Scientific Computing - Exercise Sheet 1

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1 Exercise

(a) Input: $a, b, k \in \mathbb{R}$

load(k)
load(a)
$l_1 \leftarrow$ **mult**(a, k); **load**(b)
$l_2 \leftarrow$ **add**(l_1, b)
store(l_2)

(b) Input: $a, b \in \mathbb{R}^d$, $k \in \mathbb{R}$
In the following we do not use the actual 'for' instruction because it is not contained in the instruction set. We just use it to clarify what's going on.

load(k)
for $i \leftarrow 1 : d$ **do**
    load(a_i)
    $l_{2i-1} \leftarrow$ **mult**(k, a_i)
    load(b_i)
    $l_{2i} \leftarrow$ **add**(l_{2i-1}, b_i)
    store(l_{2i})
end for

Number of instructions: $5d + 1$
In particular, we have 51 instructions for $d = 10$.

(c) Input: $a, b \in \mathbb{R}^d$, $k \in \mathbb{R}$
In the following we do not use the actual 'for' instruction because it is not contained in the instruction set. We just use it to clarify what's going on.

load(k)
load(a_1)
$l_1 \leftarrow$ **mult**(k, a_1); **load**(b_1)
$l_2 \leftarrow$ **add**(l_1, b_1); **load**(a_2)
for $i \leftarrow 2 : (d-1)$ **do**
    $l_{2i-1} \leftarrow$ **mult**(k, a_i); **store**(l_{2i-2}); **load**(b_i)
    $l_{2i} \leftarrow$ **add**(l_{2i-1}); **load**(a_{i+1})
end for

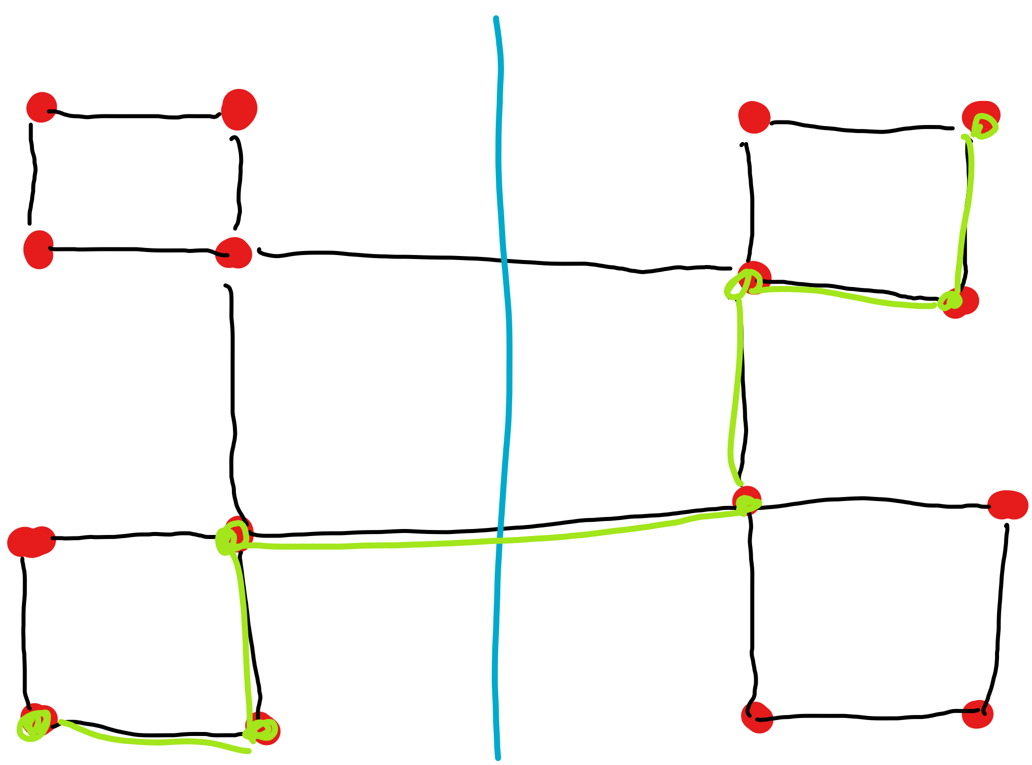
9: $l_{2d-1} \leftarrow$ **mult**(k, a_d); **store**(l_{2d-2}); **load**(b_d)
10: $l_{2i} \leftarrow$ **add**(l_{2d-1})
11: **store**(l_{2d})

Number of instructions: $2d + 3$
In particular, we have 23 instructions for $d = 10$.
On the optimality of the algorithm:
In total there are $1 + d + d = 2d + 1$ to load into registry. Only when all values are in registry all necessary calculations can be made. Additionally, one cycle to perform the last calculation and one cycle to store the result into main memory are required. therefore, an algorithm using the proposed instruction set contains at least $(2d+1)+1+1 = 2d+3$ instructions. Thus, the proposed algorithm has minimal number of instructions for the given architecture.

Both algorithms - parallel and serial - have a computation time that depends linearly on d. However, the serial algorithm has a factor of 5 while the parallel one has a factor of 2. Therefore, for large d a significant performance increase can be achieved by the parallel algorithm.

2 Exercise

a) Sketch of the 4×4 ring:



b) The network has got $4 \cdot 4 = 16$ processors. (Red nods.)

c) The bisection bandwidth is 2. (Blue cut.) There is no opportunity to split the network while cutting just one connection.

d) The longest communication part is 6. It is the number of hops from one 'outside' nod to the diagonal 'outside' nod. (Green path.)