

Introduction to 2nd-order System Response

Jonas Wagner

The University of Texas at Dallas

1 Motivation

2 Forced Response

3 Applied Example: Spring-Mass-Damper

- Review: System Modeling
- Derivation: Transfer Function and Step-Response
- Derivation/Activity: Response Comparison

Outline



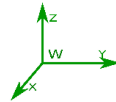
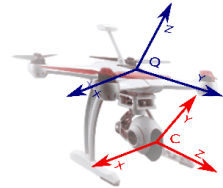
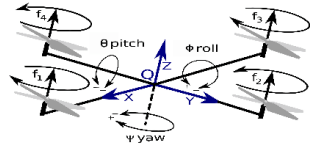
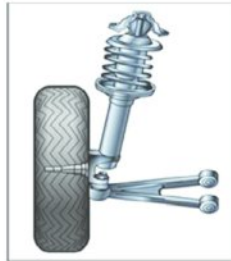
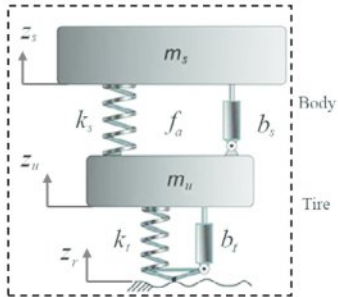
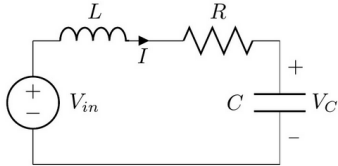
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Real-World Dynamical Systems



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Step Response - 1st vs 2nd order



Step Input: $u(t) \xrightarrow{\mathcal{L}} U(s) = \frac{1}{s}$

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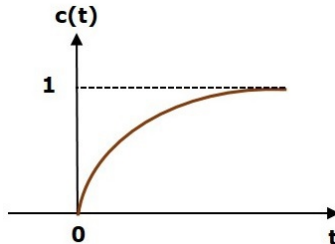
$$Y(s) = \frac{K}{\tau s + 1} \left(\frac{1}{s} \right) \xRightarrow{\mathcal{L}^{-1}} y(t) = K(1 - e^{-t/\tau})\mathbf{u}(t)$$

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$\Delta(s)$ dictates transient dynamics

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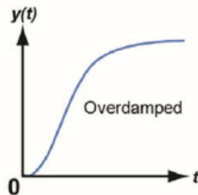
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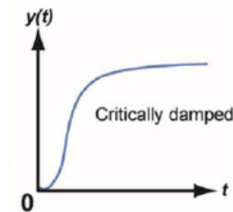


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Special
Case: $C_2 e^{-p_{1,2}t} + C_3 t e^{-p_{1,2}t}$



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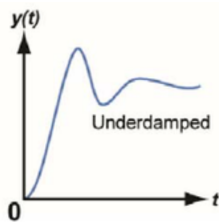
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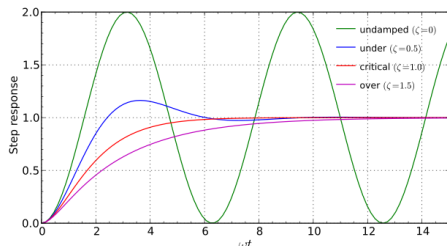
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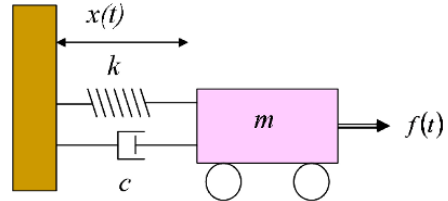
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Spring Mass-Damper System Modeling

Newton's 2nd Law:

$$F = m\mathbf{a} = m \frac{d}{dt} \mathbf{v} = m \frac{d}{dt} \left(\frac{d}{dt} \mathbf{x} \right)$$



Spring Mass Damper System [1]

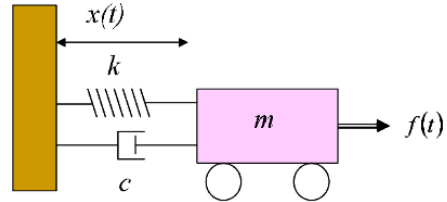
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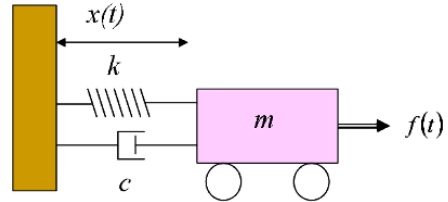
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Differential Equation: ($\mathbf{x} = x(t)$, $\mathbf{u} = f(t)$)

$$m\ddot{\mathbf{x}} + c\dot{\mathbf{x}} + k\mathbf{x} = \mathbf{u}$$

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Transfer Function Derivation

Convert Differential Equation to Laplace: ($x(t) = \dot{x}(t) = 0$)

$$f(t) = m\ddot{x}(t) + b\dot{x}(t) + kx(t) \quad \xRightarrow{\mathcal{L}} \quad F(s) = ms^2X(s) + bsX(s) + kX(s)$$



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Solve for $X(s)$ in terms of $F(s)$

$$F(s) = (ms^2 + bs + k)X(s)$$



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$$X(s) = \frac{1}{ms^2 + bs + k}F(s) = \frac{1}{\textcolor{red}{m}(s^2 + \frac{b}{m}s + \frac{k}{m})}\left(\frac{\textcolor{red}{k}}{\textcolor{red}{k}}\right)F(s)$$



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$$H(s) = \frac{X(s)}{F(s)} = \left(\frac{1}{k}\right) \frac{\frac{k}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$



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(Standard Form)



Factoring the characteristic polynomial

Apply the quadratic formula to find the roots of the characteristic polynomial:

$$\Delta(s) = ms^2 + bs + k \quad \Rightarrow \quad s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$



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$$= C_1 u(t) + 2e^{-\sigma t} \left(\frac{C_2 e^{j\omega t} + C_3 e^{-j\omega t}}{2} \right) u(t) \Leftarrow \text{Convert using Euler's Identity}$$



Activity: Response Comparison

TODO:

- 1 Experiment with different m, k , and b parameters to gain an intuitive understanding of how each parameter effects the response
- 2 Select one of each case and derive the functional form (i.e. solve for poles and coefficients)

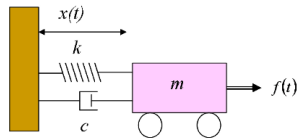
Online Tool: <https://www.sccs.swarthmore.edu/users/12/abiele1/Linear/examples/simple.html>

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$$

$$X(s) = \frac{1}{ms^2 + bs + k} F(s) = \frac{\frac{k}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}} \frac{1}{k} F(s)$$

$$H(s) = \frac{X(s)}{F(s)} = (K) \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}} \quad \zeta = \sqrt{\frac{b^2}{4mk}} \quad K = \frac{1}{k}$$





Detroit Mercy University of Michigan, Carnegie Mellon.
Introduction: System modeling.



Engineer on a Disk.
ebook: Dynamic system modeling and control.

Transfer Function

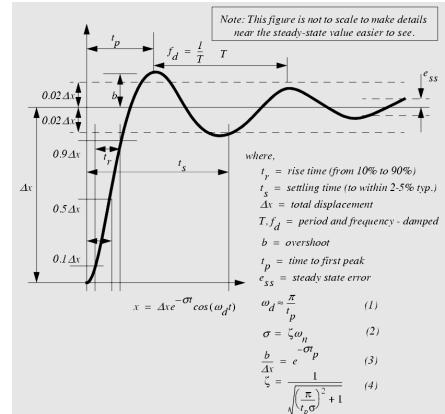
$$H(s) = \frac{Y(s)}{U(s)} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

System Poles

$$s = -\zeta\omega_0 \pm \omega_0\sqrt{1 - \zeta^2}$$

Spring Mass Damper System Parameters

$$\omega_0 = \sqrt{\frac{k}{m}} \quad \zeta = \sqrt{\frac{c^2}{4mk}}$$



2nd Order System Response [2]