



Review

Dynamic pricing and learning: Historical origins, current research, and new directions



Arnoud V. den Boer

University of Twente, P.O. Box 217, 7500 AE Enschede, Netherlands

HIGHLIGHTS

- The historical origins of research on pricing and demand estimation are sketched.
- An in-depth survey of literature on dynamic pricing and learning is given.
- Relations with methodologically related research areas are highlighted.
- Important directions for future research are identified.

ARTICLE INFO

Article history:

Received 6 February 2014

Received in revised form

17 March 2015

Accepted 17 March 2015

ABSTRACT

The topic of dynamic pricing and learning has received a considerable amount of attention in recent years, from different scientific communities. We survey these literature streams: we provide a brief introduction to the historical origins of quantitative research on pricing and demand estimation, point to different subfields in the area of dynamic pricing, and provide an in-depth overview of the available literature on dynamic pricing and learning. Our focus is on the operations research and management science literature, but we also discuss relevant contributions from marketing, economics, econometrics, and computer science. We discuss relations with methodologically related research areas, and identify directions for future research.

© 2015 Elsevier Ltd. All rights reserved.

Contents

1.	Introduction.....	2
2.	Historical origins of pricing and demand estimation	3
2.1.	Demand functions in pricing problems.....	3
2.2.	Demand estimation	3
2.3.	Practical applicability	3
3.	Dynamic pricing	3
3.1.	Dynamic pricing with dynamic demand	4
3.1.1.	Demand depends on price-derivatives	4
3.1.2.	Demand depends on price history	4
3.1.3.	Demand depends on amount of sales	4
3.2.	Dynamic pricing with inventory effects.....	4
3.2.1.	Selling a fixed, finite inventory during a finite time period.....	4
3.2.2.	Jointly determining selling prices and inventory–procurement	5
4.	Dynamic pricing and learning	5
4.1.	No inventory restrictions	5
4.1.1.	Early work	5
4.1.2.	Bayesian approaches	6

E-mail address: a.v.denboer@utwente.nl.

4.1.3.	Non-Bayesian approaches	7
4.2.	Finite inventory	8
4.2.1.	Early work	8
4.2.2.	Bayesian approaches	8
4.2.3.	Non-Bayesian approaches	9
4.3.	Machine-learning approaches	10
4.4.	Joint pricing and inventory problems	10
5.	Methodologically related areas	11
6.	Extensions and new directions	11
6.1.	Strategic consumer behavior	11
6.2.	Competition	11
6.3.	Time-varying market parameters	12
6.4.	Model misspecification	12
7.	Conclusion	12
	Acknowledgments	13
	References	13

*Pricing is an interesting problem from Economics.*¹

1. Introduction

Dynamic pricing is the study of determining optimal selling prices of products or services, in a setting where prices can easily and frequently be adjusted. This applies to vendors selling their products via Internet, or to brick-and-mortar stores that make use of digital price tags. In both cases, digital technology has made it possible to continuously adjust prices to changing circumstances, without any costs or efforts. Dynamic pricing techniques are nowadays widely used in various businesses, and in some cases considered to be an indispensable part of pricing policies.

Digital sales environments generally provide firms with an abundance of sales data. This data may contain important insights on consumer behavior, in particular on how consumers respond to different selling prices. Exploiting the knowledge contained in the data and applying this to dynamic pricing policies may provide key competitive advantages, and knowledge how this should be done is of highly practical relevance and theoretical interest. This consideration is a main driver of research on *dynamic pricing and learning*: the study of optimal dynamic pricing in an uncertain environment where characteristics of consumer behavior can be learned from accumulating sales data.

The literature on dynamic pricing and learning has grown fast in recent years, with contributions from different scientific communities: operations research and management science (OR/MS), marketing, computer science, and economics/econometrics. This survey aims at bringing together the literature from these different fields, and at highlighting some of the older (and sometimes forgotten) literature where many important results and ideas already can be found.

A few literature reviews on dynamic pricing and learning do already exist. Araman and Caldentey [2] and Aviv and Vulcano [3, Section 4] review in detail a number of recent studies, mostly from the OR/MS community; Christ [4, Section 3.2.1] contains an elaborate discussion of a selection of demand learning studies; and Chen and Chen [5] review recent research on multiple-product pricing, pricing with competition, and pricing with limited demand information. Our survey complements these publications by aiming at a larger scope, and, although our main focus is on the OR/MS literature, we also address relevant contributions from computer science, marketing, economics and econometrics.

Content. This survey reviews the literature on dynamic pricing with demand uncertainty. We discuss how this is embedded in

the literature on dynamic pricing in general, but do not review all relevant research topics associated with dynamic pricing; for this we refer to Bitran and Caldentey [6], Elmaghraby and Keskinocak [7], Talluri and van Ryzin [8], Phillips [9], Heching and Leung [10], Gönsch et al. [11], Rao [12], Chenavaz et al. [13], Deksnyte and Lydeka [14] and Özer and Phillips [15]. We focus on studies where selling price is a control variable; we only scarcely discuss learning in capacity-based revenue management [8] or learning in newsvendor/inventory control problems. Neither do we consider mechanism design [16,17] or auction theory with incomplete information (see e.g. [18,19] and the references therein), although there are some similarities with dynamic pricing and learning. Most of the studies that we discuss are written from an (online) retailer perspective; we do not consider social welfare optimization [20,21]. We do not dive into specific details associated with particular application area such as pricing in queueing or telecommunication environments [22], road pricing [23–25], or electricity pricing [26,27], to name a few. We also neglect a recent stream of *empirical* studies that aims to explain the dynamic pricing strategies of sellers by fitting models to sales data (see e.g. [28] for an example on prices of airline tickets, Sweeting [29] on prices of Major League Baseball tickets, and Huang et al. [30] on pricing for a used-car dealership). Finally, this survey focuses on studies where the *seller* learns about the *demand function*, and not on studies where *buyers* (or *sellers*) learn (typically about the quality of the product) [31–37].

Methodology. We used Google Scholar to find all relevant references that were available online October 1, 2014. We excluded double versions of the same papers, or conference papers that largely overlap with journal papers. For all papers that we found this way we looked on Google Scholar for relevant other work citing these papers. We also checked the websites of key authors in the field for relevant publications. We aimed for a comprehensive review of the dynamic-pricing-and-learning literature; for the other sections, on e.g. demand estimation or dynamic pricing under full information, we restrict to key papers and reviews.

Organization of the paper. This survey is organized as follows. We review in Section 2 some of the pioneering historical work on pricing and demand estimation, and discuss how this work initially was difficult to apply in practice. In Section 3 we sketch important references and developments for dynamic pricing in general, and in Section 4 we focus on the literature that specifically deals with dynamic pricing and learning. Connections between dynamic pricing and learning and related research areas are addressed in Section 5, and related new research directions are discussed in Section 6. The core of the paper is Section 4, while the other sections are supporting.

¹ Zhang et al. [1].

2. Historical origins of pricing and demand estimation

Dynamic pricing with learning can be regarded as the combined application of two research fields: (1) statistical learning, specifically applied to the problem of estimating demand functions, and (2) price optimization. Both these fields are already quite old, dating back more than a century. In this section we briefly describe the historical fundamentals out of which dynamic pricing and learning has emerged, by pointing to some key references on static pricing and estimating demand functions that have been important in the progress of the field.

2.1. Demand functions in pricing problems

Cournot [38] is generally acknowledged as the first to use a mathematical function to describe the price–demand relation of products, and subsequently solve the mathematical problem of determining the optimal selling price. As vividly described by Fisher [39], such an application of mathematical methods to study an economical problem was quite new and controversial at the time, and the work was neglected for several decades. Cournot showed that if $F(p)$ denotes the demand as a function of price p , where $F(p)$ is continuous, decreasing in p , and $pF(p)$ is unimodal, then the price that maximizes the revenue $pF(p)$ can be found by equating the derivative of $pF(p)$ to zero. If $F(p)$ is concave there is a unique solution, which is the optimal price (this is contained in Chapter IV of Cournot [38]). In this way, Cournot was the first to solve a “static pricing” problem by mathematical methods.

2.2. Demand estimation

To make theoretical knowledge on optimal pricing theory applicable in practical problems, one needs to have an estimate of the demand function. The first known empirical work on demand curves is the so-called King–Davenant Law [40] which relates the supply and price of corn (see [41], for an exposition on the origins of this work). More advanced research on estimating demand curves, by means of statistical techniques such as correlation and linear regression, took off in the beginning of the 20th century. Benini [42], Gini [43] and Leffeldt [44] estimate demand curves for various goods as coffee, tea, salt, and wheat, using various curve-fitting methods. Further progress on methodology was made, among others, by Moore [45,46], Wright [47] and Tinbergen [48]; the monumental work of Schultz [49] gives a thorough overview of the state-of-the-art of demand estimation in his time, accompanied by many examples. Further references and more information on the historical progress of demand estimation can be found in [50, Section II], [51, particularly section iii], [52–54]. A small sample of the many contemporary studies on demand estimation in different contexts is Berry et al. [55], McFadden and Train [56] and Bajari and Benkard [57].

2.3. Practical applicability

Estimating demand curves of various products was in first instance not aimed at profit optimization of commercial firms, but rather used to support macro-economic theories on price, supply, and demand. Application of the developed methods in practical problems was initially far away. An illustrative quote is from [58], who doubted the possibilities of applying the theory of monopoly pricing on practical problems, exactly because of the difficulty of estimating the demand curve:

It is evidently the opinion of some of the writers under discussion that the modern theory of monopoly is not only capable of

throwing considerable light on the general principles underlying an individualistic economic structure, but that it is also capable of extensive use in the analysis of particular practical economic problems, that is to say, in applied economics. Personally, I cannot but feel sceptical about this. [...] There does not seem to be any reason why a monopolist should not make a mistake in estimating the slope of the demand curve confronting him, and should maintain a certain output, thinking it was the position which maximized his profit, although he could actually have increased his profit by expanding or contracting. [58, p. 18,19].

Hawkins [59] reviews some of the attempts made by commercial firms to estimate the demand for their products. Most of these attempts were not successful, and suffered from difficulties of obtaining sufficiently many data for reliable estimates, and of changes in the quality of the product and the prices of competitors. Even a very detailed study of General Motors on automobile demand ends, somewhat ironically, with:

The most important conclusion from these analyses of the elasticity of demand for automobiles with respect to price is that no exact answer to the question has been obtained. [60, p. 137].

In view of these quotations, it is rather remarkable that dynamic pricing and learning has nowadays found its way in practice; many applications have been reported in various branches such as airline companies, the hospitality sector, car rental, retail stores, internet advertisement, and many more. A main cause for this is the fact that nowadays historical sales data is typically digitally available. This significantly reduces the efforts required to obtain sufficiently accurate estimates of the demand function. In addition, whenever products are sold via the Internet or using digital price tags, the costs associated with adjusting the prices in response to updated information or changed circumstances are practically zero. In contrast, a price-change in the pre-digital era would often induce costs, for example because a new catalog had to be printed or price tags had to be replaced. For a detailed study on such price-adjustment costs we refer to Zbaracki et al. [61] and the references therein. Slade [62] and Netessine [63] are two studies that consider dynamic pricing with costly price adjustments.

3. Dynamic pricing

In this section we discuss the literature on dynamic pricing. Because of the huge amount of literature on this subject, we cannot give a complete overview of the field. Instead, we briefly describe some of the major research streams and key references, in order to provide a context in which one can position the literature on dynamic pricing with learning discussed in Section 4. For a more elaborate overview of the vast literature on dynamic pricing, we refer to the books Talluri and van Ryzin [8], Phillips [9], Rao [12], Özer and Phillips [15], and the reviews by Bitran and Caldentey [6], Elmaghraby and Keskinocak [7], Heching and Leung [10], Gönsch et al. [11], Seetharaman [64], Chenavaz et al. [13] and Deksnite and Lydeka [14].

The literature on dynamic pricing by a monopolist firm can roughly be classified as follows:

- Models where the demand function is dynamically changing over time.
- Models where the demand function is static, but where pricing dynamics are caused by the inventory level.

In the first class of models, reviewed in Section 3.1, the demand function changes according to changing circumstances: for example, the demand function may depend on the time-derivative of price, on the current inventory level, on the amount of cumulative sales, on the firm's pricing history, et cetera. In the second class of

models, reviewed in Section 3.2, it is not the demand function itself that causes the price dynamics: a product offered in two different time periods against the same selling price is expected to generate the same amount of average demand. Instead, the price dynamics are caused by inventory effects, more concretely by changes in the marginal value of inventory. Naturally, it is also possible to study models that fall both in classes, where both the demand function is dynamically changing and the price dynamics are influenced by inventory effects; some of this literature is also reviewed in Section 3.2.

3.1. Dynamic pricing with dynamic demand

3.1.1. Demand depends on price-derivatives

Evans [65] is one of the first to depart from the static pricing setting introduced by Cournot [38]. In a study on optimal monopoly pricing, he assumes that the (deterministic) demand is not only a function of price, but also of the time-derivative of price. This models the fact that buyers do not only consider the current selling price in their decision to buy a product, but also anticipated price changes. The purpose of the firm is to calculate a price function, on a continuous time interval, that maximizes the profit. The problem is solved explicitly using techniques from calculus of variations. Various extensions to this model are made by Evans [66], Roos [67–70], Tintner [71], and Smithies [72]. Thompson et al. [73] study an extended version of the model of Evans [65], where optimal production level, investment level, and output price have to be determined. Closely connected to this work is Simaan and Takayama [74], who consider a model where supply is the control variable, and where the time-derivative of price is a function of the current supply and current price. Methods from control theory are used to derive properties of the optimal supply path.

3.1.2. Demand depends on price history

A related set of literature considers the effect of reference prices on the demand function. Reference prices are perceptions of customers about the price that the firm has been charging in the past; see [75] for a review on the subject. A difference between the reference price and the actual selling price influences the demand, and, as a result, each posted selling price does not only affect the current demand but also the future demand. Dynamic pricing models and properties of optimal pricing strategies in such settings are studied, among others, by Greenleaf [76], Kopalle et al. [77], Fibich et al. [78], Heidhues and Köszegi [79], Popescu and Wu [80] and Ahn et al. [81].

3.1.3. Demand depends on amount of sales

Another stream of literature on dynamic pricing emerged from diffusion and adoption models for new products. A key reference is Bass [82], and reviews of diffusion models are given by Mahajan et al. [83], Baptista [84], and Meade and Islam [85]. In these models, the demand for products does not only depend on the selling price, but also on the amount of cumulative sales. This allows modeling several phenomena related to market saturation, advertisement, word-of-mouth effects, and product diffusion. Robinson and Lakhani [86] study dynamic pricing in such a model, and numerically compare the performance of several pricing policies with each other. Their work stimulated much further research on optimal dynamic pricing policies, see e.g. [87–89], and the references therein. The models studied in these papers are deterministic, and somewhat related to the literature following Evans [65]: both types of pricing problems are solved by principles from optimal control theory, and the optimal pricing strategy is often given by the solution of a differential equation.

Chen and Jain [90], Raman and Chatterjee [91], and Kamrad et al. [92] extend these models by incorporating randomness in

the demand. In [90], the demand is determined by a finite-state Markov chain for which each state corresponds to a deterministic demand function that depends on price and cumulative sales. The optimal price path is characterized in terms of a stochastic differential equation, and compared with the optimal policy in a fully deterministic setting. Raman and Chatterjee [91] model uncertainty by adding a Wiener process to the (known) deterministic component of the demand function. They characterize the pricing policy that maximizes discounted cumulative profit, and compare it with the optimal price path in the fully deterministic case. Under some specific assumptions, closed form solutions are derived. Similar models that incorporate demand uncertainty are analyzed by Kamrad et al. [92]. For various settings they provide closed-form solutions of the optimal pricing policies.

3.2. Dynamic pricing with inventory effects

There are two important research streams on dynamic pricing models where the dynamics of the optimal pricing policy are caused by the inventory level: (i) “revenue management” type of problems, where a finite amount of perishable inventory is sold during a finite time period, and (ii) joint pricing and inventory–procurement problems.

3.2.1. Selling a fixed, finite inventory during a finite time period

In this stream of literature, a firm is assumed to have a certain number of products at its disposal, which are sold during a finite time period. There is no replenishment: inventory that is unsold at the end of the selling horizon is lost, and cannot be transferred to another selling season. In these problems, the dynamic nature of optimal prices is not caused by changes in the demand function, but by the fact that the marginal value of remaining inventory is changing over time. As a result, the optimal selling price is not a fixed quantity, but depends on the remaining amount of inventory and the remaining duration of the selling season.

Kincaid and Darling [93] may be the first to characterize and analyze the optimal pricing policy in such a setting. A more recent key reference is Gallego and van Ryzin [94]. They consider a continuous-time setting where demand is modeled as a Poisson process, with arrival rate that depends on the posted selling price. The pricing problem is formulated as a stochastic optimal control problem, and the optimal solution is characterized using the Hamilton–Jacobi–Bellman equation. For the exponential demand function a closed-form solution is derived. Because the optimal policy changes prices continuously, which may be undesirable in practical applications, two heuristics are proposed: one based on a deterministic version of the problem, and one based on determining the optimal fixed price. Both these heuristics are shown to be asymptotically optimal as the expected amount of sales grows large or as the length of the time horizon converges to zero. The authors further propose price heuristics – and show their asymptotic optimality – for many extensions of the problem: a discrete instead of a continuous set of feasible prices, customers who arrive according to a compound Poisson process, a demand function that depends both on price and on the time elapsed since the start of the selling season, the presence of holding costs and a discount rate, the case where initial inventory is a decision variable, and a setting where resupply, cancellations and overbookings are possible.

Numerous other extensions and variations of the model by Gallego and van Ryzin [94] have been studied: for example, settings with restrictions on the number of allowable prices or price changes [95–98], time-varying demand functions [99,100], and extensions to multiple stores [101] or multiple products [102,103] that share the same finite set of resources.

The extension to multiple products can be formulated as a dynamic program, which typically is intractable due to the curse of dimensionality. A number of papers study heuristic solutions [102, 104–109], which are typically based on static-price policies or on deterministic approximations or decompositions of the original dynamic program. Another stream of literature focuses on deriving structural properties of the price optimization problem, for various models of consumer demand [110–113].

Another important extension to Gallego and van Ryzin [94] is strategically behaving customers: customers who, when arriving at the (online) store, do not immediately decide whether to buy the product, but instead wait for a while to anticipate possible decreases in the selling price. In contrast, so-called myopic customers instantly decide whether to buy the product at the moment they arrive at the store. In such settings, the demand at a certain moment depends on the past, present, and future selling prices. Dynamic pricing in view of strategic customers has received a considerable amount of research attention in recent years; a representative sample is Su [114], Aviv and Pazgal [115], Elmaghraby et al. [116], Liu and van Ryzin [117], Levin et al. [118], Cachon and Swinney [119] and Su [120]. Reviews of these literature are given by Shen and Su [121] and Gönsch et al. [122]. These studies typically have a game-theoretic flavor, since both the firm and the strategic customers have a decision problem to solve, with contradicting interests.

3.2.2. Jointly determining selling prices and inventory–procurement

A main assumption of the literature discussed above is that the initial capacity level is fixed. In many situations in practice this is a natural condition: the number of seats in an aircraft, rooms in a hotel, tables in a restaurant, or seats in a concert hall are all fixed for a considerable time period, and modifications in the capacity occur at a completely different time scale than dynamic price changes. In many other settings, however, the initial capacity is a decision variable to the firm; in particular, when the firm can decide how many items of inventory should be produced or purchased. Pricing and inventory management can then be considered as a simultaneous optimization problem.

This research field bridges the gap between the pricing and inventory management literature. Many different settings and models are subject to study, with different types of production, holding and ordering costs, different replenishment policies (periodic or continuous), finite or infinite production capacity, different models for the demand function, et cetera. Extensive reviews of the literature on simultaneous optimization of price and inventory decisions can be found in [123,124], [7, Section 4.1], [125–128].

4. Dynamic pricing and learning

In the static monopoly pricing problem considered by Cournot [38], the demand function is deterministic and completely known to the firm. These assumptions are somewhat unrealistic in practice, and eventually it was realized that demand should be modeled as a random variable. One of the first to pursue this direction is Mills [129], who assumes that the demand is the sum of a random term with zero mean and a deterministic function of price. He studies how a monopolist firm that sells finitely many products in a single time period should optimally set its production level and selling price. Further extensions of this model and properties of pricing problems with random demand are studied by Karlin and Carr [130], Nevins [131], Zabel [132], Baron [133,134], Sandmo [135] and Leland [136]. An important research question in these studies is how the firm's optimal decisions are influenced by the demand uncertainty and by the firm's attitude towards risk (risk-neutral, risk-averse, or risk-preferred).

The papers mentioned above model demand as a random variable, but still assume that the expected demand (as a function of the selling price) is completely known by the firm. This makes these models somewhat unrealistic and not usable in practice. The common goal of the literature on dynamic pricing and learning is to develop pricing policies that take the intrinsic uncertainty about the relation between price and expected demand into account.

In the next two sections we discuss the literature on dynamic pricing and learning. Section 4.1 considers the literature on the problem of a price-setting firm with infinite inventory and unknown demand function. This basically is the monopoly pricing problem described in Section 2.1, with uncertainty about the demand function. The full-information case of this problem is static; the price dynamics are completely caused by the fact that the firm learns about the price–demand relation through accumulating sales data. Section 4.2 discusses literature on pricing policies for firms selling a fixed, finite amount of inventory, with unknown demand function. For this problem, the full-information case is already dynamic by itself, as discussed in Section 3.2, and the learning aspect of the problem provides an additional source of the price dynamics.

4.1. No inventory restrictions

4.1.1. Early work

Uppsala econometrics seminar. The first analytical work on dynamic monopoly pricing with unknown demand curve seems to have been presented on August 2, 1954, at the 16th European meeting of the Econometric Society [137], by F. Billström, H. Laadi, and S.A.O. Thore, with contributions from L.O. Friberg, O. Johansson, and H.O.A. Wold. A mimeographed report Billström et al. [138] of the presented work has not been published in a journal, but an English reprint has appeared in [139,140]. These two works discuss the problem of a monopolist facing a linear demand curve that depends on two unknown parameters. Thore [140] proposes to use a dynamic pricing rule that satisfies $\text{sign}(p_t - p_{t-1}) = \text{sign}((p_{t-1} - p_{t-2})(r_{t-1} - r_{t-2}))$, where p_t , r_t denote the price and revenue in period t . This models the following intuition: if a previous price increase led to an increase in revenue, the price will again be increased; otherwise it will be decreased. Similarly, if a previous price decrease led to an increase in revenue, the price will again be decreased; otherwise, it will be increased. In addition, Thore [140] proposes to let the magnitude of the price adjustment depend on the difference between the last two revenue observations. He specifies two pricing rules in detail, and analyzes convergence properties of the resulting dynamical systems. Billström and Thore [139] perform simulation experiments for one of these pricing rules, both in a deterministic demand setting and in a setting where a normally distributed disturbance term is added to the demand. They also extend the model to incorporate inventory replenishment, and provide a rule of thumb for the optimal choice of a constant appearing in the pricing rule.

Subsequent work. These studies emerging from the Uppsala Econometrics Seminar have not received much research attention in subsequent years. Clower [141] studies a monopolist firm facing a linear, deterministic demand function whose parameters may change over time. He discusses several price-adjustment mechanisms that may be applied by the firm to adapt its prices to changing situations. Baumol and Quandt [142] propose rules of thumb for the monopolist pricing problem, and assess their performance by a set of numerical experiments. In their Appendix A they propose a pricing rule equal to one of the rules proposed by Thore [140], although they are apparently unaware of that work. The authors investigate some convergence and stability

properties of the resulting dynamical system, both in a discrete-time and continuous-time framework. Baetge et al. [143] extend the simulation results of Billström and Thore [139] to non-linear demand curves, and study the optimal choice of a constant appearing in the pricing rules. A final study in this line of research is from [144]. He studies a model where a monopolist has to decide on price, output level in the current period and maximum output in the next period. Three decision rules are compared with each other via a computer simulation. In addition, their performance is compared with a laboratory experiment, where test subjects had to determine their optimal pricing strategy.

4.1.2. Bayesian approaches

Several authors study the dynamic pricing and learning problem within a Bayesian framework.

Work by Aoki. One of the first is Aoki [145], who applies methods from stochastic adaptive control theory. He considers a setting where the demand function depends on unknown parameters, which are learned by the decision maker in a Bayesian fashion. The purpose is to minimize (a function of) the excess demand. He shows how the optimal Bayesian policy can, in theory, be computed via dynamic programming, but that in many situations no closed-form analytical expression of the solution exists. He then proposes two approximation policies. In the first, certainty equivalent pricing (CEP), at each decision moment the price is chosen that would be optimal if the current parameter estimates were correct. In the second, called an approximation under static price expectation, the firm acts at each decision moments as if the chosen price will be maintained throughout the remainder of the selling period. Aoki [146] shows that the prices generated by (two variants of) CEP and by the static-price-expectation approximation both converge a.s. to the optimal price.

Variations and extensions to Aoki [145]. An early study along the same lines of Aoki [145] is Chong and Cheng [147]. They assume a linear demand function with two unknown parameters and normally distributed disturbance terms, and formulate the optimal pricing problem as a Bayesian dynamic program. They show that certainty equivalent pricing is the optimal policy in case the slope of the demand function is known. For the case that both slope and intercept are unknown, they propose three approximations to the optimal policy: the approximation under static price expectation from [145], and two heuristics based on approximations of the value function. Their second approximation selects at each decision epoch the price that maximizes the difference between the expected profit and a term proportional to the covariance of the parameter estimates; this reflects the exploration–exploitation trade-off seen in many later work on optimization under uncertainty.

Other studies closely related to Aoki [145] are Nguyen [148,149], Wruck [150], Lobo and Boyd [151], Chhabra and Das [152], Qu et al. [153], and Kwon et al. [154]. Nguyen [148] considers a quantity-setting monopolist firm facing random demand in multiple periods, where the demand function depends on an unknown parameter which is learned by the firm in a Bayesian fashion. Structural (monotonicity) properties of the optimal policy are derived, and its performance is compared with a myopic one-period policy. Nguyen [149, Section 5] discusses similar questions in the context of a price-setting monopolist. Wruck [150] considers optimal pricing of durable and non-durable goods in a two-period model. The support of the uniformly distributed willingness-to-pay distribution is learned by Bayesian updating a uniform prior, and the optimal price policy is determined by solving a dynamic program. Lobo and Boyd [151] consider the same setting as Chong and Cheng [147], and compare via simulation the performance of four pricing policies with each other. Chhabra and Das [152]

study the finite-time performance of standard multi-armed bandit policies and of a policy that (possibly incorrectly) assumes a linear demand function whose unknown parameters are learned by Bayesian updating of a Beta prior. Qu et al. [153] assume Bernoulli distributed demand with expectation a logit function of price, with a normal prior on the unknown parameters. Because this distributional form hampers exact calculation of posterior distributions, the authors discuss how to calculate an approximation. They propose a Bayesian-greedy price policy which cannot be computed exactly either, and show how an approximation can be calculated. A numerical study compares the pricing policies with a few alternatives. Kwon et al. [154] study optimal markdown pricing in an infinite-horizon setting with discounted rewards, where the decision variables are the initial price, the markdown price, and the time of markdown. Cumulative demand is modeled as a Brownian motion with unknown drift which is either high or low and which is learned via Bayesian updating. The authors determine the optimal time of markdown, characterize the corresponding optimal initial and markdown prices, and prove a few monotonicity properties.

Finite action set. Some literature simplifies the problem by allowing only a finite set of feasible prices. This transforms the pricing-and-learning problem to a Bayesian multi-armed bandit problem. Leloup and Deveaux [155] show for Bernoulli distributed arms that approximations to Gittins-index policies circumvent the computational problems associated with solving the full Bayesian dynamic program. Wang [156] allows two feasible prices, assumes compound-Poisson distributed rewards from a certain parametric family, and approximate the Gittins index. Cope [157] considers a general Dirichlet prior on the discretized reservation-price distribution of customers, and develops approximations to the intractable Bayesian dynamic program that are closely related to Gittins-index policies. He shows that his pricing heuristics converge to the optimal price under an average-reward criterion, and argues that their performance do not suffer much from a misspecified prior distribution.

Incomplete learning and remedies. A common theme in the references mentioned above is that it is often intractable to compute the optimal Bayesian policy, and that therefore approximations are necessary. Rothschild [158] points to a more fundamental problem of the Bayesian framework. He assumes that there are only two prices the firm can choose, with demand for each price Bernoulli distributed with unknown mean. The dynamic pricing problem is thus viewed as a two-armed bandit problem. The optimal Bayesian policy can be computed via the corresponding dynamic programming formulation. The key result of Rothschild [158] is that, under the optimal Bayesian strategy, with positive probability the price sequence converges to a price that (with hindsight) is not the optimal price. McLennan [159] derives a similar conclusion in a related setting: the set of admissible prices is continuous, and the relation between price and expected demand is one of two known linear demand curves. It turns out that, under an optimal Bayesian policy, the sequence of prices may converge with positive probability to a price different from the optimal price. This work is extended by Harrison et al. [160], who show that in several instances a myopic Bayesian policy may lead to such “incomplete learning”. They propose two modifications of the myopic Bayesian policy that avoid incomplete learning, and prove bounds on their performance. Afèche and Ata [161] derive incomplete-learning results of similar flavor in the context of pricing different types of customers in an M/M/1 queue. Cheung et al. [162] extend Harrison et al. [160] to the case with $k \in \mathbb{N}$ unknown demand functions, and where in addition at most m price changes are allowed. They propose a pricing policy, show that it achieves $\text{Regret}(T) = O(\log^{(m)} T)$ where $\log^{(m)}$ denotes the m -times iterated logarithm, and prove that any

policy has regret $\Omega(\log^{(m)} T)$.² Similar results are shown to hold in a continuous-time setting. Keskin [163] models the cumulative deviation from the expected demand at an incumbent price as the sum of a Wiener process and a drift-term proportional to the difference between selling price and incumbent price. The unknown drift-coefficient is learned by Bayesian updating of the parameters of a Gaussian prior. The author explains why a myopic policy induces incomplete learning, and characterizes the optimal learning-policy as the solution to a partial differential equation (PDE). Based on this policy, he proposes a simple pricing rule that deviates from the myopic price proportionally to the squared coefficient of variation of the posterior belief on the optimal price, and that does not require solving a PDE. Numerical illustrations suggest that the performance of this heuristic is close to optimal.

Risk-averse pricing. All studies mentioned above assume that the firm is risk-neutral and optimizes the expected revenue. Sun and Abbas [164] depart from this assumption by studying the optimal price in a Bayesian dynamic-pricing-and-learning problem with a risk-averse seller. Choi et al. [165] study a family of simple pricing policies, in a Bayesian setting, based on separating the finite time horizon in an exploration and exploitation phase. They calculate the optimal risk-averse price with respect to a number of risk measures, and provide numerical examples.

Economics and econometrics literature. The economics and econometrics literature also contains several studies on pricing and Bayesian learning. Prescott [166], Grossman et al. [167], Mirman et al. [168] consider two-period models and study the necessity and effects of price experimentation. Trefler [169] focuses on the direction of experimentation, and applies his results on several pricing problems. Rustichini and Wolinsky [170] and Keller and Rady [171] consider a setting where the market environment changes in a Markovian fashion between two known demand functions, and study properties of optimal experimentation. Balvers and Cosimano [172] consider a dynamic pricing model where the coefficients of a linear demand model change over time, and discuss the implications of active learning. Willems [173] aims to explain observed discreteness in price data. The author considers a model where the expected demand depends linearly on price via two time-varying parameters with known expectation. The author elaborates a Bayesian learning approach via dynamic programming, and discusses the differences between active and passive learning. Easley and Kiefer [174], Kiefer and Nyarko [175], Aghion et al. [176] are concerned with Bayesian learning in general stochastic control problems with uncertainty. They study the possible limits of Bayesian belief vectors, and show that in some cases these limits may differ from the true value. This implies that active experimentation is necessary to obtain strongly consistent control policies.

Related studies on optimal market design. Finally, we mention the studies Manning [177] and Venezia [178] on optimal design of market research. Manning [177] considers a monopolist firm facing a finite number of customers. By doing market research, the firm can ask n potential customers about their demand at some price p . Such market research is not for free, and the main question of the paper is to determine the optimal amount of market research. This setting is closely related to pricing rules that split the selling season in two parts (e.g. the first pricing rule proposed by Witt [144]): in the first phase, price experimentation takes place in order to learn the unknown parameters, and in the second phase of the selling season, the myopic price is used. Venezia [178]

considers a linear demand model with unknown parameters, one of which behaves like a random walk. The firm learns about these parameters using Bayes' rule. In addition, the firm can learn the true current value of this random walk by performing market research (which costs money). The author shows how the optimal market-research policy can be obtained from a dynamic program.

4.1.3. Non-Bayesian approaches

Policies without performance bounds. Despite the disadvantages of the Bayesian framework outlined above (computational intractability of the optimal solution, the results by Rothschild [158] and McLennan [159] on incomplete learning), it has taken several decades before research on pricing policies in a non-Bayesian setting took off. An early exception is Aoki [146], who proposes a pricing scheme based on stochastic approximation in a non-Bayesian framework. He proves that the prices converge almost surely to the optimal price, and compares the policy with Bayesian pricing schemes introduced in [145]. More recently, Carvalho and Puterman [179,180] and Morales-Enciso and Branke [181] propose several pricing heuristics based on approximations of an underlying finite-horizon dynamic program whose states contain historical price/demand observations (reminiscent of the dynamic-programming approximations developed by Aoki [145,146]).

Seminal paper with performance bounds. A disadvantage of the many pricing heuristics that have been proposed in the literature, both in a Bayesian and a non-Bayesian setting, is that a qualitative statement of their performance is often missing. In many studies the performance of pricing policies is only evaluated numerically, without any analytical results. This changes with the groundbreaking work of Kleinberg and Leighton [182], who quantify the performance of a pricing policy by $\text{Regret}(T)$: the expected loss in T time periods incurred by not choosing optimal prices. They consider a setting where buyers arrive sequentially to the firm, and buy only if their willingness-to-pay (WtP) exceeds the posted price. Under some additional assumptions, they show that if the WtP of the individual buyers is an i.i.d. sample of a common distribution, then there is no pricing policy that achieves $\text{Regret}(T) = o(\sqrt{T})$; in addition, there is a pricing policy that achieves $\text{Regret}(T) = O(\sqrt{T} \log T)$. In an adversarial or worst-case setting, where the WtP of individual buyers is not assumed to be i.i.d., they show that no pricing policy can achieve $\text{Regret}(T) = o(T^{2/3})$, and that there is a pricing policy with $\text{Regret}(T) = O(T^{2/3}(\log T)^{1/3})$.

Parametric approaches. Le Guen [183], Broder and Rusmevichientong [184], den Boer and Zwart [185], den Boer [186], and Keskin and Zeevi [187] take a parametric approach, using classical maximum (quasi)likelihood or least-squares estimators to estimate unknown parameters of the demand function. Le Guen [183] considers a multi-product setting with linear demand, assuming a particular structure on the unknown parameter matrices. He shows that certainty equivalent pricing augmented with price experimentation at predetermined time intervals leads to prices converging to the true optimal price, and proposes a pricing heuristic for non-linear demand functions. Assuming a single product setting with Bernoulli distributed demand, Broder and Rusmevichientong [184] show a \sqrt{T} lower bound on the regret, using information-theoretic inequalities and techniques found in [188], and show that a pricing policy that strictly separates exploration from exploitation achieves $O(\sqrt{T})$ regret. This rate can be improved to $O(\log T)$ if the demand-function is such that there are no “uninformative prices”: prices at which the expected demand given a (erroneous) parameter estimate is equal to the true expected demand (the existence of such prices plays an important role in the best achievable growth rate of the regret; cf. [160]). den Boer and Zwart [185] consider an extended class of generalized linear single-product demand models, and show in an example

² We use $f(T) = O(g(T))$ to denote $\sup_{T \in \mathbb{N}} f(T)/g(T) < \infty$, $f(T) = o(g(T))$ to denote $\limsup_{T \rightarrow \infty} f(T)/g(T) = 0$, and $f(T) = \Omega(g(T))$ to denote $g(T) = O(f(T))$, when f and g are functions on \mathbb{N} .

that certainty equivalent pricing is not strongly consistent. They propose a pricing policy that always chooses the price closest to the certainty equivalent price that guarantees a minimum amount of price dispersion. This price dispersion, measured by the sample variance of the selling prices, guarantees convergence of the prices to the optimal price, and leads to $\text{Regret}(T) = O(T^{1/2+\delta})$, for arbitrary small $\delta > 0$. den Boer [186] extends this policy to multiple products, attaining $\text{Regret}(T) = O(\sqrt{T \log T})$ for so-called canonical link functions and $\text{Regret}(T) = O(T^{2/3})$ for general link functions. Keskin and Zeevi [187] assume a linear demand function, show a \sqrt{T} lower bound on the regret using proof techniques different from [184], and generalize Broder and Rusmevichientong [184] and den Boer and Zwart [185] by providing sufficient conditions for any pricing policy to achieve $\text{Regret}(T) = O(\sqrt{T \log T})$. Assuming that the mean demand is exactly known at a certain price, they show that $\text{Regret}(T) = O(\log T)$ is attainable. Both these results are extended to the multiple-product setting, focusing on a class of so-called orthogonal pricing policies.

Robust optimization. Eren and Maglaras [189] study dynamic pricing in a robust optimization setting. They show that if an infinite number of goods can be sold during a finite time interval, it is optimal to use a price-skimming strategy. They also study settings where learning of the demand function occurs, but under the rather strong assumption that observed demand realizations are without noise. Bergemann and Schlag [190,191] and Handel et al. [192] also consider pricing in a robust framework, but their setting is static instead of dynamic. Handel and Misra [193] consider a two-period model where a monopolist sets prices based on a set of demand curves feasible with acquired sales data. The authors describe and analyze the optimal two-period pricing policy that minimizes a dynamic version of the minimax regret, and investigate how customer preferences influence the difference between dynamic and static prices.

Finite action set. If the demand model is assumed to lie in a finite set of known demand functions, the dynamic-pricing-and-learning problem can be regarded as a multi-armed bandit problem with dependent arms. This viewpoint is taken by Tehrani et al. [194], who develop a pricing policy based on the likelihood-ratio test, and show that its regret is bounded assuming that there are no uninformative prices. Their work can be viewed as a non-Bayesian counterpart to Harrison et al. [160] and Cheung et al. [162].

Variants. Pricing without demand information in a queueing model is studied by Haviv and Randhawa [195]. They consider the problem of pricing delay-sensitive customers in an unobservable M/M/1 queue. The purpose of the paper is to study the impact of ignoring arrival-rate information on the optimal pricing strategy. The authors find that a policy that ignores this information performs surprisingly well, and in some cases can still capture 99% of the optimal revenue.

Finally, we mention Jia et al. [196], who consider dynamic pricing and learning in an electricity market, where the goal is to steer the expected demand to a desired level. This particular objective is reminiscent of the multi-period control problem discussed in [197,198]. A stochastic-approximation type policy inspired by Lai and Robbins [199] is shown to achieve $O(\log T)$ regret, and in addition it is shown that no policy can achieve sub-logarithmic regret. The fact that logarithmic instead of \sqrt{T} regret can be achieved is caused by subtle differences between dynamic pricing and this multi-period control problem, which are further discussed in [185, Remark 1].

4.2. Finite inventory

We here discuss the literature on dynamic pricing and learning in the presence of finite inventory that cannot be replenished.

Most of the studies assume a finite selling season, corresponding to the models discussed in Section 3.2.1. Some studies assume an infinite time horizon, and consider the objective of maximizing total discounted reward.

4.2.1. Early work

Lazear [200] considers a simple model where a firm sells one item during at most two periods. In the first period a number of customers visit the store; if none of them buys the item, the firm adapts its prior belief on the value of the product, updates the selling price, and tries to sell the item in the second period. The author shows that the expected profit increases by having two selling periods instead of one. He extends his model in several directions, notably by allowing strategic behavior of customers who may postpone their purchase decision if they anticipate a price decrease. Sass [201] extends the model of Lazear and studies the relation between the optimal price strategy and the number of potential buyers.

4.2.2. Bayesian approaches

Unknown arrival rate, known willingness-to-pay. Aviv and Pazgal [202] start a research stream on Bayesian learning in dynamic pricing with finite inventory. Customers arrive according to a Poisson process with unknown arrival rate, and purchase a product with (known) probability $\exp(-p)$, where p is the current selling price. The unknown arrival rate is learned via Bayesian updates of a Gamma prior. The authors characterize the optimal continuous-time pricing policy by means of a differential equation. Because this equation does not always admit an explicit solution, three pricing heuristics are proposed: certainty equivalent pricing (CEP), a fixed price policy, and a naive pricing policy that ignores uncertainty on the market. Numerical experiments suggest that CEP performs quite well. Lin [203] considers a similar setting, allowing for general willingness-to-pay distributions. He proposes a pricing policy where the seller sets the price based on repeatedly updated estimates of the demand distribution, and evaluates its performance via simulations.

Araman and Caldentey [204], Farias and van Roy [205] and Mason and Välimäki [206] study the infinite-horizon discounted-reward case. Araman and Caldentey [204] assume a known willingness-to-pay distribution and a two-point prior distribution on the unknown arrival rate, propose a pricing heuristic based on an asymptotic approximation of the value function of the underlying optimal-control problem, and compare its performance numerically with CEP, static pricing, and a two-price policy. In a similar setting Farias and van Roy [205] take a finite mixture of gamma distributions as prior, propose a pricing heuristic called decay balancing, and show numerically that it frequently outperforms CEP and the heuristics of Araman and Caldentey [204]. They further show that the expected discounted reward obtained from decay balancing is at least one third of the optimal reward, and discuss an extension to multiple stores. Mason and Välimäki [206] assume that only a single item is sold, with either high or low customer arrival-rate which is learned in a Bayesian fashion. The authors study structural properties of the optimal price policy and compare it to policies that neglect learning.

Avramidis [207] observes that the number of arrivals or the number of sales is sufficient to compute the posterior arrival-rate distribution, given any prior. This means that the setting considered by Aviv and Pazgal [202], Lin [203], Araman and Caldentey [204] and Farias and van Roy [205] can be resolved without imposing a specific family of priors (Gamma, two-point discrete, finite mixture of Gammas).

Unknown arrival rate and unknown willingness-to-pay. Chen and Wang [208] consider pricing of a single asset in an infinite

horizon with discounted rewards, and assume that the willingness-to-pay distribution is unknown but equal to one of two known distributions. The authors formulate a Bayesian dynamic program and prove that the optimal prices decline over time if the hazard rates of these distributions can be ranked uniformly; a counterexample shows that this condition cannot be relaxed to first-order stochastic dominance. Sen and Zhang [209] extend the model of Aviv and Pazgal [202] by assuming that the purchase probabilities of arriving customers are not known to the firm. They assume that the demand distribution is an element of a finite known set, and consider a discrete-time setting with Bayesian learning and a gamma prior on the arrival rate. The optimal pricing policy can be explicitly calculated, and, in an extensive computational study, its performance is compared with both a full information setting and a setting where no learning occurs.

Partially observable Markov decision processes. Aviv and Pazgal [210] consider a Markov-modulated demand environment, modeled by a Markov chain where each of the finite states corresponds to a different known demand function, and where the state of the system is learned in a Bayesian fashion. The optimal pricing problem is formulated as a Partially Observable Markov Decision Process, that turns out to be computationally intractable. Various approximate solutions are proposed that rely on modifying the information structure of the problem, and their performance is evaluated in a numerical study. Chen [211] considers a similar partially observable Markov decision process. In his setting, the seller estimates the willingness-to-pay distribution of customers based on two-sided censored observations. Three near-optimal price heuristics are proposed and their performance is assessed by numerical experiments. For exponentially or Weibull distributed demand, more refined results on the behavior of the heuristics are obtained.

4.2.3. Non-Bayesian approaches

Asymptotic regime where inventory grows large. The optimal pricing problem studied by Gallego and van Ryzin [94] does often not admit an explicit solution, and the authors therefore consider an asymptotic regime where both the demand and the level of inventory grow large. They prove that the optimal revenue obtained in this asymptotic regime serves as an upper bound for the optimal expected revenue of the original problem, and show that the optimal asymptotic pricing policy is to use a static price throughout the sales horizon. This static price is the maximum of the unconstrained optimal price (the revenue-maximizing price in the case of infinite inventory) and of the clearance price (the price that induces a stock-out precisely at the end of the sales horizon).

Besbes and Zeevi [212] initiate a stream of literature that attempts to learn this optimal static price in an incomplete-information setting. For both a parametric and non-parametric setting they develop pricing policies, based on the idea of dividing the sales horizon into an “exploration” phase during which the demand function is learned and an “exploitation” phase during which the perceived optimal price is used. To establish performance bounds, they consider a sequence of problems indexed by $n \in \mathbb{N}$, where the n -th problem has initial inventory nx and demand function $n\lambda$, for some $x > 0$ and some function λ . They prove an $O(n^{-1/4}(\log n)^{1/2})$ upper bound on the relative regret of this policy in the non-parametric setting, an $O(n^{-1/3}(\log n)^{1/2})$ bound in the parametric setting, and an $O(n^{-1/2}(\log \log n)(\log n)^{1/2})$ bound in case the demand function is known up to a single unknown parameter. These upper bounds are complemented by results showing that all policies have relative regret $\Omega(n^{-1/2})$. In a non-parametric setting, Wang et al. [213] improve these upper bounds by developing a policy with relative regret $O(n^{-1/2}(\log n)^{4.5})$; apart from the logarithmic term, this policy thus achieves the asymptotically optimal regret rate. The pricing policy is based on the idea of iterative

price experimentation in a shrinking series of intervals that with high probability contain the optimal price. Lei et al. [214] provide a further improvement by even removing the logarithmic terms in the upper bound: they propose and analyze three pricing algorithms based on ideas from bisection search and stochastic approximation, and show that one of these algorithms achieves $O(n^{-1/2})$ relative regret.

Besbes and Zeevi [215] extend Besbes and Zeevi [212] to a setting where multiple products share the same finite resources. They show that a policy which separates “exploration” and “exploitation” achieves relative regret $O(n^{-1/(3+d)}(\log n)^{1/2})$, where d is the number of products. This is improved to $O(n^{-1/(3+d/s)}(\log n)^{1/2})$ if one assumes additional smoothness conditions on the demand function, including s -times differentiability, and to $O(n^{-1/3}(\log n)^{1/2})$ if the set of feasible prices is discrete and finite; this last setting is accompanied by an $\Omega(n^{-1/2})$ lower bound on the relative regret of any policy. Another variant of the problem is studied by Besbes and Maglaras [216], who study the situation where certain financial milestone constraints in terms of sales and revenues targets are imposed. They formulate a pricing policy that periodically updates its pricing decisions in order to the most stringent financial constraint, and show an $O(n^{-1/2}(\log n)^{1/2})$ bound on the relative regret. Avramidis [207] modifies the type of policy proposed by Besbes and Zeevi [212] by estimating both arrival rate and purchase probabilities, and by using the solution of the finite-time Markov decision problem in the exploitation phase instead of the solution that corresponds to the asymptotic regime. Numerical experiments suggest that these modifications lead to lower regret.

Alternative asymptotic regimes. The high-volume regime of the studies mentioned above is not applicable for settings with relatively low inventory. This motivates den Boer and Zwart [217] to study a setting with multiple, consecutive, finite selling seasons and finite inventories. They show in a parametric framework that this setting satisfies a certain endogenous-learning property, which implies that price experimentation is hardly necessary to eventually learn the optimal prices. The authors prove that the (cumulative) regret of a small modification of certainty equivalent pricing is $O(\log^2 T)$ after T selling seasons, and that any policy has regret $\Omega(\log T)$.

In [218], a firm tries to sell k items to n potential customers. The authors propose an online pricing-and-learning policy that does not require parametric assumptions on the demand distribution, and that exploits the fact that the problem is closely related to multi-armed bandit problems (despite the finite inventory, which means that the optimal price depends on the current inventory level). They show a regret bound of $O((k \log n)^{2/3})$, and provide a matching lower bound on the regret. If the ratio k/n is sufficiently small this bound is improved to $O(\sqrt{k \log n})$.

Bertsimas and Perakis [219] consider pricing in a least-squares-learning setting with a single selling season. They formulate a dynamic program that describes the optimal pricing policy but is computationally intractable, and propose approximate solutions based on state-space reductions. For an extended model with competitors and slowly varying parameters, estimation methods and price policies are discussed. No performance bounds on pricing policies are provided.

Robust optimization. A number of studies take a robust approach, where the demand function is not learned over time but assumed to lie in some known uncertainty set. Lim and Shanthikumar [220] and Thiele [221] study this in a single-product setting, and Lim et al. [222], Thiele [223] in a multi-product setting. With these robust approaches no learning takes place, despite the accumulation of sales data. Cohen et al. [224] develop an approach to dynamic pricing and learning that attempts to bridge this gap

between robust and data-driven approaches, by sampling different scenarios from historical sales data. A robust extension of Araman and Caldentey [204] and Farias and van Roy [205], where finite inventory is sold during an infinite time horizon, is studied by Li et al. [225]. Another approach that does not rely on historical demand data is Xiong et al. [226] (see also [227–229]). They model demand uncertainty using fuzzy set theory, propose different fuzzy programming models, and present an algorithm based on fuzzy simulation and a genetic algorithm to solve these problems. Dzieciuchowicz et al. [230] do not focus on determining optimal prices, but instead study the optimal timing of markdowns. The authors derive a robust optimization problem to determine the optimal markdown policy, which in some special cases can be solved exactly and in other cases can be approximated by a mixed-integer problem. The results are also extended to multiple products. Finally, Ferrer et al. [231] assume that demand is a non-random function of price, and lies in an uncertainty set of demand functions known to the decision maker. The authors introduce a measure of risk aversion, and study the relation between risk-aversion and properties of the optimal pricing policy, both theoretically and by numerical simulations.

Variants. Gallego and Talebian [232] consider a setting where a finite number of products is offered in multiple versions during a finite sales horizon. Demand is modeled by a customer choice model. The different product-versions share an unknown “core value”, which is estimated by maximum likelihood estimation. The possible time-varying arrival rate of customers is learned in a Bayesian fashion. The authors develop a pricing rule in a rolling horizon framework, and illustrate its behavior by a computational study. Somewhat related to this is Berg and Ehtamo [233], where a firm sells different versions of a product to different customer segments. The utility functions of each segment are partly unknown, and learned by the firm using stochastic gradient methods or variants thereof.

4.3. Machine-learning approaches

A considerable stream of literature on dynamic pricing and learning has emerged from the computer science community. In general, the focus of these papers is not to provide a mathematical analysis of the performance of pricing policies, but rather to design a realistic model for electronic markets and subsequently apply machine learning techniques. An advantage of this approach is that one can model many phenomena that influence the demand, such as competition, fluctuating demand, and strategic buyer behavior. A drawback is that these models are often too complex to analyze analytically, and insights on the behavior of various pricing strategies can only be obtained by performing numerical experiments.

Machine-learning techniques that have been applied to dynamic pricing problems include evolutionary algorithms [234, 235], particle swarm optimization [236], reinforcement learning and Q-learning [237–252], simulated annealing [253], Markov chain Monte Carlo methods [254], the aggregating algorithm [255] by Vovk [256], goal-directed and derivative-following strategies in simulation [257, 258], neural networks [259–263], and direct search methods [259, 264–266].

These papers all use very different models, methods, assumptions and performance metrics. This makes it hard to compare different papers with each other, which we therefore do not attempt.

4.4. Joint pricing and inventory problems

A few studies consider the problem of simultaneously determining an optimal pricing and inventory replenishment policy while learning about the demand.

Parametric approaches. Most of them consider learning in a Bayesian framework. Subrahmanyam and Shoemaker [267] assume that the unknown demand function lies in a finite known set of demand functions, and is learned in a Bayesian fashion. The optimal policy is determined by solving a dynamic program. Several numerical experiments are provided to offer insight in the properties of the pricing policy. Bitran and Wadhwa [268] and Bisi and Dada [269] study a similar type of problem, where an unknown parameter is learned in a Bayesian manner, and where the optimal decisions are determined by solving a dynamic program. Bitran and Wadhwa [268] perform extensive computational experiments, and Bisi and Dada [269] derive several structural properties of the optimal policy. Zhang and Chen [270] consider Bayesian learning of a component of the demand distribution that does not depend on the selling price, and show that the finite-horizon expected discounted profit is maximized by a so-called base stock list price policy [271, 124]. Motivated by industrial practice of fashion retailers, Choi [272] considers a scenario where the seller can order inventory and change selling prices at two distinct stages in the selling season. Information obtained from the first stage is used in a Bayesian manner to determine optimal decisions in the second stage. The author formulates a dynamic program, proves several structural properties, and carries out numerical experiments to illustrate his results. Gao et al. [273] formulate and solve a two-period two-products pricing and inventory problem with Bayesian learning of unknown demand parameters. Forghani et al. [274] allow for a single price change, formulate the optimal control problem, and show numerical examples. In [275], a manufacturer estimates the optimal capacity decision from advance sales information obtained prior to the regular selling season. Formal learning of demand parameters is not considered, but the electronic companion to the paper elaborates upon an extension to Bayesian learning.

Non-parametric and robust approaches. Burnetas and Smith [276] consider a joint pricing and inventory problem in a non-parametric setting. They propose an adaptive stochastic-approximation policy, and show that the expected profit per period converges to the optimal profit under complete information. A robust approach to the dynamic pricing and inventory control problem with multiple products is studied by Adida and Perakis [277]. The focus of that paper is the formulation of the robust optimization problem, and to study its complexity properties. Related is the work of Petrucci and Dada [278]. These authors assume that there is no demand noise, which means that the unknown parameters that determine the demand function are completely known once a demand realization is observed that does not lead to stock-out. Adida and Perakis [279] discuss several robust and stochastic optimization approaches to joint pricing and procurement under demand uncertainty, and compare these approaches with each other in a numerical study. Mahmoudzadeh et al. [280] formulate a robust control problem for joint pricing and production in a hybrid manufacturing/remufacturing system, where the coefficients of a linear demand function are unknown. Arasteh et al. [281] consider a similar setting for a joint pricing-and-inventory problem.

Variants. Lariviere and Porteus [282] consider the situation of a manufacturer that sells to a retailer. The manufacturer decides on a wholesale price offered to the retailer, and the retailer has to choose an optimal inventory replenishment policy. Both learn about a parameterized demand function in a Bayesian fashion. Properties of the optimal policy, both for the manufacturer and the retailer, are studied. Gaul and Azizi [283] assume that a product is sold in different stores. The problem is to determine optimal prices in a finite number of periods, as well as to decide if and how inventory should be reallocated between stores. Parameters of the demand function are learned by Bayesian updating, and numerical experiments are provided to illustrate the method.

5. Methodologically related areas

Dynamic pricing under uncertainty is closely related to multi-armed bandit problems. This is a class of problems that capture many essential features of optimization problems under uncertainty, including the well-known exploration–exploitation trade-off: the decision maker should properly balance the two objectives of maximizing instant reward (exploitation of current knowledge) and learning the unknown properties of the system (exploration). This trade-off between learning and instant optimization is also frequently observed in dynamic pricing problems. The literature on multi-armed bandit problems is large; some key references are Thompson [284], Robbins [285], Lai and Robbins [286], Gittins [287], Auer et al. [288]; see further Vermorel and Mohri [289], Cesa-Bianchi and Lugosi [290], and Powell [291]. If in a dynamic pricing problem the number of admissible selling prices is finite, the problem can be modeled as a classical multi-armed bandit problem. This approach is e.g. taken by Rothschild [158], Xia and Dube [253], and Cope [157]. If the set of admissible selling prices is a continuum, then the dynamic pricing problem is closely related to the continuum-armed bandit problem. This problem has recently been studied, among others, by Kleinberg [292], Auer et al. [293], Cope [294], Wang et al. [295], Goldenshluger and Zeevi [296], Rusmevichientong and Tsitsiklis [297], Filippi et al. [298], Abbasi-Yadkori et al. [299], Yu and Mannor [300], Slivkins [301], Perchet and Rigollet [302], Combes and Proutiere [303]. Pricing and learning in a time-varying market, as discussed in Section 6.3, is related to the non-stationary multi-armed bandit problem [304,305].

Another important area related to dynamic pricing and learning is the study of convergence rates of statistical estimates. Lai and Wei [306] study how the speed of convergence of least-squares linear regression estimates depend on the amount of dispersion in the explanatory variables. Their results are applied in several dynamic pricing problems with linear demand functions, such as Le Guen [183] and Cooper et al. [307]. Similarly, results on the convergence rate of maximum-likelihood estimators, as in [308,309], are crucial in the analysis of pricing policies by Besbes and Zeevi [188], Broder and Rusmevichientong [184] and den Boer and Zwart [185].

Dynamic pricing and learning can also be put in a general framework of stochastic control problems with parametric uncertainty. For various dynamic economic models, such problems have been considered by Easley and Kiefer [174], Kiefer and Nyarko [175], Marcet and Sargent [310], Wieland [311,312], Beck and Wieland [313], Han et al. [314], amongst many others. Kendrick et al. [315] review some of this literature, focusing on continuous-time optimal control problems with linear system equations, quadratic cost functions, and Gaussian additive noise terms.

Clearly, many sequential optimization problems in operations research that are studied in an online-learning setting have methodological similarities with dynamic pricing and learning: inventory control ([316–318], is just a small sample of the vast literature), online advertisement (e.g. [319]), resource allocation (e.g. [320]), assortment planning (e.g. [321–323]), product launch and product exit optimization (e.g. [324]), et cetera. These problems can also be studied in conjunction with optimal pricing. For example, Talebian et al. [325] study simultaneous pricing and assortment optimization with Bayesian demand learning, and Section 4.4 lists a number of studies that combine pricing and inventory control in an incomplete-information setting.

6. Extensions and new directions

Most of the literature discussed above studies dynamic pricing and learning for a monopolist firm that sells a single product to nonstrategic customers in a stationary market environment.

Here we review literature on a number of extensions to this setting: strategic customer behavior, competition, time-varying markets, and model misspecification. Note that the first three of these extensions can in some sense be regarded as extending the literature of Section 3.1, on dynamic pricing with dynamic demand, to an incomplete-information framework.

6.1. Strategic consumer behavior

The importance of incorporating the effect of strategically behaving customers on the dynamic pricing policy has repeatedly been recognized; cf. the papers on dynamic pricing with strategic customer behavior mentioned in Section 3.2.1. The study of dynamic pricing with both strategic customer behavior and demand learning, however, is not that well developed.

Learning of the willingness-to-pay distribution. Studies in which the demand distribution is not assumed to be known are Loguinova and Taylor [326], Levina et al. [255], Weaver and Moon [327] and Caldentey et al. [328], and Lazear [200] which is discussed in Section 4.2.1. In [326], the seller learns the willingness-to-pay in a stylized model with a single customer whose product valuation is either “high” or “low”. The authors characterize game-theoretic equilibria and study the effect of strategic behavior of the buyer. Levina et al. [255] consider a seller who learns about a complicated demand process that incorporates, among other things, strategic customer behavior. This behavior is described using a game-theoretic consumer choice model. The focus of the paper is on properties and numerical performance of an online-learning algorithm proposed by the authors. Weaver and Moon [327] and Caldentey et al. [328] take a robust-optimization approach. Inspired by sales of food and agriculture products, Weaver and Moon [327] formulate a robust pricing model with multiple products, multiple customer segments, and linear demand function, and numerically compare its performance with two “price-assurance policies”: a policy that offers refund to consumers if the price falls below their purchase price, and a policy that ensures that price will not decline sufficiently to induce refunds. Caldentey et al. [328] consider a robust formulation of the seller’s pricing problem for a discrete-time infinite-inventory setting with unknown valuations and arrival times of customers, and characterize the pricing policies that minimize the worst-case regret, both in case of myopic or strategic customers. The authors also analyze the limit of the regret and optimal pricing strategies as the length of the discrete time periods goes to zero.

No learning of the willingness-to-pay distribution. In [329–331], the unknown quality of the product is learned. This introduces a game with strategic behavior of customers, who may postpone their purchase in order to retrieve more information, induce a markdown, or influence the belief of the seller about the quality of product. In [332] consumers learn about the capacity of the seller in a repeated two-period selling season, and in [333] consumers learn their valuations while the firm learns the market size. Mersereau and Zhang [334] considers a firm that learns the relative amount of strategic customers. None of these studies involve learning the demand or willingness-to-pay distribution (given the quality of the product).

6.2. Competition

Several studies address the issue of incorporating competition in dynamic pricing and learning. This is challenging, because one generally does not know how competitors behave, and their behavior influences a firm’s own optimal pricing policy. The literature references in this section are not comprehensive, but meant to give a flavor of available approaches and results.

Parametric approaches. The already mentioned paper by Bertsimas and Perakis [219] considers a least-squares learning setting with finite inventories, multiple competitors, and linear demand functions with slowly varying parameters. The authors propose an estimation and price-optimization scheme, and illustrate the method by numerical examples. The related study Kachani et al. [335] considers multiple quantity-setting firms selling the same type of products with finite inventories. The price–demand relation is linear, with unknown parameters that may slowly vary over time. The authors formulate the optimization problem as a mathematical program with equilibrium constraints, provide some computational results, and derive a closed-form expression of a limiting Nash equilibrium in case of two firms and a selling horizon of two time periods. Kwon et al. [336] formulate a finite-time finite-capacity oligopolistic pricing problem in the language of differential variational inequalities. They consider a demand model where the time-derivative of demand is proportional to the difference between a firm's price and a moving average of prices used in the past. Unknown model parameters are estimated with a Kalman filter, which is recalculated at several moments in the sales horizon. The authors propose an algorithm to solve the resulting differential variational inequalities, and illustrate their method by a numerical example. A similar setting is considered by Li et al. [337] and Chung et al. [254], who incorporate a non-parametric functional-coefficient autoregressive time series in their demand model. Duopoly models are studied, among many others, by Coughlan and Mantrala [338] in a Bayesian framework and by Choi and Jagpal [339] in a setting with risk-averse firms.

Robust approaches. A robust optimization approach to dynamic pricing in an oligopolistic environment with demand uncertainty is taken by Perakis and Sood [340] (see also the closely related study Friesz et al. [341]). They consider a multi-period setting with fixed, finite inventory levels, and study Nash equilibrium price policies. Adida and Perakis [342] consider a joint pricing-and-inventory control problem with duopolistic competition and demand uncertainty, and formulate it as a robust optimization problem. The authors discuss existence and uniqueness of a Nash equilibrium and address various computational aspects of the problem.

Economics and econometrics literature. A large stream of economics literature is concerned with the long-term behavior of price adjustment processes in oligopolistic settings, cf. [343–357]. One usually assumes that firms are using a certain specific learning scheme, and then studies whether the selling prices converge to a Nash equilibrium. While these papers are interesting from the perspective of understanding price formation, application in business practice is hampered by the fact that one generally does not know which learning scheme competing firms are using. A remarkable early study on pricing and learning in a competitive environment is Barta and Varaiva [358], who study convergence properties of various stochastic-approximation based price-adjustment rules.

Computer science literature. Contributions from the computer science community to dynamic pricing with demand uncertainty in a competitive environment include Greenwald and Kephart [359], Dasgupta and Das [360], Tesauro and Kephart [361], Kutschinski et al. [237], Könönen [239], Jumadinova and Dasgupta [362,363].

6.3. Time-varying market parameters

Recently several studies have appeared that depart from the strong assumption of unchanging market conditions. We here only list papers that include mathematical performance analysis of the proposed pricing rules.

Besbes and Sauré [364] study the classical finite-inventory finite selling-season pricing problem in a setting with deterministic demand functions. The seller anticipates that the initial demand function d may at some moment τ change to $d^\theta \in \{d^1, \dots, d^K\}$, where (τ, θ) are random variables with known distributions and where d^1, \dots, d^K are known functions. The authors formulate the optimal pricing problem and derive conditions that induce monotonicity properties of the optimal pricing strategy. Chen and Farias [365] also study pricing in a finite-inventory setting where the market process is fluctuating over time. They propose a pricing heuristic which is based on the idea of frequently re-calculating the estimated optimal fixed price. The authors prove bounds on the performance loss relative to an optimal clairvoyant pricing strategy in an asymptotic regime where inventory and demand grows large.

In an infinite-inventory setting, Besbes and Zeevi [188] study a pricing problem where customers' willingness-to-pay (WtP) distribution changes at some unknown point in time. The WtP distribution before and after the change is assumed to be known, only the time of change is unknown to the seller. Lower bounds on the worst-case regret are derived, and pricing strategies are developed that achieve the order of these bounds. Keskin and Zeevi [366] consider a linear demand function with time-varying parameters, which are estimated with weighted least squares. They measure the amount of change in the parameter process by the quadratic variation over T periods, and assume it is bounded by BT^ν , for some known $\nu \in [0, 1]$ and unknown $B \geq 0$. An $\Omega(T^{(2+\nu)/3})$ lower bound on the regret attainable by any policy is proven, and a pricing policy that separates the time horizon into pure exploration and exploitation periods is shown to have regret $O(T^{(2+\nu)/3})$. More refined expressions are derived in case B is known. If only “bursty” changes occur, the authors show that a regret of $O(T^{1/2} \log T)$ can be achieved. den Boer [367] considers a similar set-up with linear demand and weighted least-squares estimation, but assumes that only the size of the market (the intercept) is unknown. A myopic pricing policy is considered, and upper bounds on the long-term average regret are provided. The methodology is illustrated by applying it to pricing in the Bass model, and to pricing in a competitive environment.

A particular structural form of the evolution of the market process is assumed by Wang et al. [368] and Chakravarty et al. [369]. Wang et al. [368] assume that the demand intensity over the life-cycle of an information good behaves as the density function of a Weibull distribution, whose unknown parameters are learned in a Bayesian fashion using Monte-Carlo simulation techniques. Chakravarty et al. [369] propose a simulation-based algorithm to calculate the optimal price path for a product-diffusion model introduced by Robinson and Lakhani [86].

6.4. Model misspecification

Studies that assume a parametric form of the demand functions face the risk of model misspecification. Besbes and Zeevi [370] show in a single-product setting that the loss incurred by such misspecification may in some settings be not that significant. Another form of misspecification is incorrectly assuming that there are no competitors present. Schinkel et al. [354], Tuinstra [357], Bischi et al. [371,372], Isler and Imhof [373], Cooper et al. [307], Anufriev et al. [374] study the effect of this error on the resulting equilibria in various (linear) models, elaborating on earlier work by Kirman [344,375–377]. In an airline revenue-management setting, Cooper et al. [378] show that incorrectly assuming high-fare and low-fare class passengers behave independently can be detrimental for the firm's revenue.

7. Conclusion

Dynamic pricing with incomplete demand information is a topic that has received considerable research attention in recent

years. Different scientific communities have studied characteristics of pricing policies, usually with different aims in mind: the operations research/management science literature typically focuses on finding an optimal pricing policy from the perspective of a seller, and on proving optimality properties in tractable models; the economics literature is generally more concerned with explaining price behavior and price formation observed in markets; and the computer science literature is typically not afraid to consider complicated demand models that are not tractable for mathematical analysis, but that can be handled using appropriate machine learning techniques. In spite of these different aims, there is much in overlap in the studied demand models, proposed pricing policies, and the techniques deployed to analyze the behavior of these policies. With this survey we aim to provide a comprehensive overview of these studies, and facilitate further research on this broad and lively research topic.

Acknowledgments

Part of this work was written while the author was affiliated with Centrum Wiskunde & Informatica (CWI), Eindhoven University of Technology, and University of Amsterdam. The author is supported by an NWO Veni grant. This survey is a considerable extension of the literature review in Chapter 2 of den Boer [379]. The careful reading and constructive suggestions of the editor and the referees have improved the exposition of the paper, and is kindly acknowledged.

References

- [1] Y. Zhang, F.Y.L. Chin, H.F. Ting, Competitive algorithms for online pricing, in: B. Fu, D.Z. Du (Eds.), *Computing and Combinatorics*, in: *Lecture Notes in Computer Science*, vol. 6842, Springer, Berlin, Heidelberg, 2011, pp. 391–401.
- [2] V.F. Araman, R. Caldentey, Revenue management with incomplete demand information, in: J.J. Cochran (Ed.), *Encyclopedia of Operations Research*, Wiley, 2011.
- [3] Y. Aviv, G. Vulcano, Dynamic list pricing, in: Ö. Özer, R. Phillips (Eds.), *The Oxford Handbook of Pricing Management*, Oxford University Press, London, 2012, (Chapter 23).
- [4] S. Christ, Operationalizing dynamic pricing models: Bayesian demand forecasting and customer choice modeling for low cost carriers (Ph.D. thesis), Augsburg University, 2011.
- [5] M. Chen, Z.L. Chen, Recent developments in dynamic pricing research: multiple products, competition, and limited demand information, *Prod. Oper. Manage.* (2014) Online pre-publication.
- [6] G. Bitran, R. Caldentey, An overview of pricing models for revenue management, *Manuf. Serv. Oper. Manag.* 5 (3) (2003) 203–230.
- [7] W. Elmaghraby, P. Keskinocak, Dynamic pricing in the presence of inventory considerations: research overview, current practices, and future directions, *Manag. Sci.* 49 (10) (2003) 1287–1309.
- [8] K.T. Talluri, G.J. van Ryzin, *The Theory and Practice of Revenue Management*, Kluwer Academic Publishers, Boston, 2004.
- [9] R. Phillips, *Pricing and Revenue Optimization*, Stanford University Press, Stanford, CA, 2005.
- [10] A. Heching, Y.T. Leung, Product pricing in the e-business era, in: C. An, H. Fromm (Eds.), *Supply Chain Management on Demand*, Springer, 2005, pp. 65–96. (Chapter 4).
- [11] J. Gönsch, R. Klein, C. Steinhart, Dynamic pricing—state-of-the-art, in: *Zeitschrift für Betriebswirtschaft, Ergänzungsheft 3 'Operations Research in der Betriebswirtschaft'*, 2009, pp. 1–40.
- [12] V.R. Rao (Ed.), *Handbook of Pricing Research in Marketing*, Edward Elgar, Cheltenham, UK, 2009.
- [13] R. Chenavaz, L.P. Carrier, L. Etienne, C. Paraschiv, Dynamic pricing in management science, *J. Econ. Stud. Res.* 2011 (2011).
- [14] I. Deksnite, Z. Lydeka, Dynamic pricing and its forming factors, *Int. J. Bus. Soc. Sci.* 3 (23) (2012) 213–220.
- [15] Ö. Özer, R. Phillips (Eds.), *The Oxford Handbook of Pricing Management*, Oxford University Press, London, 2012.
- [16] M. Harris, A. Raviv, A theory of monopoly pricing schemes with demand uncertainty, *Amer. Econ. Rev.* 71 (3) (1981) 347–365.
- [17] E. Maskin, J. Riley, Monopoly with incomplete information, *RAND J. Econ.* 15 (2) (1984) 171–196.
- [18] I. Segal, Optimal pricing mechanisms with unknown demand, *Amer. Econ. Rev.* 93 (3) (2003) 509–529.
- [19] M. Babaioff, L. Blumrosen, S. Dughmi, Y. Singer, Posting prices with unknown distributions, in: B. Chazelle (Ed.), *Innovations in Computer Science—ICS 2010*, Tsinghua University, Beijing, China, January 7–9, 2011. *Proceedings*, Tsinghua University Press, 2011, pp. 166–178.
- [20] M.F. Balcan, A. Blum, Y. Mansour, Item pricing for revenue maximization, in: *Proceedings of the 9th ACM Conference on Electronic Commerce, EC'08*, ACM, New York, NY, USA, 2008, pp. 50–59.
- [21] T. Chakraborty, Z. Huang, S. Khanna, Dynamic and nonuniform pricing strategies for revenue maximization, *SIAM J. Comput.* 42 (6) (2013) 2424–2451.
- [22] H. Mutlu, M. Alanyali, D. Starobinski, A. Turhan, Online pricing of secondary spectrum access with unknown demand function, *IEEE J. Sel. Areas Commun.* 30 (11) (2012) 2285–2294.
- [23] A. de Palma, R. Lindsey, S. Proost, Research challenges in modelling urban road pricing: an overview, *Transp. Policy* 13 (2) (2006) 97–105.
- [24] T. Tsekeris, S. Voß, Design and evaluation of road pricing: state-of-the-art and methodological advances, *NETNOMICS: Econ. Res. Electron. Netw.* 10 (1) (2009) 5–52.
- [25] Y. Lou, A unified framework of proactive self-learning dynamic pricing for high-occupancy/toll lanes, *Transp. A: Transp. Sci.* 9 (3) (2013) 205–222.
- [26] A. Garcia, E. Campos-Nañez, J. Reitzes, Dynamic pricing and learning in electricity markets, *Oper. Res.* 53 (2) (2005) 231–241.
- [27] M. Roozbehani, M. Dahleh, S. Mitter, Dynamic pricing and stabilization of supply and demand in modern electric power grids, in: *2010 First IEEE International Conference on Smart Grid Communications (SmartGridComm)*, Curran Associates, Inc., Redhook, NY, USA, 2010, pp. 543–548.
- [28] D. Escobari, Dynamic pricing, advance sales and aggregate demand learning in airlines, *J. Ind. Econ.* 60 (4) (2012) 697–724.
- [29] A. Sweeting, Dynamic pricing behavior in perishable goods markets: evidence from secondary markets for major league baseball tickets, *J. Polit. Econ.* 120 (6) (2012) 1133–1172.
- [30] G. Huang, H. Luo, J. Xia, Invest in information or wing it? A model of dynamic pricing with seller learning, Working Paper, Yale School of Management, 2012. Available at: https://server1.tepper.cmu.edu/seminars/docs/Infovalue_Huang.pdf.
- [31] D. Bergemann, J. Välimäki, Market diffusion with two-sided learning, *RAND J. Econ.* 28 (4) (1997) 773–795.
- [32] R. Caminal, X. Vives, Price dynamics and consumer learning, *J. Econ. Manag. Strategy* 8 (1) (1999) 95–131.
- [33] E.E. Schlee, Buyer experimentation and introductory pricing, *J. Econ. Behav. Organ.* 44 (3) (2001) 347–362.
- [34] S. Bose, G. Orosel, M. Ottaviani, L. Vesterlund, Dynamic monopoly pricing and herding, *RAND J. Econ.* 37 (4) (2006) 910–928.
- [35] M. Bhalla, Dynamic pricing under social learning with strategic consumers, Working Paper No. 367, Indian Institute of Management, Bangalore, 2012. <http://ssrn.com/abstract=2114054>.
- [36] B. Ifrach, C. Maglaras, M. Scarsini, Monopoly pricing in the presence of social learning, NET Institute Working Paper No. 12-01, 2012. <http://ssrn.com/abstract=1957924>.
- [37] A.T. Ching, T. Erdem, M.P. Keane, Invited paper—learning models: an assessment of progress, challenges, and new developments, *Mark. Sci.* 32 (6) (2013) 913–938.
- [38] A.A. Cournot, *Researches into the Mathematical Principles of the Theory of Wealth*. Translated in 1897 by N.T. Bacon, with a Bibliography of Mathematical Economics by Irving Fisher, The Macmillan Company, New York, 1838.
- [39] I. Fisher, Cournot and mathematical economics, *Quart. J. Econ.* 12 (2) (1898) 119–138.
- [40] C. Davenant, *An Essay Upon the Probable Methods of making a People Gainers in the Balance of Trade*, James Knapton, London, 1699.
- [41] J. Creedy, On the King–Davenant “law” of demand, *Scott. J. Polit. Econ.* 33 (3) (1986) 193–212.
- [42] R. Benini, Sull’uso delle formole empiriche a nell’economia applicata, *G. degli Econ.*, 2nd Ser. 35 (1907) 1053–1063.
- [43] C. Gini, Prezzi e consumi, *G. degli Econ.* 3rd Ser. 40 (1910) 99–114. 235–249.
- [44] R.A. Leffeldt, The elasticity of demand for wheat, *Econom. J.* 24 (94) (1914) 212–217.
- [45] H.L. Moore, *Economic Cycles; their Law and Cause*, The Macmillan Company, New York, 1914.
- [46] H.L. Moore, *Forecasting the Yield and the Price of Cotton*, The Macmillan Company, New York, 1917.
- [47] P.G. Wright, *The Tariff on Animal and Vegetable Oils*, The Macmillan Company, New York, 1928.
- [48] J. Tinbergen, *Bestimmung und Deutung von Angebotskurven: Ein Beispiel*, *Z. Natl.* 1 (5) (1930) 669–679.
- [49] H. Schultz, *The Theory and Measurement of Demand*, University of Chicago Press, Chicago, 1938.
- [50] G.J. Stigler, The early history of empirical studies of consumer behavior, *J. Polit. Econ.* 62 (2) (1954) 95–113.
- [51] G.J. Stigler, Henry L. Moore and statistical economics, *Econometrica* 30 (1) (1962) 1–21.
- [52] A. Brown, A. Deaton, Surveys in applied economics: models of consumer behaviour, *Econom. J.* 82 (328) (1972) 1145–1236.
- [53] C.F. Christ, Early progress in estimating quantitative economic relationships in America, *Amer. Econ. Rev.* 75 (6) (1985) 39–52.
- [54] R.W. Farebrother, Early explorations in econometrics, in: T.C. Mills, K. Patterson (Eds.), *Palgrave Handbook of Econometrics*, Volume 1: *Econometric Theory*, Palgrave MacMillan, Basingstoke, 2006.
- [55] S. Berry, J. Levinsohn, A. Pakes, Automobile prices in market equilibrium, *Econometrica* 63 (4) (1995) 841–890.

- [56] D. McFadden, K. Train, Mixed MNL models for discrete response, *J. Appl. Econometrics* 15 (1) (2000) 447–470.
- [57] P. Bajari, C.L. Benkard, Demand estimation with heterogeneous consumers and unobserved product characteristics: a hedonic approach, *J. Polit. Econ.* 113 (6) (2005) 1239–1276.
- [58] J.R. Hicks, Annual survey of economic theory: the theory of monopoly, *Econometrica* 3 (1) (1935) 1–20.
- [59] E.R. Hawkins, Methods of estimating demand, *J. Mark.* 21 (4) (1957) 428–438.
- [60] S.L. Horner, C.F. Roos, V. von Szeliski, A.T. Court, S.M. DuBrul, *The Dynamics of Automobile Demand*, General Motors Corporation, New York, 1939.
- [61] M.J. Zbaracki, M. Ritson, D. Levy, S. Dutta, M. Bergen, Managerial and customer costs of price adjustment: direct evidence from industrial markets, *Rev. Econ. Stat.* 86 (2) (2004) 514–533.
- [62] M.E. Slade, Optimal pricing with costly adjustment: evidence from retail-grocery prices, *Rev. Econom. Stud.* 65 (1) (1998) 87–107.
- [63] S. Netessine, Dynamic pricing of inventory/capacity with infrequent price changes, *European J. Oper. Res.* 174 (1) (2006) 553–580.
- [64] P.B. Seetharaman, Dynamic pricing, in: V.R. Rao (Ed.), *Handbook of Pricing Research in Marketing*, Edward Elgar, Cheltenham, UK, 2009, pp. 384–393. (Chapter 17).
- [65] G.C. Evans, The dynamics of monopoly, *Amer. Math. Monthly* 31 (2) (1924) 77–83.
- [66] G.C. Evans, The mathematical theory of economics, *Amer. Math. Monthly* 32 (3) (1925) 104–110.
- [67] C.F. Roos, A mathematical theory of competition, *Amer. J. Math.* 47 (3) (1925) 163–175.
- [68] C.F. Roos, A dynamic theory of economics, *J. Polit. Econ.* 35 (5) (1927) 632–656.
- [69] C.F. Roos, Dynamical economics, *Proc. Natl. Acad. Sci. USA* 13 (3) (1927) 145–150.
- [70] C.F. Roos, *Dynamic Economics: Theoretical and Statistical Studies of Demand, Production, and Prices*, The Principia Press, Bloomington, 1934.
- [71] G. Tintner, Monopoly over time, *Econometrica* 5 (2) (1937) 160–170.
- [72] A. Smithies, The maximization of profits over time with changing cost and demand functions, *Econometrica* 7 (4) (1939) 312–318.
- [73] R.G. Thompson, M.D. George, P.L. Brown, M.S. Proctor, Optimal production, investment, and output price controls for a monopoly firm of the Evans' type, *Econometrica* 39 (1) (1971) 119–129.
- [74] M.A. Simaan, T. Takayama, Optimum monopolist control in a dynamic market, *IEEE Trans. Syst. Man Cybern.* 6 (12) (1976) 799–807.
- [75] T. Mazumdar, S.P. Raj, I. Sinha, Reference price research: review and propositions, *J. Mark.* 69 (4) (2005) 84–102.
- [76] E.A. Greenleaf, The impact of reference price effects on the profitability of price promotions, *Mark. Sci.* 14 (1) (1995) 82–104.
- [77] P.K. Kopalle, A.G. Rao, J.L. Assunção, Asymmetric reference price effects and dynamic pricing policies, *Mark. Sci.* 15 (1) (1996) 60–85.
- [78] G. Fibich, A. Gaviot, O. Lowengart, Explicit solutions of optimization models and differential games with nonsmooth (asymmetric) reference-price effects, *Oper. Res.* 51 (5) (2003) 721–734.
- [79] P. Heidhues, B. Köszegi, Regular prices and sales, *Theor. Econ.* 9 (1) (2014) 217–251.
- [80] I. Popescu, Y. Wu, Dynamic pricing strategies with reference effects, *Oper. Res.* 55 (3) (2007) 413–429.
- [81] H.S. Ahn, M. Gümüş, P. Kaminsky, Pricing and manufacturing decisions when demand is a function of prices in multiple periods, *Oper. Res.* 55 (6) (2007) 1039–1057.
- [82] F.M. Bass, A new product growth for model consumer durables, *Manag. Sci.* 15 (5) (1969) 215–227.
- [83] V. Mahajan, E. Muller, F.M. Bass, New product diffusion models in marketing: a review and directions for research, *J. Mark.* 54 (1) (1990) 1–26.
- [84] R. Baptista, The diffusion of process innovations: a selective review, *Int. J. Econ. Bus.* 6 (1) (1999) 107–129.
- [85] N. Meade, T. Islam, Modelling and forecasting the diffusion of innovation—a 25-year review, *Int. J. Forecast.* 22 (3) (2006) 519–545.
- [86] B. Robinson, C. Lakhani, Dynamic price models for new-product planning, *Manag. Sci.* 21 (10) (1975) 1113–1122.
- [87] F. Clarke, M.N. Darrrough, J. Heineke, Optimal pricing policy in the presence of experience effects, *J. Bus.* 55 (4) (1982) 517–530.
- [88] S. Kalish, Monopolist pricing with dynamic demand and production cost, *Mark. Sci.* 2 (2) (1983) 135–159.
- [89] D.G. Clarke, R.J. Dolan, A simulation analysis of alternative pricing strategies for dynamic environments, *J. Bus.* 57 (1) (1984) S179–S200.
- [90] Y.M. Chen, D.C. Jain, Dynamic monopoly pricing under a Poisson-type uncertain demand, *J. Bus.* 65 (4) (1992) 593–614.
- [91] K. Raman, R. Chatterjee, Optimal monopolist pricing under demand uncertainty in dynamic markets, *Manag. Sci.* 41 (1) (1995) 144–162.
- [92] B. Kamrad, S.S. Lele, A. Siddique, R.J. Thomas, Innovation diffusion uncertainty, advertising and pricing policies, *European J. Oper. Res.* 164 (3) (2005) 829–850.
- [93] W.M. Kincaid, D.A. Darling, An inventory pricing problem, *J. Math. Anal. Appl.* 7 (1963) 183–208.
- [94] G. Gallego, G. van Ryzin, Optimal dynamic pricing of inventories with stochastic demand over finite horizons, *Manag. Sci.* 40 (8) (1994) 999–1020.
- [95] Y. Feng, G. Gallego, Optimal starting times for end-of-season sales and optimal stopping times for promotional fares, *Manag. Sci.* 41 (8) (1995) 1371–1391.
- [96] G.R. Bitran, S.V. Mondschein, Periodic pricing of seasonal products in retailing, *Manag. Sci.* 43 (1) (1997) 64–79.
- [97] Y. Feng, B. Xiao, Optimal policies of yield management with multiple predetermined prices, *Oper. Res.* 48 (2) (2000) 332–343.
- [98] Y. Feng, B. Xiao, A continuous-time yield management model with multiple prices and reversible price changes, *Manag. Sci.* 46 (5) (2000) 644–657.
- [99] Y. Feng, G. Gallego, Perishable asset revenue management with Markovian time dependent demand intensities, *Manag. Sci.* 46 (7) (2000) 941–956.
- [100] W. Zhao, Y.S. Zheng, Optimal dynamic pricing for perishable assets with nonhomogeneous demand, *Manag. Sci.* 46 (3) (2000) 375–388.
- [101] G. Bitran, R. Caldentey, S. Mondschein, Coordinating clearance markdown sales of seasonal products in retail chains, *Oper. Res.* 46 (5) (1998) 609–624.
- [102] G. Gallego, G. van Ryzin, A multiproduct dynamic pricing problem and its applications to network yield management, *Oper. Res.* 45 (1) (1997) 24–41.
- [103] A.J. Kleywegt, An optimal control problem of dynamic pricing, Working Paper, Georgia Institute of Technology, 2001. http://www2.isye.gatech.edu/people/faculty/Anton_Kleywegt/OptControl.pdf.
- [104] C. Maglaras, J. Meissner, Dynamic pricing strategies for multiproduct revenue management problems, *Manuf. Serv. Oper. Manag.* 8 (2) (2006) 136–148.
- [105] D. Zhang, W.L. Cooper, Pricing substitutable flights in airline revenue management, *European J. Oper. Res.* 197 (3) (2009) 848–861.
- [106] L. Dong, P. Kouvelis, Z. Tian, Dynamic pricing and inventory control of substitute products, *Manuf. Serv. Oper. Manag.* 11 (2) (2009) 317–339.
- [107] A. Erdelyi, H. Topaloglu, Using decomposition methods to solve pricing problems in network revenue management, *J. Revenue Pricing Manag.* 10 (4) (2011) 325–343.
- [108] D. Zhang, Z. Lu, Assessing the value of dynamic pricing in network revenue management, *INFORMS J. Comput.* 25 (1) (2013) 102–115.
- [109] S. Jasin, Reoptimization and self-adjusting price control for network revenue management, *Oper. Res.* 62 (5) (2014) 1168–1178.
- [110] Y. Akçay, H.P. Natarajan, S.H. Xu, Joint dynamic pricing of multiple perishable products under consumer choice, *Manag. Sci.* 56 (8) (2010) 1345–1361.
- [111] M. Suh, G. Aydin, Dynamic pricing of substitutable products with limited inventories under logit demand, *IEE Trans.* 43 (5) (2011) 323–331.
- [112] H. Li, W.T. Huh, Pricing multiple products with the multinomial logit and nested logit models: concavity and implications, *Manuf. Serv. Oper. Manag.* 13 (4) (2011) 549–563.
- [113] G. Gallego, R. Wang, Multiproduct price optimization and competition under the nested logit model with product-differentiated price sensitivities, *Oper. Res.* 62 (2) (2014) 450–461.
- [114] X. Su, Intertemporal pricing with strategic customer behavior, *Manag. Sci.* 53 (5) (2007) 726–741.
- [115] Y. Aviv, A. Pazgal, Optimal pricing of seasonal products in the presence of forward-looking consumers, *Manuf. Serv. Oper. Manag.* 10 (3) (2008) 339–359.
- [116] W. Elmaghraby, A. Gülcü, P. Keskinocak, Designing optimal pre-announced markdowns in the presence of rational customers with multi-unit demands, *Manuf. Serv. Oper. Manag.* 10 (1) (2008) 126–148.
- [117] Q. Liu, G.J. van Ryzin, Strategic capacity rationing to induce early purchases, *Manag. Sci.* 54 (6) (2008) 1115–1131.
- [118] Y. Levin, J. McGill, M. Nediak, Dynamic pricing in the presence of strategic consumers and oligopolistic competition, *Manag. Sci.* 55 (1) (2009) 32–46.
- [119] G.P. Cachon, R. Swinney, Purchasing, pricing, and quick response in the presence of strategic consumers, *Manag. Sci.* 55 (3) (2009) 497–511.
- [120] X. Su, Optimal pricing with speculators and strategic consumers, *Manag. Sci.* 56 (1) (2010) 25–40.
- [121] Z.M. Shen, X. Su, Customer behavior modeling in revenue management and auctions: a review and new research opportunities, *Prod. Oper. Manage.* 16 (6) (2007) 713–728.
- [122] J. Gönsch, R. Klein, M. Neugebauer, C. Steinhart, Dynamic pricing with strategic customers, *J. Bus. Econ.* 83 (5) (2013) 505–549.
- [123] J. Eliashberg, R. Steinberg, Marketing-production joint decision-making, in: J. Eliashberg, J.D. Lilien (Eds.), *Management Science in Marketing*, Handbooks in Operations Research and Management Science. Vol. 5, North Holland Publishing Co., Amsterdam, 1993, pp. 827–880.
- [124] A. Federgruen, A. Heching, Combined pricing and inventory control under uncertainty, *Oper. Res.* 47 (3) (1999) 454–475.
- [125] C.A. Yano, S.M. Gilbert, Coordinated pricing and production/procurement decisions: a review managing business interfaces, in: A.K. Chakravarty, J. Eliashberg (Eds.), *Managing Business Interfaces*, in: International Series in Quantitative Marketing, vol. 16, Springer, New York, 2004, pp. 65–103. (Chapter 3).
- [126] L. Chan, Z. Shen, D. Simchi-Levi, Coordination of pricing and inventory decisions: a survey and classification, in: D. Simchi-Levi, D. Wu, Z. Shen (Eds.), *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, Springer, New York, 2004, pp. 335–392.
- [127] X. Chen, D. Simchi-Levi, Joint pricing and inventory management, in: Ö. Özer, R. Phillips (Eds.), *The Oxford Handbook of Pricing Management*, Oxford University Press, London, 2012, (Chapter 30).
- [128] D. Simchi-Levi, X. Chen, J. Bramel, Integration of inventory and pricing, in: *The Logic of Logistics*, in: Springer Series in Operations Research and Financial Engineering, Springer, New York, 2014, pp. 177–209.
- [129] E.S. Mills, Uncertainty and price theory, *Quart. J. Econ.* 73 (1) (1959) 116–130.
- [130] S. Karlin, C.R. Carr, Prices and optimal inventory policies, in: K. Arrow, S. Karlin, H. Scarf (Eds.), *Studies in Applied Probability and Management Science*, Stanford University Press, Stanford, 1962, pp. 159–172.

- [131] A.J. Nevins, Some effects of uncertainty: simulation of a model of price, *Quart. J. Econ.* 80 (1) (1966) 73–87.
- [132] E. Zabel, Monopoly and uncertainty, *Rev. Econom. Stud.* 37 (2) (1970) 205–219.
- [133] D.P. Baron, Price uncertainty, utility, and industry equilibrium in pure competition, *Internat. Econom. Rev.* 11 (3) (1970) 463–480.
- [134] D.P. Baron, Demand uncertainty in imperfect competition, *Internat. Econom. Rev.* 12 (2) (1971) 196–208.
- [135] A. Sandmo, On the theory of the competitive firm under price uncertainty, *Amer. Econ. Rev.* 61 (1) (1971) 65–73.
- [136] H.E. Leland, Theory of the firm facing uncertain demand, *Amer. Econ. Rev.* 62 (3) (1972) 278–291.
- [137] B. Hansen, Report of the Uppsala meeting, August 2–4, 1954, *Econometrica* 23 (2) (1955) 198–216.
- [138] F. Billström, H. Laadi, S.A.O. Thore, L.O. Friberg, O. Johansson, Some experiments with dynamization of monopoly models, in: *Mimeo. 16th Meeting of the Econometric Society in Uppsala, 1954*.
- [139] F. Billström, S.A.O. Thore, Simulation experiments with dynamic price strategies in monopoly theory, in: H.O.A. Wold (Ed.), *Econometric Model Building. Essays on the Causal Chain Approach*, North Holland Publishing Co., Amsterdam, 1964, pp. 297–321.
- [140] S.A.O. Thore, Price strategies of an “ignorant” monopolist, in: H.O.A. Wold (Ed.), *Econometric Model Building. Essays on the Causal Chain Approach*, North Holland Publishing Co., Amsterdam, 1964.
- [141] R.W. Clower, Some theory of an ignorant monopolist, *Econom. J.* 69 (276) (1959) 705–716.
- [142] W.J. Baumol, R.E. Quandt, Rules of thumb and optimally imperfect decisions, *Amer. Econ. Rev.* 54 (2) (1964) 23–46.
- [143] J. Baetge, G. Bolenz, W. Ballwieser, R. Hömberg, P. Wullers, Dynamic price policies in monopolistic competition, in: J. Rose, C. Biliu (Eds.), *Modern Trends in Cybernetics and Systems. Proceedings of the Third International Congress of Cybernetics and Systems*, Bucharest, Romania, August 25–29, 1975, Vol. 1, Springer Verlag, 1977, pp. 401–417.
- [144] U. Witt, How can complex economical behavior be investigated? The example of the ignorant monopolist revisited, *Behav. Sci.* 31 (3) (1986) 173–188.
- [145] M. Aoki, On a dual control approach to the pricing policies of a trading specialist, in: R. Conti, A. Ruberti (Eds.), *5th Conference on Optimization Techniques Part II*, in: *Lecture Notes in Computer Science*, vol. 4, Springer, Berlin, Heidelberg, 1973, pp. 272–282. (Chapter 24).
- [146] M. Aoki, On some price adjustment schemes, *Ann. Econ. Soc. Meas.* 3 (1) (1974) 95–115.
- [147] C.Y. Chong, D. Cheng, Multistage pricing under uncertain demand, *Ann. Econ. Soc. Meas.* 4 (2) (1975) 311–323.
- [148] D. Nguyen, The monopolistic firm, random demand, and Bayesian learning, *Oper. Res.* 32 (5) (1984) 1038–1051.
- [149] D. Nguyen, Pricing Under Uncertainty, in: *International Series in Quantitative Marketing*, vol. 6, Springer, 1997, pp. 23–57. (Chapter 2).
- [150] E.G. Wruck, Dynamic pricing implications of uncertainty about demand (Ph.D. thesis), Cornell University, 1989.
- [151] M.S. Lobo, S. Boyd, Pricing and learning with uncertain demand, Working Paper, Fuqua School of Business, Duke University, 2003. Available at: http://www.stanford.edu/~boyd/papers/pdf/pric_learn_unc_dem.pdf.
- [152] M. Chhabra, S. Das, Learning the demand curve in posted-price digital goods auctions, in: *The 10th International Conference on Autonomous Agents and Multiagent Systems—Volume 1, AAMAS’11*, International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC, 2011, pp. 63–70.
- [153] H. Qu, I.O. Ryzhov, M.C. Fu, Learning logistic demand curves in business-to-business pricing, in: R. Pasupathy, S.H. Kim, A. Tolk, R. Hill, M.E. Kuhl (Eds.), *Proceedings of the 2013 Winter Simulation Conference*, IEEE, 2013.
- [154] H.D. Kwon, S.A. Lippman, C.S. Tang, Optimal markdown pricing with demand learning, *Probab. Engrg. Inform. Sci.* 26 (1) (2012) 77–104.
- [155] B. Leloup, L. Deveaux, Dynamic pricing on the Internet: theory and simulations, *Electron. Commer. Res.* 1 (3) (2001) 265–276.
- [156] X. Wang, Dynamic pricing with a Poisson bandit model, *Sequential Anal.* 26 (4) (2007) 355–365.
- [157] E. Cope, Bayesian strategies for dynamic pricing in e-commerce, *Nav. Res. Logist.* 54 (3) (2007) 265–281.
- [158] M. Rothschild, A two-armed bandit theory of market pricing, *J. Econom. Theory* 9 (2) (1974) 185–202.
- [159] A. McLennan, Price dispersion and incomplete learning in the long run, *J. Econom. Dynam. Control* 7 (3) (1984) 331–347.
- [160] J.M. Harrison, N.B. Keskin, A. Zeevi, Bayesian dynamic pricing policies: learning and earning under a binary prior distribution, *Manag. Sci.* 58 (3) (2012) 570–586.
- [161] P. Afèche, B. Ata, Bayesian dynamic pricing in queueing systems with unknown delay cost characteristics, *Manuf. Serv. Oper. Manag.* 15 (2) (2013) 292–304.
- [162] W.C. Cheung, D. Simchi-Levi, H. Wang, Dynamic pricing and demand learning with limited price experimentation, Working Paper, MIT, 2013. Available at: <http://web.mit.edu/wanghe/www/research.html>.
- [163] N.B. Keskin, Optimal dynamic pricing with demand model uncertainty: a squared-coefficient-of-variation rule for learning and earning, Working Paper, University of Chicago, Booth School of Business, 2014. Available at: <http://ssrn.com/abstract=2487364>.
- [164] Z. Sun, A.E. Abbas, Bayesian updating on price elasticity of uncertain demand, in: *2013 IEEE International Systems Conference (SysCon)*, IEEE, 2013, pp. 222–228.
- [165] T.M. Choi, P.S. Chow, T. Xiao, Electronic price-testing scheme for fashion retailing with information updating, *Int. J. Prod. Econ.* 140 (1) (2012) 396–406.
- [166] E.C. Prescott, The multi-period control problem under uncertainty, *Econometrica* 40 (6) (1972) 1043–1058.
- [167] S.J. Grossman, R.E. Kihlstrom, L.J. Mirman, A Bayesian approach to the production of information and learning by doing, *Rev. Econom. Stud.* 44 (3) (1977) 533–547.
- [168] L.J. Mirman, L. Samuelson, A. Urbano, Monopoly experimentation, *Internat. Econom. Rev.* 34 (3) (1993) 549–563.
- [169] D. Trefler, The ignorant monopolist: optimal learning with endogenous information, *Internat. Econom. Rev.* 34 (3) (1993) 565–581.
- [170] A. Rustichini, A. Wolinsky, Learning about variable demand in the long run, *J. Econom. Dynam. Control* 19 (5–7) (1995) 1283–1292.
- [171] G. Keller, S. Rady, Optimal experimentation in a changing environment, *Rev. Econom. Stud.* 66 (3) (1999) 475–507.
- [172] R.J. Balvers, T.F. Cosimano, Actively learning about demand and the dynamics of price adjustment, *Econom. J.* 100 (402) (1990) 882–898.
- [173] T. Willems, Actively learning by pricing: a model of an experimenting seller, Working Paper, 2014. Available at: <https://sites.google.com/site/twillems85/>.
- [174] D. Easley, N.M. Kiefer, Controlling a stochastic process with unknown parameters, *Econometrica* 56 (5) (1988) 1045–1064.
- [175] N.M. Kiefer, Y. Nyarko, Optimal control of an unknown linear process with learning, *Internat. Econom. Rev.* 30 (3) (1989) 571–586.
- [176] P. Aghion, P. Bolton, C. Harris, B. Jullien, Optimal learning by experimentation, *Rev. Econom. Stud.* 58 (4) (1991) 621–654.
- [177] R. Manning, Market research by a monopolist: a Bayesian analysis, *Economica* 46 (183) (1979) 301–306.
- [178] I. Venezia, Optimal investments in market research, *European J. Oper. Res.* 18 (2) (1984) 198–207.
- [179] A.X. Carvalho, M.L. Puterman, Learning and pricing in an Internet environment with binomial demand, *J. Revenue Pricing Manag.* 3 (4) (2005) 320–336.
- [180] A.X. Carvalho, M.L. Puterman, Dynamic optimization and learning: how should a manager set prices when the demand function is unknown? Technical Report, Instituto de Pesquisa Economica Aplicada—IPEA, Brasília, 2005, Discussion Papers 1117. Available at: http://www.ipea.gov.br/portal/images/stories/PDFs/TDs/td_1117.pdf.
- [181] S. Morales-Enciso, J. Branke, Revenue maximization through dynamic pricing under unknown market behaviour, in: S. Ravizza, P. Holborn (Eds.), *3rd Student Conference on Operational Research*, (SCOR 2012), Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2012, pp. 11–20.
- [182] R. Kleinberg, T. Leighton, The value of knowing a demand curve: bounds on regret for online posted-price auctions, in: *Proceedings of the 44th IEEE Symposium on Foundations of Computer Science*, 2003, pp. 594–605.
- [183] T. Le Guen, Data-driven pricing (Master’s thesis), Massachusetts Institute of Technology, 2008. Available at: <http://hdl.handle.net/1721.1/45627>.
- [184] J. Broder, P. Rusmevichientong, Dynamic pricing under a general parametric choice model, *Oper. Res.* 60 (4) (2012) 965–980.
- [185] A.V. den Boer, B. Zwart, Simultaneously learning and optimizing using controlled variance pricing, *Manag. Sci.* 60 (3) (2014) 770–783.
- [186] A.V. den Boer, Dynamic pricing with multiple products and partially specified demand distribution, *Math. Oper. Res.* 39 (3) (2014) 863–888.
- [187] N.B. Keskin, A. Zeevi, Dynamic pricing with an unknown linear demand model: asymptotically optimal semi-myopic policies, *Oper. Res.* 62 (5) (2014) 1142–1167.
- [188] O. Besbes, A. Zeevi, On the minimax complexity of pricing in a changing environment, *Oper. Res.* 59 (1) (2011) 66–79.
- [189] S.S. Eren, C. Maglaras, Monopoly pricing with limited demand information, *J. Revenue Pricing Manag.* 9 (2010) 23–48.
- [190] D. Bergemann, K.H. Schlag, Pricing without priors, *J. Eur. Econ. Assoc.* 6 (2–3) (2008) 560–569.
- [191] D. Bergemann, K.H. Schlag, Robust monopoly pricing, *J. Econom. Theory* 146 (6) (2011) 2527–2543.
- [192] B.R. Handel, K. Misra, J.W. Roberts, Robust firm pricing with panel data, *J. Econometrics* 174 (2) (2013) 165–185.
- [193] B. Handel, K. Misra, Robust new product pricing, Working Paper, University of California, Berkeley, 2014. Available at: <http://ssrn.com/abstract=2307991>.
- [194] P. Tehrani, Y. Zhai, Q. Zhao, Dynamic pricing under finite space demand uncertainty: a multi-armed bandit with dependent arms, 2012. arXiv:1206.5345 [cs.LG].
- [195] M. Haviv, R.S. Randhawa, Pricing in queues without demand information, *Manuf. Serv. Oper. Manag.* 16 (3) (2014) 401–411.
- [196] L. Jia, Q. Zhao, L. Tong, Retail pricing for stochastic demand with unknown parameters: an online machine learning approach, in: *2013 Allerton Conference on Communication, Control and Computing*, Monticello, IL, 2013.
- [197] J.B. Taylor, Asymptotic properties of multiperiod control rules in the linear regression model, *Internat. Econom. Rev.* 15 (2) (1974) 472–484.
- [198] T.W. Anderson, J.B. Taylor, Some experimental results on the statistical properties of least squares estimates in control problems, *Econometrica* 44 (6) (1976) 1289–1302.
- [199] T.L. Lai, H. Robbins, Iterated least squares in multiperiod control, *Adv. in Appl. Math.* 3 (1) (1982) 50–73.

- [200] E.P. Lazear, Retail pricing and clearance sales, *Amer. Econ. Rev.* 76 (1) (1986) 14–32.
- [201] T.R. Sassi, A note on optimal price cutting behavior under demand uncertainty, *Rev. Econ. Stat.* 70 (2) (1988) 336–339.
- [202] Y. Aviv, A. Pazgal, Pricing of short life-cycle products through active learning, Working Paper, Olin School of Business, Washington University, St. Louis, 2005. Available at: https://www.researchgate.net/publication/228674522_Dynamic_pricing_of_short_life-cycle_products_through_active_learning.
- [203] K.Y. Lin, Dynamic pricing with real-time demand learning, *European J. Oper. Res.* 174 (1) (2006) 522–538.
- [204] V.F. Araman, R. Caldentey, Dynamic pricing for nonperishable products with demand learning, *Oper. Res.* 57 (5) (2009) 1169–1188.
- [205] V.F. Farias, B. van Roy, Dynamic pricing with a prior on market response, *Oper. Res.* 58 (1) (2010) 16–29.
- [206] R. Mason, J. Välimäki, Learning about the arrival of sales, *J. Econom. Theory* 146 (4) (2011) 1699–1711.
- [207] A.N. Avramidis, Learning in revenue management: exploiting estimation of arrival rate and price response, Working Paper, School of Mathematics, University of Southampton, 2013. Available at: <http://www.personal.soton.ac.uk/aa1w07/v35.pdf>.
- [208] Y. Chen, R. Wang, Learning buyers' valuation distribution in posted-price selling, *Econom. Theory* 14 (2) (1999) 417–428.
- [209] A. Sen, A.X. Zhang, Style goods pricing with demand learning, *European J. Oper. Res.* 196 (3) (2009) 1058–1075.
- [210] Y. Aviv, A. Pazgal, A partially observed Markov decision process for dynamic pricing, *Manag. Sci.* 51 (9) (2005) 1400–1416.
- [211] L. Chen, Bayesian dynamic pricing with two-sided censored customer willingness-to-pay data, Working Paper, The Fuqua School of Business, Duke University, 2014. Available at: <https://faculty.fuqua.duke.edu/~lc91/More/>.
- [212] O. Besbes, A. Zeevi, Dynamic pricing without knowing the demand function: risk bounds and near-optimal algorithms, *Oper. Res.* 57 (6) (2009) 1407–1420.
- [213] Z. Wang, S. Deng, Y. Ye, Close the gaps: a learning-while-doing algorithm for a class of single-product revenue management problems, *Oper. Res.* 62 (2) (2014) 318–331.
- [214] Y. Lei, S. Jasin, A. Sinha, Near-optimal bisection search for nonparametric dynamic pricing with inventory constraint, Working Paper, University of Michigan, Ross School of Business Paper No. 1252, 2014. Available at: <http://ssrn.com/abstract=2509425>.
- [215] O. Besbes, A. Zeevi, Blind network revenue management, *Oper. Res.* 60 (6) (2012) 1537–1550.
- [216] O. Besbes, C. Maglaras, Dynamic pricing with financial milestones: feedback-form policies, *Manag. Sci.* 58 (9) (2012) 1715–1731.
- [217] A.V. den Boer, B. Zwart, Dynamic pricing and learning with finite inventories, Working Paper, University of Twente, 2014. Available at: <http://www.utwente.nl/ewi/sor/about/staff/boer/>.
- [218] M. Babaioff, S. Dughmi, R. Kleinberg, A. Slivkins, Dynamic pricing with limited supply, *ACM Trans. Econ. Comput.* (2014) forthcoming.
- [219] D. Bertsimas, G. Perakis, Dynamic pricing: a learning approach, in: *Mathematical and Computational Models for Congestion Charging*, Springer, New York, 2006, pp. 45–79.
- [220] A.E.B. Lim, J.G. Shanthikumar, Relative entropy, exponential utility, and robust dynamic pricing, *Oper. Res.* 55 (2) (2007) 198–214.
- [221] A. Thiele, Single-product pricing via robust optimization, Working Paper, Lehigh University, 2006. Available at: http://www.optimization-online.org/DB_FILE/2006/01/1319.pdf.
- [222] A.E.B. Lim, J.G. Shanthikumar, T. Watwail, Robust multi-product pricing, Working Paper, University of California, Berkeley, 2008. Available at: <http://dx.doi.org/10.2139/ssrn.1078012>.
- [223] A. Thiele, Multi-product pricing via robust optimisation, *J. Revenue Pricing Manag.* 8 (2009) 67–80.
- [224] M. Cohen, R. Lobel, G. Perakis, Dynamic pricing through data sampling, Working Paper, Massachusetts Institute of Technology, 2013. Available at: <http://ssrn.com/abstract=2376667>.
- [225] G. Li, Y. Xiong, Z. Xiong, Robust dynamic pricing over infinite horizon in the presence of model uncertainty, *Asia-Pac. J. Oper. Res.* 26 (6) (2009) 779–804.
- [226] Y. Xiong, G. Li, K.J. Fernandes, Dynamic pricing model and algorithm for perishable products with fuzzy demand, *Appl. Stoch. Models Bus. Ind.* 26 (6) (2010) 758–774.
- [227] G. Li, W. Li, Dynamic pricing of perishable products with random fuzzy demand, in: *2010 International Conference on Management Science & Engineering*, IEEE, 2010, pp. 191–199.
- [228] G. Li, Z. Xiong, Y. Zhou, Y. Xiong, Dynamic pricing for perishable products with hybrid uncertainty in demand, *Appl. Math. Comput.* 219 (20) (2013) 10366–10377.
- [229] Ö. Coşgun, Y. Ekinci, S. Yanik, Fuzzy rule-based demand forecasting for dynamic pricing of a maritime company, *Knowl.-Based Syst.* 70 (2014) 88–96.
- [230] M. Dzieciuchowicz, D. Caro, A. Thiele, Robust timing of markdowns. Technical Report, Lehigh University, Department of Industrial and Systems Engineering, Bethlehem, PA, 2010, http://www.optimization-online.org/DB_HTML/2010/11/2804.html.
- [231] J.C. Ferrer, D. Oyarzún, J. Vera, Risk averse retail pricing with robust demand forecasting, *Int. J. Prod. Econ.* 136 (1) (2012) 151–160.
- [232] G. Gallego, M. Talebian, Demand learning and dynamic pricing for multi-version products, *J. Revenue Pricing Manag.* 11 (2012) 303–318.
- [233] K. Berg, H. Ehtamo, Continuous learning methods in two-buyer pricing problem, *Math. Methods Oper. Res.* 75 (1) (2012) 287–304.
- [234] S. Shakya, F. Oliveira, G. Owusu, Analysing the effect of demand uncertainty in dynamic pricing with EAs, in: M. Bramer, M. Petridis, F. Coenen (Eds.), *Research and Development in Intelligent Systems XXV. Proceedings of AI-2008, The Twenty-Eighth SGA International Conference on Innovative Techniques and Applications of Artificial Intelligence*, Springer, London, 2009, pp. 77–90.
- [235] S. Ramezani, P.A.N. Bosman, H. La Poutre, Adaptive strategies for dynamic pricing agents, in: *2011 IEEE/WIC/ACM International Conferences on Web Intelligence and Intelligent Agent Technology*, IEEE, 2011, pp. 323–328.
- [236] P.B. Mullen, C.K. Monson, K.D. Seppi, S.C. Warnick, Particle swarm optimization in dynamic pricing, in: *2006 IEEE International Conference on Evolutionary Computation*, IEEE, 2006, pp. 1232–1239.
- [237] E. Kutschinski, T. Uthmann, D. Polani, Learning competitive pricing strategies by multi-agent reinforcement learning, *J. Econom. Dynam. Control* 27 (11–12) (2003) 2207–2218.
- [238] C.V.L. Raju, Y. Narahari, K. Ravikumar, Learning dynamic prices in electronic retail markets with customer segmentation, *Ann. Oper. Res.* 143 (1) (2006) 59–75.
- [239] V. Kōnönen, Dynamic pricing based on asymmetric multiagent reinforcement learning, *Int. J. Intell. Syst.* 21 (1) (2006) 73–98.
- [240] V.L.R. Chinthalapati, N. Yadati, R. Karumanchi, Learning dynamic prices in multiseller electronic retail markets with price sensitive customers, stochastic demands, and inventory replenishments, *IEEE Trans. Syst. Man Cybern. Part C Appl. Rev.* 36 (1) (2006) 92–106.
- [241] M. Schwind, Dynamic Pricing and Automated Resource Allocation for Complex Information Services, in: *Lecture Notes in Economics and Mathematical Systems*, vol. 589, Springer, Berlin, Heidelberg, 2007.
- [242] Y. Cheng, Dynamic pricing decision for perishable goods: a Q-learning approach, in: *4th International Conference on Wireless Communications, Networking and Mobile Computing*, 2008, WiCOM'08, IEEE, 2008, pp. 1–5.
- [243] W. Han, L. Liu, H. Zheng, Dynamic pricing by multiagent reinforcement learning, in: *2008 International Symposium on Electronic Commerce and Security*, IEEE, 2008, pp. 226–229.
- [244] D. Vengerov, A gradient-based reinforcement learning approach to dynamic pricing in partially-observable environments, *Future Gener. Comput. Syst.* 24 (7) (2008) 687–693.
- [245] Y. Cheng, Real time demand learning-based Q-learning approach for dynamic pricing in E-retailing setting, in: *International Symposium on Information Engineering and Electronic Commerce IEEC'09*, 2009, pp. 594–598.
- [246] W. Jintian, Z. Lei, Application of reinforcement learning in dynamic pricing algorithms, in: *IEEE International Conference on Automation and Logistics*, 2009, ICAL'09, IEEE, 2009, pp. 419–423.
- [247] W. Han, A dynamic pricing algorithm by Bayesian Q-learning, in: *Second International Conference on Computer Modeling and Simulation*, 2010, ICCMS'10, Vol. 2, 2010, pp. 515–519.
- [248] A. Collins, L. Thomas, Comparing reinforcement learning approaches for solving game theoretic models: a dynamic airline pricing game example, *J. Oper. Res. Soc.* 63 (1) (2012) 1165–1173.
- [249] A. Collins, L. Thomas, Learning competitive dynamic airline pricing under different customer models, *J. Revenue Pricing Manag.* 12 (5) (2013) 416–430.
- [250] I. Dogan, A.R. Güner, A reinforcement learning approach to competitive ordering and pricing problem, *Expert Syst.* 37 (16–17) (2013) 8141–8161.
- [251] R. Rana, F.S. Oliveira, Real-time dynamic pricing in a non-stationary environment using model-free reinforcement learning, *Omega* 47 (2014) 116–126.
- [252] R. Rana, F.S. Oliveira, Dynamic pricing policies for interdependent perishable products or services using reinforcement learning, *Expert Syst. Appl.* 42 (1) (2015) 426–436.
- [253] C.H. Xia, P. Dube, Dynamic pricing in e-services under demand uncertainty, *Prod. Oper. Manage.* 16 (6) (2007) 701–712.
- [254] B.D. Chung, J. Li, T. Yao, C. Kwon, T.L. Friesz, Demand learning and dynamic pricing under competition in a state-space framework, *IEEE Trans. Eng. Manage.* 59 (2) (2012) 240–249.
- [255] T. Levina, Y. Levin, J. McGill, M. Nediak, Dynamic pricing with online learning and strategic consumers: an application of the aggregating algorithm, *Oper. Res.* 57 (2) (2009) 327–341.
- [256] V.G. Vovk, Aggregating strategies, in: *Proceedings of the Third Annual Workshop on Computational Learning Theory*, COLT'90, Morgan Kaufmann Publishers, San Francisco, 1990, pp. 371–386.
- [257] J.M. DiMicco, P. Maes, A. Greenwald, Learning curve: a simulation-based approach to dynamic pricing, *Electron. Commer. Res.* 3 (3–4) (2003) 245–276.
- [258] M.J. Vázquez-Gallo, M. Estévez, S. Egidio, Active learning and dynamic pricing policies, *Am. J. Oper. Res.* 4 (2) (2014) 90–100.
- [259] C.H. Brooks, S. Fay, R. Das, J.K. MacKie-Mason, J.O. Kephart, E.H. Durfee, Automated strategy searches in an electronic goods market: learning and complex price schedules, in: *Proceedings of the 1st ACM Conference on Electronic Commerce*, ACM, 1999, pp. 31–40.
- [260] D. Kong, One dynamic pricing strategy in agent economy using neural network based on online learning, in: *WI'04 Proceedings of the 2004 IEEE/WIC/ACM International Conference on Web Intelligence*, 2004, pp. 98–102.
- [261] T.K. Ghose, T.T. Tran, Dynamic pricing in electronic commerce using neural network, in: G. Babin, P. Kropf, M. Weiss (Eds.), *E-Technologies: Innovation in an Open World*, Vol. 26, Springer, Berlin, Heidelberg, 2009, pp. 227–232.

- [262] S. Shakya, M. Kern, G. Owusu, C.M. Chin, Neural network demand models and evolutionary optimisers for dynamic pricing, *Knowl.-Based Syst.* 29 (2012) 44–53.
- [263] G. Liu, H. Wang, An online sequential feed-forward network model for demand curve prediction, *J. Inf. Comput. Sci.* 10 (10) (2013) 3063–3069.
- [264] C.H. Brooks, R. Das, J.O. Kephart, J.K. MacKie-Mason, R.S. Gazzale, Information bundling in a dynamic environment, in: *Proceedings of the IJCAI-01 Workshop on Economic Agents, Models, and Mechanisms*, Seattle, WA, Vol. 205, 2001.
- [265] J.O. Kephart, C.H. Brooks, R. Das, J.K. MacKie-Mason, R. Gazzale, E.H. Durfee, Pricing information bundles in a dynamic environment, in: *Proceedings of the 3rd ACM conference on Electronic Commerce*, ACM, New York, 2001, pp. 180–190.
- [266] C.H. Brooks, R.S. Gazzale, R. Das, J.O. Kephart, J.K. MacKie-Mason, E.H. Durfee, Model selection in an information economy: choosing what to learn, *Comput. Intelligence* 18 (4) (2002) 566–582.
- [267] S. Subrahmanyam, R. Shoemaker, Developing optimal pricing and inventory policies for retailers who face uncertain demand, *J. Retail.* 72 (1) (1996) 7–30.
- [268] G.R. Bitran, H.K. Wadhwa, A methodology for demand learning with an application to the optimal pricing of seasonal products, *Working Paper, WP#3898-96*, MIT Sloan School of Management, 1996. Available at: <http://dspace.mit.edu/handle/1721.1/2619>.
- [269] A. Bisi, M. Dada, Dynamic learning, pricing, and ordering by a censored newsvendor, *Nav. Res. Logist.* 54 (4) (2007) 448–461.
- [270] J.L. Zhang, J. Chen, Bayesian solution to pricing and inventory control under unknown demand distribution, *Oper. Res. Lett.* 34 (5) (2006) 517–524.
- [271] E.L. Porteus, Stochastic inventory theory, in: D.P. Heyman, M.J. Sobel (Eds.), *Handbooks in Operations Research and Management Science*, Vol. 2, Elsevier, Amsterdam, 1990, pp. 605–652. (Chapter 12).
- [272] T.M. Choi, Pre-season stocking and pricing decisions for fashion retailers with multiple information updating, *Int. J. Prod. Econ.* 106 (1) (2007) 146–170.
- [273] J.-j. Gao, Y. Zheng, X. Mao, A joint decision model of dynamic pricing and dynamic inventory in an apparel supply chain with demand learning, in: *2010 International Conference on Logistics Systems and Intelligent Management*, IEEE, 2010, pp. 692–696.
- [274] K. Forghani, A. Mirzazadeh, M. Rafiee, A price-dependent demand model in the single period inventory system with price adjustment, *J. Ind. Eng.* 2013 (2013) 1–9.
- [275] T. Boyaci, Ö. Özer, Information acquisition for capacity planning via pricing and advance selling: when to stop and act? *Oper. Res.* 58 (5) (2010) 1328–1349.
- [276] A.N. Burnetas, C.E. Smith, Adaptive ordering and pricing for perishable products, *Oper. Res.* 48 (3) (2000) 436–443.
- [277] E. Adida, G. Perakis, A robust optimization approach to dynamic pricing and inventory control with no backorders, *Math. Program. B* 107 (1) (2006) 97–129.
- [278] N.C. Petrucci, M. Dada, Dynamic pricing and inventory control with learning, *Nav. Res. Logist.* 49 (3) (2002) 303–325.
- [279] E. Adida, G. Perakis, Dynamic pricing and inventory control: robust vs. stochastic uncertainty models—a computational study, *Ann. Oper. Res.* 181 (1) (2010) 125–157.
- [280] M. Mahmoudzadeh, S.J. Sadjadi, S. Mansour, Robust optimal dynamic production/pricing policies in a closed-loop system, *Appl. Math. Model.* 37 (16–17) (2013) 8141–8161.
- [281] A. Arasteh, A. Aliahmadi, A. Norang, Inventory policies and dynamic pricing under possibility and rivals, *Int. J. Adv. Manuf. Technol.* 72 (5–8) (2014) 627–642.
- [282] M.A. Lariviere, E.L. Porteus, Manufacturer-retailer contracting under an unknown demand distribution, *Working Paper*, Duke University/Stanford University, 1995.
- [283] W. Gaul, A.D. Azizi, A demand learning data based approach to optimize revenues of a retail chain, in: H. Locarek-Junge, C. Weihs (Eds.), *Classification as a Tool for Research*, in: *Studies in Classification, Data Analysis, and Knowledge Organization*, Springer, Berlin, Heidelberg, 2010, pp. 683–691. (Chapter 75).
- [284] W.R. Thompson, On the likelihood that one unknown probability exceeds another in view of the evidence of two samples, *Biometrika* 25 (3–4) (1933) 285–294.
- [285] H. Robbins, Some aspects of the sequential design of experiments, *Bull. Amer. Math. Soc.* 58 (5) (1952) 527–535.
- [286] T.L. Lai, H. Robbins, Asymptotically efficient adaptive allocation rules, *Adv. in Appl. Math.* 6 (1) (1985) 4–22.
- [287] J.C. Gittins, Multi-armed Bandit Allocation Indices, in: *Wiley-Interscience Series in Systems and Optimization*, Wiley, New York, 1989.
- [288] P. Auer, N. Cesa-Bianchi, P. Fischer, Finite-time analysis of the multi-armed bandit problem, *Mach. Learn.* 47 (2) (2002) 235–256.
- [289] J. Vermorel, M. Mohri, Multi-armed bandit algorithms and empirical evaluation, in: *Proceedings of the European Conference of Machine Learning*, Porto, Portugal (October 3–7), in: *Springer Lecture Notes in Computer Science*, vol. 3720, 2005, pp. 437–448.
- [290] N. Cesa-Bianchi, G. Lugosi, *Prediction, Learning, and Games*, Cambridge University Press, New York, 2006.
- [291] W.B. Powell, The knowledge gradient for optimal learning, in: J.J. Cochran (Ed.), *Encyclopedia for Operations Research and Management Science*, Wiley, New York, 2010.
- [292] R. Kleinberg, Nearly tight bounds for the continuum-armed bandit problem, in: L.K. Saul, Y. Weiss, L. Bottou (Eds.), *Advances in Neural Information Processing Systems 17*, MIT Press, Cambridge, US, 2005, pp. 697–704.
- [293] P. Auer, R. Ortner, C. Szepesvári, Improved rates for the stochastic continuum-armed bandit problem, in: N. Bshouty, C. Gentile (Eds.), *Learning Theory*, in: *Lecture Notes in Computer Science*, vol. 4539, Springer, Berlin, Heidelberg, 2007, pp. 454–468.
- [294] E.W. Cope, Regret and convergence bounds for a class of continuum-armed bandit problems, *IEEE Trans. Automat. Control* 54 (6) (2009) 1243–1253.
- [295] Y. Wang, J.Y. Audibert, R. Munos, Algorithms for infinitely many-armed bandits, in: D. Koller, D. Schuurmans, Y. Bengio, L. Bottou (Eds.), *Advances in Neural Information Processing Systems 21*, Curran Associates, Inc., 2009, pp. 1729–1736.
- [296] A. Goldenshluger, A. Zeevi, Woodroofe's one-armed bandit problem revisited, *Ann. Appl. Probab.* 19 (4) (2009) 1603–1633.
- [297] P. Rusemievichentong, J.N. Tsitsiklis, Linearly parameterized bandits, *Math. Oper. Res.* 35 (2) (2010) 395–411.
- [298] S. Filippi, O. Cappé, A. Garivier, C. Szepesvári, Parametric bandits: the generalized linear case, in: J. Lafferty, C.K.I. Williams, J. Shawe-Taylor, R.S. Zemel, A. Culotta (Eds.), *Advances in Neural Information Processing Systems 23*, Curran Associates, Inc., 2010, pp. 586–594.
- [299] Y. Abbasi-Yadkori, D. Pal, C. Szepesvári, Improved algorithms for linear stochastic bandits, in: J. Shawe-Taylor, R.S. Zemel, P. Bartlett, F.C.N. Pereira, K.Q. Weinberger (Eds.), *Advances in Neural Information Processing Systems 24*, 2011, pp. 2312–2320.
- [300] J.Y. Yu, S. Mannor, Unimodal bandits, in: L. Getoor, T. Scheffer (Eds.), *Proceedings of the 28th International Conference on Machine Learning*, Bellevue, Washington, USA, Omnipress, 2011, pp. 41–48.
- [301] A. Slivkins, Contextual bandits with similarity information, 2011. [arXiv:0907.3986v5 \[cs.DS\]](https://arxiv.org/abs/0907.3986v5).
- [302] V. Perchet, P. Rigollet, The multi-armed bandit problem with covariates, *Ann. Statist.* 41 (2) (2013) 693–721.
- [303] R. Combes, A. Protuer, Unimodal bandits: regret lower bounds and optimal algorithms, in: *Proceedings of the 31th International Conference on Machine Learning*, ICML 2014, Beijing, China, 21–26 June 2014, 2014, pp. 521–529.
- [304] A. Garivier, E. Moulines, On upper-confidence bound policies for switching bandit problems algorithmic learning theory, in: J. Kivinen, C. Szepesvári, E. Ukkonen, T. Zeugmann (Eds.), *Algorithmic Learning Theory*, 22nd International Conference, ALT 2011, Espoo, Finland, October 5–7, 2011, in: *Lecture Notes in Computer Science*, vol. 6925, Springer, Berlin, Heidelberg, 2011, pp. 174–188. (Chapter 16).
- [305] O. Besbes, Y. Gur, A. Zeevi, Non-stationary stochastic optimization, *Working Paper*, Columbia Business School, 2014. <http://ssrn.com/abstract=2296012>.
- [306] T.L. Lai, C.Z. Wei, Least squares estimates in stochastic regression models with applications to identification and control of dynamic systems, *Ann. Statist.* 10 (1) (1982) 154–166.
- [307] W.L. Cooper, T. Homem de Mello, A.J. Kleywegt, Learning and pricing with models that do not explicitly incorporate competition, *Working Paper*, University of Minnesota, 2014. Available at: <http://www.menet.umn.edu/~billcoop/competitive2014.pdf>.
- [308] A. Borovkov, *Mathematical Statistics*, Gordon and Breach Science Publishers, Amsterdam, 1998.
- [309] A.V. den Boer, B. Zwart, Mean square convergence rates for maximum quasi-likelihood estimators, *Stoch. Syst.* 4 (2014) 1–29.
- [310] A. Marcat, T.J. Sargent, Convergence of least squares learning mechanisms in self-referential linear stochastic models, *J. Econom. Theory* 48 (2) (1989) 337–368.
- [311] V. Wieland, Learning by doing and the value of optimal experimentation, *J. Econom. Dynam. Control* 24 (4) (2000) 501–534.
- [312] V. Wieland, Monetary policy, parameter uncertainty and optimal learning, *J. Monetary Econ.* 46 (2000) 199–228.
- [313] G.W. Beck, V. Wieland, Learning and control in a changing economic environment, *J. Econom. Dynam. Control* 26 (9–10) (2002) 1359–1377.
- [314] J. Han, T.L. Lai, V. Spivakovskiy, Approximate policy optimization and adaptive control in regression models, *Comput. Econ.* 27 (4) (2006) 433–452.
- [315] D.A. Kendrick, H.M. Amman, M.P. Tucci, Learning about learning in dynamic economic models, *Discussion Paper 08-20*, Tjalling C. Koopmans Research Institute, Utrecht School of Economics, 2008.
- [316] H. Scarf, Bayes solutions of the statistical inventory problem, *Ann. Math. Statist.* 30 (2) (1959) 490–508.
- [317] K.S. Azoury, Bayes solution to dynamic inventory models under unknown demand distribution, *Manag. Sci.* 31 (9) (1985) 1150–1160.
- [318] W.T. Huh, R. Levi, P. Rusemievichentong, J.B. Orlin, Adaptive data-driven inventory control with censored demand based on Kaplan–Meier estimator, *Oper. Res.* 59 (4) (2011) 929–941.
- [319] H. Nazerzadeh, A. Saberi, R. Vohra, Dynamic pay-per-action mechanisms and applications to online advertising, *Oper. Res.* 61 (1) (2013) 98–111.
- [320] D.F. Ciocan, V. Farias, Model predictive control for dynamic resource allocation, *Math. Oper. Res.* 37 (3) (2012) 501–525.
- [321] F. Caro, J. Gallien, Dynamic assortment with demand learning for seasonal consumer goods, *Manag. Sci.* 53 (2) (2007) 276–292.
- [322] P. Rusemievichentong, Z.-J.M. Shen, D.B. Shmoys, Dynamic assortment optimization with a multinomial logit choice model and capacity constraint, *Oper. Res.* 58 (6) (2010) 1666–1680.
- [323] D. Sauré, A. Zeevi, Optimal dynamic assortment planning with demand learning, *Manuf. Serv. Oper. Manag.* 15 (3) (2013) 387–404.
- [324] G.J. Hitsch, An empirical model of optimal dynamic product launch and exit under demand uncertainty, *Mark. Sci.* 25 (1) (2006) 25–50.

- [325] M. Talebian, N. Boland, M. Savelsbergh, Pricing to accelerate demand learning in dynamic assortment planning, *European J. Oper. Res.* 237 (2) (2014) 555–565.
- [326] O. Loginova, C.R. Taylor, Price experimentation with strategic buyers, *Rev. Econ. Des.* 12 (3) (2008) 165–187.
- [327] R.D. Weaver, Y. Moon, Pricing perishables, in: G. Schiefer and U. Rickert, (Eds.), 2011 International European Forum, February 14–18, 2011, Innsbruck-Igls, Austria, 2011, pp. 212–231.
- [328] R. Caldentey, Y. Liu, I. Lobel, Intertemporal pricing without priors, Working Paper, New York University (NYU), 2013. <http://ssrn.com/abstract=2357083>.
- [329] H. Chade, V.V. de Serio, Pricing, learning, and strategic behavior in a single-sale model, *Econom. Theory* 19 (2) (2002) 333–353.
- [330] B. Ifrach, C. Maglaras, M. Scarsini, Bayesian social learning from consumer reviews, Working Paper, 2013. Available at: <http://ssrn.com/abstract=2293158>.
- [331] M. Yu, L. Debo, R. Kapuscinski, Strategic waiting for consumer-generated quality information: dynamic pricing of new experience goods, Working Paper 13-78, University of Chicago, Booth School of Business, 2013. Available at: <http://ssrn.com/abstract=2334423>.
- [332] Q. Liu, G.J. van Ryzin, Strategic capacity rationing when customers learn, *Manuf. Serv. Oper. Manag.* 13 (1) (2011) 89–107.
- [333] R. Swinney, Selling to strategic consumers when product value is uncertain: the value of matching supply and demand, *Manag. Sci.* 57 (10) (2011) 1737–1751.
- [334] A.J. Mersereau, D. Zhang, Markdown pricing with unknown fraction of strategic customers, *Manuf. Serv. Oper. Manag.* 14 (3) (2012) 355–370.
- [335] S. Kachani, G. Perakis, C. Simon, Modeling the transient nature of dynamic pricing with demand learning in a competitive environment, in: T.L. Friesz (Ed.), *Network Science, Nonlinear Science and Infrastructure Systems*, in: International Series in Operations Research & Management Science, vol. 102, Springer Science+Business Media, New York, 2007, pp. 223–267.
- [336] C. Kwon, T.L. Friesz, R. Mookherjee, T. Yao, B. Feng, Non-cooperative competition among revenue maximizing service providers with demand learning, *European J. Oper. Res.* 197 (3) (2009) 981–996.
- [337] J. Li, T. Yao, H. Gao, A revenue maximizing strategy based on Bayesian analysis of demand dynamics, in: D.A. Field, T.J. Peters (Eds.), *Proceedings of the 2009 SIAM Conference on "Mathematics for Industry"*, Society for Industrial and Applied Mathematics, Philadelphia, PA, 2010, pp. 174–181.
- [338] A.T. Coughlan, M.K. Mantrala, Dynamic competitive retail pricing behavior with uncertainty and learning, *Manag. Decis. Econ.* 15 (1) (1994) 3–20.
- [339] S. Chan Choi, S. Jagpal, Duopoly pricing under risk aversion and parameter uncertainty, *J. Prod. Brand Manag.* 13 (5) (2004) 359–368.
- [340] G. Perakis, A. Sood, Competitive multi-period pricing for perishable products: a robust optimization approach, *Math. Program.* 107 (1) (2006) 295–335.
- [341] T.L. Friesz, C. Kwon, T.I. Kim, L. Fan, T. Yao, Competitive robust dynamic pricing in continuous time with fixed inventories, 2012. [arXiv:1208.4374 \[math.OC\]](https://arxiv.org/abs/1208.4374).
- [342] E. Adida, G. Perakis, Dynamic pricing and inventory control: uncertainty and competition, *Oper. Res.* 58 (2) (2010) 289–302.
- [343] R.M. Cyert, M.H. DeGroot, Bayesian analysis and duopoly theory, *J. Polit. Econ.* 78 (5) (1970) 1168–1184.
- [344] A.P. Kirman, Learning by firms about demand conditions, in: R.H. Day, T. Graves (Eds.), *Adaptive Economics*, Academic Press, New York, 1975, pp. 137–156.
- [345] P. Aghion, M. Paz Espinosa, B. Jullien, Dynamic duopoly with learning through market experimentation, *Econom. Theory* 3 (3) (1993) 517–539.
- [346] L.J. Mirman, L. Samuelson, A. Urbano, Duopoly signal jamming, *Econom. Theory* 3 (1) (1993) 129–149.
- [347] A. Fishman, N. Gandal, Experimentation and learning with networks effects, *Econom. Lett.* 44 (1–2) (1994) 103–108.
- [348] J.E. Harrington, Experimentation and learning in a differentiated-products duopoly, *J. Econom. Theory* 66 (1) (1995) 275–288.
- [349] D. Bergemann, J. Valimaki, Market experimentation and pricing. Cowles Foundation Discussion Paper 1122, Cowles Foundation for Research in Economics at Yale University, 1996. <http://cowles.econ.yale.edu/P/cd/d11a/d1122.pdf>.
- [350] A.G. Gallego, Oligopoly experimentation of learning with simulated markets, *J. Econ. Behav. Organ.* 35 (3) (1998) 333–355.
- [351] M.D. Alepuz, A. Urbano, Duopoly experimentation: Cournot competition, *Math. Social Sci.* 37 (2) (1999) 165–188.
- [352] S. Rassenti, S.S. Reynolds, V.L. Smith, F. Szidarovszky, Adaptation and convergence of behavior in repeated experimental Cournot games, *J. Econ. Behav. Organ.* 41 (2) (2000) 117–146.
- [353] P. Belleflamme, F. Bloch, Price and quantity experimentation: a synthesis, *Int. J. Ind. Organiz.* 19 (10) (2001) 1563–1582.
- [354] M.P. Schinkel, J. Tuinstra, D. Vermeulen, Convergence of Bayesian learning to general equilibrium in mis-specified models, *J. Math. Econom.* 38 (4) (2002) 483–508.
- [355] G. Keller, S. Rady, Price dispersion and learning in a dynamic differentiated-goods duopoly, *RAND J. Econ.* 34 (1) (2003) 138–165.
- [356] M. Dimitrova, E.E. Schlee, Monopoly, competition and information acquisition, *Int. J. Ind. Organiz.* 21 (10) (2003) 1623–1642.
- [357] J. Tuinstra, A price adjustment process in a model of monopolistic competition, *Int. Game Theory Rev.* 6 (3) (2004) 417–442.
- [358] S. Barta, P. Varaiwa, Microeconomics: stochastic models of price adjustment, *Ann. Econ. Soc. Meas.* 5 (3) (1976) 267–281.
- [359] A.R. Greenwald, J.O. Kephart, Shopbots and pricebots, in: A. Moukas, C. Sierra, F. Ygge (Eds.), *Agent Mediated Electronic Commerce II*, Springer, 1999, pp. 1–23.
- [360] P. Dasgupta, R. Das, Dynamic pricing with limited competitor information in a multi-agent economy, in: P. Scheuermann, O. Etzion (Eds.), *Cooperative Information Systems*, in: Lecture Notes in Computer Science, vol. 1901, Springer, Berlin, Heidelberg, 2000, pp. 299–310.
- [361] G. Tesaurio, J.O. Kephart, Pricing in agent economies using multi-agent Q-learning, *Auton. Agents Multi-Agent Syst.* 5 (1) (2002) 289–304.
- [362] J. Jumadinova, P. Dasgupta, Firefly-inspired synchronization for improved dynamic pricing in online markets, in: *Second IEEE International Conference on Self-Adaptive and Self-Organizing Systems*, 2008, SASO'08, IEEE, 2008, pp. 403–412.
- [363] J. Jumadinova, P. Dasgupta, Multi-attribute regret-based dynamic pricing, in: W. Ketter, H. La Poutré, N. Sadeh, O. Shehory, W. Walsh (Eds.), *Agent-Mediated Electronic Commerce and Trading Agent Design and Analysis*, in: Lecture Notes in Business Information Processing, vol. 44, Springer, Berlin, Heidelberg, 2010, pp. 73–87.
- [364] O. Besbes, D. Sauré, Dynamic pricing strategies in the presence of demand shocks, *Manuf. Serv. Oper. Manag.* 16 (4) (2014) 513–528.
- [365] Y. Chen, V.F. Farias, Simple policies for dynamic pricing with imperfect forecasts, *Oper. Res.* 61 (3) (2013) 612–624.
- [366] N.B. Keskin, A. Zeevi, Chasing demand: learning and earning in a changing environment, Working Paper, University of Chicago, Booth School of Business, 2013. Available at: <http://ssrn.com/abstract=2389750>.
- [367] A.V. den Boer, Tracking the market: dynamic pricing and learning in a changing environment, Working Paper, University of Twente, 2014. Available at: <http://www.utwente.nl/ewi/sor/about/staff/boer/>.
- [368] Z. Wang, S.K. Mukhopadhyay, D.Q. Yao, Bayesian pricing strategy for information goods, *Int. J. Oper. Res.* 17 (4) (2013) 399–416.
- [369] S. Chakravarty, S. Padakandla, S. Bhatnagar, A simulation-based algorithm for optimal pricing policy under demand uncertainty, *Int. Trans. Oper. Res.* 21 (5) (2014) 737–760.
- [370] O. Besbes, A. Zeevi, On the (surprising) sufficiency of linear models for dynamic pricing with demand learning, *Manag. Sci.* (2014) forthcoming.
- [371] G.I. Bischi, C. Chiarella, M. Kopel, The long run outcomes and global dynamics of a duopoly game with misspecified demand functions, *Int. Game Theory Rev.* 06 (03) (2004) 343–379.
- [372] G.I. Bischi, A.K. Naimzada, L. Sbragia, Oligopoly games with local monopolistic approximation, *J. Econ. Behav. Organ.* 62 (3) (2007) 371–388.
- [373] K. Isler, H. Imhof, A game theoretic model for airline revenue management and competitive pricing, *J. Revenue Pricing Manag.* 7 (4) (2008) 384–396.
- [374] M. Anufriev, D. Kopányi, J. Tuinstra, Learning cycles in Bertrand competition with differentiated commodities and competing learning rules, *J. Econom. Dynam. Control* 37 (12) (2013) 2562–2581.
- [375] A. Kirman, On mistaken beliefs and resultant equilibria, in: R. Frydman, E.S. Phelps (Eds.), *Individual Forecasting and Aggregate Outcomes*, Cambridge University Press, New York, 1983, pp. 147–166.
- [376] A.P. Kirman, Learning in oligopoly: theory, simulation, and experimental evidence, in: A.P. Kirman, M. Salmon (Eds.), *Learning and Rationality in Economics*, Basil Blackwell, Cambridge, MA, 1995, pp. 127–178.
- [377] V. Brousseau, A. Kirman, Apparent convergence of learning processes in mis-specified games, in: B. Dutta, D. Mookherjee, T. Parthasarathy, T.E.S. Raghavan, D. Ray, S. Tijs (Eds.), *Game Theory and Economic Applications*, Springer-Verlag, Berlin, 1992, pp. 303–331.
- [378] W.L. Cooper, T. Homem de Mello, A.J. Kleywegt, Models of the spiral-down effect in revenue management, *Oper. Res.* 54 (5) (2006) 968–987.
- [379] A.V. den Boer, Dynamic pricing and learning (Ph.D. thesis), Vrije Universiteit Amsterdam, 2013.