Research Proposal for 2025 SURF: Extension of the Co-Quantum Dynamics Model to Atoms of Higher Spin

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2 Introduction

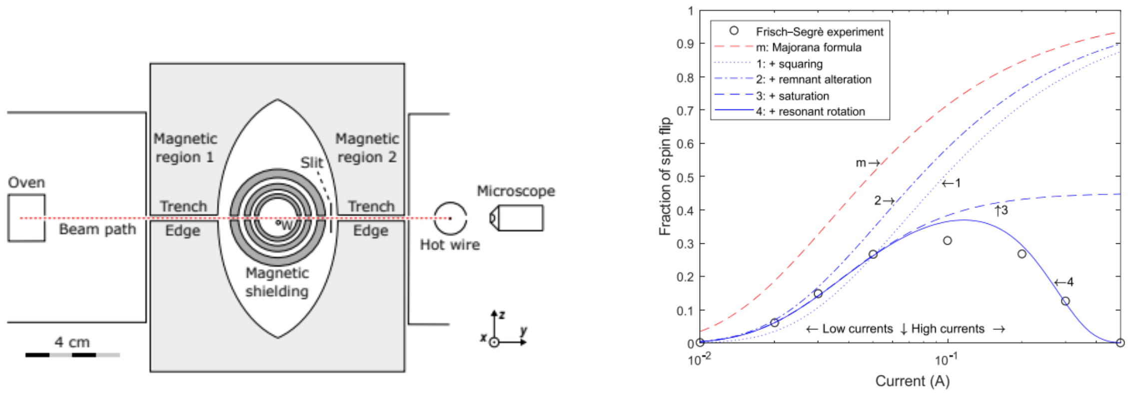
For the 2025 SURF program, I will be conducting research on a model of atomic spin called the co-quantum dynamics (CQD) theory, as proposed in Ref. [1]. The model is currently used to model a multi-stage Stern-Gerlach experiment, with spin-$\frac{1}{2}$ atoms. This research project will try to extend the model to accommodate atoms of higher electron spin.

3 Literature Review

3.1 Co-Quantum Dynamics

Co-Quantum Dynamics (CQD) [1][2][3][4] is a semiclassical model that describes atomic spin dynamics and wave function collapse. Currently, the model succeeds in providing a precise description of the spin-flip phenomena in the multi-stage Stern–Gerlach experiment, in particular the Frisch-Segrè experiment of 1933 [5], without fitting parameters.

CQD models the interaction between electron and nuclear magnetic moments (μ_e and μ_n), treating μ_n as a ”co-quantum” which guides the collapse of μ_e, but does not itself collapse upon measurement of atomic spin.



(a) (b)

Figure 1: (a) Original setup of the Frisch-Segrè experiment from Ref. [1]. (b) Result of current simulation using the CQD model, taken from Ref. [2]. The horizontal axis is current that induces the central magnetic field; the vertical axis is the spni-flip fraction.

Figure 1(a) illustrates the apparatus used in the Frisch-Segrè experiment. As an atom traverses the central magnetic field, the model incorporates induction effects and torque-averaged magnetic dynamics instead of traditional self-averaging to describe its evolution. A key postulate is a semiclassical branching condition, where the relative orientation of μ_e and μ_n dictates the collapse direction of μ_e. Denoting the electron and nuclear co-quantum state as $|\mu_\mathrm{e}©\mu_\mathrm{n}\rangle$, we have [3].

$$|\mu_\mathrm{e}©\mu_\mathrm{n}\rangle = \frac{1-\mathrm{sgn}(\theta_\mathrm{e}-\theta_\mathrm{n})}{2}|+z\rangle + \frac{1+\mathrm{sgn}(\theta_\mathrm{e}-\theta_\mathrm{n})}{2}|-z\rangle. \tag{1}$$

The result of the Frishc-Segrè experiment is closely replicated by the current model, Cf. Figure 1(b) [2], including the signature fact that the spin-flip fraction curve bends down at higher current values.

3.2 Schwinger Oscillator Model

Julian Schwinger showed that the algebra of a system of two simultaneous harmonic oscillators is equivalent to an angular momentum algebra [6].

Consider two independent oscillators, with the annihilation and creation operators given respectively by $a_\pm$ and $a_\pm^\dagger$. Define the number operators as usual: $N_\pm \equiv a_\pm^\dagger a_\pm$.

If we assume the usual commutation relations among $a_\pm$, $a_\pm^\dagger$, and $N_\pm$,

$$[a_\pm, a_\pm^\dagger] = 1, \quad [N_\pm, a_\pm] = -a_\pm, \quad [N_\pm, a_\pm^\dagger] = a_\pm^\dagger, \tag{2}$$

we will obtain

$$|n_+, n_-\rangle = \frac{(a_+^\dagger)^{n_+}(a_-^\dagger)^{n_-}}{\sqrt{n_+!}\sqrt{n_-!}}|0,0\rangle. \tag{3}$$

Then define

$$J_+ \equiv \hbar a_+^\dagger a_-, \quad J_- \equiv \hbar a_-^\dagger a_+, \quad J_z \equiv \left(\frac{\hbar}{2}\right)(a_+^\dagger a_+ - a_-^\dagger a_-). \tag{4}$$

The operators in equation (4) can be shown to satisfy the usual angular-momentum commutation relations. As far as the transformation properties under rotations are concerned, any spin-j object can be understood as a composite system of $2j$ spin-$\frac{1}{2}$ particles.

4 Objectives

4.1 Research Problems

Extending the CQD model to higher-spin systems could provide deeper insights into quantum measurement processes. This research aims to explore the feasibility of generalizing CQD theory to higher-spin atoms. To achieve this, we will conduct the following:

1. Conduct a comprehensive literature review to identify relevant experimental data on higher-spin atomic systems.
2. Develop theoretical models for atoms with higher spins and simulate their dynamics.

By addressing these objectives, this study seeks to refine the theoretical foundation of CQD and expand its applicability to a broader range of quantum systems.

4.2 Significance of Research

Co-quantum dynamics theory has provided good results in predicting the result of the Frisch-Segrè experiment, which, as a multi-stage Stern-Gerlach experiment, is of great importance in quantum mechanical development. What is even better is the lack of fitting parameters, showing that this model agrees with quantum-mechanical calculations. Extending the model to possibly accommodate atoms of higher spins may provide a new perspective for understanding atomic spin.

5 Work Plan

The methods involved in the research project are outlined below:

1. Collect and organize existing experimental data on atoms with spin higher than $\frac{1}{2}$. The amount and quality of the data collected could be used to decide the type of experiment that could be used to verify step 2, which is an extension of the model to atoms with higher spins.

2. Although the current model predicts the result of the Frisch-Segrè experiment very well, it has limited applications for atoms of higher spin. Section 3.2 hints at the fact that we can extend the model for spin-$\frac{1}{2}$ particles to particles with higher spin. We will investigate possible ways to modify the current model to accommodate atoms of higher spin, for example by adopting an appropriate generalization of the branching condition equation (1). Atoms with nuclear spin 0 is a special case that is not considered by the current model, this may also be considered. Using the data organized in step 1, we will compare the theoretical calculations with experimental data.

The research will be guided by principal investigator Professor Lihong V. Wang. The proposed research will happen in the summer of 2025. This research will be performed in the Caltech Optical Imaging Laboratory.

6 References

[1] Wang, Lihong V. (2023). *Multi-stage Stern–Gerlach experiment modeled.*

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[3] He, Zhe; Titimbo, Kelvin; Garrett, David C.; Kahraman, S. Süleyman; Wang, Lihong V. (2023). *Numerical modeling of the multi-stage Stern–Gerlach experiment by Frisch and Segrè using co-quantum dynamics via the Schrödinger equation.*

[4] Titimbo, Kelvin; Garrett, David C.; Kahraman, S. Süleyman; He, Zhe; Wang, Lihong V. (2023). *Numerical modeling of the multi-stage Stern–Gerlach experiment by Frisch and Segrè using co-quantum dynamics via the Bloch equation.*

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[6] Sakurai, J. J.; Napolitano, Jim (2020). *Modern Quantum Mechanics 3e.*