Theory of Computer Games

Homework #1

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1. For the game 2048, estimate its state-space complexity and game-tree complexity. You may state assumptions for the estimation.

State - space Complexity

There are 16 positions on a board.

For each of the positions, there can be different values, which can be 2, 4, 8, 16 ... 1024, which is 12 possibilities for each tile.

There are 16 tiles, so we get $16^{12} = 2^{48}$ possible board states, and this most likely includes a few unreachable ones.

Game - Tree Complexity

I'll make a few assumptions for the sake of simplicity

- There are always 15 open squares
- You always have 4 moves (up, right, down, left)
- Once the total num of all tiles on the board reaches 2048, it will take the minimum number of combinations to get a single 2048

  (e.g. If placing a 2 makes the sum 2048, the combinations will be

  $2 \to 4 \to 8 \to 16 \to 32 \to 64 \to 128 \cdots \to 2048$

  $\Rightarrow$ this takes 10 moves in total
- 2 will always get placed, never a 4

  (I assumed this for the sake of calculating the upper bound)
- I will not consider the possibility the fact of the generation of the duplicate boards

to reach 2048, there needs to be $2048/2 = 1024$ tiles placed.

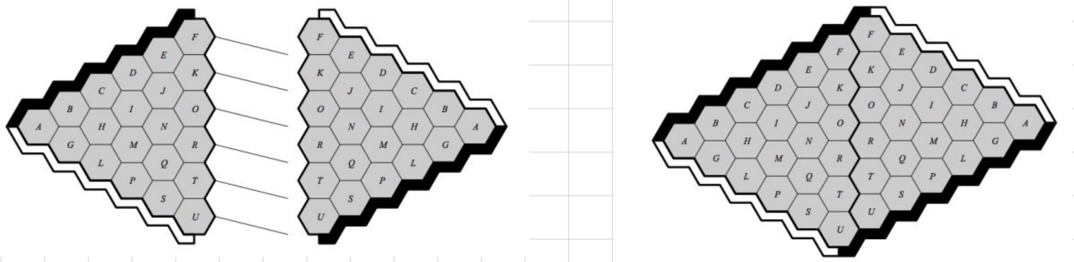
We start with 2 randomly placed tiles, then repeatly make a move and another tile gets placed So there's about 1022 "turns" until we get
(a turn consisting of making a move and a tile getting placed)
a sum of 2048, then there's another 10 turns to get a 2048 tile.

In each turn, we have 4 moves, and there can be one of two tiles placed in one of the 15 positions (30 possibilities), so thats $4 \times 30 = 120$ possibilities

This will in total gives us 120^{1032} possible states. #

* if we assume a 4 will always get placed, we get 120^{519} states.

2. Prove that for N x (N+1) Hex the longer-side wins.



Let's say if the white is the longer side. White should always place the piece at the position which is symmetric to the Black's last piece. Since Black plays first, when White ends a run, the board must be symmetrically filled. Consider to the distance of sides, when the length of Black is N, the length of White is also N. Thus, when Black almost reaches its win, the White would have already won the game.

On the other hand, if the longer side is Black, the first piece of Black can be placed at a random position. Since having more pieces is an advantage in Hex, the condition will not become worse, since the additional pieces only connects our pieces, or cut the pieces of the opponent.

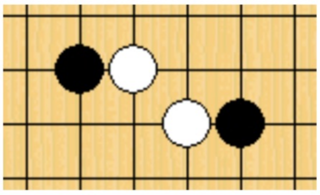
In this case, the Black should always place the piece as the position of the symmetric of the White's last piece. Somehow, if the position is already occupied, we place a piece at another random position.

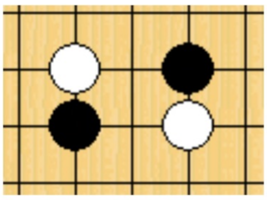
For the same reason which longer side is White case, when the length of White is N, the length of Black is also N, and Black must win.

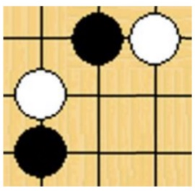
3. Prove that the theoretical value of Connect(m,n,k,p,q+1) for Black is not worse than that in Connect(m,n,k,p,q).

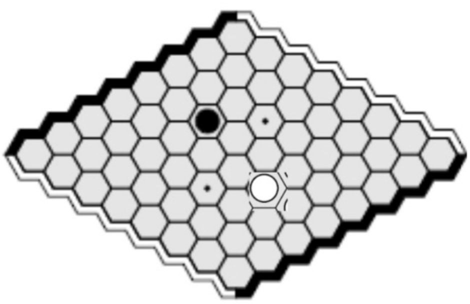
Let us suppose the Black has a winning strategy "S" in Connect(m,n.k.p.q). The strategy can be applied to Connect(m, n, k, p. q+1) since the additional move will not baffle the Black's situation in the game. If the best move of "S" of a state is the same as the addition move, we will make a random move again. In this case, the theoretical value of Connect(m,n.k.p.q+1) will never be worse than the value of Connect(m,n.k.p.q).

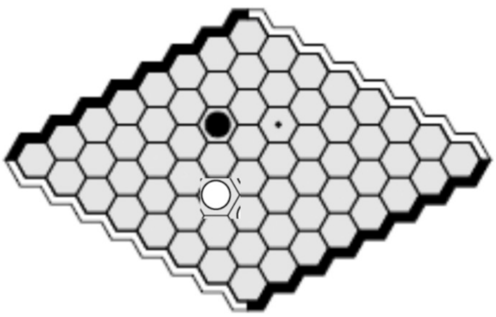
4. The game Connect(6,2,2) and Hex can both be ultra-weakly solved with no win for White. Please solve the following four positions ultra-weakly. If you think it is impossible to solve it, answer "no" and explain why.


(a)


(b)


(c)


(d)


(e)

(a)

Since the current situation is not symmetric, we cannot apply the strategy stealing, which will make the values remains unknown.

(b)

The situation is symmetric, and the extra pieces will not harm the play in Connect (m, n, k, q, p). This makes the Black will never lose with strategy stealing. since the White does not have a winning strategy.

(c)

Not symmetric. hence we are not able to apply the stealing strategy, which makes the value remains unknown.

(d)

The situation is not symmetric, making the value remains unknown because we cannot apply the strategy stealing.

e)

The situation is symmetric, and the extra pieces do not harm the player in Hex. Then Black will never lose with strategy stealing. Besides, since there is no draw for Hex, we can say that Black will always wins.