

## Assignment 3

(SYSM 6v80.001 / MECH 6v29.001 – Multiagent Robotic Systems)

- The submission deadline is **March 22** (Tuesday) 5:00pm (central).

### Problem 1:

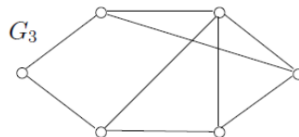
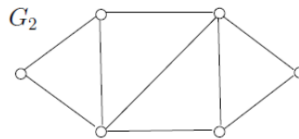
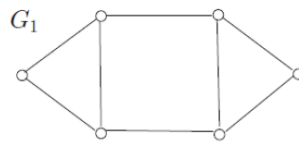
(10 points)

State a summary of Notes 11, 12 and 13 in very brief words, or by creating a concept map (flow diagram). The whole purpose is to make sure that we are clear about the bigger picture and reiterate why we are doing and discussing the specific topics in the class. Do not merely write the topics, instead create connections between topics to clarify the information flow.

### Problem 2:

(5 + 5 + 5 points)

Consider a two-dimensional plane ( $d = 2$ ). Which of the following graphs are (generically) rigid? Use the rigidity test (based on the rank of the rigidity matrix) to support your answer.



### Problem 3:

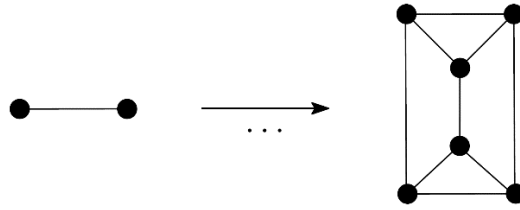
(5 + 5 + 5 points)

We can “grow” a minimally rigid graph by adding nodes one by one using Henneberg construction.

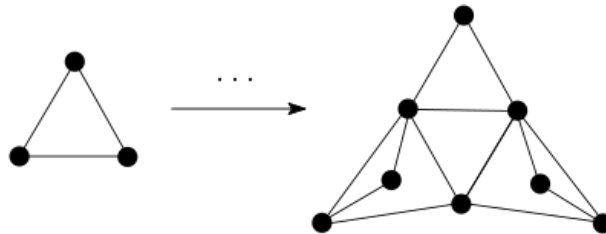
**(A)** Briefly state the two rules of Henneberg construction (vertex addition and vertex split) by consulting pages 16 and 17 of Notes 12\_3 uploaded at the elearning page. You can also consult page 6 of the reference [1] (which is also uploaded at the elearning course page).

[1] B. Anderson, et al. "Rigid graph control architectures for autonomous formations." *IEEE Control Systems Magazine* 28.6 (2008): 48-63.

**(B)** Starting with the seed graph on the left, obtain the minimally rigid graph on the right by applying the Henneberg construction rules. Clearly state the rule used in each step.



(C) Repeat the same problem (as in part (B)) for the following.



#### Problem 4:

(10 points)

Let's consider two trees  $T_1 = (V, E_1)$  and  $T_2 = (V, E_2)$  that have the same vertex sets containing  $N$  vertices, but possibly different edge sets. Is it possible that the union of two such trees result in a minimally rigid graph? If yes, explain and construct an example. If not, explain why not.

#### Problem 5:

(15 points)

In class we saw that if we place a total of  $(P + 2)$  agents on a line, with  $P$  agents at location 0, one agent at location  $\Delta$ , and one agent at location  $2\Delta$ , then if the network is a  $\Delta$ -disk proximity graph it will get disconnected under the linear consensus equation if  $P > 2$ .

(A) Show that this particular configuration is indeed the worst-case scenario from a connectivity point-of-view.

(B) Is the following theorem valid: A  $\Delta$ -disk proximity graph network of 4 agents will always stay connected under the consensus equation if it starts connected.

#### Problem 6:

(15 points)

On elearning course page (assignment 3 folder), four Matlab files have been added. The files are

- hw3.m,
- plotsol.m,
- loadnetwork.m, and
- disk.m

Download these four files and open up hw3.m. By running hw3.m you will see the system attempt a rendezvous using three different initial network configurations. Your job is to try and modify the consensus equation in some clever way so that the following objectives are met.

**(A)** Make sure that the network stays connected for all three initial networks. (I am not looking for a proof here – just try and be clever and produce some appropriate control law that works in simulation.)

**(B)** Make sure that the agents do not run into each other, i.e., that collisions are avoided, at the same time as the network remains connected. Again, I do not want a proof I want the simulations to work.