Scientific Computing - Exercise Sheet 1

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1 Exercise

(a) **Input**: $\mathbf{A} \in \mathbb{R}^{n \times n} \quad b, x_0 \in \mathbb{R}^n$

$h_0 = \mathbf{A}x_0$ (matrix multiplication)
$r_0 = b - h_0$ (1.1 loop)
$p_0 = r_0$ (1.2 loop)
for $k = 1, 2, \ldots$ **do**
  $h_{k-1} = \mathbf{A}p_{k-1}$ (matrix multiplication)
  $\gamma_{k-1} = p_{k-1}^T \cdot h_{k-1}$ (2.1 loop)
  $\beta_{k-1} = r_{k-1}^T \cdot r_{k-1}$ (2.2 loop)
  $\delta_{k-1} = r_{k-1}^T \cdot h_{k-1}$ (2.3 loop)
  $\zeta_{k-1} = h_{k-1}^T \cdot h_{k-1}$ (2.4 loop)
  $\alpha_{k-1} = \frac{\beta^{k-1}}{\gamma^{k-1}}$
  $\beta_k = \beta_{k-1} - 2\alpha_{k-1}\delta_{k-1} + \alpha_{k-1}^2\zeta_{k-1}$
  $x_k = x_{k-1} + \alpha_{k-1}p_{k-1}$ (2.5 loop)
  $r_k = r_{k-1} - \alpha_{k-1}h_{k-1}$ (2.6 loop)
  $p_k = r_k + \frac{\beta_k}{\beta_{k-1}}p_{k-1}$ (2.7 loop)
end for

All sections which are marked to be a loop can be somehow parallelized. The minimal number of loop within the iteration loops is 2, since the loops 2.1, 2.2, 2.3, 2.4 and the loops 2.5, 2.6, 2.7 can be summarized to one loop respectively.

(b) **Input**: $\mathbf{A} \in \mathbb{R}^{n \times n} \quad b, x_0 \in \mathbb{R}^n$

$h_0 = \mathbf{A}x_0$ (matrix multiplication)
begin parallel private(i, b, h_0) **shared**(r_0, p_0)
$r_0 = b - h_0$ (1.1 loop)
$p_0 = r_0$ (1.2 loop)
end parallel
for $k = 1, 2, \ldots$ **do**
  $h_{k-1} = \mathbf{A}p_{k-1}$ (matrix multiplication)
  begin parallel private$(i, r_{k-1}, p_{k-1}, h_{k-1})$ **shared**$(\gamma_{(k-1)}, \beta_{(k-1)}, \delta_{(k-1)}, \zeta_{(k-1)})$
  $\gamma_{k-1} = p_{k-1}^T \cdot h_{k-1}$ (2.1 loop)

$\beta_{k-1} = r_{k-1}^T \cdot r_{k-1}$ (2.2 loop)
$\delta_{k-1} = r_{k-1}^T \cdot h_{k-1}$ (2.3 loop)
$\zeta_{k-1} = h_{k-1}^T \cdot h_{k-1}$ (2.4 loop)
end parallel
$\alpha_{k-1} = \frac{\beta^{k-1}}{\gamma^{k-1}}$
$\beta_k = \beta_{k-1} - 2\alpha_{k-1}\delta_{k-1} + \alpha_{k-1}^2\zeta_{k-1}$
begin parallel private$(i, x_{k-1}, \alpha_{k-1}, p_{k-1}, r_{k-1}, h_{k-1}, r_k, \beta_k, \beta_{k-1})$ **shared**(x_k, r_k, p_k)
$x_k = x_{k-1} + \alpha_{k-1}p_{k-1}$ (2.5 loop)
$r_k = r_{k-1} - \alpha_{k-1}h_{k-1}$ (2.6 loop)
$p_k = r_k + \frac{\beta_k}{\beta_{k-1}}p_{k-1}$ (2.7 loop)
end parallel
end for

Here i denotes the counter, which is needed for the loop operation. Notice that for the Vectormultiplications in the loops 2.1, 2.2, 2.3 and 2.4 there normally also reduce operations should be included.

(c) **Input**: $\mathbf{A} \in \mathbb{R}^{n\times n} \quad b, x_0 \in \mathbb{R}^n$

$h_0 = \mathbf{A}x_0$ (matrix multiplication)
$r_0 = b - h_0$ (1.1 loop)
barrier
send(r_0)
recv(r_0)
$p_0 = r_0$ (1.2 loop)
barrier
send(p_0)
recv(p_0)
for $k = 1, 2, \ldots$ **do**
$h_{k-1} = \mathbf{A}p_{k-1}$ (matrix multiplication)
$\gamma_{k-1} = p_{k-1}^T \cdot h_{k-1}$ (2.1 loop)
$\beta_{k-1} = r_{k-1}^T \cdot r_{k-1}$ (2.2 loop)
$\delta_{k-1} = r_{k-1}^T \cdot h_{k-1}$ (2.3 loop)
$\zeta_{k-1} = h_{k-1}^T \cdot h_{k-1}$ (2.4 loop)
barrier
send$(\gamma_{k-1}, \beta_{k-1}, \delta_{k-1}, \zeta_{k-1})$
recv$(\gamma_{k-1}, \beta_{k-1}, \delta_{k-1}, \zeta_{k-1})$
$\alpha_{k-1} = \frac{\beta^{k-1}}{\gamma^{k-1}}$
$\beta_k = \beta_{k-1} - 2\alpha_{k-1}\delta_{k-1} + \alpha_{k-1}^2\zeta_{k-1}$
send(α_{k-1}, β_k)
recv(α_{k-1}, β_k)
$x_k = x_{k-1} + \alpha_{k-1}p_{k-1}$ (2.5 loop)
$r_k = r_{k-1} - \alpha_{k-1}h_{k-1}$ (2.6 loop)
$p_k = r_k + \frac{\beta_k}{\beta_{k-1}}p_{k-1}$ (2.7 loop)
barrier

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    send(x_k, r_k, p_k)
    recv(x^k, r^k, p^k)
end for
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