Models of formation

preferential attachment

Barabasi-Albert

Local-Attachment

Vertex-copying

Small World

Regular networks

SYSM 6302

CLASS 11

Barabasi-Albert Model $BA(n,q)$

Initialization: start with a clique of q nodes $\left(\frac{q(q-1)}{2} \text{ edges}\right)$

At each "time" step: add a new node v_t with q connections made to existing nodes.

Edge between v_t and v_i ($i<t$) with probability $\dfrac{k_i}{\sum_{j=1}^{t-1} k_j}$

a node with 3 times the degree of another is 3 times more likely to receive an edge

$$n = q + t$$
$$m = \frac{q(q-1)}{2} + qt$$
$$c = \frac{2m}{n} \sim 2q \quad \text{(for large enough } t\text{)}$$

Preferential Attachment

"the rich get richer"

$\Rightarrow$ Preferential attachment is a model of network formation that leads to scale-free degree behavior.

$$P_k \sim k^{-\alpha}, \quad \alpha = 3$$

BA properties

Avg. shortest path: $\ell = \dfrac{\ln(n)}{\ln(\ln(n))}$

(slightly) shorter average shortest path than ER model

Clustering Coefficient: $C = \dfrac{[\ln(n)]^2}{n}$

decreases with $n \to \infty$

Directed version: typically $\underset{\text{new}}{\bullet} \longrightarrow \underset{\text{old}}{\bullet}$

Extensions: nonlinear preferential attachment
preferential attachment based not on degree

Local Attachment

$LA(n, q, q_r)$, $q_r \leq q$

Inherently directed

Initialization: Start with a clique of $q+1$ nodes

At each "time" step: ① Add q_r edges randomly with uniform probability

② Add $q - q_r$ edges randomly (uniformly) to the outbound neighbors of the nodes connected to in ①

$p_k \sim k^{-r-2}$, $r = q_r / (q - q_r)$ $\Rightarrow$ Local attachment is also a mechanism that leads to scale free degree distributions

$\rightarrow$ Can exhibit high clustering

$\rightarrow$ diameter $d \sim \dfrac{\ln(n)}{\ln(\ln(n))}$

Vertex-Copying (Duplication-Divergence Model) $DD(n,p)$

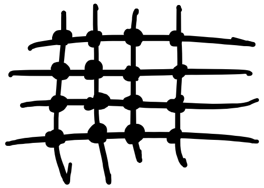
Initialization: Begin with two connected nodes

At each "time" step: ① Randomly (uniformly) select a node and duplicate it

② Retain the original edges with probability p

(if no edges are retained, discard & redo)

Vertex-copying is another (biologically-inspired) model that can generate scale free degree distributions.

Regular Networks: all nodes have the same degree 

+ Random Networks

Small World Networks $SW(n, q, p)$, q must be even

① place nodes in a ring and connect each node to $q/2$ nodes to its "left" & "right."

② randomly (uniformly) connect nodes with probability p

Combines clustering (regular networks) with short average paths (random graphs)

$$\text{clustering } C = \frac{3(q-2)}{4(q-1)} \to \frac{3}{4} \text{ as } q \to \infty$$