Assignment 1

(SYSM 6v80.001 / MECH 6v29.001 – Multiagent Robotic Systems)

- The submission deadline is **10 February 2022 (Friday)** 5:00 PM (CT).
- Please either upload your solution (in the form of a pdf file) at the elearning web page, or give a hard copy to the instructor (preferred option).
- Each problem has **10 points**.

Problem 1

State a summary of first **five** lectures, preferably by creating a concept map diagram (flow diagram). The whole purpose is to make sure that we are clear about the ***bigger picture***, and reiterate why are we doing and discussing the specific topics in the class. Do not merely write the topics, instead create connections between topics to clarify the flow of information.

Problem 2

Go to the following youtube link, and watch a TED talk by Dr. Magnus Egerstedt titled ***"Swarm robotics – From local rules to global behaviors"***.

https://www.youtube.com/watch?v=ULKyXnQ9xWA

Then, write down a one paragraph summary, just highlighting the main point or the 'take-away' message of the talk. Be brief and to the point.

Problem 3

In class we saw that an undirected graph G is connected if and only if its Laplacian's second smallest eigen value, λ_2 is non-zero. Using a similar argument as the one in class, show that the number of connected components (i.e. connected subgraphs that are disconnected from each other) is equal to the number of zero eigen values of the Laplacian.

Problem 4

Let the subspace S be

$$S = \text{span}\{\mathbf{1}\}^{\perp},$$

i.e.,

$$x \in S \Leftrightarrow x^T\mathbf{1} = 0$$

Show that S is $L-$invariant, i.e., $LS \subseteq S$ (i.e., $Lx \in S$, $\forall\ x \in S$), where L is the Laplacian of an undirected, connected graph.

Problem 5

$K_{1,6}$ is a star graph with one central node and six leaf nodes as shown in Fig. 1.

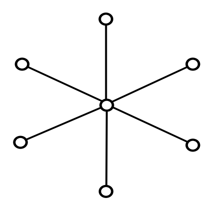

Figure 1: Star graph, $K_{1,6}$.

Your task is to show that $K_{1,6}$ can never be an induced subgraph of a Δ-disk proximity graph.

Problem 6

If $l_{i,j}$ is the shortest path distance (number of edges one needs to follow) between vertices v_i and v_j, the diameter of the graph is defined as

$$\text{diam}(G) = \max_{v_i, v_j \in V} l_{i,j}$$

Similarly, if we let l_i^* (known as the eccentricity of vertex v_i) be the longest distance to any vertex from the vertex v_i, i.e.,

$$l_i^* = \max_{v_j \in V} l_{i,j}$$

then the radius of a graph is defined as

$$\text{radius}(G) = \min_{v_i \in V} l_i^*$$

Find the radius and diameter of the following graphs.

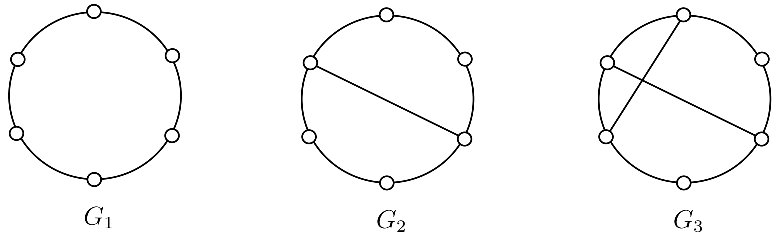

Figure 2:

Problem 7

Following are some undirected networks on four nodes with the same initial positions. In which of these networks, nodes converge fastest under the distributed consensus dynamics? Explain your answer.

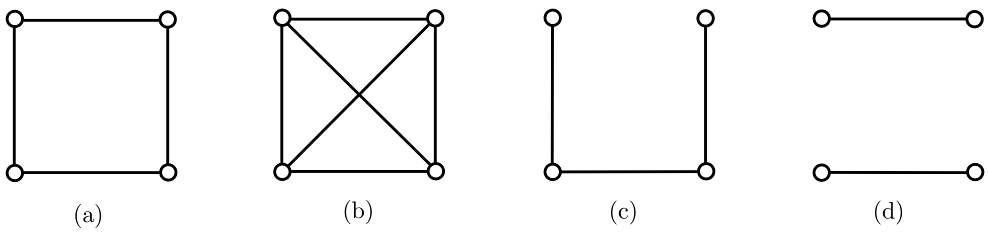

Figure 3:

Problem 8

What is the *necessary and sufficient* condition for the consensus to happen in the case of static directed networks? Derive this condition.