Scientific Computing - Exercise Sheet 1

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1 Exercise

Notation:

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(a) Input: \mathbf{A} \in \mathbb{R}^{n \times n} b, x_0 \in \mathbb{R}^n
  1: for i = 1, 2, ..., n do
                 h^{(0)} = \mathbf{A}x^{(0)}
                 r^{(0)} = b - h^{(0)}
  3:
                 p^{(0)} = r^{(0)}
  5: end for
  6: for k = 1, 2, \dots do
                 \gamma_{(k-1)} = 0
                 \beta_{(k-1)} = 0
  8:
                 \delta_{(k-1)} = 0
  9:
10:
                 \zeta_{(k-1)} = 0
                 \textbf{begin parallel private}(i,\gamma_{(k-1)},\beta_{(k-1)},\delta_{(k-1)},\zeta_{(k-1)}) \textbf{ shared}(A,r^{(k-1)},p^{(k-1)},h^{(k-1)})
11:
                 \begin{aligned} \mathbf{reduce}(+:\gamma_{(k-1)},\beta_{(k-1)},\delta_{(k-1)},\zeta_{(k-1)}) \\ \mathbf{for} \ i &= 1,2,\dots,n \ \mathbf{do} \\ h_i^{(k-1)} &= \mathbf{A} p_i^{(k-1)} \end{aligned} 
12:
13:
14:
                       \begin{split} & \gamma_{(k-1)} = \gamma_{(k-1)} + p_i^{(k-1)} h_i^{(k-1)} \\ & \beta_{(k-1)} = \beta_{(k-1)} + r_i^{(k-1)} r_i^{(k-1)} \\ & \delta_{(k-1)} = \delta_{(k-1)} + r_i^{(k-1)} h_i^{(k-1)} \\ & \zeta_{(k-1)} = \zeta_{(k-1)} + h_i^{(k-1)} h_i^{(k-1)} \end{split}
15:
16:
17:
18:
                 end for
19:
                 end parallel
20:
                 \alpha_{(k-1)} = \frac{\beta^{(k-1)}}{\gamma^{(k-1)}}
21:
                 \beta_{(k)} = \beta_{(k-1)} - 2\alpha_{(k-1)}\delta_{(k-1)} + \alpha_{(k-1)}^2\zeta_{(k-1)}
22:
                 begin parallel default(shared) private(i)
23:
                for i = 1, 2, ..., n do x_i^{(k)} = x_i^{(k-1)} + \alpha_{(k-1)} p_i^{(k-1)} r_i^{(k)} = r_i^{(k-1)} - \alpha_{(k-1)} h_i^{(k-1)} p_i^{(k)} = r_i^{(k)} + \frac{\beta_{(k)}}{\beta_{(k-1)}} p_i^{(k-1)}
24:
25:
26:
27:
```

28: end for

29: end parallel

30: end for

(b) Comment: We need a way to sum the result calculated on different processors: There needs to be a reduce operation.