Time Series Analysis

Fall 2018

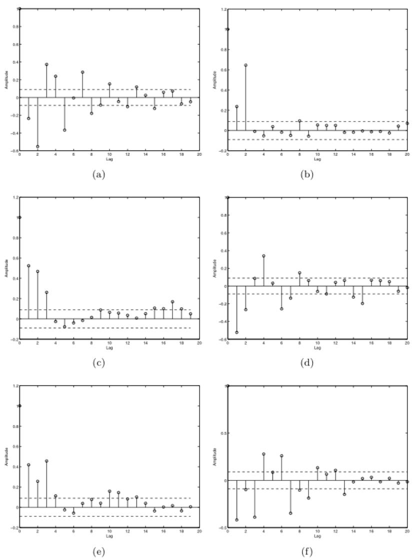
Andreas Jakobsson

Identification

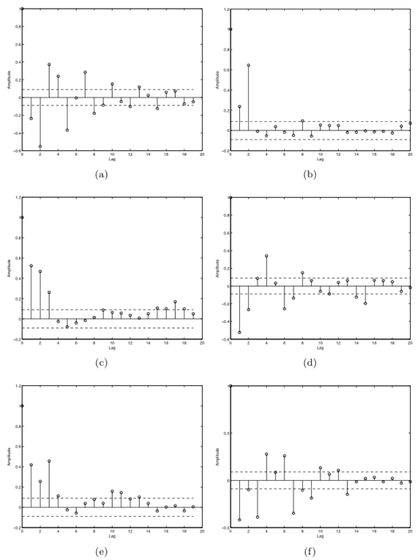
Characteristics for the autocorrelation functions:

	ACF $\rho(k)$	PACF ϕ_{kk}
$AR(p)$	Damped exponential and/or sine functions	$\phi_{kk} = 0$ for $k > p$
$MA(q)$	$\rho(k) = 0$ for $k > q$	Dominated by damped exponential and or/sine functions
$ARMA(p, q)$	Damped exponential and/or sine functions after lag $q - p$	Dominated by damped exponential and/or sine functions after lag $p - q$

Identification



Identification

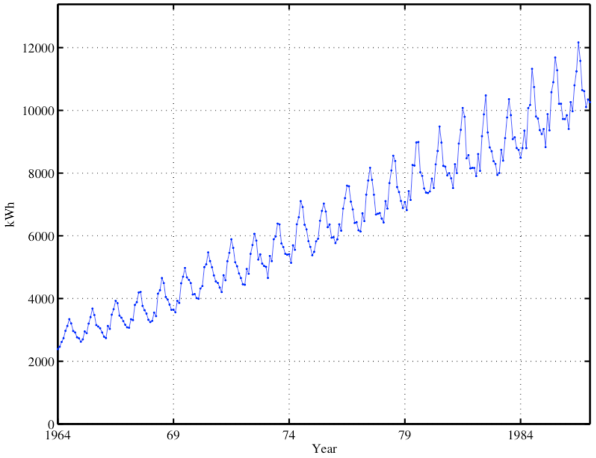


AR(2)

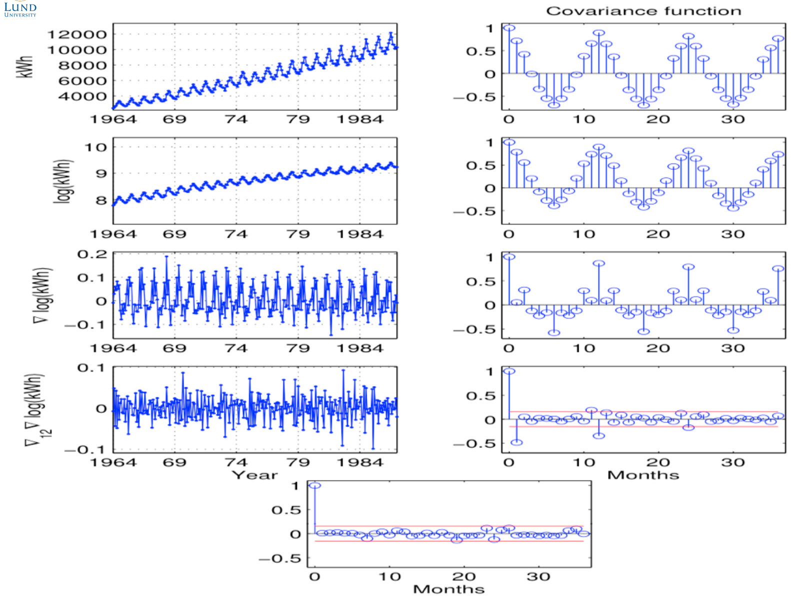
MA(3)

ARMA(1,4)

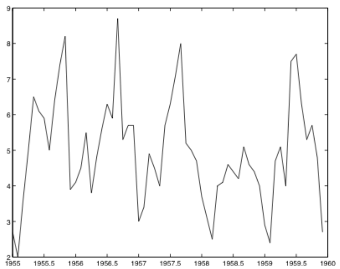
Electricity consumption in Australia



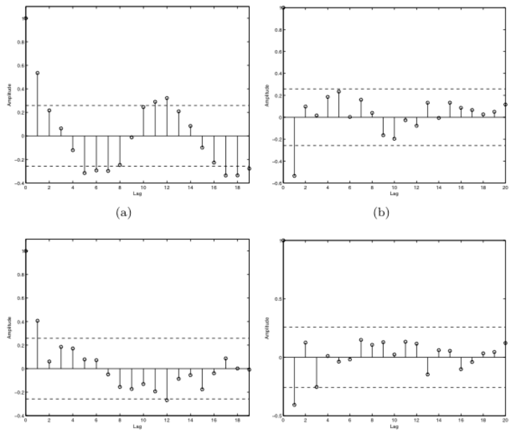
$$\nabla_{12}\nabla \log y_t = \left(1 - 0.71B\right)\left(1 - 0.67B^{12}\right)e_t$$



Oxidant levels in Los Angeles



Oxidant levels in Los Angeles

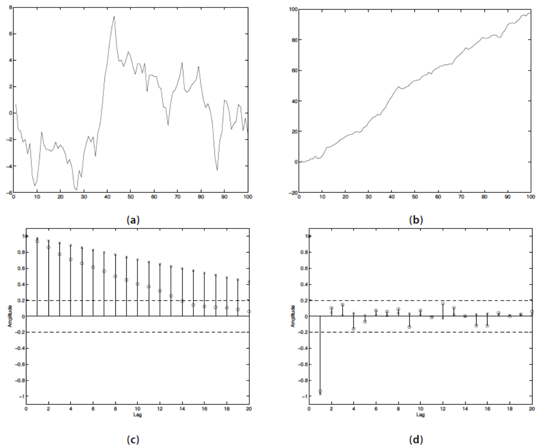


(a) (b)

(c) (d)

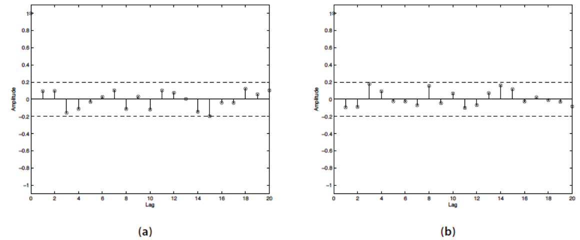
Testing for a non-zero mean

$\nabla y_t = e_t$



$\nabla y_t = e_t + 1$

Testing for a non-zero mean



$\nabla y_t = e_t$ $\quad$ $\nabla y_t = e_t + 1$

Reject the hypothesis that $m_y = \hat{m}_y$, with significance α, if

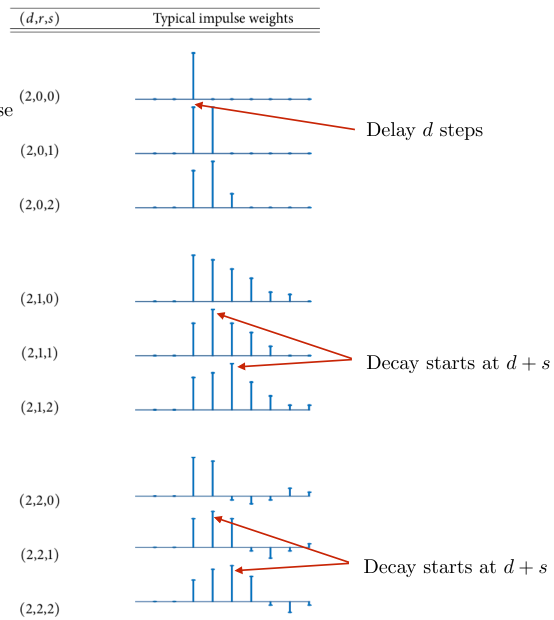
$$N\left(\hat{m}_y - m_y\right)\hat{\sigma}_y^{-2}\left(\hat{m}_y - m_y\right) > t^2_{N-1}(\alpha/2)$$

Use the provided function `testMean`.

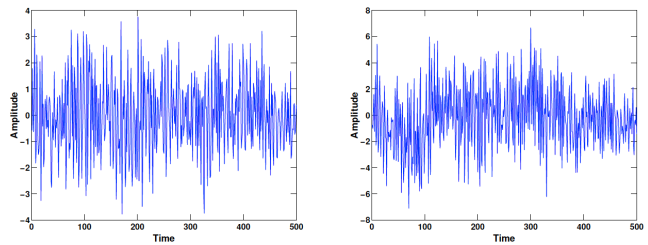
Finite impulse response

Exponential decay

Damped exponential or sinusoidal



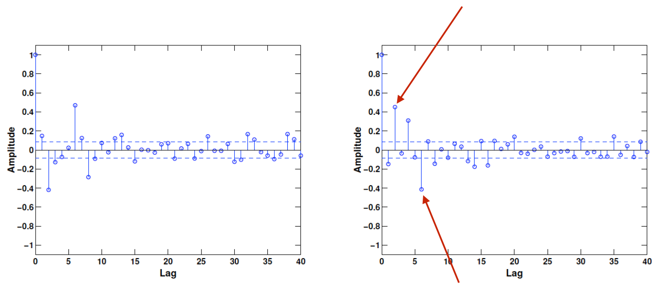
Identifying a Box-Jenkins model



x_t $\quad$ y_t

Input and output of the simulated process in Example 4.21.

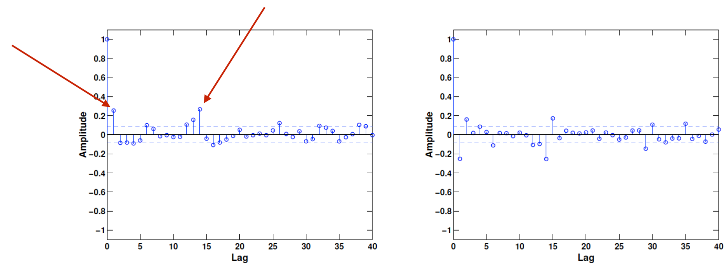
Begin by modeling the input signal, x_t.



ACF PACF

There seems to be strong dependencies for order 2 and 6, as well as, perhaps, at 4. In order to have a simple model, we begin with trying

$$A_3(z) = 1 + a_2 z^{-2} + a_6 z^{-6}$$



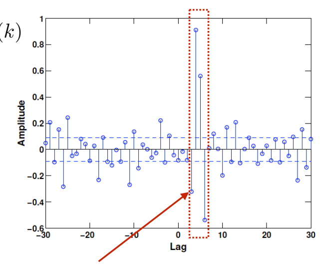
ACF PACF

We estimate the parameters of this model and examine the ACF and PACF of the residual (above). Looking at the ACF, there seems to be strong dependencies at lag 1 and 14. We thus modify our model to

$$A_3(z) = 1 + a_2 z^{-2} + a_6 z^{-6}$$
$$C_3(z) = 1 + c_1 z^{-1} + c_{14} z^{-14}$$

We re-estimating *all* parameters and examine the residuals. These are now deemed to be white.

$\frac{\sigma_\epsilon}{\sigma_w}\rho_{\epsilon,w}(k)$



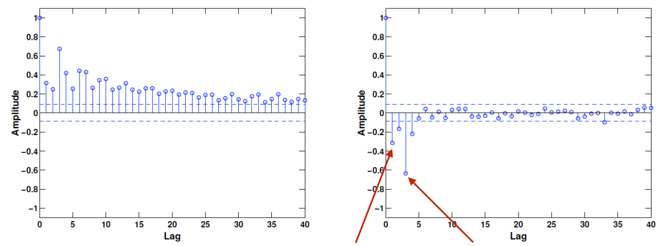
We form the "white" input signal and the corresponding output

$$w_t = \frac{A_3(z)}{C_3(z)} x_t \quad \text{and} \quad \epsilon_t = \frac{A_3(z)}{C_3(z)} y_t$$

and then estimate the transfer function from w_t to ϵ_t as

$$h_k = \frac{\sigma_\epsilon}{\sigma_w}\rho_{\epsilon,w}(k)$$

The delay suggests $d = 3$. The impulse response seems to "ring", so we try $r = 2$. There seems to be 4 dominant components, i.e., $s = 3$.



We pretend that the additive noise is white, and estimate the parameters detailing the model

$$y_t = \frac{B(z) z^{-d}}{A_2(z)} x_t + \tilde{e}_t$$

where $B(z)$ and $A_2(z)$ are of order s and r, respectively. We then compute the ACF and PACF of the residual $\tilde{e}_t$ (above).

We form a model of the residual, beginning with using just a_1 and a_3. Examining the resulting residual suggest that we also needs c_1. This yields a white residual. Finally, we *re-estimate* all coefficients.

This week

We will cover

- Identification. Estimation.
- Reading instructions: Ch. 4, 5.1-5.2
- Problems: 4.1-4.4