INCENTIVE-BASED ENERGY ALLOCATION VIA GAME THEORY

Michael Daley, Johnathan Diaz, Jonathan Karlsen, Samantha Lorenc, Yumilka Polanco

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Game Theory is the methodology of using mathematical tools to model and analyze situations of interactive decision making. In simpler terms Game Theory is a process that is use to understand and evaluate a decision that is needed to be made.

In a game in which Game Theory can be applied there has to be at least one player. The player is the person who is either playing the game or making a decision.

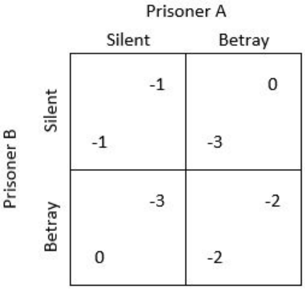
The players in a game have the end goal, which is the choice with the most favorable outcome. Players also form different strategies that they determine to be the best choice for them to make.

A strategy in terms of Game Theory is the set of steps that a player would take to try to win or obtain a more favorable outcome.

For a player a winning strategy is one that will get that player the their most favorable outcome. In a game with multiple players one players winning strategy could negatively impact the strategy of another player. The reason for this is because the players in a game are working independently of each other, so the players do not know what decisions their fellow players are making.

PRISONER’S DILEMMA

The prisoner’s dilemma is a well known Game Theory example. There are two prisoners being charged with a crime, the prisoners are the players in this situation. They are given two possible choices to make, either to remain silent or betray the other prisoner and help the police. This game is an example of two players independently making a decision for their own personal gain. In this scenario the best strategy for both players would be to betray each other.



There is an energy company that supplies energy to a housing development. At the housing development there is a manager that oversees the energy that is used by the tenants that live there. The manager would benefit from the tenants using less energy because the energy company offers an incentive for saving energy. The tenants are offered an incentive by the manager, in an attempt to get the tenants to use less energy.

Players

The first model is based into two players which are two tenants. After we sets up this model, we generate to 100 tenants.

Each tenant uses the appropriate energy by playing the best strategy.

Interest

The main interest is to save energy and maximum the utility for each tenant.

Utility

Utility is the amount of satisfaction that a person will get from the consumption of a services.Utility function is widely used in the rational choice theory to analyze human behavior. It is based its beliefs upon individuals' preferences. In other words, the order in which tenants choose one service over another can establish that tenants assign a higher value to the first services. Ordinal utility measures how tenants rank one services versus another.

Nash Equilibrium

Nash equilibrium is a concept within game theory where the optimal outcome of a game is where there is no incentive to deviate from their initial strategy. Overall, an individual can receive no incremental benefit from changing actions, assuming other players remain constant in their strategies.

Decision

We would each know a lot about the other tenant's actions. We would expect these two strategies to form a Nash equilibrium. Suppose exactly this: The production quantities q_1 and q_2 form a Nash equilibrium.

THE EQUILIBRIUM FOR THE TWO OCCUPANTS SCENARIO

Both want to get maximum utility :$u_1 = (100 - q_1 - q_2)q_1$, $u_2 = (100 - q_1 - q_2)q_2$, but if their combined consumption ≥ 100 there will be a power outage.

Occupant 1 wants to use 50 and wants occupant 2 to use 0 (for a utility of 2500), but occupant 2 wants the opposite, therefore they will meet at an equilibrium (if both occupants are rational players)

Nash equilibrium: neither occupant will benefit from deviating from the equilibrium

NASH EQUILIBRIUM FOR THE TWO OCCUPANTS

To find the equilibrium:

$u_1 = (100 - q_1 - q_2)q_1$

$u_1' = 100 - 2q_1 - q_2 = 0$

$q_1 = \frac{100-q_2}{2}$ and $q_2 = \frac{100-q_1}{2}$ from u_2'

$q_1 = \frac{100}{3}$ and $q_2 = \frac{100}{3}$

$u_1 = \frac{10000}{9}$ and $u_2 = \frac{10000}{9}$

$U_1 = (100 - q_1 - q_2 - q_3 - \cdots - q_{100})q_1$

$U_1' = 100 - 2q_1 - q_2 - q_3 - \cdots - q_{100}$

$q_1 = (100 - q_2 - q_3 - \cdots - q_{100})/2$

From the observations: $q_1 = q_2 = q_3 = \cdots = q_{100}$,

$q_1 = (100 - 99q_1)/2$

$q_1 = 100/101.$

Here all the players will use 0.98 and get a utility of 1.96

OBSERVATIONS FROM THE TWO OCCUPANT SCENARIO

At the equilibrium, both occupants use the same amount of power.

At the equilibrium, both occupants get the same utility

$q_n = \frac{100}{n+1}$, $U_n = (100 - n \cdot \frac{100}{n+1}) \cdot \frac{100}{n+1}$

$U'_n = -\frac{20000}{(n+1)^3}$

MODELING THE SECOND ENERGY ALLOCATION PROBLEM

Now we will look at a different modeling of the energy allocation problem.

Motivation- The main difference from the first energy allocation problem is that the manager is trying to gain a massive profit by limiting options and using contracts, the manager is trying to narrow the options to attract more customers who would fall into the 2 categories.

Tenants are divided into 2 types: high (H) or low (L)

Just like the first problem we will be looking at another real life situation where you must choose a data plan for your cell phone.

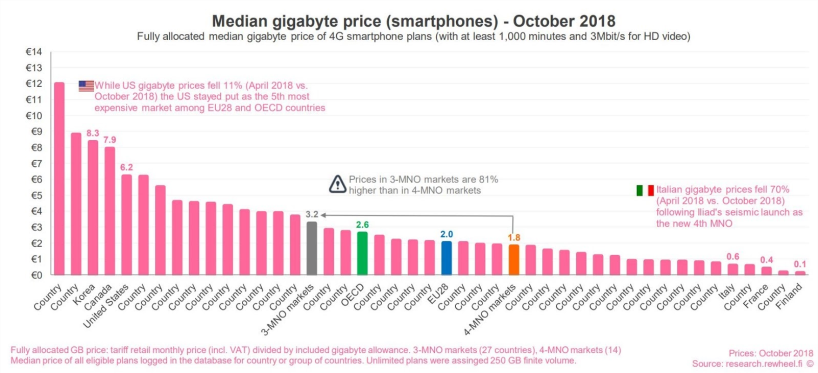
Your 2 options are a cheap basic plan where you must note down all your usage so you don’t reach your data cap for the rest of the month and have to spend a additional charge for how much you go over your limit.

Or an expensive/deluxe option that you know you wont reach the end of the data cap for but buy it just for the peace of mind.

AT&T PREPAID CELL PHONE DATA PLAN 2018

	$35/mo.	$45/mo.	$65/mo.	$85/mo.
With AutoPay discount[2]	$30/mo.	$40/mo.	$60/mo.	$75/mo.
Data	1GB[3]	6GB[3]	Unlimited[4]	Unlimited[5]
Mobile hotspot	✓	✓		6GB[6]
Stream HD video **HD quality when available w/Stream Saver turned off**[7]	✓	✓		✓
Unlimited talk & text to Mexico & Canada[8]		✓	✓	✓
Use talk, text & data in Mexico & Canada[8]		✓	✓	✓

WHY SHOULD YOU CARE ABOUT THE DATA PLAN PRICE



Well for one reason the US in 2016 were the 6th highest for average wireless data plan pricing.

Utility functions for each (Tenant and Manager):

- $u_H = a_H V(q_H) - P_H$
- $u_L = a_L V(q_L) - P_L$
- $u_M = \theta(P_H - cq_H) + (1-\theta)(P_L - cq_L)$

where

- a_t: scalar
- $V(q_t)$: satisfaction of the tenant
- q_t: quantity consumed
- P_t : price
- c: constant
- $t \in \{L, H\}$

If we only work with one H-type tenant, or one L-type tenant, we can solve for q_H and q_L: $q_H = \frac{a_H}{c} - 1$ and $q_L = \frac{a_L}{c} - 1$.

There are several conditions for the game:

The manager will offer price plans (q_H, P_H) and (q_L, P_L)

1. $a_H V(q_H) - P_H \geq 0$
2. $a_L V(q_L) - P_L \geq 0$
3. $a_H V(q_H) - P_H \geq a_H V(q_L) - P_L$
4. $a_L V(q_L) - P_L \geq a_L V(q_H) - P_H$

$a_H > a_L$

From the second condition we have $u_L = a_L V(q_L) - P_L = 0$, so we can solve for P_L and then solve for the price when the H-type tenant will cheat.

$$P_H = a_H[V(q_H) - V(q_L)] + a_L V(q_L)$$

The price, P_H, has to be set less than the above equation to prevent the H-type tenant from cheating.

The L-type tenant will have a hard time cheating because they will be paying more money for energy that they are not using or need; therefore, their cheating utility function will be less than their honest utility function.

The manager wants to maximize their profits (utility function).
$T = \theta[P_H - cq_H] + (1-\theta)[P_L - cq_L]$, since there are two variables, q_H and q_L, we can solve for q_H and q_L. $V'(q_H) = \frac{c}{a_H}$ and $V'(q_L) = \frac{(1-\theta)c}{\theta(a_L - a_H) + (1-\theta)a_L}$

In order to solve further, we can simplify what we know about the H-type tenant:

- Will not cheat
- Will pick q_H

So, the manager will charge the H-type $P_H = a_H V(q_H)$ and will charge the L-type $P_L = a_L V(q_L)$ (similar argument). $q_H = \frac{a_H}{c} - 1$, $q_L = \frac{a_L}{c} - 1$

Using the values for q_H and q_L will allow for the manager to maximize his profits/utility.

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