Numerical Approaches - Exercise Sheet 1

Exercise 1. (Epsilon) (max. 3 Points)

In numerical analysis epsilon ε is defined as the smallest machine number for which the statement

$$1 + \varepsilon \neq 1$$

holds. One way to determine ε has already been shown in the lecture. The Python script-file `oldeps.py` as well as the MATLAB script `oldeps.m` can be found on our website at STiNE.

a) Save the script in your working directory and run it in your preferred Python or MATLAB environment; record the result.

b) Find another way to determine ϵ by using a `while`-loop and write your own MATLAB or Python script.

c) Compare the results of `oldeps` and your own script.

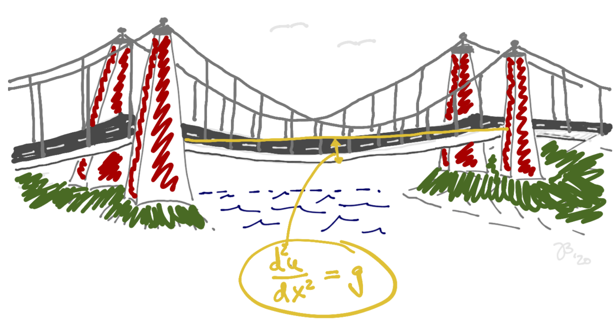
Exercise 2. (Linear System of Equations) (max. 6 Points)

We want to derive a linear system of equations from a differential equation. Here we describe the displacement of a suspension bridge by the force of gravity. This can be described by the following differential equation (simplifying the suspension bridge to a one-dimensional rope):

$$-\frac{d^2u}{dx^2} = g, \quad u \in [a,b] \subset \mathbb{R}, \tag{1}$$

$$u(a) = u(b) = 0, \tag{2}$$

where the bridge spans an area of $[a,b]$ and we assume that the level of the bridge on both sides of the suspension area is normalized to zero. Furthermore, we assume that the force on the bridge is only given by gravity and we normalize $g \equiv -10$ (pointing downwards).



a) Discretize equation (1) by a finite difference approach. Use a forward and a backward finite difference approximation consecutively to show that

$$\frac{d^2u}{dx^2} \approx \frac{1}{h^2}\left[u(x+h) - 2u(x) + u(x-h)\right].$$

b) In order to simplify writing, generate a grid function. The grid with $N + 1$ gridpoints will range from a to b with grid size $h = \frac{b-a}{N}$. What are the grid points x_i?

c) Now, reformulate the above discretized differential operator with respect to the grid points. You may write $u(x_i) = u_i$. From this derive a linear system of equations, with coefficients a_{ij}, unknown values of our function of displacements u_i at grid points, and a right hand side $g_i \equiv -10$.

d) Solve the linear system of equations, using the Python command `solve` from module `numpy.linalg` or the MATLAB backslash operator "\". If possible, plot the result. You may assume the following values:

- $a = 0$, $b = 1$,
- $N = 100$ or $N = 1000$.

The solutions are due on Wednesday, December 02, 2020 Please submit your solutions to the Moodle system.