Scientific Computing - Exercise Sheet 2

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1 Exercise

(a) Computation for t^{ν}_{total}, where $\nu = 0.01$ and $p = 10^k$ for $k \in \{1, \dots, 5\}$:

$$\begin{aligned} t^{\nu}_{total} &= \nu t_1^N + (1-\nu)t_p^N \\ &= 10^{-2} \cdot 10^5 h + (1 - 10^{-2}) \cdot 10^5 h \cdot 10^{-k} \\ &= 1000h + 0.99 \cdot 10^{5-k} h \end{aligned}$$

Values for different k:

k	t^{ν}_{total}
1	10900h
2	1990h
3	1099h
4	1009.9h
5	1000.99h

(b) The theoretical speedup and the theoretical efficiency are the following values:

$$\begin{aligned} S_p^T &= \frac{t_1^N}{t_p^N} = 10^k, \qquad \text{for } k \in \{1, \dots, 5\} \\ E_p^T &= \frac{t_1^N}{p t_p^N} = 1, \end{aligned}$$

where we still use $p = 10^k$ for $k \in \{1, \dots, 5\}$.
We have to exchange the theoretical computing time t_p^N where we assumed that every part of the algorithm can be parallelized with the real computing time t^N_{total} which include that we can't parallelize just 99% of the algorithm. Therefore we have a look at the effective speedup S_p^E and efficiency E_p^E. It holds

$$\begin{aligned} S_p^E &= \frac{t_1^N}{t^{\nu}_{total}} = \frac{10^5}{t^{\nu}_{total}} = \frac{10^5}{1000h + 0.99 \cdot 10^{5-k} h} \\ E_p^E &= \frac{t_1^N}{p t^{\nu}_{total}} = \frac{10^5}{p t^{\nu}_{total}} = \frac{10^5}{p(1000h + 0.99 \cdot 10^{5-k} h)} \end{aligned}$$

Values for different k:

k	S_p	E_p
1	9,17	0.917
2	50.25	0.5025
3	90.99	0.09099
4	99.02	0.009902
5	99.9	0.000999

(c) It holds:

$$t_{total}^{\nu} = \nu t_1^N + (1-\nu) t_p^N = t_s^N + (1-\nu) \underbrace{\frac{t_1^N}{p}}_{\to 0} \xrightarrow[p\to\infty]{} t_s^N = 1000.$$

Using that we compute the limit of the speedup and the efficiency:

$$S_p^E = \frac{t_1^N}{t_{total}^{\nu}} \xrightarrow[p\to\infty]{} \frac{t_1^N}{t_s^N} = \frac{1}{\nu} = 100$$
$$E_p^E = \frac{t_1^N}{p t_{total}^{\nu}} \xrightarrow[p\to\infty]{} 0,$$

where the last limit results from the convergence of t_{total}^{ν} to a positive number such that $p t_{total}^{\nu} \to \infty$ when $p \to \infty$.

(d) Now the communication time overhead t_c is increasing by 1 when p increases by 10. Since we look on $p = 10^k$, we can identify $t_c = \frac{p}{10}$. It follows

$$S_p^c = \frac{t_1^N}{t_p^N + t_c} = \frac{10^5}{\frac{10^5}{p} + \frac{p}{10}} \xrightarrow[k\to\infty]{} 0,$$

where $t_1^N = 10^5$.
Now we look at the speedup while insert values $p = 10^k$, $k \in \{1, \ldots, 5\}$:

k	S_p^c
1	9.99
2	99.01
3	500
4	99.01
5	9.99

We can see that for p=1000 we have a maximum and then the speedup decays (See figure 1.).

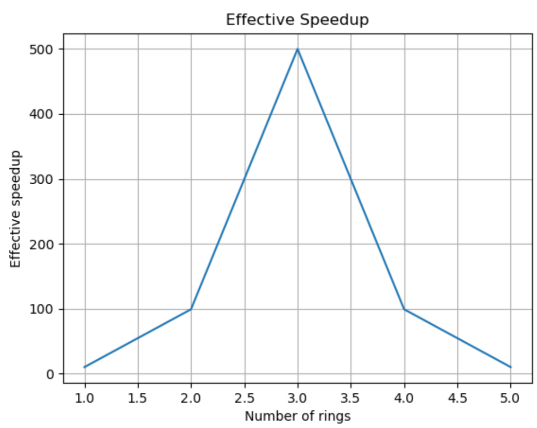

Figure 1: Effective speedup depending on the number of rings