3
D3 = [
    -1   0   1   0   0   0   0   0   0   0
     1  -1   0   0   0   0   0   1  -1   0
     0   1  -1  -1   0   0   0   0   0   1
     0   0   0   1   1  -1   0  -1   0   0
     0   0   0   0   0   1  -1   0   0   0
     0   0   0   0  -1   0   1   0   1  -1
];
N3 = size(D3,1);
M3 = size(D3,2);

A3 = zeros(N3);
for i = 1:M3
    idx_out = find(D3(:,i)==-1,1);
    idx_in = find(D3(:,i)==1,1);
    A3(idx_out,idx_in) = 1;
end
A3
```

```
A3 = 6×6
```

```
     0     1     0     0     0     0
     0     0     1     0     0     1
     1     0     0     1     0     0
     0     1     0     0     1     0
     0     0     0     0     0     1
     0     0     1     1     0     0
```

```
DeltaIn3 = diag(sum(D3==-1,2))
```

```
DeltaIn3 = 6×6
     1     0     0     0     0     0
     0     2     0     0     0     0
     0     0     2     0     0     0
     0     0     0     2     0     0
     0     0     0     0     1     0
     0     0     0     0     0     2
```

```
DeltaOut3 = diag(sum(D3==1,2));
L3 = DeltaIn3 - A3
```

```
L3 = 6×6
     1    -1     0     0     0     0
     0     2    -1     0     0    -1
    -1     0     2    -1     0     0
     0    -1     0     2    -1     0
     0     0     0     0     1    -1
     0     0    -1    -1     0     2
```

```
L_rank_3 = rank(L3)
```

```
L_rank_3 = 5
```

```
L_balenced_3 = sum(L3,1)
```

```
L_balenced_3 = 1×6
     0     0     0     0     0     0
```