# Scientific Computing - Exercise Sheet 2

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## 1 Exercise

(a) Computation for $t^{\nu}_{total}$, where $\nu = 0.01$ and $p = 10^k$ for $k \in \{1, \dots, 5\}$:

$$\begin{aligned} t^{\nu}_{total} &= \nu t^N_1 + (1-\nu) t^N_p \\ &= 10^{-2} \cdot 10^5 h + (1 - 10^{-2}) \cdot 10^5 h \cdot 10^{-k} \\ &= 1000h + 0.99 \cdot 10^{5-k} h \end{aligned}$$

Values for different $k$:

| $k$ | $t^{\nu}_{total}$ |
|---|---|
| 1 | 10900h |
| 2 | 1990h |
| 3 | 1099h |
| 4 | 1009.9h |
| 5 | 1000.99h |

(b) The theoretical speedup and the theoretical efficiency are the following values:

$$\begin{aligned} S^T_p &= \frac{t^N_1}{t^N_p} = 10^k, \qquad \text{for } k \in \{1, \dots, 5\} \\ E^T_p &= \frac{t^N_1}{p t^N_p} = 1, \end{aligned}$$

where we still use $p = 10^k$ for $k \in \{1, \dots, 5\}$.
We have to exchange the theoretical computing time $t^N_p$ where we assumed that every part of the algorithm can be parallelized with the real computing time $t^N_{total}$ which include that we can't parallelize just 99% of the algorithm. Therefore we have a look at the effective speedup $S^E_p$ and efficiency $E^E_p$. It holds

$$\begin{aligned} S^E_p &= \frac{t^N_1}{t^{\nu}_{total}} = \frac{10^5}{t^{\nu}_{total}} = \frac{10^5}{1000h + 0.99 \cdot 10^{5-k} h} \\ E^E_p &= \frac{t^N_1}{p t^{\nu}_{total}} = \frac{10^5}{p t^{\nu}_{total}} = \frac{10^5}{p(1000h + 0.99 \cdot 10^{5-k} h)} \end{aligned}$$

Values for different $k$:

| $k$ | $S_p$ | $E_p$ |
|---|---|---|
| 1 | 9,17 | 0.917 |
| 2 | 50.25 | 0.5025 |
| 3 | 90.99 | 0.09099 |
| 4 | 99.02 | 0.009902 |
| 5 | 99.9 | 0.000999 |

(c) It holds:

$$t_{total}^{\nu} = \nu t_1^N + (1-\nu)t_p^N = t_s^N + (1-\nu)\underbrace{\frac{t_1^N}{p}}_{\to 0} \xrightarrow[p\to\infty]{} t_s^N = 1000.$$

Using that we compute the limit of the speedup and the efficiency:

$$\begin{aligned} S_p^E &= \frac{t_1^N}{t_{total}^{\nu}} \xrightarrow[p\to\infty]{} \frac{t_1^N}{t_s^N} = \frac{1}{\nu} = 100 \\ E_p^E &= \frac{t_1^N}{p t_{total}^{\nu}} \xrightarrow[p\to\infty]{} 0, \end{aligned}$$

where the last limit results from the convergence of $t_{total}^{\nu}$ to a positive number such that $p t_{total}^{\nu} \to \infty$ when $p \to \infty$.

(d) Now the communication time overhead $t_c$ is increasing by 1 when $p$ increases by 10. Since we look on $p = 10^k$, we can identify $t_c = \frac{p}{10}$. It follows

$$S_p^c = \frac{t_1^N}{t_p^N + t_c} = \frac{10^5}{\frac{10^5}{p} + \frac{p}{10}} \xrightarrow[k\to\infty]{} 0,$$

where $t_1^N = 10^5$.
Now we look at the speedup while insert values $p = 10^k$, $k \in \{1, \dots, 5\}$:

| $k$ | $S_p^c$ |
|---|---|
| 1 | 9.99 |
| 2 | 99.01 |
| 3 | 500 |
| 4 | 99.01 |
| 5 | 9.99 |

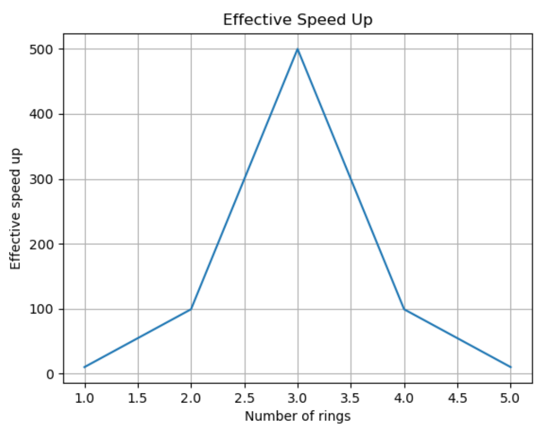


Figure 1: Effective speedup depending on the number of rings