On Quantum Darkness

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(Dated: June 9, 2025)

Quantum darkness, embodied by dark states, originates from interference that produces subradiant atomic modes or multimode light fields undetectable by measurements. We outline the underlying mechanisms and implementations of dark and bright states in multi-emitter platforms, demonstrate control of localized dark states in waveguide QED for efficient energy storage and retrieval, and reinterpret classical double-slit interference as arising from bright–dark mode superpositions.

I. INTRODUCTION

The idea of 'quantum darkness' is encapsulated in the physics of dark states. In quantum optics, dark states arise from quantum interference, resulting in states that either decay more slowly, like subradiant atomic states, or are completely non-interacting with detectors, like certain multimode light field states. By introducing the mechanism behind dark states and their physical implementation, we bring forth a short dive into the world of superradiance and subradiance, which are fundamental collective phenomena that emerge when multiple quantum emitters, such as atoms or qubits, interact coherently with an electromagnetic environment.

In one example, we will explore the control of localized dark states in waveguide QED, where chains of two-level emitters coupled to a one-dimensional waveguide exhibit dark states due to interference, and we will simulate a system that enables efficient energy storage and retrieval. In another example, we will use the concept of bright and dark states to introduce another viewpoint to interpret the quantum origin of classical light interference.

II. THEORY

A. Fundamental Concepts of Dark States and Bright States

Collective states arise when multiple subsystems, such as qubits or photon modes, interact in a way that their individual excitations cannot be treated independently. Instead, the system shows joint behaviors resulting from coherent superpositions. This coherence allows the system to behave collectively, leading to bright states or dark states. In general, bright states are those that can exchange energy with an external field or detector. On the contrary, dark states are those that do not couple to the external fields or detectors, and thus remain invisible to certain kinds of measurements or decay channels.

The concept of dark and bright states is not limited to radiation fields. While dark states are discussed in cavity QED, they also appear in many-body atomic qubits. This makes them powerful tools for quantum memory, manipulation, and sensing.

Let us introduce Dicke Model. [1] The Dicke Model describes the coupling between a single-mode light cavity and N two-level systems. The Hilbert space of the cavity is spanned by Fock states $|n\rangle$, and that of the two level system is defined by spin operators $\hat{\vec{\sigma_j}} = (\hat{\sigma}_j^x, \hat{\sigma}_j^y, \hat{\sigma}_j^z)$. The Hamiltonian of the Dicke Model is

$$\hat{\mathcal{H}}_{\text{Di}} = \omega_c \hat{a}^\dagger \hat{a} + \omega_z \sum_{j=0}^{N} \hat{\sigma}_j^z + \frac{2\lambda}{\sqrt{N}} (\hat{a} + \hat{a}^\dagger) \sum_{j=0}^{N} \hat{\sigma}_j^x \tag{1}$$

where ω_c is the cavity frequency and $\hbar\omega_z$ is the energy gap. The last term describes the coupling between the two-level systems and the cavity, where $\lambda_{j,s}$ is the coupling strength and $\hat{a} + \hat{a}^\dagger$ corresponds to electric fields. Additionally, $\hat{\sigma}^x = \frac{1}{2}(\hat{\sigma}^- + \hat{\sigma}^+)$, where $\hat{\sigma}$ is the lowering operator. Hence the coupling term can be written as two terms: a **co-rotating** terms proportional to $\hat{a}\hat{\sigma}^+ + \hat{a}^\dagger\hat{\sigma}^-$, conserving the excitations, and a **counter-rotating** term proportional to $\hat{a}\hat{\sigma}^- + \hat{a}^\dagger\hat{\sigma}^+$.

For open Dicke Model, the master equation in Lindblad form is

$$\dot{\hat{\rho}} = -i[\hat{\mathcal{H}}_{\text{Di}}, \hat{\rho}] + \sum_\alpha \gamma_\alpha (\hat{L}_\alpha \hat{\rho} \hat{L}_\alpha^\dagger - \frac{1}{2}[\hat{L}_\alpha^\dagger \hat{L}_\alpha, \hat{\rho}]) \tag{2}$$

where $\hat{L}_\alpha$ is the the Lindblad operator of the decay channel α and γ_α is the associated decay rate. In the collective decay, $\hat{L} = \sum_j \hat{\sigma}_j$.

B. Dark States in Waveguide QED

1. Collective Hamiltonian and Master Equation

In this model, [2] we consider a chain of N qubits, each located at x_m and having two internal states $|e_m\rangle$

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and $|g_m\rangle$, with transition frequency ω_0. They are coupled to a 1-D waveguide, which mediates both coherent qubit-qubit interactions and collective dissipation by photons. The coupling is determined by the single-qubit decay rate γ, and for qubit separations of integer multiples of the guided wavelength λ_0, a degenerate family of dark states emerge.

The Hamiltonian in (1) can be written as $\hat{\mathcal{H}}_{\rm Di} = \hat{\mathcal{H}}_b + \hat{\mathcal{H}}_s + \hat{\mathcal{H}}_{\rm int}$, where the coupling term $\hat{\mathcal{H}}_{\rm int} = \frac{2\lambda}{\sqrt{N}}(\hat{a} + \hat{a}^\dagger)\sum_{j=0}^{N}(\hat{\sigma}_j^- + \hat{\sigma}_j^+)$. But now consider continuous bath modes, we replace it with $\hat{\mathcal{H}}_{\rm int} = \hbar\sum_{j,s}\int d\omega\lambda_{j,s}(\hat{a}_s(\omega) + \hat{a}_s^\dagger(\omega))(\hat{\sigma}_j^- + \hat{\sigma}_j^+)$. Moving it to the interaction picture and applying the Born-Markov and rotating-wave approximation, we trace over the waveguide degrees of freedom, and assume the bath remains in vacuum state. This results in the master equation[2]

$$\dot{\hat{\rho}} = -i[\hat{\mathcal{H}}_{\rm eff}, \hat{\rho}] + \sum_{m,n=1}^{N}\gamma_{m,n}\hat{\sigma}_m^-\hat{\rho}\hat{\sigma}_n^+ \tag{3}$$

where $\hat{\sigma}_m^- = |g_m\rangle\langle e_m|$ is the lowering operator for m-th qubit. The interaction and dissipation between qubits are described by $J_{m,n} = \frac{\gamma}{2}\sin(k_0|x_m - x_n|)$ and $\gamma_{m,n} = \gamma\cos(k_0|x_m - x_n|)$, with k_0 being the wave vector of the guided mode. The non-Hermitian collective Hamiltonian governing the coherent part of the evolution is [2]

$$\hat{\mathcal{H}}_{\rm eff} = \sum_{m,n=1}^{N}(J_{m,n} - i\frac{\gamma_{m,n}}{2})\hat{\sigma}_m^+\hat{\sigma}_n^- \tag{4}$$

To simplify and understand the emergence of the dark states, we divide the system into two parts: a group of M qubits and the remaining $N - M$ qubits. For the case when the qubit separation satisfies $d = n\lambda_0$, we define collective operators for each group[2]: $\hat{\mathcal{S}}_1 = \frac{1}{\sqrt{M}}\sum_{j=1}^{M}\hat{\sigma}_j^-$ and $\hat{\mathcal{S}}_2 = \frac{1}{\sqrt{N-M}}\sum_{j=M+1}^{N}\hat{\sigma}_j^-$. Then the collective Hamiltonian for $d = n\lambda_0$ can be written as

$$\begin{aligned}\hat{\mathcal{H}}_{\rm eff} = &-i\frac{M\gamma}{2}\hat{\mathcal{S}}_1^\dagger\hat{\mathcal{S}}_1 - i\frac{(N-M)\gamma}{2}\hat{\mathcal{S}}_2^\dagger\hat{\mathcal{S}}_2 \\ &- i\Gamma(\hat{\mathcal{S}}_1^\dagger\hat{\mathcal{S}}_2 + \hat{\mathcal{S}}_2^\dagger\hat{\mathcal{S}}_1)\end{aligned} \tag{5}$$

where the enhanced collective coupling rate is $2\Gamma = \sqrt{M(N-M)}\gamma$. The last term in the Hamiltonian describes the dissipative coupling between symmetric superpositions of the two groups.

2. *Single Excitation*

Consider a particular configuration when the qubit separation satisfies $d = n\lambda_0$, referred to as the **mirror configuration**. In this case, $J_{m,n} = 0$ and $\lambda_{m,n} = \lambda$. Assume there is only one bright state $|\Psi_S^{(1)}\rangle = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}\hat{\sigma}_j^+|G\rangle$, where $|G\rangle = |g\rangle^{\bigotimes N}$, and $(N-1)$ perfectly dark states. Since the dark subspace consists of all orthogonal combinations, i.e. $\langle\Psi_S^{(1)}|\Psi_D^{(1)}\rangle = 0$ and $\langle\Psi_{D,i}^{(1)}|\Psi_{D,j}^{(1)}\rangle = \delta_{ij}$, one can construct **highly localized dark states** by Gram-Schmidt Process:

$$|\Psi_D^{(1)}\rangle = \frac{1}{\sqrt{N}}(\sqrt{N-1}\hat{\sigma}_1^+ - \hat{\mathcal{S}}_2^\dagger)|G\rangle \tag{6}$$

where $\hat{\mathcal{S}}_2^\dagger = \frac{1}{\sqrt{N-1}}\sum_{j=2}^{N}\hat{\sigma}_j^-$ is the collective excitation operator for qubits 2 to N. We can find $\langle\hat{\sigma}_1^+\hat{\sigma}_1^-\rangle = 1 - \frac{1}{N}$, which means a large fraction of excitation is concentrated in the first qubit. Meanwhile, we can see that the dissipation term $\langle\Psi_D^{(1)}|\sum_{m,n}\lambda_{m,n}\hat{\sigma}_m^+\hat{\sigma}_n^-|\Psi_D^{(1)}\rangle = 0$, so the excitation is protected from radiative decay. Additionally, the coherent exchange interaction term $J_{m,n}$ is also 0, hence $\langle\hat{\mathcal{H}}_{\rm eff}\rangle = 0$, implying there is neither energy exchange between qubits nor collective dissipation effects due to coupling mediated by waveguide.

3. *Dark State Preparation via Local Coherent Driving*

To prepare the single-excitation dark state $|\Psi_D^{(1)}\rangle$, one applies an external, resonant pulsed drive localized on qubit 1 (any other qubit would work equally well), described by $\hat{\mathcal{H}}_d(t) = \Omega_d(t)\left(\hat{\sigma}_1^- + \hat{\sigma}_1^+\right)$, where $\Omega_d(t)$ is a time-dependent Rabi frequency. This drive couples the ground state both to the bright state $|\Psi_S^{(1)}\rangle$ and to the dark state $|\Psi_D^{(1)}\rangle$ with asymmetrical coupling strengths,

$$\langle\Psi_S^{(1)}|\hat{\mathcal{H}}_d(t)|G\rangle = \Omega_d(t)\sqrt{1/N} \tag{7}$$

and

$$\langle\Psi_D^{(1)}|\hat{\mathcal{H}}_d(t)|G\rangle = \Omega_d(t)\sqrt{1-1/N}. \tag{8}$$

In the $N \gg 1$ limit the latter term dominates, yielding near-unit-fidelity preparation of the dark state.

C. Dark and Bright States: The Substratum Underlying Classical Interference

To begin with, a two-level atom functions as an electric field sensor and the field is detected when the sensor is excited from its ground state. The interaction Hamiltonian between the field and the sensor takes the form (in the rotating-wave approximation): [3]

$$\hat{\mathcal{H}}_{\rm fs} = \hat{\mathbf{E}}^{(+)}(\mathbf{r},t)\,\hat{\sigma}^+ + \hat{\mathbf{E}}^{(-)}(\mathbf{r},t)\,\hat{\sigma}^-, \tag{9}$$

with $\hat{\mathbf{E}}^{(+)}$ $(\hat{\mathbf{E}}^{(-)})$ the positive (negative) frequency parts of the electric field operator $\hat{\mathbf{E}}$, and $\hat{\sigma}^+$ $(\hat{\sigma}^-)$ the raising (lowering) operators of the sensor atom.

Since the absorption rate of a photon in state $|\Phi\rangle$ at position $\mathbf{r}$ and time t by the sensor atom is proportional to [4]

$$\langle\Phi|\hat{\mathbf{E}}^{(-)}(\mathbf{r},t)\,\hat{\mathbf{E}}^{(+)}(\mathbf{r},t)\,|\Phi\rangle\,, \tag{10}$$

it follows that any photon in state $|\Phi\rangle$ satisfying $\hat{\mathbf{E}}^{(+)}(\mathbf{r},t)\,|\Phi\rangle = 0$, or equivalently $\hat{\mathcal{H}}_{\text{fs}}\,|\Phi\rangle\,|g\rangle = 0$, is undetectable, thus identifying it as a perfectly dark state (PDS).

1. *The Two-mode Collective States of Light*

Whereas single-mode fields admit only the vacuum as the PDS, two-mode fields admit an enlarged set of such states. To demonstrate this, consider two modes A and B with annihilation operators $\hat{a}$ and $\hat{b}$ (and the corresponding creation operators $\hat{a}^\dagger$ and $\hat{b}^\dagger$), and a relative phase θ between them. In this case, the Hamiltonian (9) becomes $\hat{\mathcal{H}}_{\text{fs}} = g\left(\hat{a} + \hat{b}e^{i\theta}\right)\hat{\sigma}^+ + h.c.$, where $g = g(\mathbf{r})$ is the coupling constant between the sensor and each field mode-here taken to be identical for both modes for simplicity-and $\mathbf{r}$ denotes the sensor location. For any $n \in \mathbb{N}_0$ with $n \le N$, define the N-excitation collective

$$|\psi_n^N(\theta)\rangle \equiv \frac{\left(\hat{a}^\dagger + \hat{b}^\dagger e^{-i\theta}\right)^n \left(-\hat{a}^\dagger e^{i\theta} + \hat{b}^\dagger\right)^{N-n}}{\sqrt{2^N n!\,(N-n)!}}\,|0,0\rangle_{a,b}\,. \tag{11}$$

Then, one can show that

$$\hat{\mathcal{H}}_{\text{fs}}\,|\psi_n^N\rangle\,|g\rangle = g\sqrt{2n}\,|\psi_{n-1}^{N-1}\rangle\,|e\rangle \tag{12}$$

and $\{|\psi_n^N(\theta)\rangle\}_{n=0}^N$ constitutes a complete basis for the N-excitation subspace.

Specializing Eq. 11 to $n = 0$ and $n = N$ and then applying the binomial theorem yields

$$|\psi_0^N(\theta)\rangle = \sqrt{\frac{N!}{2^N}}\sum_{m=0}^{N}\frac{(-1)^m\,e^{im\theta}}{\sqrt{m!\,(N-m)!}}\,|m,N-m\rangle_{a,b} \tag{13}$$

and

$$|\psi_N^N(\theta)\rangle = e^{-iN\theta}\sqrt{\frac{N!}{2^N}}\sum_{m=0}^{N}\frac{e^{im\theta}}{\sqrt{m!\,(N-m)!}}\,|m,N-m\rangle_{a,b}\,. \tag{14}$$

It is evident that Eq. 13 yields the PDS for the subspace of N photons, whereas Eq. 14 exhibits the maximal transition rate $g\sqrt{2N}$, thus recognized as the maximally superradiant state (MSS) or the bright state.

2. *Double Slit Interference*

In this context, we examine the correspondence between quantum-mechanical dark and bright states and the classical phenomena of destructive and constructive interference. To illustrate this, a coherent state is sent through a double slit so that one component propagates from slit 1 at position $\mathbf{d}_1$ and the other from slit 2 at position $\mathbf{d}_2$, both with equal amplitude α and characterized by wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively. In the detection plane, the joint field is the product of two coherent states, each picking up its respective phase factor (not normalized):

$$|e^{-i\mathbf{k}_1\cdot\mathbf{d}_1}\alpha, e^{-i\mathbf{k}_2\cdot\mathbf{d}_2}\alpha\rangle = e^{-|\alpha|^2}\sum_{N=0}^{\infty} C_N\,|\chi^N(\delta\phi)\rangle\,, \tag{15}$$

with the coefficient

$$C_N = \sqrt{2^N/N!}\left(e^{-i\mathbf{k}_2\cdot\mathbf{d}_2}\alpha\right)^N \tag{16}$$

and the phase–dependent state

$$|\chi^N(\delta\phi)\rangle = \sqrt{\frac{N!}{2^N}}\sum_{m=0}^{N}\frac{e^{-im\delta\phi}e^{im\theta}}{\sqrt{m!\,(N-m)!}}\,|m,N-m\rangle\,, \tag{17}$$

and $\delta\phi$ is the phase difference accumulated between the two paths from the slits to the sensor atom, and can be written as $-(\mathbf{k}_2\cdot\mathbf{d}_2 - \mathbf{k}_1\cdot\mathbf{d}_1) + \theta$. When $\delta\phi = 2l\pi$ ($l \in \mathbb{Z}$), the two modes are in phase and the state reduces to the MSS, $|\chi^N(2l\pi)\rangle \propto |\psi_N^N(\theta)\rangle$, which corresponds to classical constructive interference. In contrast, when $\delta\phi = (2l+1)\pi$, the two modes are out of phase, resulting in PDS, $|\chi^N((2l+1)\pi)\rangle = |\psi_0^N(\theta)\rangle$, related to destructive interference. Crucially, however, the average number of photons

$$\langle\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b}\rangle(\delta\phi) = 2|\alpha|^2 \tag{18}$$

remains independent of $\delta\phi$. In other words, even at points where classical interference predicts zero intensity (complete destructive interference), the quantum description with coherent states still yields photons everywhere on the screen.

However, it should be noted that destructive interference does not guarantee that a field remains undetectable. For example, consider the state

$$\left(|0\rangle_a + |1\rangle_a\right)\left(|0\rangle_b - |1\rangle_b\right)/2. \tag{19}$$

Despite each mode exhibiting a non-zero mean electric field, their exact phase opposition leads to complete cancellation of the net average field.

III. PROTOCOLS

A. Preparation of dark states

Here we shall outline a protocol to prepare dark states, which will later be retrieved and manipulated for the release of energy in a controlled manner. The method proposed by the authors used superconducting qubits in a

1D transmission line, implemented by transmon qubits coupled to a common coplanar waveguide.

A *transmon qubit* is a type of superconducting charge qubit designed to have reduced sensitivity to charge noise, This is realized by coupling the system to a resonator, such as a waveguide. The 1D transmon chain exhibits multilevel properties, and we may write the effective Hamiltonian as

$$\hat{\mathcal{H}}_{\text{eff}} = \sum_{m,n} \left(J_{m,n} - i\frac{\Gamma_{m,n}}{2} \right) \hat{a}_m^\dagger \hat{a}_n - \frac{U}{2} \sum_m \hat{n}_m (\hat{n}_m - \mathbb{I}), \tag{20}$$

where the (bosonic) creation and annihilation operators $\hat{a}_m$ creates/annihilates an excitation on the mth site. Transmons behave like anharmonic oscillators, and the resulting dark state is:

$$|\Psi_D^{(1)}\rangle = \frac{1}{\sqrt{N}} (\sqrt{N-1}\hat{a}_1^\dagger - \hat{\mathcal{S}}_2^\dagger) |G\rangle \,. \tag{21}$$

On the other hand, the two-excitation dark states become

$$|\Phi_D^{(2)}\rangle = \frac{\sqrt{2}}{N} \left((\hat{a}_1^\dagger)^2 - 2\sqrt{N-1}\hat{a}_1^\dagger S_2^\dagger + (S_2^\dagger)^2 \right) |G\rangle \,. \tag{22}$$

As before, the choice of using the first transmon is arbitrary, and any transmon is equally valid for storage. There is another solution, given by a form analogous to the two-qubit case, but with the substitution $\hat{\sigma}_m \to \hat{a}_m$:

$$|\Psi_{\mathcal{D}}^{(2)}\rangle = \frac{\sqrt{N-3}}{\sqrt{N-1}} \left((\tilde{S}_1^\dagger)^2 - \frac{\sqrt{2}\,\tilde{S}_1^\dagger \tilde{S}_2^\dagger}{\sqrt{N-2}} + \frac{(\tilde{S}_2^\dagger)^2}{\sqrt{N-3}} \right) |G\rangle \,. \tag{23}$$

If the first two transmons are driven by an external drive the general state is a superposition $c_1 |\Phi_{\mathcal{D}}^{(2)}\rangle + c_2 |\Psi_{\mathcal{D}}^{(2)}\rangle$. The effect for the qubit case extends to transmons, although the collective dark and bright states have a slightly different population distribution due to their *multilevel structure*.

B. An Exciting Example: The storage and release of excitations

Building on the theory described above, we can establish a simple protocol for controlled storage and release of two excitations. Drive a chain of N qubits in the ground state into the dark state $|\Psi_{\mathcal{D}}^{(2)}\rangle$ with a coherent pulse. The chain is set up such that $d \sim \lambda_0$ is satisfied. Moreover, we can tune qubits on and off resonance on-chip using what is known as flux lines.

Weakly coupled control lines can be used to realize the drive, allowing us to excite single qubits selectively. This is an effective way to prepare dark states, and driving the resulting dark states would release the stored photons via decay.

IV. RESULTS

A. Probing Dark States

The probability amplitudes $|\langle e_n e_m|\Psi\rangle|^2$ to find two excitations at sites e_n and e_m, display complex patterns.

The following simulation displays four different correlation patterns: checkerboard, fermionic, dark states, and uncorrelated. We replicate the result in the paper and extend it to higher N's by restricting to the two-excitation subspace, so we only need to work with a $\sim \binom{N}{2}$-dimensional Hamiltonian matrix instead of a 2^N-dimensional one. Results are shown in fig. (1), and include the following:

1. A checkerboard pattern appears when $k_0 d = (2n+1)\pi/2$, due to the fact that coherent nearest-neighbor and dissipative next-nearest-neighbor interactions are zero in $\mathcal{H}_{\text{eff}}$. Probing demands high resolution and low noise level.

2. For general fermionic states with $k_0 d \neq (2n+1)\pi/2$, the pattern is complex but appears symmetric.

3. Dark states satisfy $k_0 d = 2n\pi$ (Cf. eqn.(23)). Compared to ordinary states, which exhibit complicated correlation, dark states are easier to probe.

4. We noticed that when $k_0 d = n\pi$ for $n \in \mathbb{N}$, the pattern becomes very uncorrelated.

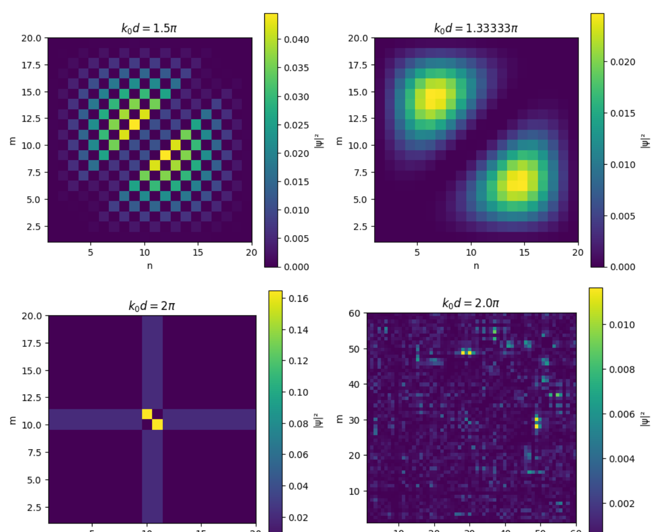

FIG. 1. The first three figures follow the paper and show correlation patterns for $N = 20$ qubits, as explained above. The fourth figures shows the disappearance of correlation at $k_0 d = 2\pi$, at $N = 60$ qubits.

For a large number of qubits, namely $N > 50$ and $k_0 d = (6n-1)\pi/6$, dimerized correlations with low decay rate appear. This implies that *qualitative behavior of the* *correlations does not scale with the number of qubits* (fig (2)).

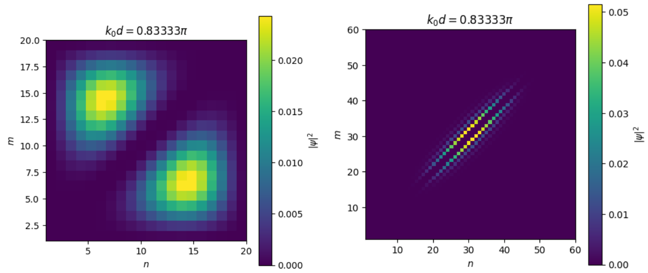

FIG. 2. As the number of qubits increase from 20 to 60, a distinct dimerized pattern appears, and the fermionic correlation turns into an extremely dark state with dimerized correlation.

B. Simulation of Excitation Release

As inspired by Holzinger et al. (2022) [2], we ran a simulation for the storage-and-release protocol in a 1D chain of 16 qubits. Firstly, a pulse drives the two qubits into the dark state $|Psi_{\mathcal{D}}^{(2)}\rangle$. At $\gamma t = 12$, we detune the remaining $N-2$ qubits, leading to a burst of energy released via a bright channel. This is shown in figure (3).

V. CONCLUSION

Collective states arise when multiple subsystems interact with complex correlations. One notable emergent effect is subradiance, also known as dark states. In the first section, we introduced Dicke's model for dark states, which describes light and matter interaction with single-mode light coupled to a set of two-level systems. Then we expounded on the relationship between Dicke states and the interference experiment in the context of bright and dark states.

Finally, we simulated correlation patterns in 1D systems composed of 20 to 60 qubits. We found that distinct patterns emerge for different wave numbers k_0, and new states may appear as the system size scales. Also, a protocol for controlled storage-and-release of multiple photons into a waveguide is proposed. As mentioned in the paper [2], this has applications to multiphoton sources or quantum repeaters, pointing to highly practical applications for dark states.

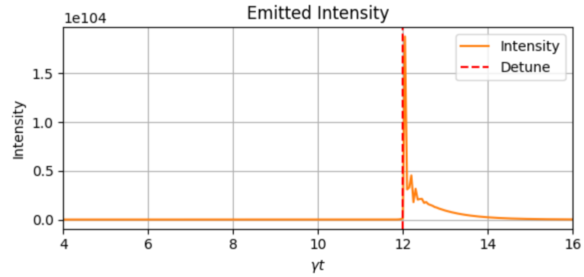

FIG. 3. The preparation phase is not simulated, and the time period $4 \leq \gamma t < 12$ shows a stable dark state stored in the system of qubits. At $\gamma t = 12$, photons are released by a drive.

VI. AUTHOR CONTRIBUTIONS

Below are the respectively contributions of our group members:

Section	Member 1	Member 2
Introduction	Shao-Kai Huang	Shuo-Ching Huang
Theory - A	Yuan-Heng Wang	
Theory - B	Yuan-Heng Wang	Shuo-Ching Huang
Theory - C	Shuo-Ching Huang	
Protocols	Shao-Kai Huang	
Results	Shao-Kai Huang	
Conclusion	Shao-Kai Huang	

TABLE I. Contributors for each section of the written report. Member 1 is the main contributor.

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