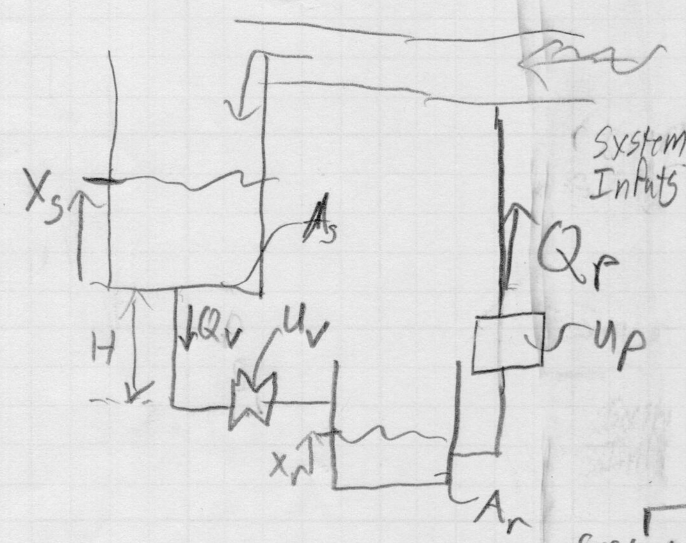
Seperator Testbed - Tank Level Estimator | Jonas Wager | 2020-10-23 | 1



*H = Seperator Tank Height

*A_s ≡ Seperator Tank Area

*A_r ≡ Resivoir Tank Area

System Inputs:
- Q_p = Pump Flow Rate
- u_v = Valve Flow Control signal

*$K_{v,r}$ = Valve Flow Coeficent

*γ = Water Head Pressur coefficent

System States:
- X_s = Seperator Tank Height
- X_r = Resivoir Tank Height

*System Parameters

Seperator Tank simplified Non-linear Dynamics:

$$f_1(x,u) = \dot{X}_s = \frac{1}{A_s}\left(Q_p - u_v K_{v,r}\sqrt{\gamma(x_s + H - x_r)}\right)$$

$$f_2(x,u) = \dot{x}_r = \frac{1}{A_r}\left(-Q_p + u_v K_{v,r}\sqrt{\gamma(x_s + H - x_r)}\right)$$

$$g_1(x,u) = Y_1 = X_s$$

$$g_2(x,u) = Y_2 = X_r$$

$$g_3(x,u) = Y_3 = Q_p$$

$$g_4(x,u) = Y_4 = u_v$$

Measurments already Coverted to Real-world Values/units

$$X = \begin{bmatrix} X_s \\ X_r \end{bmatrix} \quad U = \begin{bmatrix} Q_p \\ u_v \end{bmatrix}$$

measured

$$\overset{*}{X} = \begin{bmatrix} \overset{*}{X}_s \\ \overset{*}{X}_r \end{bmatrix} \quad \overset{*}{U} = \begin{bmatrix} \overset{*}{Q}_p \\ \overset{*}{u}_v \end{bmatrix}$$

Estimate

$$\hat{X} = \begin{bmatrix} \hat{X}_s \\ \hat{X}_r \end{bmatrix} \quad \hat{U} = \begin{bmatrix} \hat{Q}_p \\ \hat{u}_v \end{bmatrix}$$

$$Y = \begin{bmatrix} X_s \\ X_r \\ Q_p \\ u_v \end{bmatrix} + \underbrace{V}_{\mathcal{N}(0,R)} + \underbrace{r}_{\text{Attacks}}$$

x^- ← Pre measurement

x^+ ← Post measurment

Estimator Proposal:

Initilize: measured input

$$\hat{X}(0) = \begin{bmatrix} X_s(0) \\ X_r(0) \end{bmatrix} \qquad \hat{U}(0) = \begin{bmatrix} \overset{*}{Q}_p(0) \\ \overset{*}{Q}_v(0) \end{bmatrix}$$

1) Dynamics Guess/Update:

@ t_0

$$\dot{\hat{X}}(t^-) = f(\hat{X}^+(t), \hat{U}^+(t)) \qquad \hat{X}^-(t+\Delta t) = \dot{\hat{X}}^-(t)(\Delta t) + \hat{X}^-(t)$$

2) Measurment Update/Guess:

@ t_0

$$\dot{\overset{*}{X}}(t) = \frac{f(\hat{X}(t), \overset{*}{U}(t)) + f(\overset{*}{X}(t), \hat{U}(t))}{2}$$

3) Check Error State: Detector?

@ t_1

$$\dot{\hat{X}}^-(t) \overset{?}{=} \dot{\overset{*}{X}}(t) \qquad \hat{X}^-(t) \overset{?}{=} \overset{*}{X}(t)$$

↓ Filter

$$\dot{\hat{X}}^+(t) = \frac{\dot{\hat{X}}^-(t) + \dot{\overset{*}{X}}(t)}{2}$$

4) Estimate Update: Acount for uncertainty?

constant? tuned

@ t_1

$$\hat{X}^+(t+\Delta t) = \hat{X}^-(t+\Delta t) + K\left(\overset{*}{X}(t+\Delta t) - \hat{X}(t+\Delta t)\right)\Delta T$$

This May be a route to go... but is too complicated for right now...

simple Non-Linear $\rightarrow$ Linear CT $\rightarrow$ DT Method!

Linearization: @ $\hat{X}$

$$\dot{X} = f(X,U) = \begin{bmatrix} f_1(x_s, x_r, Q_p, u_v) \\ f_2(x_s, x_r, Q_p, u_v) \end{bmatrix}$$

Let

$$x = X - \hat{X} = \begin{bmatrix} x_s - \hat{x}_s \\ x_r - \hat{x}_r \end{bmatrix} \qquad u = U - \hat{U} = \begin{bmatrix} Q_p - \hat{Q}_p \\ u_v - \hat{u}_v \end{bmatrix}$$

$$\dot{x} = Ax + Bu \qquad \dot{X} \approx f(\hat{X}, \hat{U}) + \left.\frac{\partial f}{\partial X}\right|_{\hat{X}}(x) + \left.\frac{\partial f}{\partial U}\right|_{\hat{U}} u$$

$$y = Cx + Du \qquad A = \left.\frac{\partial f}{\partial X}\right|_{\hat{X},\hat{U}} \quad B = \left.\frac{\partial f}{\partial U}\right|_{\hat{X},\hat{U}}$$

Then,

$$A = \left.\frac{\partial f}{\partial X}\right|_{\hat{X}} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_s} & \frac{\partial f_1}{\partial x_r} \\ \frac{\partial f_2}{\partial x_s} & \frac{\partial f_2}{\partial x_r} \end{bmatrix}\right|_{\substack{x_s = \hat{x}_s,\ Q_p = \hat{Q}_p \\ x_r = \hat{x}_r,\ u_v = \hat{u}_v}} =$$

$$B = \left.\frac{\partial f}{\partial U}\right|_{\hat{X},\hat{U}} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial Q_p} & \frac{\partial f_1}{\partial u_v} \\ \frac{\partial f_2}{\partial Q_p} & \frac{\partial f_2}{\partial u_v} \end{bmatrix}\right|_{\substack{x_s = \hat{x}_s,\ Q_p = \hat{Q}_p \\ x_r = \hat{x}_r,\ u_v = \hat{U}_v}}$$

$$C = \begin{bmatrix} I_2 \\ 0 \end{bmatrix}_{4\times2}$$

$$D = \begin{bmatrix} 0 \\ I_2 \end{bmatrix}_{4\times2}$$

Linearization: Cont.

$$f_1 = \frac{1}{A_s}\left(Q_P - u_v K_{u,v}\sqrt{\gamma(x_s + H - x_r)}\right)$$

$$f_2 = \frac{1}{A_r}\left(-Q_P + u_v K_{u,v}\sqrt{\gamma(x_s + H - x_r)}\right)$$

$$A_{11} = \left.\frac{\partial f_1}{\partial x_s}\right|_{\check{x}} = \frac{-u_v K_{u,v}}{A_s}\left.\left(\frac{1}{2\sqrt{\gamma(x_s + H - x_r)}}\right)\right|_{\check{x}} = \frac{-\check{u}_v K_{u,v}}{2A_s\sqrt{\gamma(\check{x}_s + H - \check{x}_r)}}$$

$$A_{12} = \left.\frac{\partial f_1}{\partial x_r}\right|_{\check{x}}^{\check{u}} = \frac{-u_v K_{u,v}}{A_s}\left.\left(\frac{-1}{2\sqrt{\gamma(x_s + H - x_r)}}\right)\right|_{\check{x}}^{\check{u}} = \frac{\check{u}_v K_{u,v}}{2A_s\sqrt{\gamma(\check{x}_s + H - \check{x}_r)}}$$

$$A_{21} = \left.\frac{\partial f_2}{\partial x_s}\right|_{\check{x}}^{\check{u}} = \frac{u_v K_{u,v}}{A_r}\left.\left(\frac{1}{2\sqrt{\gamma(x_s + H - x_r)}}\right)\right|_{\check{x}}^{\check{u}} = \frac{\check{u}_v K_{u,v}}{2A_r\sqrt{\gamma(\check{x}_s + H - \check{x}_r)}}$$

$$A_{22} = \left.\frac{\partial f_2}{\partial x_s}\right|_{\check{x}}^{\check{u}} = \frac{u_v K_{u,v}}{A_r}\left.\left(\frac{-1}{2\sqrt{\gamma(x_s + H - x_r)}}\right)\right|_{\check{x}}^{\check{u}} = \frac{-\check{u}_v K_{u,v}}{2A_r\sqrt{\gamma(\check{x}_s + H - \check{x}_r)}}$$

$$B_{11} = \left.\frac{\partial f_1}{\partial Q_P}\right|_{\check{x}}^{\check{u}} = \frac{1}{A_s} \qquad B_{12} = \frac{-K_{u,v}\sqrt{\gamma(\check{x}_s + H - \check{x}_r)}}{A_s}$$

$$B_{21} = \left.\frac{\partial f_2}{Q_P}\right|_{\check{x}}^{\check{u}} = \frac{-1}{A_r} \qquad B_{22} = \frac{K_{u,v}\sqrt{\gamma(\check{x}_s + H - \check{x}_r)}}{A_r}$$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Discritization:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \xrightarrow[\Delta t]{\text{Sample + Hold}} \begin{aligned} x_{k+1} &= Fx_k + Gu_k \\ y_k &= Cx_k + Du_k \end{aligned}$$

$$F = e^{A\Delta t} = \exp\left\{\begin{bmatrix} A_{11}\Delta t & A_{12}\Delta t \\ A_{21}\Delta t & A_{22}\Delta t \end{bmatrix}\right\}$$

$$G = F\int_0^{\Delta t} e^{-A\tau}\, d\tau\, B = F\left[I - e^{-A\Delta t}\right]A^{-1}B$$

Theoretically Constant Matrices @ $\check{X}_e$

Luenberger Observer:

$(F - LC)$ must be stable

$$\hat{x}_{k+1} = F\hat{x}_k + L(y_k - \hat{y}_k) + Gu_k$$
$$\hat{y}_k = C\hat{x}_k + Du_k$$

$$\hat{x}_{k+1} = F\hat{x}_k + L(y_k - C\hat{x}_k - Du_k) + Gu_k$$
$$= (F - LC)\hat{x}_k + (G - LD)u_k + Ly_k$$

Let $\hat{u}_k = \begin{bmatrix} u_k \\ y_k \end{bmatrix}$

$$\hat{x}_{k+1} = \underbrace{(F - LC)}_{\hat{A}}\hat{x}_k + \underbrace{[G - LD \;\vdots\; L]}_{\hat{B}}\hat{u}_k$$

$$\hat{y} = \underbrace{C}_{\hat{C}}\hat{x}_k + \underbrace{[D \;\vdots\; 0_{4\times4}]}_{\hat{D}}\hat{u}_k$$