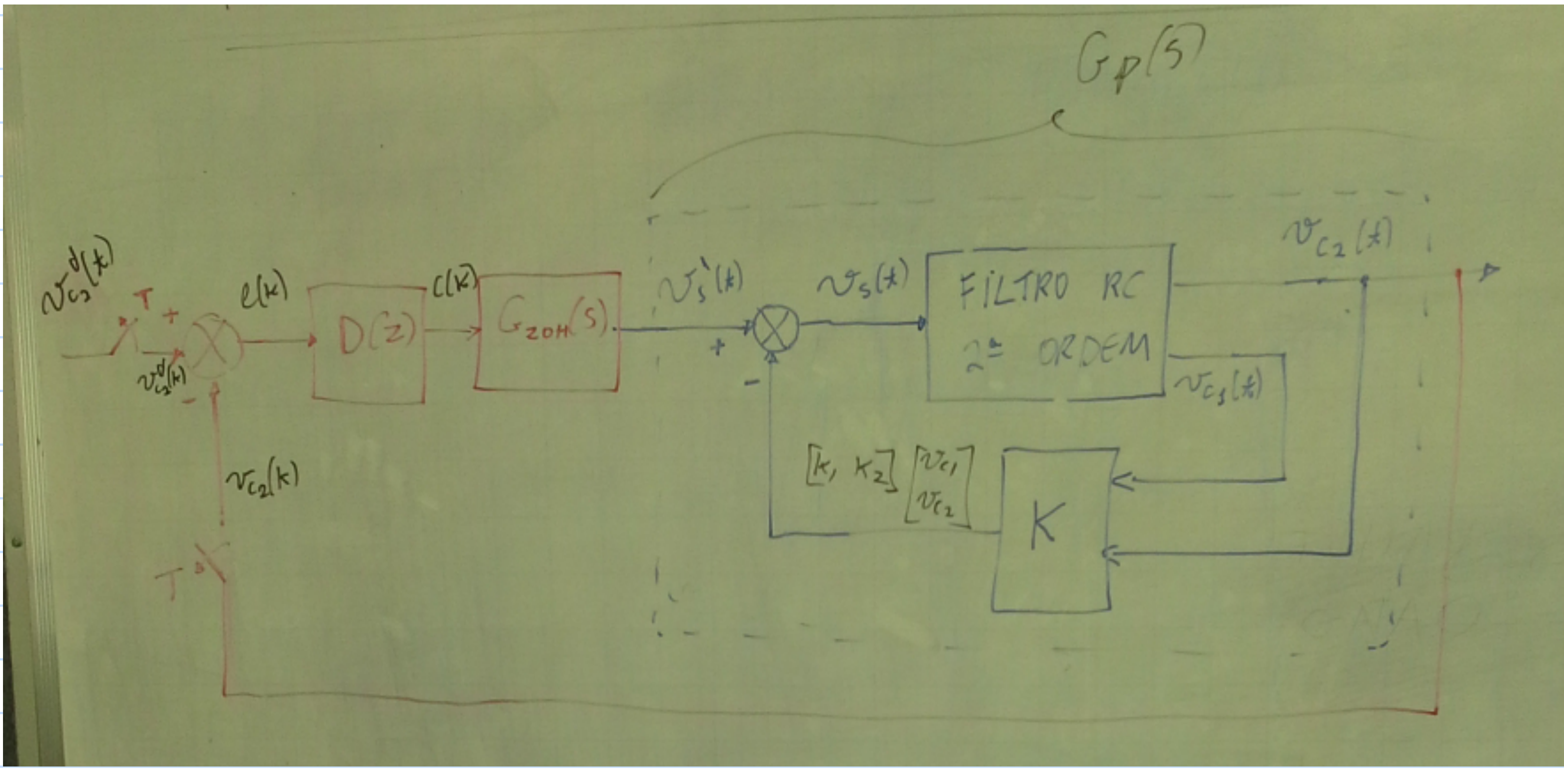
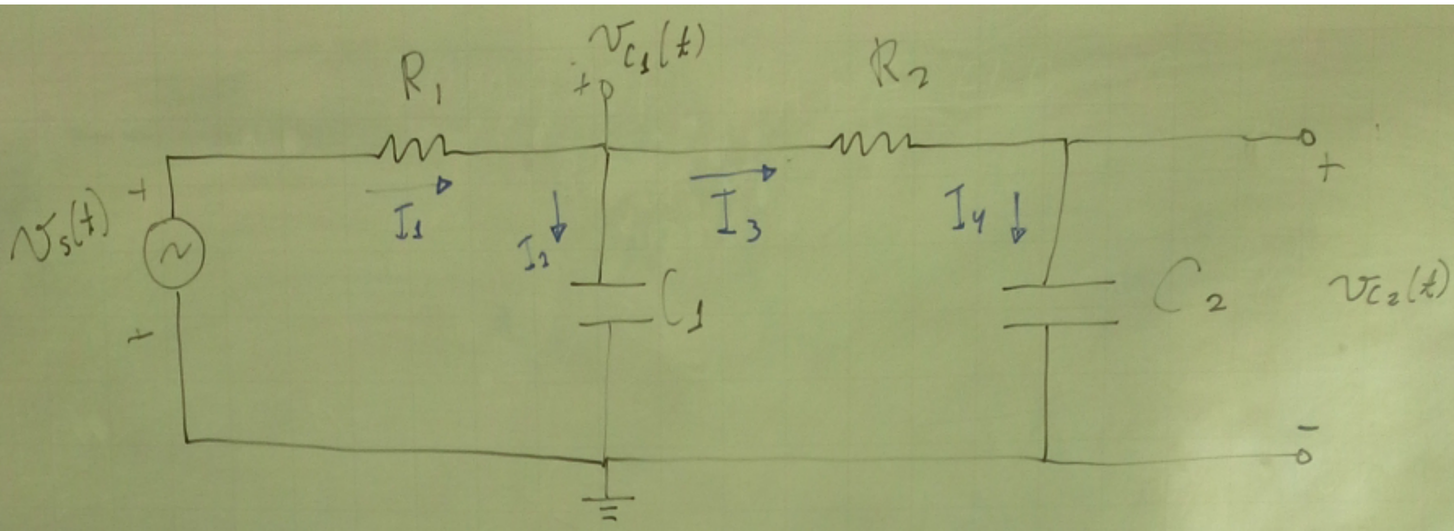


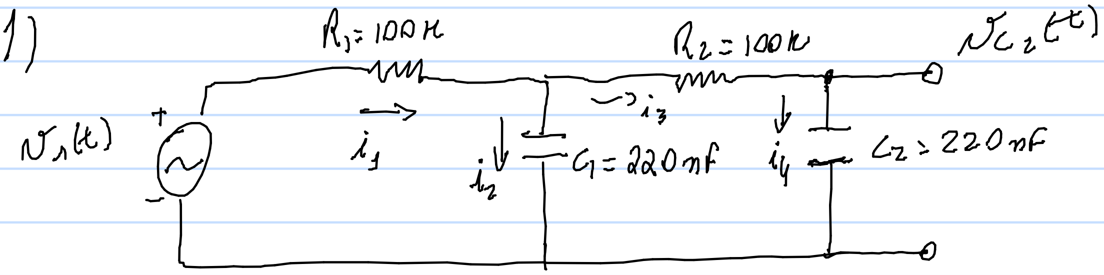


$$I_1 = I_2 + I_3 \longrightarrow \boxed{\frac{V_s - V_{c_1}}{R_1} = \frac{V_{c_1}(s)}{\frac{1}{C_1 s}} + \frac{V_{c_1} - V_{c_2}}{R_2}}$$

$$I_3 = I_4 \Longrightarrow \boxed{\frac{V_{c_1} - V_{c_2}}{R_2} = \frac{V_{c_2}(s)}{1/C_2 s}}$$

$$\boxed{y = v_{c_2}}$$

1)



KCL: (I) $i_1 = i_2 + i_3 = \dfrac{V_s - V_{C_1}}{R_1} = \dfrac{V_{C_1}}{1/C_1 s} + \dfrac{V_{C_1} - V_{C_2}}{R_2}$

(II) $i_3 = \dfrac{V_{C_1} - V_{C_2}}{R_2} = \dfrac{V_{C_2}}{1/C_2 s}$

$Y = V_{C_2}$

Variáveis de estado: V_{C_1} e V_{C_2}

$$V_{C_1} = \left(\frac{V_s - V_{C_1}}{R_1} - \frac{(V_{C_1} - V_{C_2})}{R_2}\right) \cdot \frac{1}{C_1 s}$$

por (I)

$$V_{C_1} = \frac{V_s - 2V_{C_1} + V_{C_2}}{R C_1 s},$$

$R_1 = R_2 = R$

Como $C_1 = C_2 = C$

$$V_{C_1} = \frac{V_s - 2V_{C_1} + V_{C_2}}{RCs}$$

$$RCs \cdot V_{C_1} = V_s - 2V_{C_1} + V_{C_2}$$

$$RCs\, V_{C_1} + 2V_{C_1} = V_s + V_{C_2}$$

$$\boxed{V_{C_1} = \frac{V_s + V_{C_2}}{RCs + 2}} \quad \text{(III)}$$

- $\dfrac{V_{C_2}}{1/C_2 s} = \dfrac{V_{C_1} - V_{C_2}}{R_2}$ (por II)

$$V_{C_2} = \frac{V_{C_1} - V_{C_2}}{R_2 C_2 s}$$

$$\boxed{V_{C_2} = \frac{V_{C_1}}{RCs + 1}} \quad \text{(IV)}$$

$$\bullet \quad V_{c_1} = \frac{V_s + V_{c_2}}{RCs + 2}$$

$$RCsV_{c_1} + 2V_{c_1} = V_s + V_{c_2}$$

$$RC\,\dot{V}_{c_1}(t) + 2V_{c_1}(t) = V_s(t) + V_{c_2}(t)$$

$$\boxed{\dot{V}_{c_1}(t) = \left(-2V_{c_1}(t) + V_{c_2}(t) + V_s(t)\right)\frac{1}{RC}} \quad (V)$$

$$\bullet \quad V_{c_2} = \frac{V_{c_1}}{RCs+1} \Rightarrow RCsV_{c_2} + V_{c_2} = V_{c_1} \overset{\mathcal{L}^{-1}}{\Rightarrow} RC\,\dot{V}_{c_2}(t) + V_{c_2}(t) = V_{c_1}(t)$$

$$\boxed{\dot{V}_{c_2}(t) = \left(-V_{c_2}(t) + V_{c_1}(t)\right)\cdot\frac{1}{RC}} \quad (VI)$$

$y = V_{c_2}(t)$
$u = V_s(t)$

$$\begin{bmatrix}\dot{V}_{c_1}(t)\\ \dot{V}_{c_2}(t)\end{bmatrix} = \frac{1}{RC}\begin{bmatrix}-2 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}V_{c_1}(t)\\ V_{c_2}(t)\end{bmatrix} + \frac{1}{RC}\begin{bmatrix}1\\ 0\end{bmatrix}\cdot V_s(t)$$

$$y = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}V_{c_1}(t)\\ V_{c_2}(t)\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}\cdot V_s(t)$$

2)

$$G(s) = C(sI - A)^{-1}B + D$$

$\Downarrow$ Magica do Matlab:

$$G(s) = \frac{2066}{s^2 + 136{,}4s + 2066}$$

Polos: $-119{,}0015$
$-17{,}3621$

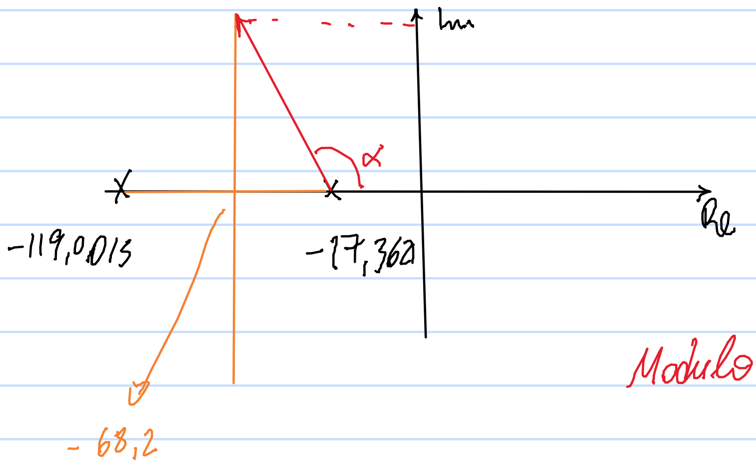
$$G(s) = \frac{2066}{(s+119{,}0015)(s+17{,}3621)}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Bigg\} \quad \omega_n = 45{,}4532$$
$$\zeta = 1{,}5$$

4)

$$G(s) = \frac{2066}{s^2 + 136{,}4s + 2066}$$

Polos: $-119{,}0015$
$-17{,}3621$



Modulo:

$$1 = \left| \frac{\cancel{(s+17{,}3621)}}{(s+\sigma+j\omega)(s+\sigma-j\omega)} \cdot \frac{2066}{(s+119{,}0015)\cancel{(s+17{,}3621)}} \right|$$

$$0 \leq \frac{2066}{(s+\sigma \pm j\omega)(s+119{,}0015)} \leq 1$$

$$2066 \leq (s+\sigma \pm j\omega)(s+119{,}0015)$$

$$G(s) = \frac{2066}{s^2 + 136{,}4s + 2066}$$

$$t_s = 2s \Rightarrow t_s = \frac{4}{\sigma} = 2 \Rightarrow \sigma = 2 \text{ rad/s}$$

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\sigma = \xi \omega_n = 2 \Rightarrow \boxed{\omega_n = \frac{2}{\xi}} \qquad t_p = \frac{\pi}{\omega_d}$$

$$M.P. = e^{-\xi \omega_n t} = 0{,}2$$

$$M.P = 0{,}2 = e^{-\xi \cancel{\omega_d}/\sqrt{1-\xi^2} \cdot \frac{\pi}{\cancel{\omega_d}}} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\ln 0{,}2 = \frac{-\xi\pi}{\sqrt{1-\xi^2}} \Rightarrow \xi = \sqrt{\frac{1}{1+\frac{\pi^2}{(\ln 0{,}2)^2}}} = 0{,}4559$$

$$\omega_n = \frac{45{,}4533}{0{,}4559} = 99{,}7002 \text{ rad/s}$$

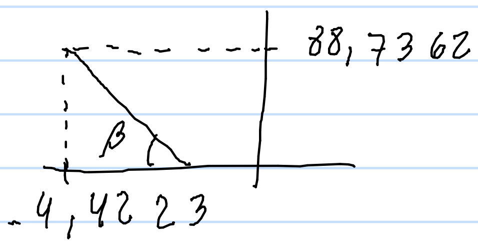
$$\sigma = \xi \omega_n = 4{,}4223$$

$$\omega_d = \omega_n \sqrt{1-\xi^2} = 88{,}7362$$

$$E.C = (s + \sigma \pm j\,\omega_d)$$

$$\boxed{E.C = s + 4{,}4223 \pm j\,88{,}7362}$$

$$D(s)\,G(s) = -1 = 1 \angle 180^\circ$$



$$\beta = tg^{-1} \frac{88{,}7362}{4{,}4223} = 87{,}1469^\circ$$

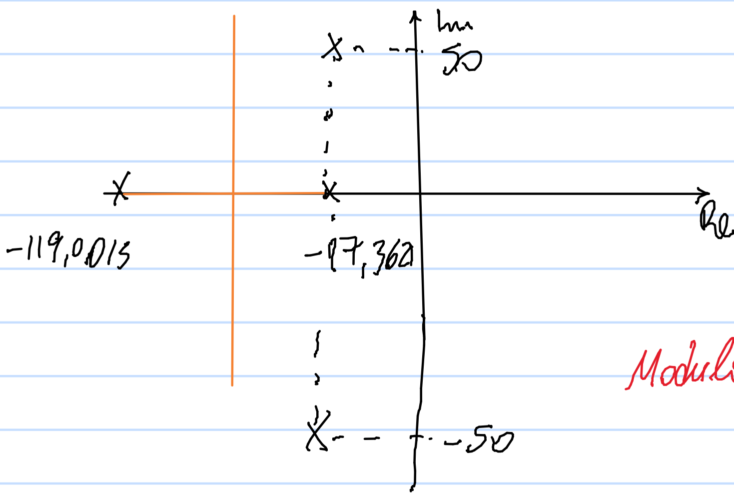
- Contribuição da planta no ponto:

$G(s = -4{,}4223 + j88{,}7362) =$

$$G(s) = \frac{2066}{(-4{,}4223 + j\,88{,}7362)^2 + 136{,}4(-4{,}4223 + j\,88{,}7362) + 2066}$$

$$= 0{,}1589 \angle -119{,}4536$$

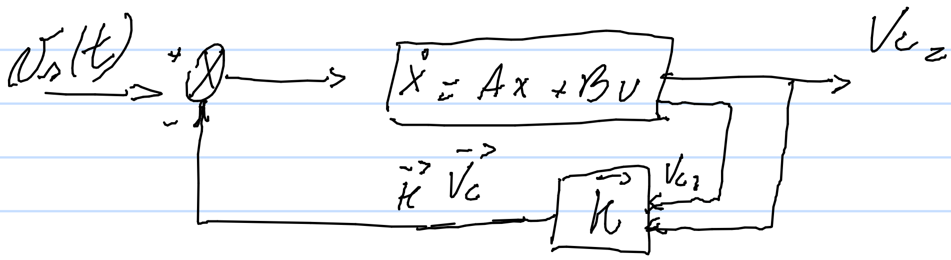
Ignorando tudo:



Modulo:

Novos Polos: $-17,3621 \pm j50$

$$K = [-2,2361 \quad 2,5920]$$



$$v_s(t) = \vec{K}\cdot\vec{V_c} = [-2,2361 \quad 2,5920]\begin{bmatrix} v_{c1} \\ v_{c2} \end{bmatrix}$$

8) $G_p(s) = \dfrac{2066}{s^2 + 34,72s + 2801}$ $\rightarrow$ $\tau_{Gp} = 0,0332$

9) $\rightarrow$ $T = \dfrac{1}{5} \cdot 0,0332 = 0,00664$

$$G(z) = \frac{0,04187\,z + 0,03871}{z^2 - 1,685\,z + 0,7941} \quad \text{(Matlab)}$$

10) Final do arq. projeto_alocacao_polos.m

11) PID Tool

Dahlin :

$$\frac{C(Z)}{E(Z)} = \frac{2,9742\,(Z^2 - 1.685Z + 0,7941)}{Z^2 - 0,0742\,Z - 0,9258}$$

$$(Z^2 - 0,0742\,Z - 0,9258)\,C(Z)$$

$$= \left(2,9742\,(Z^2 - 1.685Z + 0,7941)\right) E(Z)$$

$$Z^2 C(Z) - 0,0742\,Z\,C(Z) - 0,9258\,C(Z) =$$

$$2,9742\,Z^2 E(Z) - 5,01162\,Z\,E(Z) + 2,3618\,E(Z)$$

$$\Downarrow\ Z^{-1}$$

$$C_{k+2} - 0,0742\,C_{k+1} - 0,9258\,C_k = 2,9742\,E_{k+2} - 5,01152\,E_{k+1} + 2,3618\,E_k$$

$$C_k = 0,0742\,C_{k-1} + 0,9258\,C_{k-2} + 2,9742\,E_k - 5,01152\,E_{k-1} + 2,3618\,E_{k-2}$$