Scientific Computing - Exercise Sheet 2

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1 Exercise

(a) Computation for t^{ν}_{total}, where $\nu = 0.01$ and $p = 10^k$ for $k \in \{1, \dots, 5\}$:

$$\begin{aligned} t^{\nu}_{total} &= \nu t^N_1 + (1-\nu) t^N_p \\ &= 10^{-2} \cdot 10^5 h + (1 - 10^{-2}) \cdot 10^5 h \cdot 10^{-k} \\ &= 1000h + 0.99 \cdot 10^{5-k} h \end{aligned}$$

Values for different k:

k	t^{ν}_{total}
1	10900h
2	1990h
3	1099h
4	1009.9h
5	1000.99h

(b) The theoretical speedup and the theoretical efficiency are the following values:

$$\begin{aligned} S_p &= \frac{t^N_1}{t^N_p} = 10^k, \qquad \text{for } k \in \{1, \dots, 5\} \\ E_p &= \frac{t^N_1}{p t^N_p} = 1. \end{aligned}$$

We have to exchange the theoretical computing time t^N_p where we assumed that every part of the algorithm can be parallelized with the real computing time t^N_{total} which include that we can't parallelize just 99% of the algorithm. Therefore it holds

$$\begin{aligned} S_p &= \frac{t^N_1}{t^{\nu}_{total}} = \frac{10^5}{t^{\nu}_{total}} \\ E_p &= \frac{t^N_1}{p t^{\nu}_{total}} = \frac{10^5}{p t^{\nu}_{total}}. \end{aligned}$$

Values for different k:

k	S_p	E_p
1		
2		
3		
4		
5		

(c) It holds:

$$t^{\nu}_{total} = \nu t^N_1 + (1-\nu)t^N_p = t^N_s + (1-\nu)\underbrace{\frac{t^N_1}{p}}_{\to 0} \xrightarrow[p\to\infty]{} t^N_s = 1000.$$

Using that we compute the limit of the speedup and the efficiency:

$$S_p = \frac{t^N_1}{t^{\nu}_{total}} \xrightarrow[p\to\infty]{} \frac{t^N_1}{t^N_s} = \frac{1}{\nu} = 100$$
$$E_p = \frac{t^N_1}{pt^{\nu}_{total}} \xrightarrow[p\to\infty]{} 0,$$

where the last limit comes up because of the convergence of t^{ν}_{total} to a positive number such that $pt^{\nu}_{total} \to \infty$ when $p \to \infty$.

(d) Now the communication time overhead t_c is increasing by 1 when p increases by 10. Since we look on $p = 10^k$, we can identify $t_c = \frac{p}{10}$. It follows

$$S^c_p = \frac{t^N_1}{t^N_p + t_c} = \frac{10^5}{\frac{10^5}{p} + \frac{p}{10}} \xrightarrow[k\to\infty]{} 0$$

Now we look at the speedup while insert values $p = 10^k$, $k \in \{1, \ldots, 5\}$:

k	S^c_p
1	
2	
3	
4	
5	