



LUND
UNIVERSITY

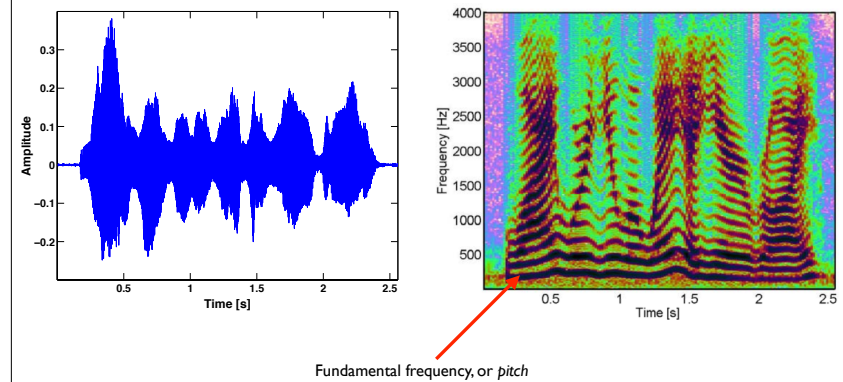
Time Series Analysis

Fall 2018

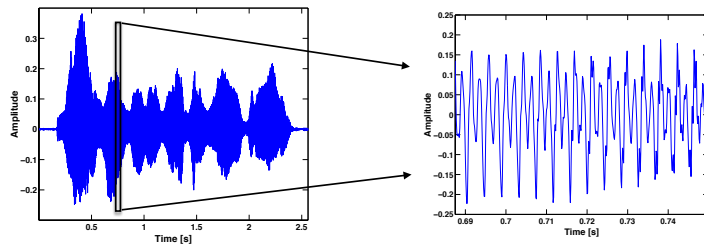
Andreas Jakobsson



”Why were you away a year, Roy?”



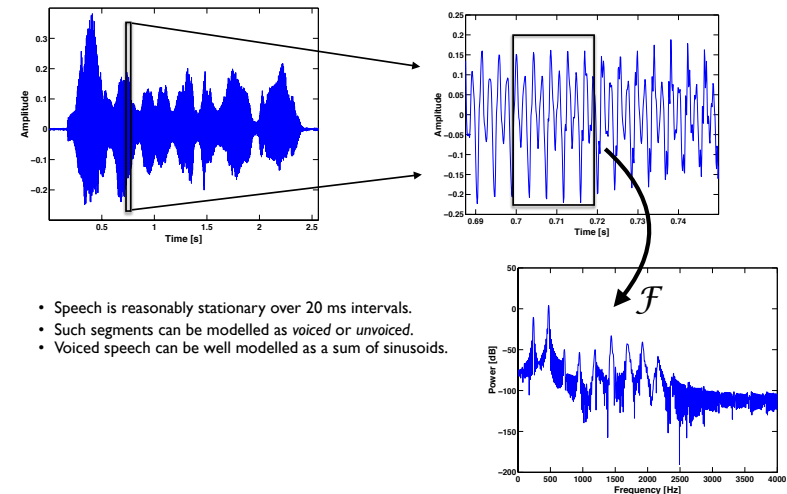
”Why were you away a year, Roy?”



- Speech is reasonably stationary over 20 ms intervals.
- Such segments can be modelled as *voiced* or *unvoiced*.

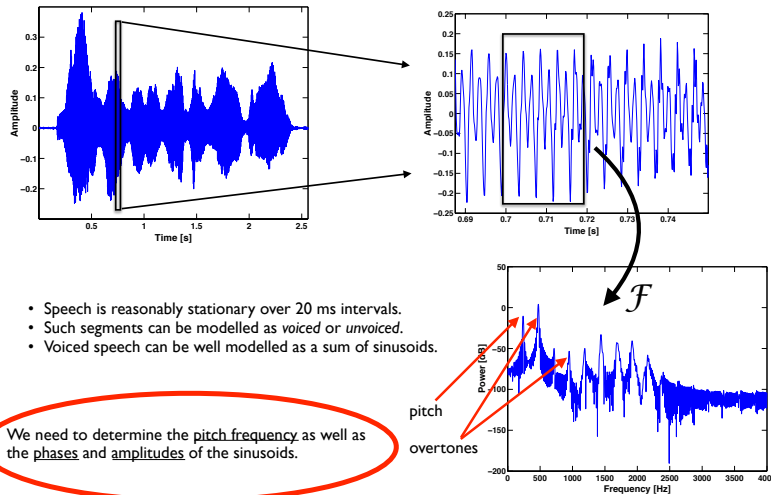


”Why were you away a year, Roy?”



- Speech is reasonably stationary over 20 ms intervals.
- Such segments can be modelled as *voiced* or *unvoiced*.
- Voiced speech can be well modelled as a sum of sinusoids.

"Why were you away a year, Roy?"



Estimating the unknown parameters

Using a complex-valued notation, the signal may be expressed as

$$y_t = \sum_{k=1}^d \alpha_k e^{i\omega_0 k t} + e_t$$

$$= \begin{bmatrix} e^{i\omega_0 t} & \dots & e^{i\omega_0 d t} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_d \end{bmatrix} + e_t$$

$$= \mathbf{x}_t^T \boldsymbol{\theta} + e_t$$

Gathering N samples in a vector

$$\mathbf{y} = \begin{bmatrix} y_1 & \dots & y_N \end{bmatrix}^T$$

$$= \begin{bmatrix} e^{i\omega_0} & \dots & e^{i\omega_0 d} \\ \vdots & \ddots & \vdots \\ e^{i\omega_0 N} & \dots & e^{i\omega_0 d N} \end{bmatrix} \boldsymbol{\theta} + \mathbf{e}$$

$$= \mathbf{X} \boldsymbol{\theta} + \mathbf{e}$$

Estimating the unknown parameters

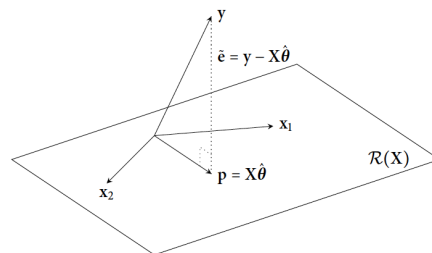
The signal can thus be expressed as

$$\mathbf{y} = \mathbf{X} \boldsymbol{\theta} + \mathbf{e}$$

where the regressor \mathbf{X} depends on the unknown pitch ω_0 . Assuming for the moment that we know ω_0 , the (unknown) complex-valued amplitudes $\boldsymbol{\theta}$ may be estimated as

$$\hat{\boldsymbol{\theta}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$$

where $\mathbf{p} = \mathbf{X} \hat{\boldsymbol{\theta}} = \mathbf{X} (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$ is the projection of \mathbf{y} onto \mathbf{X} .



Estimating the unknown parameters

However, we don't know ω_0 . To solve this, find the ω_0 that minimize the difference between \mathbf{y} and its projection, i.e.,

$$\hat{\omega}_0 = \arg \min_{\omega_0} \left\| \mathbf{y} - \mathbf{X} \hat{\boldsymbol{\theta}} \right\|_2^2$$

$$= \arg \min_{\omega_0} \mathbf{y}^T (\mathbf{I} - \boldsymbol{\Pi}_{\mathbf{X}}) \mathbf{y}$$

$$= \arg \max_{\omega_0} \mathbf{y}^T \boldsymbol{\Pi}_{\mathbf{X}} \mathbf{y}$$

where $\boldsymbol{\Pi}_{\mathbf{X}} = \mathbf{X} (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T$ is the projection matrix onto \mathbf{X} .

