Separator Testbed Documentation

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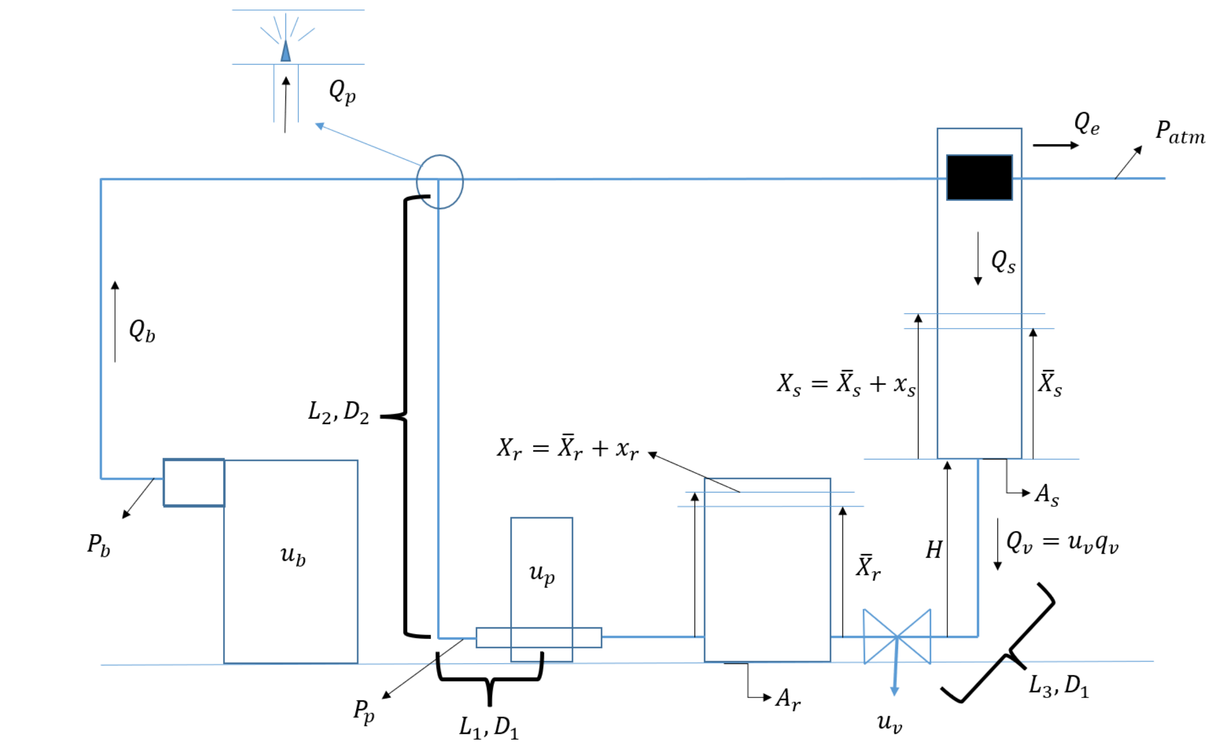

Figure 1: A schematic diagram of the separator testbed and declaration of the variables and parameters.

States

X_{s}	(m)	Height of water in the separator tank measured from the base
X_{r}	(m)	Height of water in the raw water tank measured from the base
x_{s}	(m)	Height of water in separator tank relative to $\bar{X}_{\mathsf{s}}$
x_{r}	(m)	Height of water in raw water tank relative to $\bar{X}_{\mathsf{r}}$
Q_{p}	(m^3/hr)	Flow rate passing the nozzle
q_{v}	(m^3/hr)	Flow rate passing the valve in saturated condition $(u_{\mathsf{v}} = 1)$
Q_{b}	(m^3/hr)	Air flow rate

Inputs

u_{v}	$[0, 1]$	Fraction of maximum flow rate of the motorized valve (0 being closed)
u_{p}	$[0, 1]$	Fraction of maximum RPM of the pump. (0 being off)
u_{b}	$[0, 1]$	Fraction of maximum RPM of the blower.(0 being off)

Flows

Q_{e}	(m^3/hr)	Flow of air to the environment.
Q_{s}	(m^3/hr)	Flow of water out of the separator tank and into the raw water tank.

Pressures

P_{b}	(bar)	Pressure in the pipe due to the blower
P_{atm}	(bar)	Atmospheric pressure

Parameters

H	(m)	Height of separator tanks base.
L_1	(m)	Length of horizontal portion of pipe connecting the pump and nozzle.
D_1	(m)	Diameter of horizontal portion of pipe connecting the pump and nozzle.
L_2	(m)	Length of vertical portion of pipe connecting the pump and nozzle.
D_2	(m)	Diameter of vertical portion of pipe connecting the pump and nozzle.
L_3	(m)	Length of pipes connecting the tanks.
A_s	(m^2)	Cross-sectional area of the separator tank (and chamber)
A_r	(m^2)	Cross-sectional area of the raw water tank
$K_{v,n}$	$(m^3/bar.hr)$	Nozzle flow coefficient
$K_{v,v}$	$(m^3/bar.hr)$	Valve flow coefficient
$Q_\mathsf{v}^\mathsf{max}$	0.454 m^3/hr	Maximum flow rate of the valve

Chapter 1

Dynamics

The separator testbed can be operated in continuous (using a PID controller) or batch (using a bang-bang controller) mode, where the height of the water level in the separator tank (X_{s}) is the controlled state using the percent of valve opening u_{v} as the input. A schematic diagram and list of variables is included in Figure 1.

1.1 Nonlinear Model

The nonlinear dynamics can be determined by a conservation of flow/mass. We assume the cross-sectional areas of the tanks are constant along their entire heights. The dynamics of the height of the water levels in the **s**eparator (X_{s}) and **r**aw water (X_{r}) tanks then become

$$\dot{X}_{\mathsf{s}} = \frac{Q_{\mathsf{s}} - Q_{\mathsf{v}}}{A_{\mathsf{s}}}, \tag{1.1}$$

$$\dot{X}_{\mathsf{r}} = \frac{Q_{\mathsf{v}} - Q_{\mathsf{p}}}{A_{\mathsf{r}}}, \tag{1.2}$$

where Q_α, $\alpha \in \{\mathsf{v}, \mathsf{s}, \mathsf{p}\}$, are the flows of water through the **v**alve, into the separator, and out of the **p**ump, respectively, and A_α, $\alpha \in \{\mathsf{s}, \mathsf{r}\}$, are the cross-sectional areas of the separator and raw water tanks, respectively.

The flow through the valve and through the nozzle can be quantified by the *flow factor*, K_v, which treats each as the flow through an orifice. The flow factor is a relative measure of a device's efficiency at allowing fluid flow,

$$K_v = Q\sqrt{\frac{\mathsf{SG}}{\Delta P}}. \tag{1.3}$$

SG is the specific gravity (which for water is 1), Q is the fluid flow, and ΔP is pressure difference across the orifice. The flow factor is specifically for metric units (m^3·h^{-1}·bar$^{-0.5}$); the standard unit analog is *flow coefficient*, C_v measured in gallons per minute (gpm), with $K_v = 0.865 C_v$.

1.1.1 Spray Nozzle

From the flow factor equation (1.3) for the nozzle, where the change in pressure across the nozzle is given by the difference between the water line pressure (behind the nozzle) and the air line pressure (in front of the **n**ozzle),

$$\underbrace{P_{\mathsf{p}} + \gamma X_{\mathsf{r}} - P_f}_{\text{water line pressure}} - \underbrace{P_{\mathsf{b}}}_{\text{air line pressure}} - \underbrace{\gamma L_2}_{\text{elevation difference}} = \left(\frac{Q_{\mathsf{p}}}{K_{v,\mathsf{n}}}\right)^2. \tag{1.4}$$

Here the water pressure is composed of the pump pressure P_{p}, the water head pressure in the raw water tank ($\gamma = 0.098$ bar/m for water), and the pressure loss due to friction in the water pipe. The air line pressure is

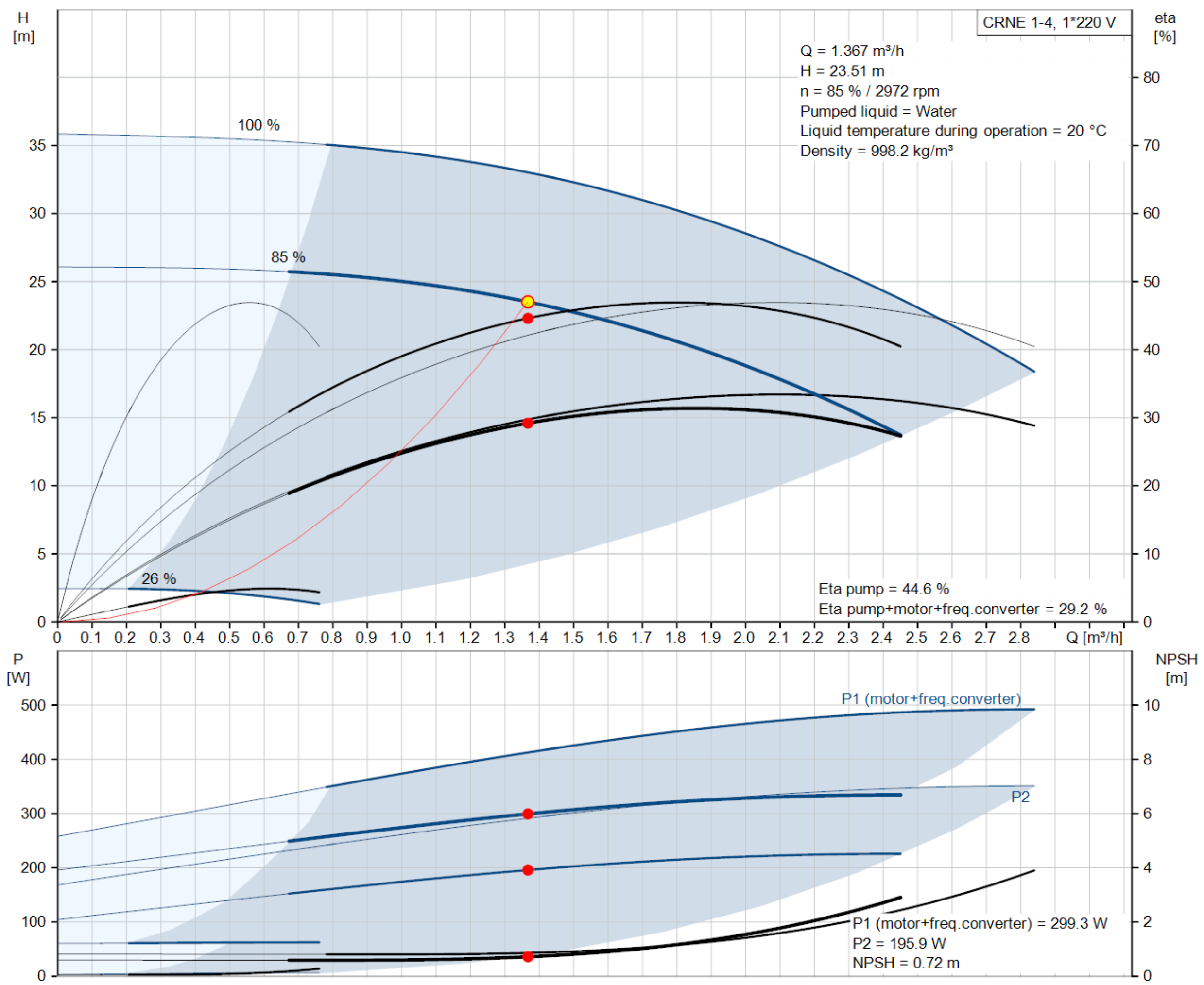

Figure 1.1: Performance curve of the Grundfos CRNE 1-4 A-FGJ-G-E-HQQE - 96553965 pump. **I think this is the wrong pump curve..replace it**

composed of the blower pressure P_{b}. From the nozzle product data sheet the flow coefficient of the nozzle is, $K_{v,\text{n}}$,

$$K_{v,\text{n}} = 0.865 C_v = 0.865 \times 0.166 \text{ (gpm/psi)} = 0.144 \ \ \text{m}^3 \cdot \text{h}^{-1} \cdot \text{bar}^{-0.5}.$$

Like an electrical circuit where the current draw from a battery, with a given voltage, is determined by the resistive load, the flow rate Q_{p} of the pump, with pressure P_{p}, is determined by the resistance of the nozzle. The relationship between the pressure (voltage) drop across the nozzle and the flow factor ($K_{v,\text{n}}$, resistance) yields the flow rate Q_{p}. Unlike a ideal battery (voltage source), the pump's pressure is not constant and is interrelated with the pump speed and flow rate (see Section 1.1.2).

1.1.2 Pump

The pump datasheet provides a performance curve of the pump which plots relationship between the pump flow rate Q_{p}, pump pressure P_{p}, and speed of the pump motor $u_{\text{p}} N_{\text{p}}^{\text{max}}$, see Figure 1.1. Here the input $u_{\text{p}} \in [0, 1]$ commands the current setting of the motor speed, and $N_{\text{p}}^{\text{max}}$ is the maximum RPM of the pump motor. To enable an analytic model we approximate this surface with a quadratic equation,

$$u_{\mathsf{p}} \approx aP_{\mathsf{p}}^2 + bP_{\mathsf{p}}Q_{\mathsf{p}} + cQ_{\mathsf{p}}^2 + dP_{\mathsf{p}} + eQ_{\mathsf{p}} + g. \tag{1.5}$$

We should document these coefficients.

1.1.3 Blower

Similar to the pump, the blower performance curves provide the relationship between air flow rate (Q_{b}), pressure (P_{b}), and the speed of the blower motor $u_{\mathsf{b}}N_{\mathsf{b}}^{\mathsf{max}}$.

In the blower path the diameter of pipe is large and the fluid is air, therefore, the pressure loss due to friction is very small and in addition pressure loss of the current of air through the separator is assumed to be small, therefore,

$$P_{\mathsf{b}} \approx P_{\mathsf{atm}},$$

where P_{atm} is atmospheric pressure. Therefore we can rewrite (1.7) as,

$$P_{\mathsf{p}} + \gamma X_{\mathsf{r}} - P_{\mathsf{atm}} - P_f - \gamma L_2 = \left(\frac{Q_{\mathsf{p}}}{K_{v,\mathsf{n}}}\right)^2 \tag{1.6}$$

1.1.4 Separator

For simplicity we initially assume the separator performs perfectly and does not pass water to the environment, this implies $Q_{\mathsf{s}} = Q_{\mathsf{p}}$ (all the water sprayed in by the pump goes directly into the separator) and $Q_{\mathsf{e}} = Q_{\mathsf{b}}$ (all the air from the blower passes through the separator and leaves to the environment). Currently this flow conservation model does not include the more subtle dynamics of the water being channeled through the separator or the displacement of the water from the main separator vessel to the sensor chamber - both of which may induce some minor delays and/or nonlinearities.

To capture the efficiency of the separator, we need to **get the separator curves from Dr. Dani Fadda**. These depict the capacity of the separator to extract water at different pressures, temperatures, air flow rates, and amount of water in the air, so it would provide the fraction $Q_{\mathsf{s}}/Q_{\mathsf{p}}$ as a function of pressure, flow, temperature, and amount of water.

1.1.5 Valve

The flow factor equation can describe the operation of the control valve with

$$\underbrace{\gamma(X_{\mathsf{s}} + H)}_{\text{head pressure of separator}} - \underbrace{\gamma X_{\mathsf{r}}}_{\text{head pressure of raw tank}} - \underbrace{\gamma H_f}_{\text{friction}} = \left(\frac{q_{\mathsf{v}}}{K_{v,\mathsf{v}}}\right)^2. \tag{1.7}$$

where the flow factor for the control valve is given in documentation as

$$K_{v,\mathsf{v}} = 0.865C_v = 0.865 \times 2.5 \text{ (GPM/psi)} = 2.1625 \text{ m}^3\text{/bar·hr}.$$

Thus q_{v} represents the maximum flow given the current heights of the water in the two tanks, i.e., the fully-open flow. When the valve is operated in partially opened flow, the actual flow Q_{v} can be expressed as a fraction of the fully-open flow,

$$Q_{\mathsf{v}} = u_{\mathsf{v}}q_{\mathsf{v}} \quad \text{and} \quad q_{\mathsf{v}} = K_{v,\mathsf{v}}\sqrt{\gamma(X_{\mathsf{s}} + H - X_{\mathsf{r}} - H_f)}, \tag{1.8}$$

where u_{v} represents the percentage open the valve is (in terms of flow).

1.1.6 Simplified Nonlinear Equations

At this point, we can compose a simplified set of equations, now assuming the following:

(i) The frictional forces are negligible.

(ii) The separator performs perfectly, so $Q_{\mathsf{p}} = Q_{\mathsf{s}}$.

(iii) The blower has no effect since the separator performs perfectly. In practice, we set it to be a constant flow, i.e., $Q_{\mathsf{b}} = Q_{\mathsf{e}}$ is constant.

(iv) The pump is set at a constant flow, i.e., Q_{p} is constant.

Where it is mainly item (i) that permits the removal of all friction terms. This makes (1.1)-(1.2) become

$$\dot{X}_{\mathsf{s}} = \frac{1}{A_{\mathsf{s}}}\left(\quad Q_{\mathsf{p}} - u_{\mathsf{v}} K_{v,\mathsf{v}} \sqrt{\gamma (X_{\mathsf{s}} + H - X_{\mathsf{r}})}\right), \tag{1.9}$$

$$\dot{X}_{\mathsf{r}} = \frac{1}{A_{\mathsf{r}}}\left(-Q_{\mathsf{p}} + u_{\mathsf{v}} K_{v,\mathsf{v}} \sqrt{\gamma (X_{\mathsf{s}} + H - X_{\mathsf{r}})}\right), \tag{1.10}$$

1.1.7 Friction Forces (H_f and P_f)

On this work, H_f and P_f are computable from the presented formula, Based on the Reynolds number Re,

$$Re = \frac{4Q_{\mathsf{v}}}{\pi \nu D} \tag{1.11}$$

where D is the pipe's diameter and ν is the viscosity. The maximum allowable flow rate for the installed valve is 0.454 m^3/hr which means we are dealing with a laminar flow. The Darcy-Weisbasch coefficient, f, for the laminar flow is,

$$f = \frac{64}{Re}, \tag{1.12}$$

so the lost in pressure due to friction in laminar flow would be:

$$\Delta P_{friction} = 8f \frac{LQ^2}{\pi^2 D^4} \tag{1.13}$$

where for the pipes connecting the pump to nozzle, because we have two types of pipes with different diameters D_1 with length L_1 and diameter D_2 with length L_2 we have two different Darcy-Weisbasch coefficients f_1 and f_2, therefore, for this line,

$$P_f = 8f_1 \frac{L_1 Q_{\mathsf{p}}^2}{\pi^2 D_1^4} + 8f_2 \frac{L_2 Q_{\mathsf{p}}^2}{\pi^2 D_2^4} = \frac{128\nu Q_{\mathsf{p}}}{\pi D_1^2} \frac{L_1}{D_1} + \frac{128\nu Q_{\mathsf{p}}}{\pi D_2^2} \frac{L_2}{D_2} \tag{1.14}$$

for the pipes which connects the two tanks through the valve, L_3 is the length of pipe between the separator tank and raw tank with diameter D_1 and for $H_f = \dfrac{\Delta P_{friction}}{\gamma}$,

$$H_f = \frac{128\nu Q_{\mathsf{v}}}{\pi D_1^2 \gamma} \frac{L_3}{D_1} \tag{1.15}$$

But due to the existence of two elbows in the path of water between the tanks and one elbow between pump and nozzle, based on the fluid mechanics charts we should define,

$$\frac{L_e}{D} = 2 \times 20 + \frac{L_3}{D_1} \quad \text{for the path between the tanks}$$

$$\frac{L_e}{D} = \frac{1}{2} \times 20 + \frac{L_1}{D_1} \quad \text{and} \quad \frac{L_e}{D} = \frac{1}{2} \times 20 + \frac{L_2}{D_2} \quad \text{for the path between the pump and nozzle}$$

and replacement of $\dfrac{L}{D}$ by $\dfrac{L_e}{D}$, in both of the ways results in,

$$\begin{aligned} H_f &= \frac{128\nu Q_{\mathsf{v}}}{\pi D_1^2\gamma}(40+\frac{L_3}{D_1}) \\ P_f &= \frac{128\nu Q_{\mathsf{p}}}{\pi D_1^2}\left(10+\frac{L_1}{D_1}\right)+\frac{128\nu Q_{\mathsf{p}}}{\pi D_2^2}\left(10+\frac{L_2}{D_2}\right) \end{aligned} \tag{1.16}$$

1.1.8 Complete Equations

Therefore can write (1.6) as,

$$\frac{Q_{\mathsf{p}}^2}{K_{v,n}^2}+\frac{128\nu Q_{\mathsf{p}}}{\pi D_1^2}\left(10+\frac{L_1}{D_1}\right)+\frac{128\nu Q_{\mathsf{p}}}{\pi D_2^2}\left(10+\frac{L_2}{D_2}\right)=\Delta P_{\mathsf{p}}+\gamma X_{\mathsf{r}}-P_{\mathsf{atm}}-\gamma L_2 \tag{1.17}$$

and we can write (1.8) as,

$$Q_{\mathsf{v}}=u_{\mathsf{v}}q_{\mathsf{v}} \quad \text{and} \quad q_{\mathsf{v}}=K_{v,v}\sqrt{\gamma(X_{\mathsf{s}}+H-X_{\mathsf{r}}-\frac{128\nu Q_{\mathsf{v}}}{\pi D_1^2\gamma}(40+\frac{L_3}{D_1}))} \tag{1.18}$$

1.2 Linear Model

For steady state condition which, we can linearize the system around the equilibrium points, we first assume we design u_{p} and u_{v} in a way that equilibrium point happens. First we can define the perturbations $x_{\mathsf{r}}=X_{\mathsf{r}}-\bar{X}_{\mathsf{r}}$, $x_{\mathsf{s}}=X_{\mathsf{s}}-\bar{X}_{\mathsf{s}}$, where $\bar{X}_{\mathsf{r}},\bar{X}_{\mathsf{s}}$ are the expected values of elevations at equilibrium and certainly $Q_{\mathsf{v}}=Q_{\mathsf{p}}$ (when the perturbations, δu_{v} and δu_{p} are relatively small) as a necessary condition for equilibrium. To compute the equilibrium point we should first solve for $(Q_{\mathsf{p}},\ Q_{\mathsf{v}},\ \Delta P_{\mathsf{p}})$ as a function of $(u_{\mathsf{p}},\ u_{\mathsf{v}},\ X_{\mathsf{r}},\ X_{\mathsf{s}})$ through the three independent equations (1.5), (1.17), (1.18), then we can solve the (1.1), (1.2) as an ODE equation in MATLAB given the inputs u_{p} and u_{v} and initial conditions $X_{\mathsf{s}}(0),\ X_{\mathsf{r}}(0)$. we will perform this solution numerically in MATLAB and we can find the equilibrium point and the equlibrium time, T.

Now that we know the equilibrium values for elevations and flow rates we can linearize the model around this points. From (1.1) and (1.2),

$$\begin{aligned} \dot{x}_{\mathsf{s}} &= \frac{\delta Q_{\mathsf{p}}-\delta Q_{\mathsf{v}}}{A_{\mathsf{s}}} \\ \dot{x}_{\mathsf{r}} &= \frac{\delta Q_{\mathsf{v}}-\delta Q_{\mathsf{p}}}{A_{\mathsf{r}}} \end{aligned} \tag{1.19}$$

On the other hand from, (1.5) we know,

$$\begin{aligned} &\delta u_{\mathsf{p}}=A\delta(\Delta P_{\mathsf{p}})+B\delta Q_{\mathsf{p}} \\ &\quad\text{where:} \qquad A=2a\Delta P_{\mathsf{p}}(T)+bQ_{\mathsf{p}}(T)+d \\ &\quad\text{and,} \qquad B=b\Delta P_{\mathsf{p}}(T)+2cQ_{\mathsf{p}}(T)+e \end{aligned} \tag{1.20}$$

and from (1.18) we know,

$$\begin{aligned} &G\delta Q_{\mathsf{v}}=C\delta u_{\mathsf{v}}+D(x_{\mathsf{s}}-x_{\mathsf{r}}) \\ &\quad\text{where:} \quad C=q_{\mathsf{v}}(T) \\ &\quad\text{and,} \quad D=\frac{\gamma u_v(T)}{2\sqrt{\gamma\left(X_s(T)+H-X_r(T)-\frac{128\nu Q_{\mathsf{v}}(T)}{\pi D_1^2\gamma}(40+\frac{L_3}{D_1})\right)}} \\ &\quad\text{and,} \quad G=1+\frac{128\nu}{\pi D_1^2\gamma}(40+\frac{L_3}{D_1})D \end{aligned} \tag{1.21}$$

and from (1.17) we know,

$$
\begin{aligned}
&E\delta Q_{\mathsf{p}} = \delta(\Delta P_{\mathsf{p}}) + \gamma x_{\mathsf{r}} \\
&\qquad \text{where:} \quad E = \frac{2Q_{\mathsf{p}}(T)}{K_{v,n}^2} + \frac{128\nu}{\pi D_1^2}\left(10 + \frac{L_1}{D_1}\right) + \frac{128\nu}{\pi D_2^2}\left(10 + \frac{L_2}{D_2}\right)
\end{aligned}
\tag{1.22}
$$

by combination of (1.20), (1.21) and (1.22) we can solve for δQ_{v} and δQ_{p},

$$
\begin{aligned}
\delta Q_{\mathsf{p}} &= \frac{\delta u_{\mathsf{p}} + A\gamma x_{\mathsf{r}}}{AE + B} \\
\delta Q_{\mathsf{v}} &= \frac{C}{G}\delta u_{\mathsf{v}} + \frac{D}{G}(x_{\mathsf{s}} - x_{\mathsf{r}})
\end{aligned}
\tag{1.23}
$$

applying (1.23) in (1.19),

$$
\begin{bmatrix} \dot{x}_{\mathsf{s}} \\ \dot{x}_{\mathsf{r}} \end{bmatrix} = \begin{bmatrix} -\frac{D}{GA_{\mathsf{s}}} & \frac{GA\gamma+D}{GA_{\mathsf{s}}} \\ \frac{D}{GA_{\mathsf{r}}} & -\frac{GA\gamma+D}{GA_{\mathsf{r}}} \end{bmatrix} \begin{bmatrix} x_{\mathsf{s}} \\ x_{\mathsf{r}} \end{bmatrix} + \begin{bmatrix} \frac{1}{A_{\mathsf{s}}(AE+B)} & -\frac{C}{GA_{\mathsf{s}}} \\ -\frac{1}{A_{\mathsf{r}}(AE+B)} & \frac{C}{GA_{\mathsf{r}}} \end{bmatrix} \begin{bmatrix} \delta u_{\mathsf{p}} \\ \delta u_{\mathsf{v}} \end{bmatrix}
\tag{1.24}
$$

Now we can introduce these perturbations as a linear system where its states $(x_{\mathsf{s}},\ x_{\mathsf{r}})$ are excited by the pump and valve perturbations $\delta u_{\mathsf{p}}, \delta u_{\mathsf{v}}$.

	1	2	3	4	5	6	7	8
u_{v}	(0,1]	(0,1]	(0,1]	(0,1]	0	0	0	0
u_{p}	(0,1]	(0,1]	0	0	(0,1]	(0,1]	0	0
u_{b}	(0,1]	0	(0,1]	0	(0,1]	0	(0,1]	0

The relation between pressures P_{b} and atmosphere pressure P_{atm} can be computed from the separator catalogue but we have justified that their difference is very small. Application of Table 1.2 on the presented analysis provides all the eight conditions for the test bed, here we have provided a brief description of these cases:

- First case: This one is completely equivalent to the proposed analysis.
- Second case: If $u_b = 0$ and $u_p \in (0, 1]$ it is not applicable here because the water inserts to blower which results in damage.
- Third case: This case addresses the process of filling up the raw tank which does not hold equilibrium and we just provide the non-linear modeling.
- Fourth case: This case is exactly like case 3 just the blower is off, which doesn't change the governing equations of the system. because when pump is off the blower does independent of water circle.
- Fifth Case: This case presents the process of separator tank filling, which doesn't hold equilibrium and we present just the nonlinear modeling
- Sixth Case: Not applicable , water goes in blower and results in damage.
- Seventh Case: In this case just the blower works and we should search for pressures.
- Eighth case: All of the states will stay constant.