

In the tables provided, the leftmost 5 columns are the variables `c`, `x`, `y`, `i`, `j` in `valid_sequence()`; separated by a blank column, the remaining variables are those in `valid_layers()`. `c[i]` replaces `ci` in the specification to denote the i^{th} character in sequence `c`, where index `i` starts from 1. Ellipses (...) are used to denote those neglected runs when 1) **for all** fails with one exception, denoted in red or 2) **exists** succeeds with one success, denoted in green. Light grey are used to denote those irrelevant information; in 4.1e onwards, some of these are omitted for clarity.

```

valid_layer(i, j, m, n) =
(
  j = i and
  c[i] = "N"
) or
(
  n - m + 1 = 2 (j - i + 1) and
  j != i and
  for all x (
    m <= x <= n and
    exists t
    implies (x - m = 3t - 1)
  ) implies c[x] = "S"
  and
  for all y (
    i <= y <= j and
    c[y] != "S"
  ) implies (
    (
      c[y] = "B" and
      for all w (
        2(y-i) <= w <= 3(y-i)-1
      ) implies c[m+w] = "B"
    ) or
    (
      c[y] = "N" and
      for all k (
        2(y-i) <= k <= 3(y-i)-1
      ) implies c[m+3k] = "N"
    )
  )
)

valid_sequence(m, n) =
(
  m = n and
  c[m] = "B"
) or
(
  c[m] = "N" and
  exists d implies (
    d >= 3 and
    for all c (3 <= c <= d) implies (
      exists x,y,i,j (
        y-x+1 = 2^(c-3) × 3 and
        j-i+1 = 2^(c-2) × 3 and
        j <= n and
        i = y + 1
      ) implies valid_layer(x,y,i,j)
    )
  )
)

```

- a. **FALSE.** There does not exist a d in which for all c there exists a pair $\{x, y, i, j\}$ where all of which are non-negative integer.
- b. **FALSE.** There does not exist a d in which for all c there exists a pair $\{x, y, i, j\}$ that returns true in `valid_layer()`.

														w	m+w	c[m+w]
c	x	y	i	j	n-m+1	j-i+1	x	t	c[x]	y	c[y]	2(y-i)	3(y-i)-1	k	m+3k	c(m+3k)
3	2	4	5	10	6	6	10	2	S							
	2	4	5	10				9	1.7							
	2	4	5	10				8	1.3							
	2	4	5	10				7	1	S						
	2	4	5	10				6	0.7							
	2	4	5	10				5	0.3							
	2	4	5	10						4	S					
	2	4	5	10						3	N	2	2	2	11	
	2	4	5	10						2	N	...				
1	3	4	9		6	6	9	2	B							
	1	3	4	9				8	1.7							
	1	3	4	9				7	1.3							
	1	3	4	9				6	1	N						
	1	3	4	9				5	0.7							
	1	3	4	9				4	0.3							

c. **TRUE.** There exists a d ($d = 3$) in which for all c (c in $[3, 3]$) there exists a pair $\{x = 14, y = 16, i = 17, j = 22\}$ that returns true in `valid_layer()`.

c	x	y	i	j	n-m+1	j-i+1	x	t	c[x]	y	c[y]	2(y-i)	3(y-i)-1	w	m+w	c[m+w]
					k	m+3k	c[m+3k]									
3	17	19	20	25	6	6	25	2	S							
	17	19	20	25			24	1.7								
	17	19	20	25			23	1.3								
	17	19	20	25			22	1	S							
	17	19	20	25			21	0.7								
	17	19	20	25			20	0.3								
	17	19	20	25						19	S					
	17	19	20	25						18	N	2	2	2	26	
	17	19	20	25						17	N	...				
16	18	19	24		6	6	24	2	N							
	16	18	19	24			23	1.7								
	16	18	19	24			22	1.3								
	16	18	19	24			21	1	...							
	16	18	19	24			20	0.7								
	16	18	19	24			19	0.3								
15	17	18	23		6	6	23	2	N							
	15	17	18	23			22	1.7								
	15	17	18	23			21	1.3								
	15	17	18	23			20	1	...							
	15	17	18	23			19	0.7								
	15	17	18	23			18	0.3								
14	16	17	22		6	6	22	2	S							
	14	16	17	22			21	1.7								
	14	16	17	22			20	1.3								
	14	16	17	22			19	1	S							
	14	16	17	22			18	0.7								
	14	16	17	22			17	0.3								
	14	16	17	22						16	S					
	14	16	17	22						15	N	2	2	2	23	N
	14	16	17	22						14	N	0	-1	TRUE		
...												

1d. **FALSE.** There does not exist a d in which for all c there exists a pair $\{x, y, i, j\}$ that returns true in `valid_layer()`.

c	x	y	i	j	n-m+1	j-i+1	x	t	c[x]	y	c[y]	2(y-i)	3(y-i)-1	w	m+w	c[m+w]
					k	m+3k	c[m+3k]									
3	2	4	5	10	6	6	10	2	S							
	2	4	5	10			9	1.7								
	2	4	5	10			8	1.3								
	2	4	5	10			7	1	S							
	2	4	5	10			6	0.7								
	2	4	5	10			5	0.3								
	2	4	5	10						4	S					
	2	4	5	10						3	B	2	2	2	7	S
	2	4	5	10						2	N	...				
1	3	4	9		6	6	9	2	B							
	1	3	4	9			8	1.7								
	1	3	4	9			7	1.3								
	1	3	4	9			6	1	...							
	1	3	4	9			5	0.7								
	1	3	4	9			4	0.3								

1e. **FALSE.** There does not exists a d in which for all c there exists a pair {x, y, i, j} that returns true in valid_layer().

c	x	y	i	j		n-m+1	j-i+1	x	t	c[x]	y	c[y]	2(y-i)	3(y-i)-1	k	m+3k	c[m+3k]
3	14	16	17	22		6	6	22	2	S							
	14	16	17	22				21	1.7								
	14	16	17	22				20	1.3								
	14	16	17	22				19	1	S							
	14	16	17	22				18	0.7								
	14	16	17	22				17	0.3								
	14	16	17	22							16	S					
	14	16	17	22							15	B	2	2	2	19	S
	14	16	17	22							14	B	...				
13	15	16	21			6	6	21	2	B							
13	15	16	21					18	1	...							
12	14	15	20			6	6	20	2	B							
12	14	15	20					17	1	...							
11	13	14	19			6	6	19	2	S							
11	13	14	19					16	1	S							
11	13	14	19								13	S					
11	13	14	19								12	N	2	2	2	20	B
11	13	14	19								11	B	...				
10	12	13	18			6	6	18	2	B							
10	12	13	18					15	1	...							
9	11	12	17			6	6	17	2	B							
9	11	12	17					14	1	...							
8	10	11	16			6	6	16	2	S							
8	10	11	16					13	1	S							
8	10	11	16								10	S					
8	10	11	16								9	B	2	2	2	13	S
8	10	11	16								8	B	...				
7	9	10	15			6	6	15	2	B							
7	9	10	15					12	1	...							
6	8	9	14			6	6	14	2	B							
6	8	9	14					11	1	...							
5	7	8	13			6	6	13	2	S							
5	7	8	13					10	1	S							
5	7	8	13								7	S					
5	7	8	13								6	B	2	2	2	10	S
5	7	8	13								5	N	...				
4	6	7	12			6	6	12	2	N							
4	6	7	12					9	1	...							
3	5	6	11			6	6	11	2	B							
3	5	6	11					8	1	...							
2	4	5	10			6	6	10	2	S							
2	4	5	10					7	1	S							
2	4	5	10								4	S					
2	4	5	10								3	B	2	2	2	7	S
2	4	5	10								2	N	...				
1	3	4	9			6	6	9	2	B							
1	3	4	9					6	1	...							
...																	

1f. **FALSE.** There does not exists a d in which for all c there exists a pair {x, y, i, j} that returns true in valid_layer(). (Ref: Appendix.)

Question 4.2

In `valid_sequence()`, there are two main cases:

1. when the sequence starts with B as the only letter, which will then pass the predicate, or;
2. when the sequence starts with N, and the sequence has at least 9 letters, which will then pass `x, y, i, j` to `valid_layer()`.

Each pass to `valid_layer()` will only concern a sub-sequence between `x` to `j` with a length of $9 \times 2^{c-3}$, where `c` starts from 3. The sequence is further divided into two sub-sub-sequences between `x` to `y` and `i` to `j`, where `x` to `y` occupies one-third of the sub-sequence and `i` to `j` the remaining two-thirds.

In the following example, ABCDEF123456789012XXXXXX is the sub-sequence (with `c = 4`),

```

NXXXXXABCDEF123456789012XXXXXX
      x      yi          j

```

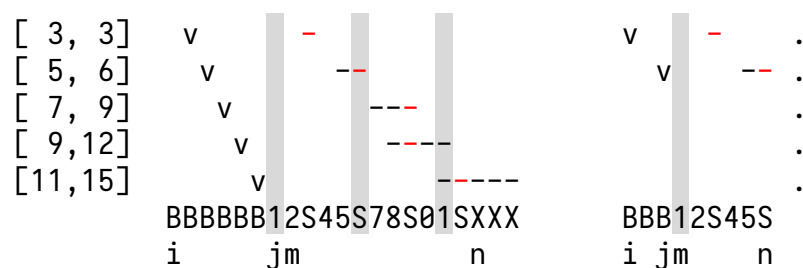
In `valid_layer()`, given the call condition from `valid_sequence()`, the following sections have fixed outputs:

- | | |
|---|-------|
| 1. <code>j = i</code> and <code>c[i] = "N"</code> | FALSE |
| 2. <code>n - m + 1 = 2 (j - i + 1)</code> | TRUE |
| 3. <code>j != i</code> | TRUE |

for all `x` (`m <= x <= n` and **exists** `t` **implies** (`x - m = 3t - 1`)) **implies** `c[x] = "S"` checks whether the latter sub-sub-sequence is all composed of multiples of `XXS`.

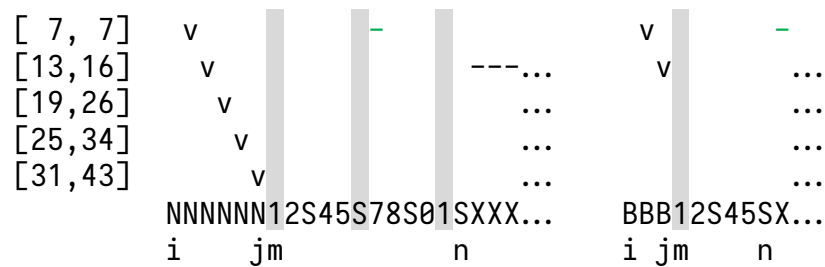
for all `y` (`i <= y <= j` and `c[y] != "S"`) **implies** (...) checks whether the letters, excluding `S`, of the former sub-sub-sequence satisfy a condition. The condition depends on whether the letter is `E` or `N`; however, the condition is almost guaranteed to fail.

For example, say if `c = {3, 4}`, the first sub-sequence is all `B`, "`v`" marks `y` position, and "`-`" marks the range of `m+w` that the condition checks,



If the **for all** `x` () **implies** () predicate passes, the condition **for all** `w` () **implies** `c[m+w] = "B"` **always** fails.

Similarly, for example, say if $c = \{3, 4\}$, the first sub-sequence is all N, “v” marks y position, and “-” marks the range of $m+3k$ that the condition checks,



If the **for all** x () **implies** () predicate passes, the condition **for all** k () **implies** $c[m+3k] = \text{"N"}$ always fails **with the only exception** of $k = 5$.

In short, the `valid_layer()` call within `valid_sequence()` is problematic in a sense that the only time it returns TRUE is when

1. an “N” situates at the second ($i+1$) position of the first sub-sub-sequence AND an “N” situates at the seventh ($m+4$) position of the second sub-sub-sequence, or;
2. there are all “B”s or “N”s within the range of $m+w$ or $m+3k$, respectively, that is out of bound of the sub-sequence.

Therefore, given the above conclusion, I cannot think of what `valid_sequence(m, n)` will check in data structure. The whole question is rendered as obsolete since the combined use of these two logic specifications has no apparent meaningful outcome.

Question 4a

(So much time than planned have been spent on Q4 to decipher the logic behind these specification that I have to forfeit this problem. I am sure I have the ability to finish this bonus problem but at the same time regretful to come to this conclusion since HW3 has been too overwhelming. Especially in Q4, it appears to be designed only for the sake of homework, but not applicable in the foreseeable future. I apologize in advance if there were any error in my reasoning in Q4, but too much is too much.)

Appendix

Fig. 1.1

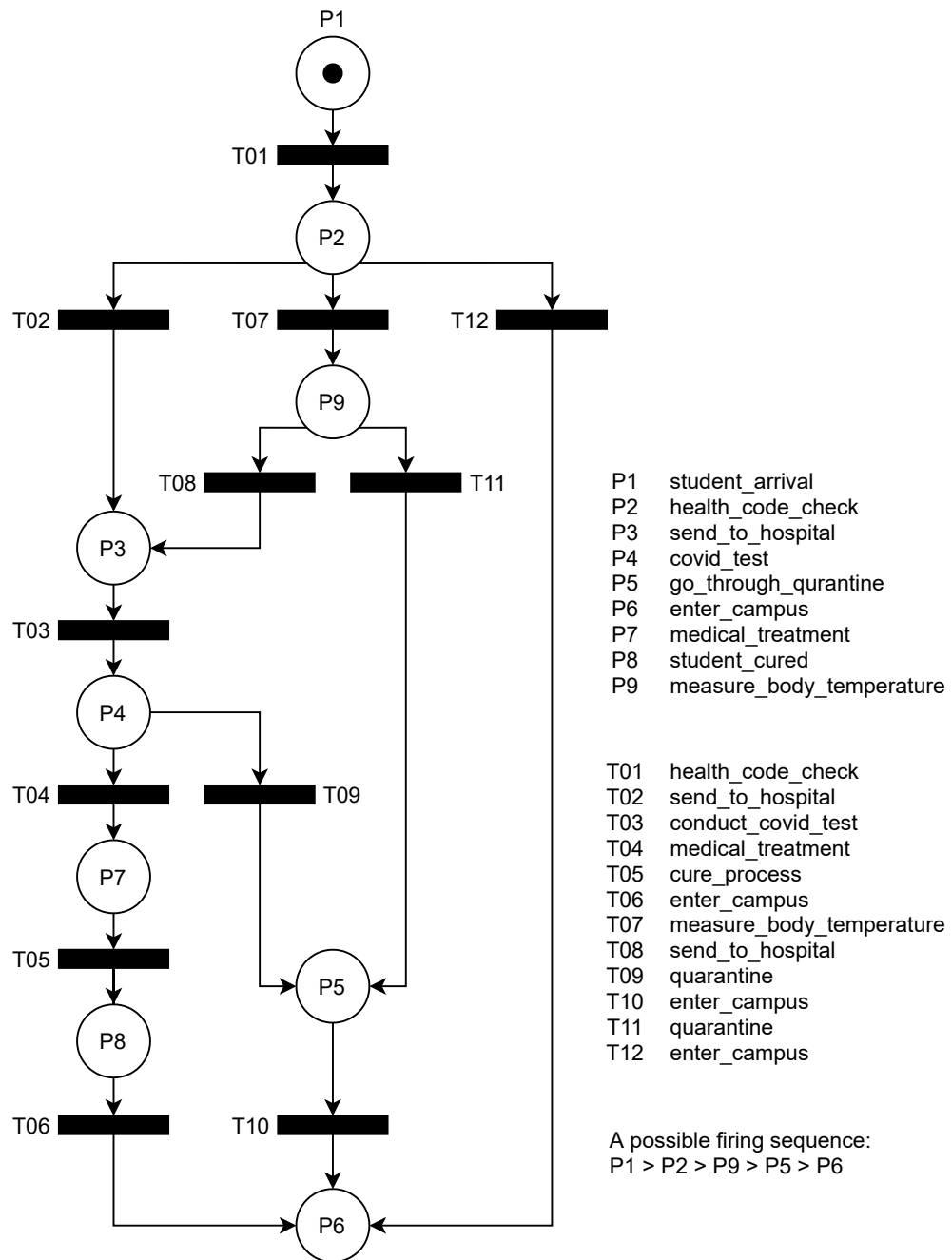


Fig. 1.2

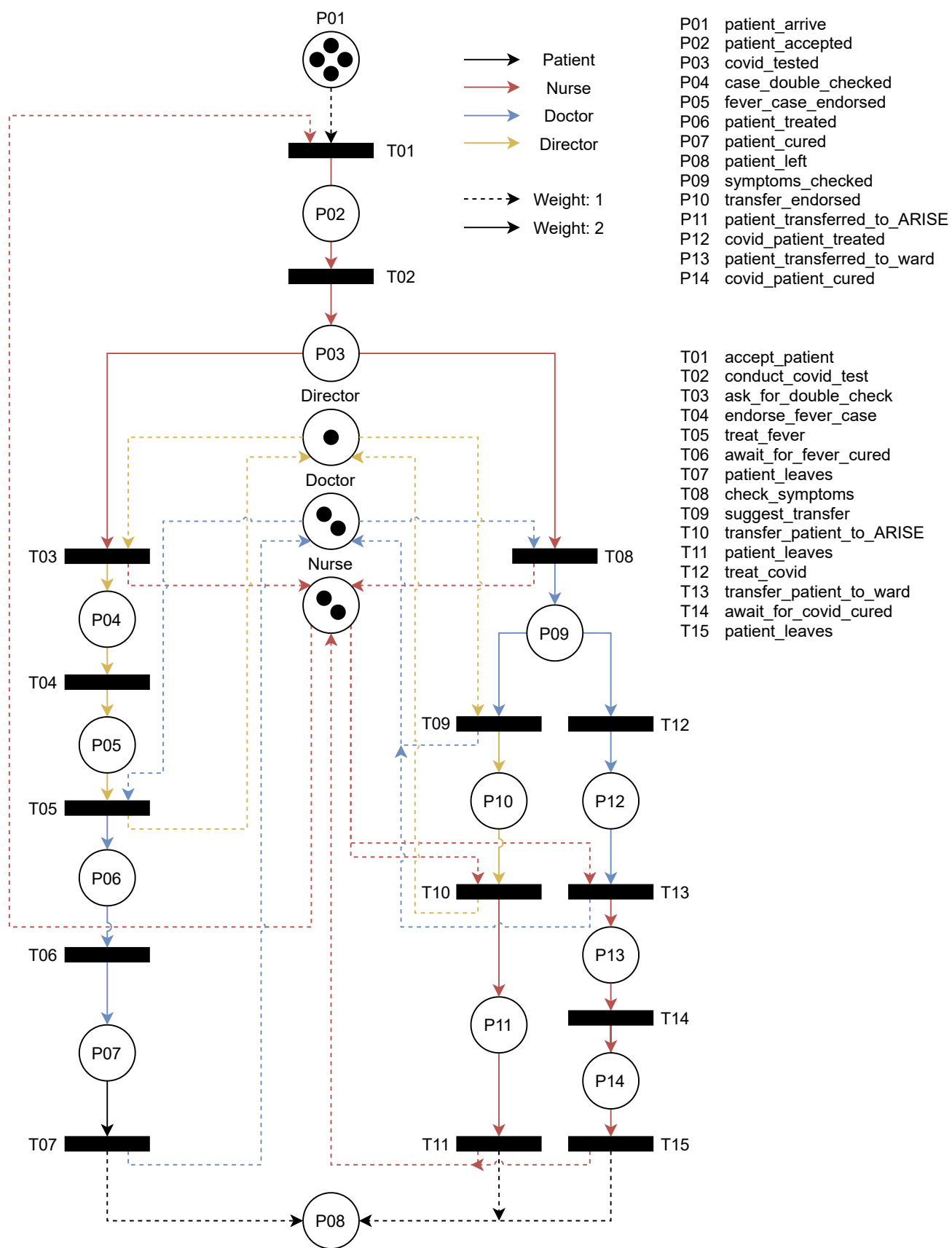
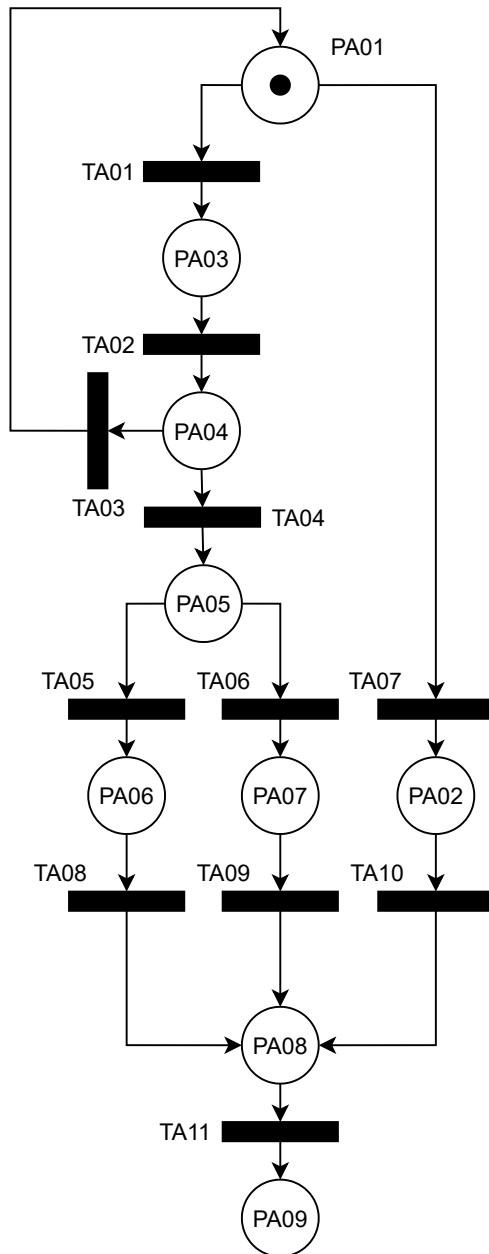


Fig. 2.1



P*01 *_turn_ready
 P*02 *_attack
 P*03 *_critical_attack
 P*04 check_^_HP
 P*05 *_critical_attack_judgement
 P*06 *_critical_attack_fails
 P*07 *_critical_attack_succeeds
 P*08 *_action_end
 P*09 *_turn_wait

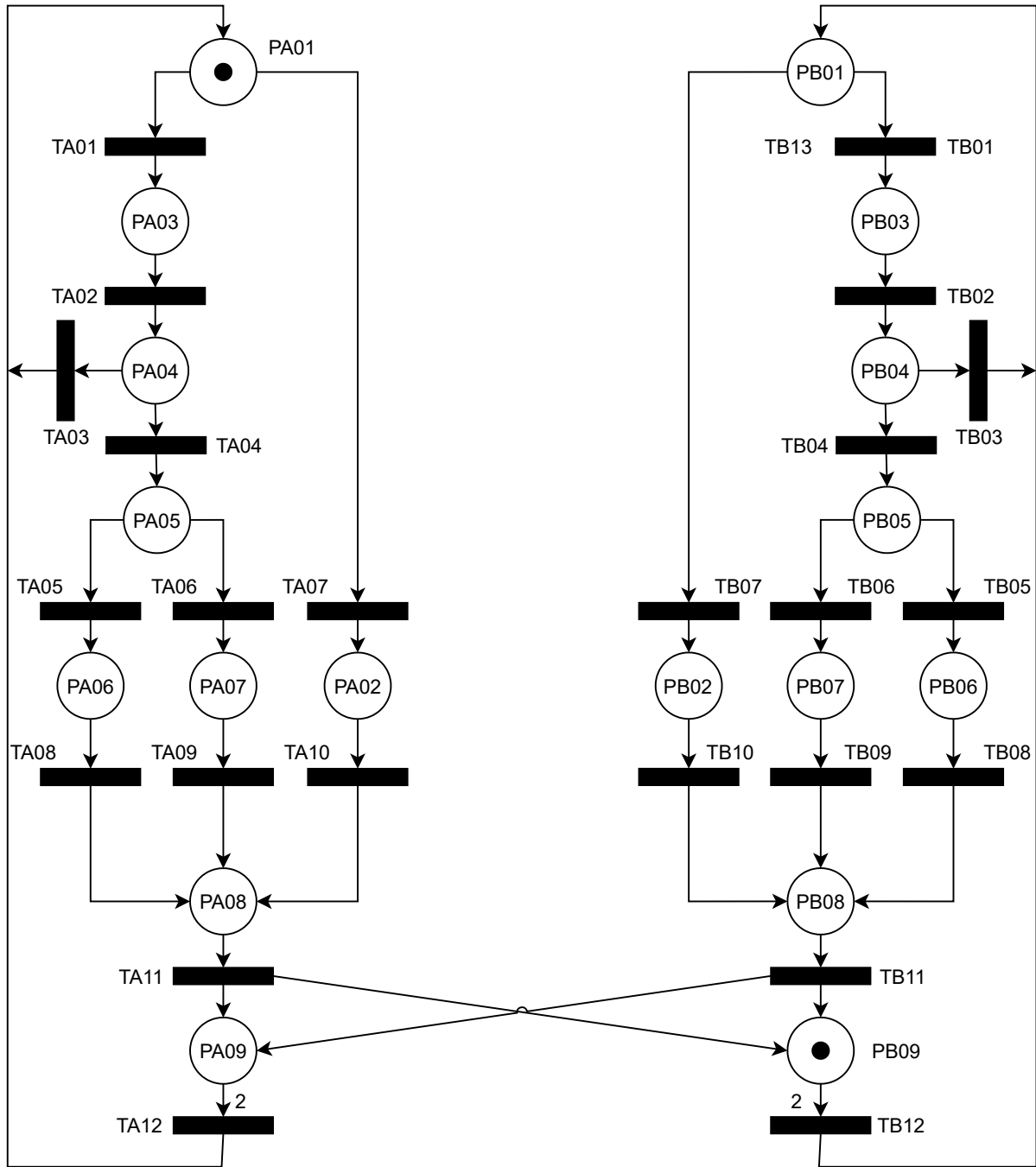
* A / B
 ^ B / A

T*01 *_chooses_crit
 T*02 check_^_HP
 T*03 return_*_ready
 T*04 judge_*_crit_valid
 T*05 fails_*_crit
 T*06 succeeds_*_crit
 T*07 *_chooses_attack
 T*08 distribute_crit_fails_damage
 T*09 distribute_crit_succeeds_damage
 T*10 distribute_attack_damage
 T*11 ready_*

* A / B
 ^ B / A

A possible firing sequence:
 PA01 > PA02 > PA08 > PA09

Fig. 2.2



P*01 *_turn_ready
P*02 *_attack
P*03 *_critical_attack
P*04 check_^_HP
P*05 *_critical_attack_judgement
P*06 *_critical_attack_fails
P*07 *_critical_attack_succeeds
P*08 *_action_end
P*09 *_turn_wait

* A / B
^ B / A

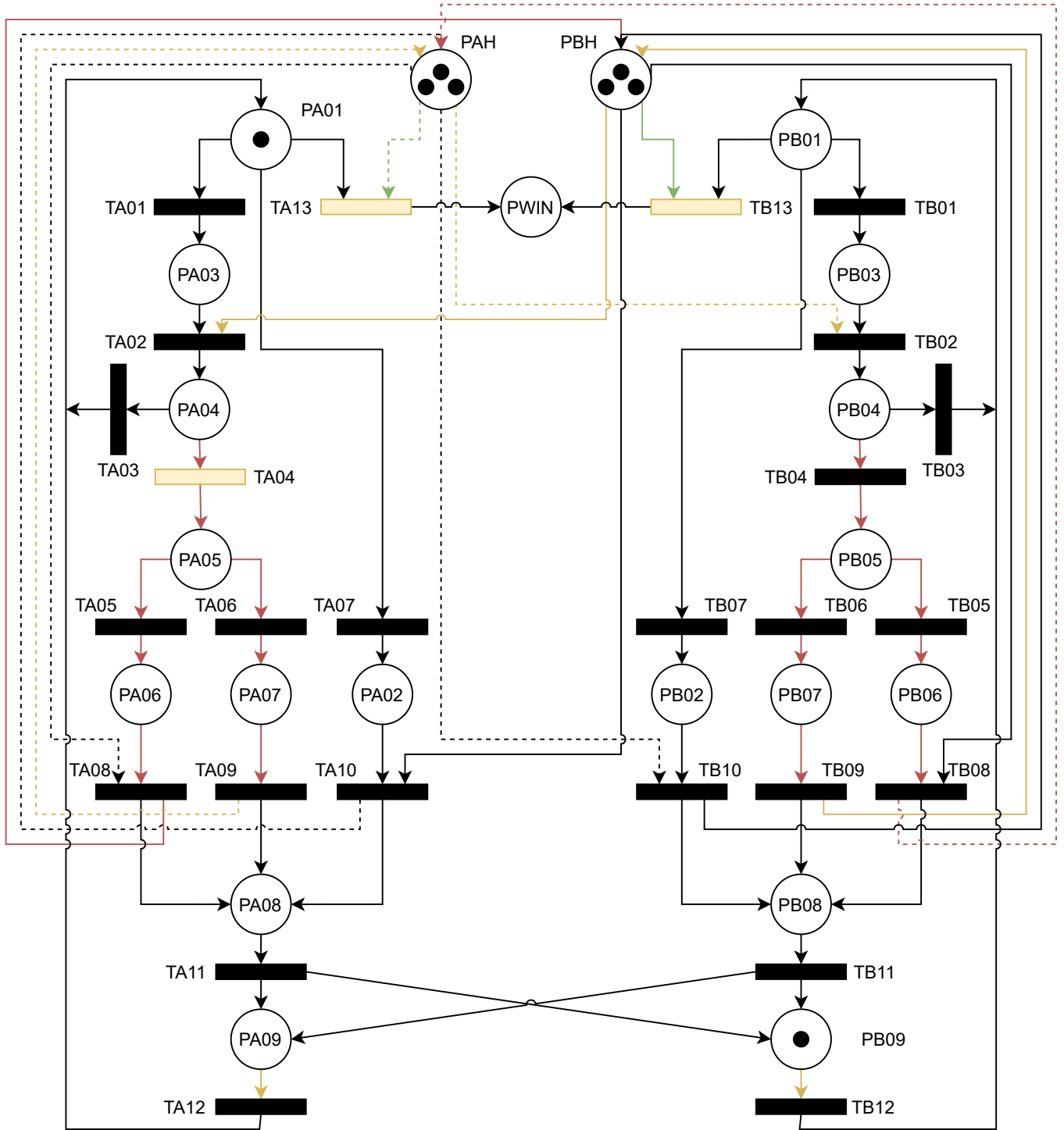
T*01 *_chooses_crit
T*02 check_^_HP
T*03 return*_ready
T*04 judge*_crit_valid
T*05 fails*_crit
T*06 succeeds*_crit
T*07 *_chooses_attack
T*08 distribute_crit_fails_damage
T*09 distribute_crit_succeeds_damage
T*10 distribute_attack_damage
T*11 ready_*
T*12 restart*_round

* A / B
^ B / A

A possible firing sequence:

PA01
PA02 > PB09
PA09 PB01
PB03
PB04
PB05
PB06
PB08
PB09

Fig. 2.3



P*01 * _turn_ready
P*02 * _attack
P*03 * _critical_attack
P*04 check_ ^_HP
P*05 * _critical_attack_judgement
P*06 * _critical_attack_fails
P*07 * _critical_attack_succeeds
P*08 * _action_end
P*09 * _turn_wait
P*H * _health
PWIN game_ends

* A / B
^ B / A

T*01 * _chooses_crit
T*02 check_ ^_HP
T*03 return_ *_ready
T*04 judge_ *_crit_valid
T*05 fails_ *_crit
T*06 succeeds_ *_crit
T*07 * _chooses_attack
T*08 distribute_crit_fails_damage
T*09 distribute_crit_succeeds_damage
T*10 distribute_attack_damage
T*11 ready_ *
T*12 restart_ *_round
T*13 * _wins

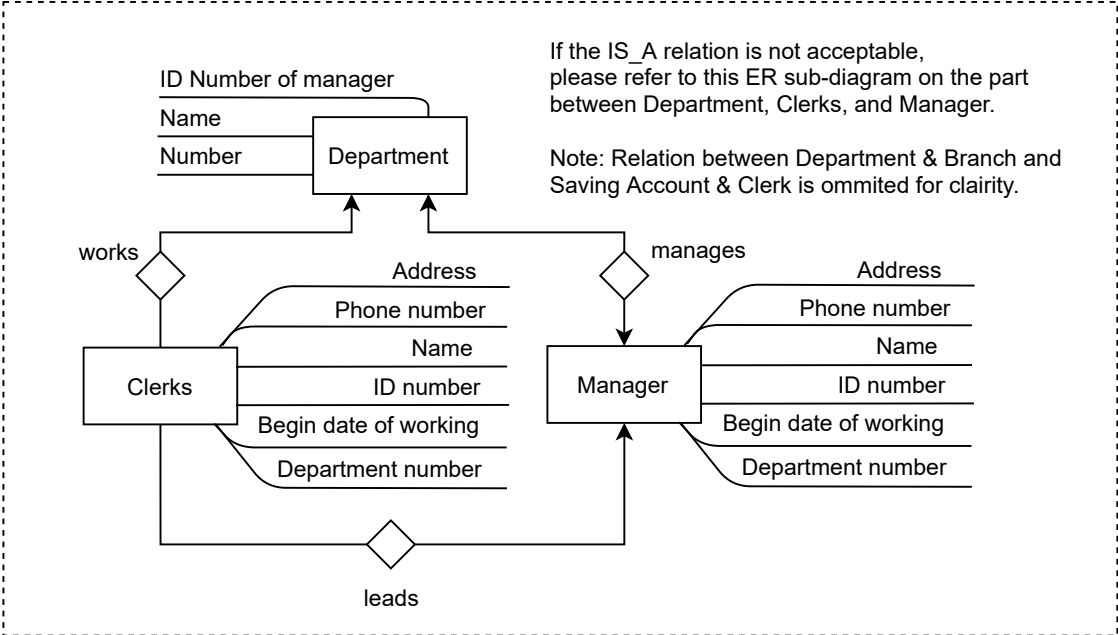
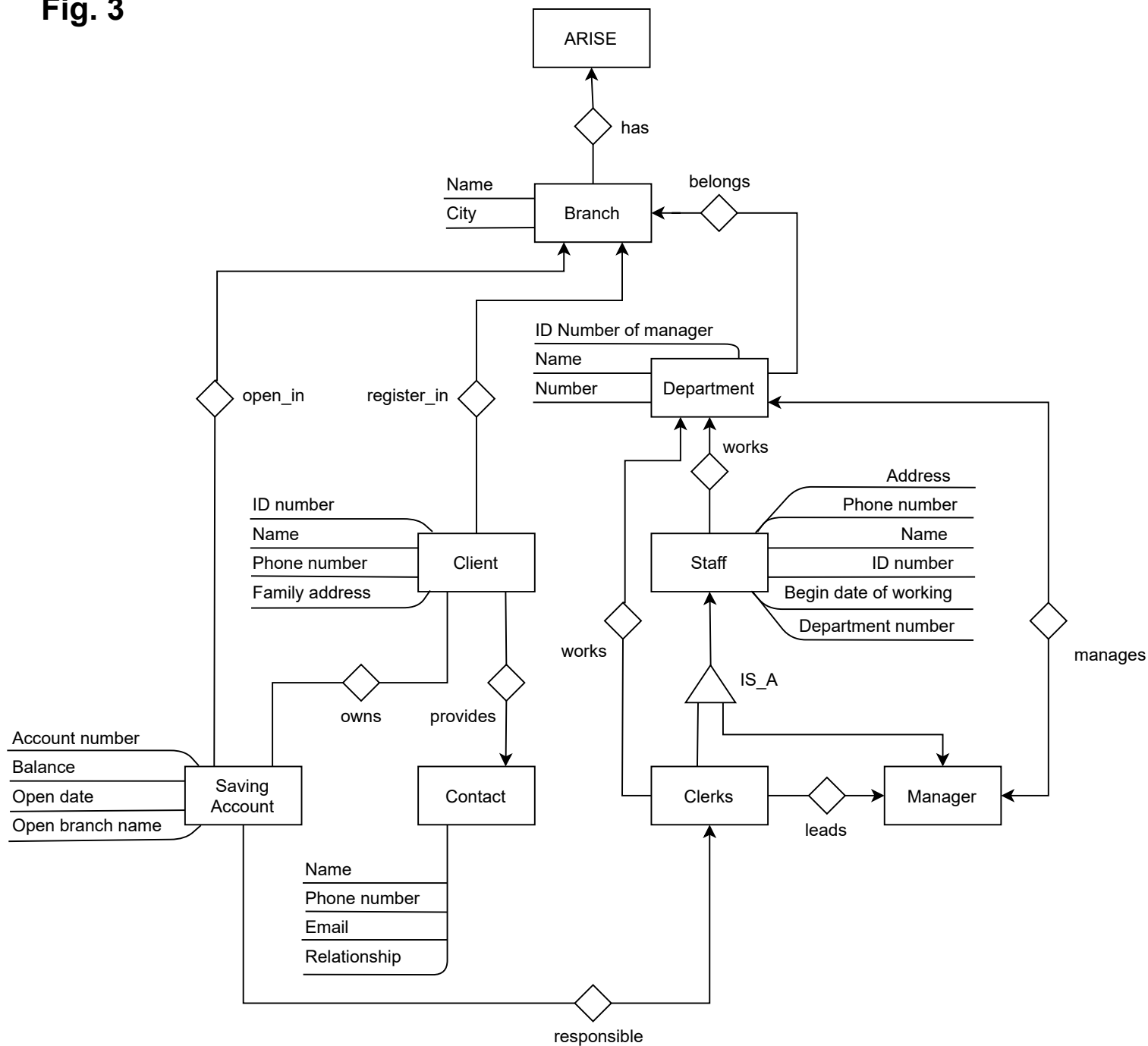
* A / B
^ B / A

→ Weight: 1
→ Weight: 2
→ Weight: 3
→ Weight: 6

-----> A HP
-----> B HP

Priority: 2
Priority: 1

Fig. 3



c	x	y	i	j	n-m+1	j-i+1	x	t	c[x]	y	c[y]	2(y-i)	3(y-i)-1	w	m+w	c[m+w]
3	38	40	41	46	6	6	46	2	S							
	38	40	41	46			45	1.7								
	38	40	41	46			44	1.3								
	38	40	41	46			43	1	S							
	38	40	41	46			42	0.7								
	38	40	41	46			41	0.3								
	38	40	41	46						40	S					
	38	40	41	46						39	B	2	2	2	43	S
	38	40	41	46						38	B	...				
37	39	40	45		6	6	45	2	B							
	37	39	40	45			44	1.7								
	37	39	40	45			43	1.3								
	37	39	40	45			42	1	...							
	37	39	40	45			41	0.7								
	37	39	40	45			40	0.3								
36	38	39	44		6	6	44	2	B							
	36	38	39	44			43	1.7								
	36	38	39	44			42	1.3								
	36	38	39	44			41	1	...							
	36	38	39	44			40	0.7								
	36	38	39	44			39	0.3								
35	37	38	43		6	6	43	2	S							
	35	37	38	43			42	1.7								
	35	37	38	43			41	1.3								
	35	37	38	43			40	1	S							
	35	37	38	43			39	0.7								
	35	37	38	43			38	0.3								
	35	37	38	43						37	S					
	35	37	38	43						36	B	2	2	2	40	S
	35	37	38	43						35	B	...				
34	36	37	42		6	6	42	2	B							
	34	36	37	42			39	1	...							
33	35	36	41		6	6	41	2	B							
	33	35	36	41			38	1	...							
32	34	35	40		6	6	40	2	S							
	32	34	35	40			37	1	S							
	32	34	35	40						34	S					
	32	34	35	40						33	B	2	2	2	37	S
	32	34	35	40						32	B	...				
31	33	34	39		6	6	39	2	B							
	31	33	34	39			36	1	...							
30	32	33	38		6	6	38	2	B							
	30	32	33	38			35	1	...							
29	31	32	37		6	6	37	2	S							
	29	31	32	37			34	1	S							
	29	31	32	37						31	S					

29	31	32	37					30	B	2		2	2	34	S
29	31	32	37					29	B	...					
28	30	31	36	6	6	36	2	B							
28	30	31	36			33	1	...							
27	29	30	35	6	6	35	2	B							
27	29	30	35			32	1	...							
26	28	29	34	6	6	34	2	S							
26	28	29	34			31	1	S							
26	28	29	34					28	S						
26	28	29	34					27	B	2		2	2	31	S
26	28	29	34					26	B	...					
25	27	28	33	6	6	33	2	B							
25	27	28	33			30	1	...							
24	26	27	32	6	6	32	2	B							
24	26	27	32			29	1	...							
23	25	26	31	6	6	31	2	S							
23	25	26	31			28	1	S							
23	25	26	31					25	S						
23	25	26	31					24	N	2		2	2	32	B
23	25	26	31					23	N	...					
22	24	25	30	6	6	30	2	B							
22	24	25	30			27	1	...							
21	23	24	29	6	6	29	2	B							
21	23	24	29			26	1	...							
20	22	23	28	6	6	28	2	S							
20	22	23	28			25	1	S							
20	22	23	28					22	S						
20	22	23	28					21	B	2		2	2	25	S
20	22	23	28					20	B	...					
19	21	22	27	6	6	27	2	B							
19	21	22	27			24	1	...							
18	20	21	26	6	6	26	2	B							
18	20	21	26			23	1	...							
17	19	20	25	6	6	25	2	S							
17	19	20	25			22	1	S							
17	19	20	25					19	S						
17	19	20	25					18	B	2		2	2	22	S
17	19	20	25					17	B	...					
16	18	19	24	6	6	24	2	N							
16	18	19	24			21	1	...							
15	17	18	23	6	6	23	2	N							
15	17	18	23			20	1	...							
14	16	17	22	6	6	22	2	S							
14	16	17	22			19	1	S							
14	16	17	22					16	S						
14	16	17	22					15	B	2		2	2	19	S
14	16	17	22					14	B	...					
13	15	16	21	6	6	21	2	B							

