Assignment 5

(SYSM 6v80.001 / MECH 6v29.001 – Multiagent Robotic Systems)

- The submission deadline is **20 April 2022 (Wednesday)** 5:00 PM (CT).

Problem 1 — 10 points

State a summary of ***Notes 16.1–17.4***, (which include the topics of lloyd's algorithm, network controllability (upper and lower bounds)) preferably by creating a concept map diagram (flow diagram). The whole purpose is to make sure that we are clear about the ***bigger picture***, and reiterate why are we doing and discussing the specific topics in the class. Do not merely write the topics, instead create connections between topics to clarify the flow of information.

Problem 2 — 10 points

Consider the graph in Figure 1, and the following statement:

"Three leader nodes are sufficient to make the graph completely controllable."

Using the *zero forcing method* (Notes 17.3), discuss if the statement is true or false? If it is true, find a set of three leader nodes making the network completely controllable and justify your choice. If the statement is false, discuss why?

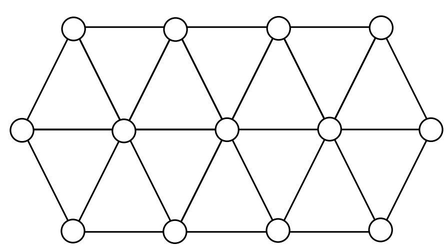

Figure 1: Caption

Problem 3 — 20 points

(Each part has 5 pints.)

(A) Consider the graph in Figure 2. Let $V_L = \{v_4, v_5\}$ be a leader set. Using *zero forcing* method, what is the minimum rank of the controllability matrix?

(B) Consider a graph with unity edge weights and $V_L \subset V$ be a zero forcing set of the graph. If we change the edge weights from unity to arbitrary positive values, what would be the rank of the controllability matrix with the same set of leaders V_L? Please explain your answer.

(C) Can you find a set of three leaders through which the graph in Figure 2 becomes completely controllable? If yes, which three nodes can be leaders?

(D) This part is about finding the *worst* leaders from the perspective of zero forcing method. Can you identify two different sets of leader nodes, each of which containing four nodes, such that the zero forcing based bound on the rank of the controllability matrix is four with each of those leader sets.

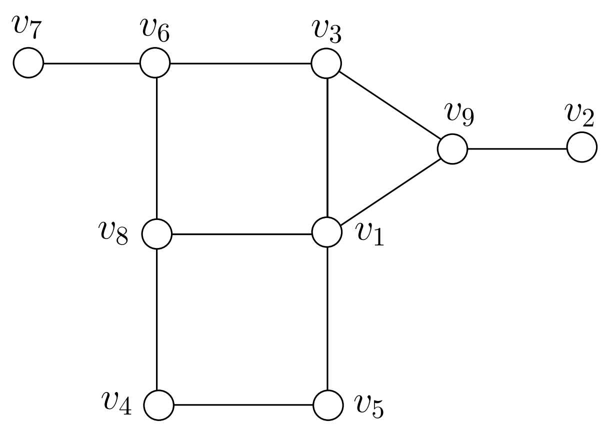

Figure 2: Graph for Problem 3.

Problem 4 — 10 points

In Problem 3(A), you computed the zero forcing based controllability bound for $V_L = \{v_4, v_5\}$. Now compute the *distance-based* bound on the rank of the controllability matrix (as we discussed in the class and also explained in Notes 17.4). Which one is a better bound here?

Problem 5 — 10 points

(Each part has 5 points.)

Consider the graph in Figure 3, in which **only black** node is a leader node.

Part - A How many *external equitable partitions (EEP)* does the graph have? Which of these EEPs is the maximal leader-invariant (LIEEP), and based on that comment if the system is completely controllable or not.

Part - B Now consider that **both black and gray nodes** are leaders, and repeat the same as in Part A.

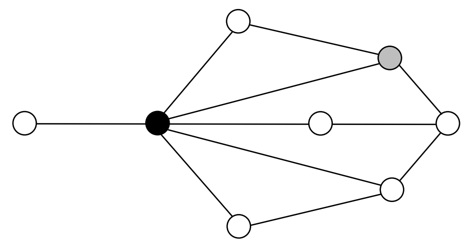

Figure 3: A leader-follower network in Problem 2.

Problem 6 — 10 points

Let G be a leader-follower network with a single leader ℓ_G. Similarly, H is another leader follower network with a single leader ℓ_H and n follower nodes. Now we obtain a new graph M by connecting H and n copies of G as follows: Connect a copy of G to H by replacing a follower node i in H by the leader node ℓ_G in G. See Figure 4 for the example.

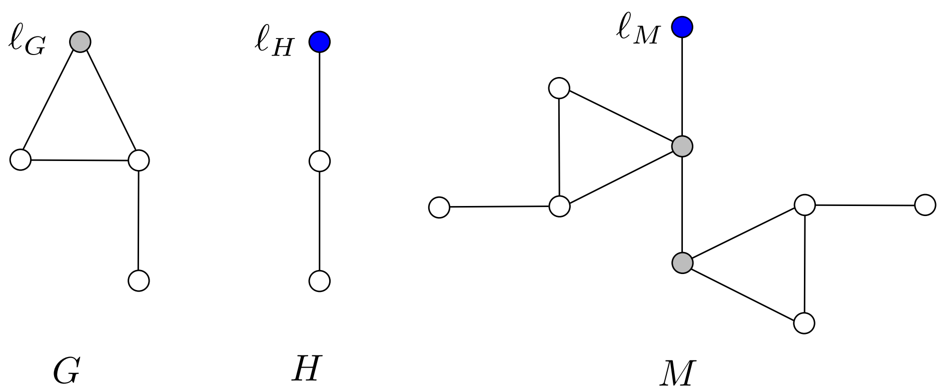

Figure 4: Construction of M in Problem 3.

Either prove or disprove (by giving a counter example) the following statement:
"***If*** G ***and*** H ***has trivial maximal LIEEPs, then*** M ***also has a trivial maximal LIEEP***"

(Recall that trivial maximal LIEEP means that each node in the graph is in a singleton cell.)