

# Scientific Computing - Exercise Sheet 2

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## 1 Exercise

- (a) Computation for  $t_{total}^\nu$ , where  $\nu = 0.01$  and  $p = 10^k$  for  $k \in \{1, \dots, 5\}$ :

$$\begin{aligned} t_{total}^\nu &= \nu t_1^N + (1 - \nu) t_p^N \\ &= 10^{-2} \cdot 10^5 h + (1 - 10^{-2}) \cdot 10^5 h \cdot 10^{-k} \\ &= 1000h + 0.99 \cdot 10^{5-k} h \end{aligned}$$

Values for different  $k$ :

$k$	$t_{total}^\nu$
1	10900h
2	1990h
3	1099h
4	1009.9h
5	1000.99h

- (b) The theoretical speedup and the theoretical efficiency are the following values:

$$\begin{aligned} S_p^T &= \frac{t_1^N}{t_p^N} = 10^k, \quad \text{for } k \in \{1, \dots, 5\} \\ E_p^T &= \frac{t_1^N}{p t_p^N} = 1, \end{aligned}$$

where we still use  $p = 10^k$  for  $k \in \{1, \dots, 5\}$ .

We have to exchange the theoretical computing time  $t_p^N$  where we assumed that every part of the algorithm can be parallelized with the real computing time  $t_{total}^\nu$  which include that we can't parallelize just 99% of the algorithm. Therefore we have a look at the effective speedup  $S_p^E$  and efficiency  $E_p^E$ . It holds

$$\begin{aligned} S_p^E &= \frac{t_1^N}{t_{total}^\nu} = \frac{10^5}{t_{total}^\nu} = \frac{10^5}{1000h + 0.99 \cdot 10^{5-k} h} \\ E_p^E &= \frac{t_1^N}{p t_{total}^\nu} = \frac{10^5}{p t_{total}^\nu} = \frac{10^5}{p(1000h + 0.99 \cdot 10^{5-k} h)} \end{aligned}$$

Values for different  $k$ :

$k$	$S_p$	$E_p$
1	9.17	0.917
2	50.25	0.5025
3	90.99	0.09099
4	99.02	0.009902
5	99.9	0.000999

(c) It holds:

$$t_{total}^\nu = \nu t_1^N + (1 - \nu) t_p^N = t_s^N + (1 - \nu) \underbrace{\frac{t_1^N}{p}}_{\rightarrow 0} \xrightarrow{p \rightarrow \infty} t_s^N = 1000.$$

Using that we compute the limit of the speedup and the efficiency:

$$S_p = \frac{t_1^N}{t_{total}^\nu} \xrightarrow{p \rightarrow \infty} \frac{t_1^N}{t_s^N} = \frac{1}{\nu} = 100$$

$$E_p = \frac{t_1^N}{p t_{total}^\nu} \xrightarrow{p \rightarrow \infty} 0,$$

where the last limit results from the convergence of  $t_{total}^\nu$  to a positive number such that  $p t_{total}^\nu \rightarrow \infty$  when  $p \rightarrow \infty$ .

(d) Now the communication time overhead  $t_c$  is increasing by 1 when  $p$  increases by 10. Since we look on  $p = 10^k$ , we can identify  $t_c = \frac{p}{10}$ . It follows

$$S_p^c = \frac{t_1^N}{t_p^N + t_c} = \frac{10^5}{\frac{10^5}{p} + \frac{p}{10}} \xrightarrow{k \rightarrow \infty} 0,$$

where  $t_1^N = 10^5$ .

Now we look at the speedup while insert values  $p = 10^k$ ,  $k \in \{1, \dots, 5\}$ :

$k$	$S_p^c$
1	9.99
2	99.01
3	500
4	99.01
5	9.99