

# Scientific Computing - Exercise Sheet 1

Jonathan Hellwig, Jule Schütt, Mika Tode, Giuliano Taccogna

19.04.2021

## 1 Exercise

- (a) **Input:**  $\mathbf{A} \in \mathbb{R}^{n \times n}$   $b, x_0 \in \mathbb{R}^n$
- 1:  $h_0 = \mathbf{A}x_0$  (matrix multiplication)
  - 2:  $r_0 = b - h_0$  (1.1 loop)
  - 3:  $p_0 = r_0$  (1.2 loop)
  - 4: **for**  $k = 1, 2, \dots$  **do**
  - 5:    $h_{k-1} = \mathbf{A}p_{k-1}$  (matrix multiplication)
  - 6:    $\gamma_{k-1} = p_{k-1}^T \cdot h_{k-1}$  (2.1 loop)
  - 7:    $\beta_{k-1} = r_{k-1}^T \cdot r_{k-1}$  (2.2 loop)
  - 8:    $\delta_{k-1} = r_{k-1}^T \cdot h_{k-1}$  (2.3 loop)
  - 9:    $\zeta_{k-1} = h_{k-1}^T \cdot h_{k-1}$  (2.4 loop)
  - 10:    $\alpha_{k-1} = \frac{\beta_{k-1}}{\gamma_{k-1}}$
  - 11:    $\beta_k = \beta_{k-1} - 2\alpha_{k-1}\delta_{k-1} + \alpha_{k-1}^2\zeta_{k-1}$
  - 12:    $x_k = x_{k-1} + \alpha_{k-1}p_{k-1}$  (2.5 loop)
  - 13:    $r_k = r_{k-1} - \alpha_{k-1}h_{k-1}$  (2.6 loop)
  - 14:    $p_k = r_k + \frac{\beta_k}{\beta_{k-1}}p_{k-1}$  (2.7 loop)
  - 15: **end for**

All sections which are marked to be a loop can be somehow parallelized. The minimal number of loop within the iteration loops is 2, since the loops 2.1, 2.2, 2.3, 2.4 and the loops 2.5, 2.6, 2.7 can be summarized to one loop respectively.

- (b) **Input:**  $\mathbf{A} \in \mathbb{R}^{n \times n}$   $b, x_0 \in \mathbb{R}^n$
- 1:  $h_0 = \mathbf{A}x_0$  (matrix multiplication)
  - 2: **begin parallel private**( $i, b, h_0$ ) **shared**( $r_0, p_0$ )
  - 3:  $r_0 = b - h_0$  (1.1 loop)
  - 4:  $p_0 = r_0$  (1.2 loop)
  - 5: **end parallel**
  - 6: **for**  $k = 1, 2, \dots$  **do**
  - 7:    $h_{k-1} = \mathbf{A}p_{k-1}$  (matrix multiplication)
  - 8:   **begin parallel private**( $i, r_{k-1}, p_{k-1}, h_{k-1}$ ) **shared**( $\gamma_{(k-1)}, \beta_{(k-1)}, \delta_{(k-1)}, \zeta_{(k-1)}$ )
  - 9:    $\gamma_{k-1} = p_{k-1}^T \cdot h_{k-1}$  (2.1 loop)

```

10:  $\beta_{k-1} = r_{k-1}^T \cdot r_{k-1}$  (2.2 loop)
11:  $\delta_{k-1} = r_{k-1}^T \cdot h_{k-1}$  (2.3 loop)
12:  $\zeta_{k-1} = h_{k-1}^T \cdot h_{k-1}$  (2.4 loop)
13: end parallel
14:  $\alpha_{k-1} = \frac{\beta_{k-1}}{\gamma_{k-1}}$ 
15:  $\beta_k = \beta_{k-1} - 2\alpha_{k-1}\delta_{k-1} + \alpha_{k-1}^2\zeta_{k-1}$ 
16: begin parallel private( $i, x_{k-1}, \alpha_{k-1}, p_{k-1}, r_{k-1}, h_{k-1}, r_k, \beta_k, \beta_{k-1}$ )
    shared( $x_k, r_k, p_k$ )
17:  $x_k = x_{k-1} + \alpha_{k-1}p_{k-1}$  (2.5 loop)
18:  $r_k = r_{k-1} - \alpha_{k-1}h_{k-1}$  (2.6 loop)
19:  $p_k = r_k + \frac{\beta_k}{\beta_{k-1}}p_{k-1}$  (2.7 loop)
20: end parallel
21: end for

```

Here  $i$  denotes the counter, which is needed for the loop operation. Notice that for the Vectormultiplications in the loops 2.1, 2.2, 2.3 and 2.4 there normally also reduce operations should be included.

(c) **Input:**  $\mathbf{A} \in \mathbb{R}^{n \times n}$   $b, x_0 \in \mathbb{R}^n$

```

1:  $h_0 = \mathbf{A}x_0$  (matrix multiplication)
2:  $r_0 = b - h_0$  (1.1 loop)
3: barrier
4: send( $r_0$ )
5: recv( $r_0$ )
6:  $p_0 = r_0$  (1.2 loop)
7: barrier
8: send( $p_0$ )
9: recv( $p_0$ )
10: for  $k = 1, 2, \dots$  do
11:  $h_{k-1} = \mathbf{A}p_{k-1}$  (matrix multiplication)
12:  $\gamma_{k-1} = p_{k-1}^T \cdot h_{k-1}$  (2.1 loop)
13:  $\beta_{k-1} = r_{k-1}^T \cdot r_{k-1}$  (2.2 loop)
14:  $\delta_{k-1} = r_{k-1}^T \cdot h_{k-1}$  (2.3 loop)
15:  $\zeta_{k-1} = h_{k-1}^T \cdot h_{k-1}$  (2.4 loop)
16: barrier
17: send( $\gamma_{k-1}, \beta_{k-1}, \delta_{k-1}, \zeta_{k-1}$ )
18: recv( $\gamma_{k-1}, \beta_{k-1}, \delta_{k-1}, \zeta_{k-1}$ )
19:  $\alpha_{k-1} = \frac{\beta_{k-1}}{\gamma_{k-1}}$ 
20:  $\beta_k = \beta_{k-1} - 2\alpha_{k-1}\delta_{k-1} + \alpha_{k-1}^2\zeta_{k-1}$ 
21: send( $\alpha_{k-1}, \beta_k$ )
22: recv( $\alpha_{k-1}, \beta_k$ )
23:  $x_k = x_{k-1} + \alpha_{k-1}p_{k-1}$  (2.5 loop)
24:  $r_k = r_{k-1} - \alpha_{k-1}h_{k-1}$  (2.6 loop)
25:  $p_k = r_k + \frac{\beta_k}{\beta_{k-1}}p_{k-1}$  (2.7 loop)
26: barrier

```

```
27:   send( $x_k, r_k, p_k$ )  
28:   recv( $x^k, r^k, p^k$ )  
29: end for
```