percolation

robustness

cascading failure

SYSM 6302

CLASS 12

Percolation

→ node (site) percolation - removing a fraction of nodes in a network (and the adjacent edges)

→ edge (bond) percolation - removing a fraction of the edges in a network

Removal can represent failure / destruction / death.

Removal isn't always a bad thing: vaccination "removes" people from a contagion network

Occupation probability ϕ

$\phi = 1$: no nodes removed, "occupied" nodes are functional

$\phi = 0$: all nodes removed

As $\phi = 1 \rightarrow \phi = 0$, there is a **percolation threshold** at which point a giant component (giant cluster) dissolves.

	Disease Spread	Internet
ABOVE:	Epidemic!	Most nodes can communicate
PERCOLATION THRESHOLD		
BELOW:	disease is confined to small sections of population	Cannot reach all nodes!

Percolation can be random (mimicing failures)

or can be strategic, e.g. by degree (mimicing attacks)

The configuration model captures most of the major percolation properties in an analytic framework.

↳ We will look at them empirically through Albert 2000

Practical Percolation

→ Instead make a fixed number of nodes (r) occupied

$$p_r = \binom{n}{r} \phi^r (1-\phi)^{n-r}$$

probability that r nodes are occupied given occupation probability ϕ

$$S(\phi) = \sum_{r=0}^{n} p_r s_r = \sum_{r=0}^{n} \binom{n}{r} \phi^r (1-\phi)^{n-r} s_r$$

↑ expected size of largest component as a function of ϕ

expected size of largest component as a function of r ↑

s_{r+1} is a minor update from s_r → it involves adding one more occupied node

↑ cluster change is minimal & with careful book keeping can be an easy update

Cascading Failures

→ A single failure can lead to successive failures →
- Power grids
- Biochemical cascades
- Finance (systemic risk)
- traffic
- ...

↳ often linked to capacities

↳ fundamentally due to an underlying process