Assignment 2

(SYSM 6v80.001 / MECH 6v29.001 – Multiagent Robotic Systems)

- The submission deadline is **01 March 2022 (Tuesday)** 5:00 PM (CT).
- Each problem has **10 points**.

Problem 1

State a summary of ***lectures 6–10***, preferably by creating a concept map diagram (flow diagram). The whole purpose is to make sure that we are clear about the ***bigger picture***, and reiterate why are we doing and discussing the specific topics in the class. Do not merely write the topics, instead create connections between topics to clarify the flow of information.

Problem 2

Recall that if x_i is scalar, with its derivative given by the consensus equation

$$\dot{x}_i = \sum_{j \in \mathcal{N}_i} (x_j - x_i), \quad i = 1, 2, \cdots, N$$

this can be written as

$$\dot{x} = -Lx$$

where L is the Laplacian of the undirected graph, and $x = [x_1 \; x_2 \; \cdots \; x_N]^T$.
Part-A If instead

$$\dot{x} = -L^2 x$$

What are the corresponding node level dynamics, that is, find

$$\dot{x}_i = ???$$

Part-B Can you give a graph-theoretic interpretation to your answer in (A)?

Problem 3

Consider a leader-follower network with two leaders and two followers, as shown in the Figure 1. Assume that leaders and followers are on the real line, and the underlying network graph is a path graph with the end nodes being the static leaders. Moreover, let the dynamics be given by the following:

$$\dot{x}_1 = \alpha_1 \left((x_3 - x_1) + (x_2 - x_1) \right)$$

$$\dot{x}_2 = \alpha_2 \left((x_1 - x_2) + (x_4 - x_2) \right)$$

$$\dot{x}_3 = \dot{x}_4 = 0.$$

Where do x_1 and x_2 end up as $t \to \infty$ if $x_3 = \beta$ and $x_4 = \gamma$?

Figure 1: Leader-follower network.

Problem 4

Given an undirected network containing a total of N nodes. There is a single anchor node that is connected to every one of the follower nodes, that is, anchor has a degree of $N-1$. Find simple expression for the following quantities.

1. ℓ,
2. $L_f \mathbf{1}$,

where L_f is the matrix obtained from the partition of Laplacian matrix as we discussed in the class.

Also, relate your answers to where followers end up as $t \leftarrow \infty$.

Problem 5

Consider the (undirected) network in Figure 2. At any instance of time, exactly one of the nodes x and y would be included in the network. So, basically, we will get a switched system. Assuming all nodes implement the consensus dynamics, will they converge at one point (centroid of initial states) asymptotically? Explain.

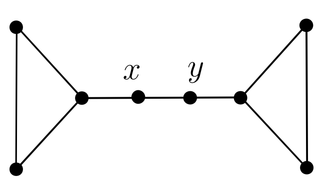

Figure 2:

Problem 6

Assume we are running the (directed) consensus protocol over the three static graphs below. In which of these cases does the protocol drive the state to span$\{\mathbf{1}\}$? When does it drive the state to

$$\frac{1}{N}\mathbf{1}\mathbf{1}^T x(0),$$

i.e. to the initial centroid? (Explain the reasons for your answer - don't just state the answer.)

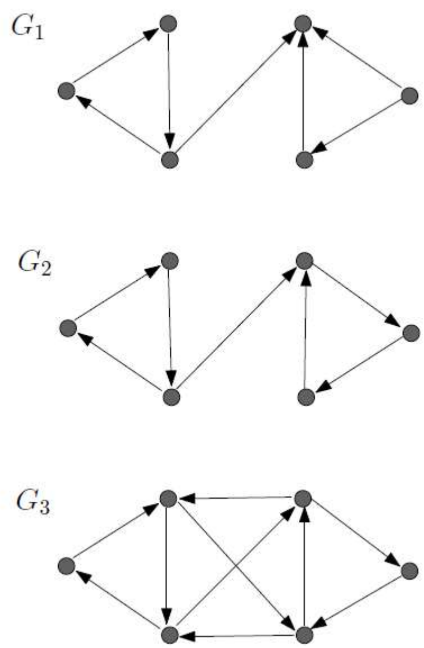

Figure 3:

Problem 7

Show that a graph that is weakly connected and balanced is also strongly connected. (I do not expect a formal proof. Just discuss your main argument.)