Introduction to 2nd-order System Response

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The University of Texas at Dallas

Outline

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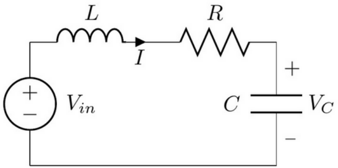
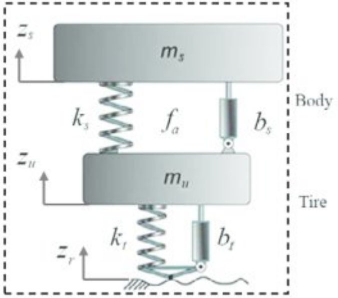
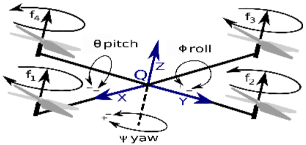
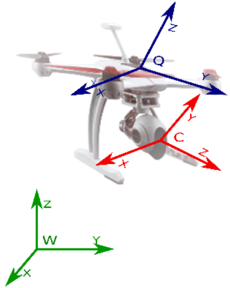
Real-World Dynamical Systems









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Step Response - 1st vs 2nd order

Step Input: $\mathbf{u}(t) \overset{\mathcal{L}}{\Rightarrow} U(s) = \frac{1}{s}$

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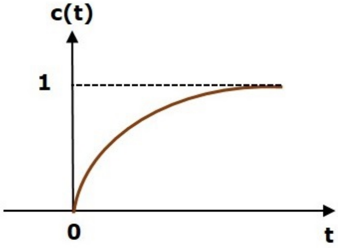

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$\Delta(s)$ dictates transient dynamics

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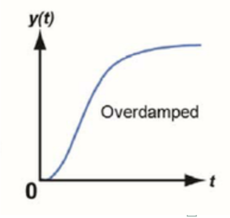

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3 distinct cases:

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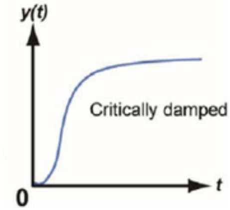

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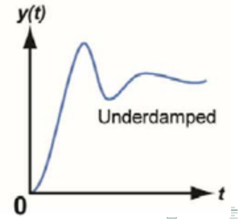

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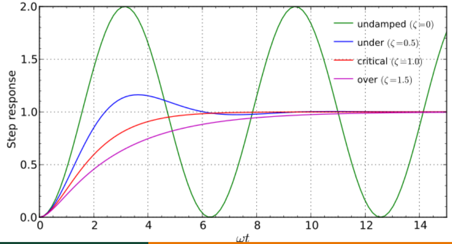

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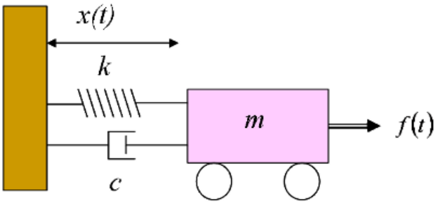
Spring Mass-Damper System Modeling

Newton's 2nd Law:

$$F = m\mathbf{a} = m\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v} = m\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}\right)$$



Spring Mass Damper System [1]

Activity: `https://www.sccs.swarthmore.edu/users/12/abiele1/Linear/examples/simple.html`

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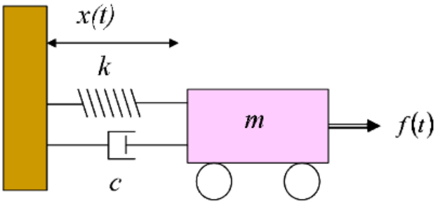

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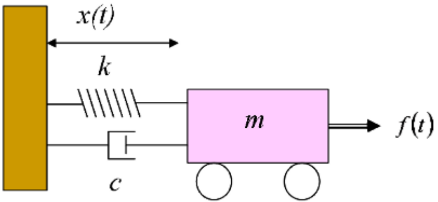

Spring Mass Damper System [1]

Differential Equation: $(\mathbf{x} = x(t),\ \mathbf{u} = f(t))$

$$m\ddot{\mathbf{x}} + c\dot{\mathbf{x}} + k\mathbf{x} = \mathbf{u}$$

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Transfer Function Derivation

Convert Differential Equation to Laplace: $(x(t) = \dot{x}(t) = 0)$

$$f(t) = m\ddot{x}(t) + b\dot{x}(t) + kx(t) \quad \overset{\mathcal{L}}{\Rightarrow} \quad F(s) = ms^2X(s) + bsX(s) + kX(s)$$

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Solve for $X(s)$ in terms of $F(s)$

$$F(s) = (ms^2 + bs + k)X(s)$$

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$$X(s) = \frac{1}{ms^2 + bs + k}F(s) = \frac{1}{m(s^2 + \frac{b}{m}s + \frac{k}{m})}\left(\frac{k}{k}\right)F(s)$$

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Transfer Function:

$$H(s) = \frac{X(s)}{F(s)} = \left(\frac{1}{k}\right)\frac{\frac{k}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

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(Hook's Law)

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(Standard Form)

Factoring the characteristic polynomial

Apply the quadratic formula to find the roots of the characteristic polynomial:

$$\Delta(s) = ms^2 + bs + k \quad \Rightarrow \quad s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

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3. **Underdamped**: $b^2 < 4mk \Rightarrow p_{1,2} = \sigma \pm j\omega \Rightarrow (s + \sigma \pm j\omega) = ((s + \sigma)^2 + \omega^2)$

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$$\overset{\mathcal{L}^{-1}}{\Rightarrow} x(t) = \left(C_1 + C_2 e^{-p_1 t} + C_3 e^{-p_2 t}\right) u(t)$$

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$$= C_1 u(t) + 2e^{-\sigma t} \left(\frac{C_2 e^{j\omega t} + C_3 e^{-j\omega t}}{2}\right) u(t) \Leftarrow \textbf{Convert using Euler's Identity}$$

Activity: Response Comparison

TODO:

1. Experiment with different m,k, and b parameters to gain an intuitive understanding of how each parameter effects the response
2. Select one of each case and derive the functional form (i.e. solve for poles and coefficients)

Online Tool: `https://www.sccs.swarthmore.edu/users/12/abiele1/Linear/examples/simple.html`

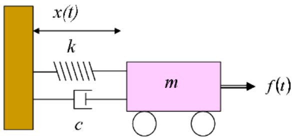
Lecture Overview

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$$

$$X(s) = \frac{1}{ms^2 + bs + k}F(s) = \frac{\frac{k}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}}\frac{1}{k}F(s)$$

$$H(s) = \frac{X(s)}{F(s)} = (K)\frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}} \quad \zeta = \sqrt{\frac{b^2}{4mk}} \quad K = \frac{1}{k}$$



Bibliography I

- Detroit Mercy University of Michigan, Carnegie Mellon.
  Introduction: System modeling.
- Engineer on a Disk.
  ebook: Dynamic system modeling and control.

TLDR: Second-Order System Dynamics

Transfer Function

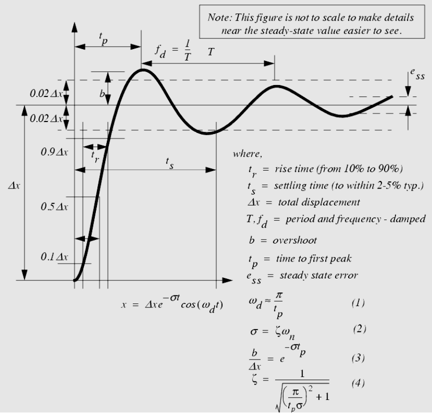
$$H(s) = \frac{Y(s)}{U(s)} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

System Poles

$$s = -\zeta\omega_0 \pm \omega_0\sqrt{1-\zeta^2}$$

Spring Mass Damper System Parameters

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad \zeta = \sqrt{\frac{c^2}{4mk}}$$



2nd Order System Response [2]