

Scientific Computing - Exercise Sheet 2

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1 Exercise

- (a) Computation for t_{total}^ν , where $\nu = 0.01$ and $p = 10^k$ for $k \in \{1, \dots, 5\}$:

$$\begin{aligned} t_{total}^\nu &= \nu t_1^N + (1 - \nu) t_p^N \\ &= 10^{-2} \cdot 10^5 h + (1 - 10^{-2}) \cdot 10^5 h \cdot 10^{-k} \\ &= 1000h + 0.99 \cdot 10^{5-k} h \end{aligned}$$

Values for different k :

| k | t_{total}^ν |
|-----|-----------------|
| 1 | 10900h |
| 2 | 1990h |
| 3 | 1099h |
| 4 | 1009.9h |
| 5 | 1000.99h |

- (b) The theoretical speedup and the theoretical efficiency are the following values:

$$\begin{aligned} S_p^T &= \frac{t_1^N}{t_p^N} = 10^k, \quad \text{for } k \in \{1, \dots, 5\} \\ E_p^T &= \frac{t_1^N}{p t_p^N} = 1, \end{aligned}$$

where we still use $p = 10^k$ for $k \in \{1, \dots, 5\}$.

We have to exchange the theoretical computing time t_p^N where we assumed that every part of the algorithm can be parallelized with the real computing time t_{total}^ν which include that we can't parallelize just 99% of the algorithm. Therefore we have a look at the effective speedup S_p^E and efficiency E_p^E . It holds

$$\begin{aligned} S_p^E &= \frac{t_1^N}{t_{total}^\nu} = \frac{10^5}{t_{total}^\nu} = \frac{10^5}{1000h + 0.99 \cdot 10^{5-k} h} \\ E_p^E &= \frac{t_1^N}{p t_{total}^\nu} = \frac{10^5}{p t_{total}^\nu} = \frac{10^5}{p(1000h + 0.99 \cdot 10^{5-k} h)} \end{aligned}$$

Values for different k :

| k | S_p | E_p |
|-----|-------|----------|
| 1 | 9.17 | 0.917 |
| 2 | 50.25 | 0.5025 |
| 3 | 90.99 | 0.09099 |
| 4 | 99.02 | 0.009902 |
| 5 | 99.9 | 0.000999 |

(c) It holds:

$$t_{total}^\nu = \nu t_1^N + (1 - \nu) t_p^N = t_s^N + (1 - \nu) \underbrace{\frac{t_1^N}{p}}_{\rightarrow 0} \xrightarrow{p \rightarrow \infty} t_s^N = 1000.$$

Using that we compute the limit of the speedup and the efficiency:

$$S_p = \frac{t_1^N}{t_{total}^\nu} \xrightarrow{p \rightarrow \infty} \frac{t_1^N}{t_s^N} = \frac{1}{\nu} = 100$$

$$E_p = \frac{t_1^N}{p t_{total}^\nu} \xrightarrow{p \rightarrow \infty} 0,$$

where the last limit results from the convergence of t_{total}^ν to a positive number such that $p t_{total}^\nu \rightarrow \infty$ when $p \rightarrow \infty$.

(d) Now the communication time overhead t_c is increasing by 1 when p increases by 10. Since we look on $p = 10^k$, we can identify $t_c = \frac{p}{10}$. It follows

$$S_p^c = \frac{t_1^N}{t_p^N + t_c} = \frac{10^5}{\frac{10^5}{p} + \frac{p}{10}} \xrightarrow{k \rightarrow \infty} 0,$$

where $t_1^N = 10^5$.

Now we look at the speedup while insert values $p = 10^k$, $k \in \{1, \dots, 5\}$:

| k | S_p^c |
|-----|---------|
| 1 | 9.99 |
| 2 | 99.01 |
| 3 | 500 |
| 4 | 99.01 |
| 5 | 9.99 |