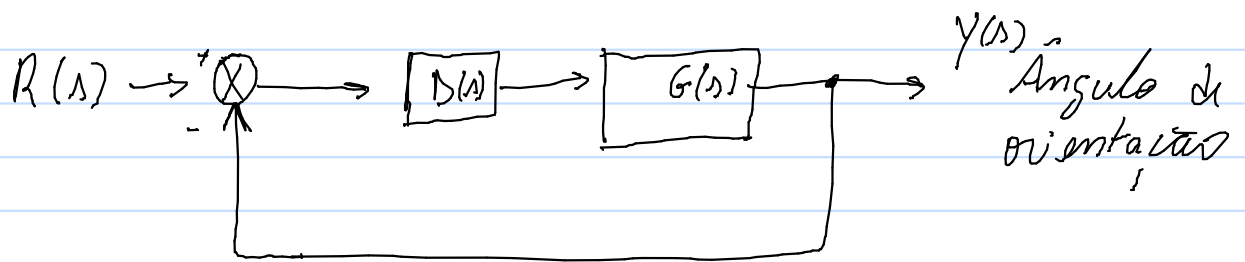


1) Controle de orientação de um satélite espacial



2)

$$G(s) = \frac{10}{(s+1)(s+9)}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -10 & -9 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \frac{10}{s^2 + 10s + 9}$$

$$\Leftrightarrow s^2 Y(s) + 10s Y(s) + 9 Y(s) = 10 U(s)$$

$$\ddot{y}(t) + 10\dot{y}(t) + 9y(t) = 10 \cdot u(t)$$

Considerando a entrada do sistema a posição desejada, e a saída a velocidade angular e o ângulo, temos

$$\ddot{\theta}(t) + 10\dot{\theta}(t) + 9\theta(t) = 10 u(t)$$

Entrada: ? torque
Saída: ? θ

$$3) \frac{Y(s)}{U(s)} = \frac{10}{s^2 + 10s + 9} \Rightarrow \text{poles } 9 \text{ e } 1$$

$$s_1 \rightarrow 9: \tau = \frac{1}{9}$$

$$s_2 \rightarrow 1: \tau = 1s$$

Malha aberta:

$$K_0 = \frac{10}{9}$$

$$\omega_n = 3$$

$$2\zeta\omega_n = 10$$

$$\zeta = \frac{10}{6} = 1,6667$$

↓
Freq Natu. $\bar{\omega}$ amortec.

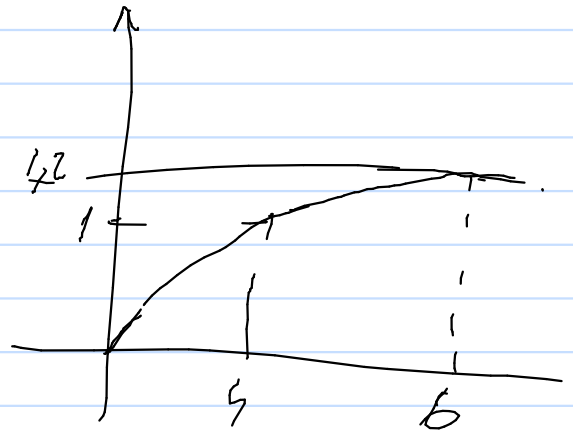
$$G(s) = 1,1111 \cdot \frac{9}{s^2 + 10s + 9}$$

$$\tau = \frac{1}{\sum \omega_n} \Rightarrow \tau = 0,1999s$$

$$M.A: \quad t_s: 6,45s$$

$$0,63 \times \text{Amplitude} = 0,702$$

$$\tau = 1,1$$



Sobre amortecido

$$\frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

τ dominante: Mais lento: $1s$

3)

$$T = 20\% \quad \tau \Rightarrow T = 0,222s$$

4)

$$G(z) = \frac{0,1288z + 0,06241}{z^2 - 0,9365z + 0,1086}$$

(Vai dar ruim)

$$G(z) = \frac{(z + 0,4845)}{(z - 0,8009)(z - 0,1356)}$$

$$5) \quad G(z) = \frac{0,1288z + 0,106241}{z^2 - 0,9365z + 0,1086}$$

$$G(z) = \frac{(z + 0,4845)}{(z - 0,8009)(z - 0,1356)}$$

Fazendo $z = \frac{1 + (T/2)w}{1 - (T/2)w} \Rightarrow$

$$G(w) = \frac{\left(\frac{1 + 0,111w}{1 - 0,111w} \right) + 0,4845}{\left[\left(\frac{1 + 0,111w}{1 - 0,111w} \right) - 0,8009 \right] \left[\left(\frac{1 + 0,111w}{1 - 0,111w} \right) - 0,1356 \right]}$$

Matlab:

$$G(w) = \frac{-0,03246w^2 - 0,5498w + 7,588}{w^2 + 7,853w + 6,829} \quad \left\{ \begin{array}{l} w_{z1} = -25,9451 \\ w_{z2} = 7,0090 \\ w_{p1} = -6,8574 \\ w_{p2} = -0,9959 \end{array} \right.$$

Critério RH:

Polinômio Característico: $w^2 + 7,853w + 6,829$

w^2	1	6,829
w^1	7,853	0
w^0	6,829	

Não houve troca de sinal, sistema só possui polos com parte real negativa. Estável.

$$T(w) = \frac{k_c G(w)}{1 + k_c G(w)}$$

$$1 + k_c \left(\frac{-0,03246w^2 - 0,5498w + 7,588}{w^2 + 7,853w + 6,829} \right)$$

$$T(w) = \frac{k_c G(w)}{1 + k_c G(w)}, \text{ Como } G(w) \text{ é SLIT 2ª ordem} \downarrow$$

$$1 + k_c \left(\frac{-0,03246w^2 - 0,5498w + 7,588}{w^2 + 7,853w + 6,829} \right) > 0$$

$$w^2 + 7,853w + 6,829 - 0,03246w^2 k_c - 0,5498w k_c + 7,588k_c > 0$$

$$(1 - 0,03246k_c)w^2 + (7,853 - 0,5498k_c)w + 7,588k_c + 6,829 > 0$$

$$\begin{cases} 1 - 0,03246k_c > 0 & k_c < 30,8071 \\ 7,853 - 0,5498k_c > 0 & \Rightarrow k_c < 14,2834 \\ 6,829 + 7,588k_c > 0 & k_c > -0,8999 \end{cases}$$

$$0 < k_c < 14,2834$$

6) verificação sobre o k_c

$$7) l_{so} = \lim_{z \rightarrow 1} \frac{z}{1 + k G(z)} = 0,05$$

$$\frac{1}{1 + k \left(\frac{0,1288 + 0,0624z}{1 - 0,9365z + 0,1086z^2} \right)} = 0,05$$

$$\frac{1}{1 + 1,11098k} = 0,05$$

$$0,05 + 0,0555k = 1$$

$$k = \frac{1 - 0,05}{0,0555} \Rightarrow k = 17,1171$$

Fora da faixa

$$\frac{1}{1 + 1,11098k} = 0$$

p/ $k = 1$

$$\phi = 0,4737$$

$$\phi = 47,37\%$$

$$p/ k = 14,2 < 14,2834$$

$$\phi = 0,05960$$

$$\phi = 5,96\%$$

4) $P/ \quad T = 10\% T \Rightarrow T = 0,11$

$$G(z) = \frac{0,04293z + 0,02981}{z^2 - 1,267z + 0,3329}$$

$$G(z) = \frac{z + 0,6944}{(z - 0,8958)(z - 0,3716)}$$

5)

$$G(w) = G(z) \left| \begin{array}{l} z = \frac{1 + (T/2)w}{1 - (T/2)w} \end{array} \right.$$

$$G(w) = \frac{-0,005045w^2 - 0,4168w + 9,247}{w^2 + 9,329w + 8,322}$$

• RH

w^2	1	8,322
w^1	9,329	
w^0	8,322	

Não houve troca de sinal, sistema só possui polos com parte real negativa. Estável.

$$T(w) = \frac{K_c G(w)}{1 + K_c G(w)} \quad \text{importante que } 1 + K_c G(w) > 0$$

$$w^2 + 9,329w + 8,322 + K_c[-0,005045w^2 - 0,4168w + 9,247] > 0$$

$$w^2(1 - 0,005045K_c) + w(9,329 - 0,4168K_c) + 9,247K_c + 8,322 > 0$$

$$\left\{ \begin{array}{l} 1 - 0,005045K_c > 0 \Rightarrow K_c < 198,2161 \\ 9,329 - 0,4168K_c > 0 \Rightarrow K_c < 22,3824 \\ 8,322 + 9,247K_c > 0 \Rightarrow K_c > -0,8999 \end{array} \right.$$

$$0 < K_c < 22,3824$$

6) OK (Roots)

$$0 < K_c < 22,3824$$

$$G(z) = \frac{0,04293z + 0,02981}{z^2 - 1,267z + 0,3329}$$

$$7) I_{ss} = \lim_{z \rightarrow 1} \frac{z}{1 + K_c G(z)} = 0,05$$

$$\frac{1}{1 + K_c \left(\frac{0,04293 + 0,02981}{1 - 1,267 + 0,3329} \right)} = 0,05$$

$$= \frac{1}{1 + 1,1038 K_c} = 0,05 \Rightarrow 0,05519 K_c + 0,05 = 1$$

$$K_c = 17,2133$$

Dentro da faixa

$$\hookrightarrow e = 0,4999$$

Erros mínimos

$$\frac{1}{1 + 1,1038 K_c} = e$$

$$p/ K = 1$$

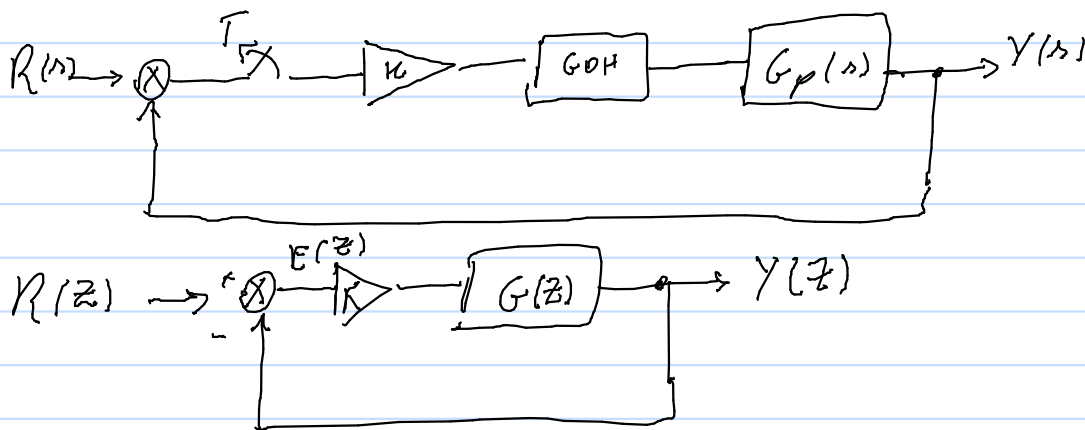
$$e = 0,4753$$

$$e = 47,53\%$$

$$p/ K = 22,3 < 22,3824$$

$$e = 0,0390$$

$$e = 3,90\%$$



$$E(z) = R(z) - Y(z)$$

$$G_{mf}(z) = \frac{Y(z)}{R(z)}$$

$$\Rightarrow R(z) = \frac{Y(z)}{G_{mf}(z)}$$

8) Dead Beat

$$D(z) = \frac{1}{G(z)} \cdot \frac{z^{-k}}{1 - z^{-k}}, \text{ mas } k = 1$$

$$D(z) = \frac{1}{G(z)} \cdot \frac{z^{-1}}{1 - z^{-1}} \cdot \frac{z}{z}$$

$$D(z) = \frac{1}{G(z)} \cdot \frac{1}{z - 1}$$

p/ $T = 0,11$

$$D(z) = \frac{23,296 (z - 0,8958)(z - 0,3716)}{(z + 0,6944)(z - 1)}$$

Zeros: $0,8958$; $0,3716$

Poles: $-0,6944$, 1

$$M(z) = \frac{D(z) G(z)}{1 + D G(z)} = \frac{(z - 0,8958)(z - 0,3716)(z + 0,6944)}{z(z - 0,3716)(z - 0,8958)(z + 0,6944)}$$

9) Dahlin

$$T_d(z) = \frac{1 - e^{-T/\tau_d}}{1 - e^{-T/\tau_d} z^{-1}} z^{-k}, \quad k \gg 1$$

$$q = e^{-T/\tau_d}$$

$$D(z) = \frac{1}{G(z)} \frac{(1 - q) z^{-k}}{1 - q z^{-1} - (1 - q) z^{-k}}, \quad k \gg 1, \quad \begin{matrix} \tau_1 G(z) = 0,999 \\ \tau_2 G(z) = 0,1111 \end{matrix}$$

Portanto, quero $\tau_d = 0,1111$ e $k = 1$,

$$D(z) = \frac{1}{G(z)} \frac{(1 - q)}{z - q - (1 - q)}$$

$$D(z) = \frac{1}{G(z)} \frac{1 - q}{z - 1}$$

Alocação de Pólos:

$$G(z) = \frac{(z + 0,6944)}{(z - 0,8958)(z - 0,3716)}$$

↙

$$T = 0,9999$$

$$\rightsquigarrow T = 0,1111$$

Quero anular o polo mais lento.

Resposta $G(z)$

$$D(z) = \frac{(z - 0,8958)(z - 0,3716)}{(z - 0,504)(z - 1)}$$