*H = Seperator Tank Height
*A_s ≡ Seperator Tank Area
*A_r ≡ Resivoir Tank Area

System Inputs $\begin{cases} Q_p = \text{Pump Flow Rate} \\ u_v = \text{Valve Flow (Control signal)} \end{cases}$

*K_{vv} = Valve Flow Coeficient
*γ = Water Head Pressur Coefficient

System States $\begin{cases} X_s = \text{Seperator Tank Height} \\ X_r = \text{Resivoir Tank Height} \end{cases}$

*System Parameters

Seperator Tank simplified Non-linear Dynamics:

$$f_1(x,u) = \dot{X}_s = \frac{1}{A_s}\left(Q_p - u_v K_{vv}\sqrt{\gamma(x_s + H - x_r)}\right)$$

$$f_2(x,u) = \dot{x}_r = \frac{1}{A_r}\left(-Q_p + u_v K_{vv}\sqrt{\gamma(x_s + H - x_r)}\right)$$

$$g_1(x,u) = Y_1 = X_s$$

$$g_2(x,u) = Y_2 = X_r$$

$$g_3(x,u) = Y_3 = Q_p$$

$$g_4(x,u) = Y_4 = u_v$$

$$X = \begin{bmatrix} X_s \\ X_r \end{bmatrix} \qquad \tilde{X} = \begin{bmatrix} \tilde{X}_s \\ \tilde{X}_r \end{bmatrix}_{measured} \qquad \hat{X} = \begin{bmatrix} \hat{X}_s \\ \hat{X}_r \end{bmatrix}_{Estimate}$$

$$U = \begin{bmatrix} Q_r \\ u_v \end{bmatrix} \qquad \tilde{U} = \begin{bmatrix} \tilde{Q}_r \\ \tilde{u}_v \end{bmatrix} \qquad \hat{U} = \begin{bmatrix} \hat{Q}_r \\ \hat{u}_v \end{bmatrix}$$

$$Y = \begin{bmatrix} X_s \\ X_r \\ Q_p \\ u_v \end{bmatrix} + V + r \quad \text{Attacks}$$

$$\sim(0, R)$$

x^- ← Pre measurement

x^+ ← Post measurement

Estimator Proposal:

Initilize: measured   input

$$\hat{X}(0) = \begin{bmatrix} X_s(0) \\ X_r(0) \end{bmatrix} \qquad \hat{U}(0) = \begin{bmatrix} \overset{*}{Q}_r(0) \\ \overset{*}{U}_v(0) \end{bmatrix}$$

@ t_0

① Dynamics Quess/update:

$$\dot{\hat{X}}(t^-) = f(\hat{X}^+(t), \hat{U}^+(t)) \qquad \hat{X}^-(t+\Delta t) = \dot{\hat{X}}(t)\,(\Delta t)$$

@ t_0

② Measurment update/Quess:

$$\overset{*}{\dot{X}}(t) = \frac{f(\hat{X}(t), \overset{*}{U}(t)) + f(\overset{*}{X}(t), \hat{U}(t))}{2}$$

@ t_1

③ Check Error state: Detector?

$$\dot{\hat{X}}^-(t) \overset{?}{=} \overset{*}{\dot{X}}(t) \qquad \hat{X}^-(t) \overset{?}{=} \overset{*}{X}(t)$$

$$\dot{\hat{X}}^+(t) \overset{?}{=} \frac{\dot{\hat{X}}^-(t) + \overset{*}{\dot{X}}(t)}{2} \qquad \downarrow \text{Filter}$$

④ Estimate update: Acount for uncertainty?
                        constant? tuned

@ t_1

$$\hat{X}^+(t+\Delta t) = \hat{X}^-(t+\Delta t) + K\left(\overset{*}{X}(t+\Delta t) - \hat{X}^-(t+\Delta t)\right)$$