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Revised Supplementary Release on Backward Equations *p*(*h*,*s*) for Region 3, Equations as a Function of *h* and *s* for the Region Boundaries, and an Equation *T*sat(*h*,*s*) for Region 4 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam

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This revised supplementary release replaces the corresponding supplementary release of 2004, and contains 34 pages, including this cover page.

This revised supplementary release has been authorized by the International Association for the Properties of Water and Steam (IAPWS) at its meeting in Moscow, Russia, 22-27 June, 2014, for issue by its Secretariat. The members of IAPWS are: Britain and Ireland, Canada, the Czech Republic, Germany, Japan, Russia, Scandinavia (Denmark, Finland, Norway, Sweden), and the United States, and associate members Argentina & Brazil, Australia, France, Greece, Italy, New Zealand, and Switzerland.

The backward equations $p(h,s)$ for Region 3, the equations as a function of *h* and *s* for the region boundaries, and the equation $T_{sat}(h,s)$ for the two-phase (wet steam) region, provided in this release are recommended as a supplement to "The IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (IAPWS-IF97) [1, 2]. Further details concerning the equations of this revised supplementary release can be found in the corresponding article by H.-J. Kretzschmar et al. [3].

This revision consists of edits to clarify descriptions of how to determine the region or subregion; the property calculations are unchanged.

Further information concerning this supplementary release, other releases, supplementary releases, guidelines, technical guidance documents, and advisory notes issued by IAPWS can be obtained from the Executive Secretary of IAPWS or from http://www.iapws.org.

Contents

1 Nomenclature

Thermodynamic quantities:

f	Specific Helmholtz free energy
h	Specific enthalpy
p	Pressure
s	Specific entropy
T	Absolute temperature [a]
v	Specific volume
Δ	Difference in any quantity
η	Reduced enthalpy, $\eta = h/h^*$
θ	Reduced temperature $\theta = T/T^*$
π	Reduced pressure, $\pi = p/p^*$
ρ	Density
σ	Reduced entropy, $\sigma = s/s^*$
ω	Reduced volume, $\omega = v/v^*$
x	Vapor fraction

Root-mean-square value:

$$\Delta z_{\mathrm{RMS}} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (\Delta z_n)^2}$$

where Δz_n can be either absolute or percentage difference between the corresponding quantities z; N is the number of Δz_n values (100 million points uniformly distributed over the range of validity in the p-T plane).

Superscripts:

97	Quantity or equation of IAPWS-IF97
01	Equation of IAPWS-IF97-S01
03	Equation of IAPWS-IF97-S03
*	Reducing quantity
'	Saturated liquid state
"	Saturated vapor state

Subscripts:

1	Region 1
2	Region 2
2a	Subregion 2a
2b	Subregion 2b
2c	Subregion 2c
3	Region 3
3a	Subregion 3a
3b	Subregion 3b
3ab	Boundary between subregions 3a and 3b
4	Region 4
5	Region 5
B23	Boundary between regions 2 and 3
B13	Boundary between regions 1 and 3
c	Critical point
it	Iterated quantity
max	Maximum value of a quantity
RMS	Root-mean-square value of a quantity
sat	Saturation state
tol	Tolerance, range of accepted value of a quantity

[a] Note: T denotes absolute temperature on the International Temperature Scale of 1990 (ITS-90).

2 Background

The IAPWS Industrial Formulation 1997 for the thermodynamic properties of water and steam (IAPWS-IF97) [1, 2] contains basic equations, saturation equations and equations for the most often used backward functions $T(p,h)$ and $T(p,s)$ valid in the liquid region 1 and the vapor region 2; see Figure 1. IAPWS-IF97 was supplemented by "Backward Equations for Pressure as a Function of Enthalpy and Entropy $p(h,s)$ to the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" [4, 5], which is referred to here as IAPWS-IF97-S01. These equations are valid in region 1 and region 2. An additional supplementary release "Backward Equations for the Functions $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$ for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" [6, 7], which is referred to here as IAPWS-IF97-S03, was adopted by IAPWS in 2003.
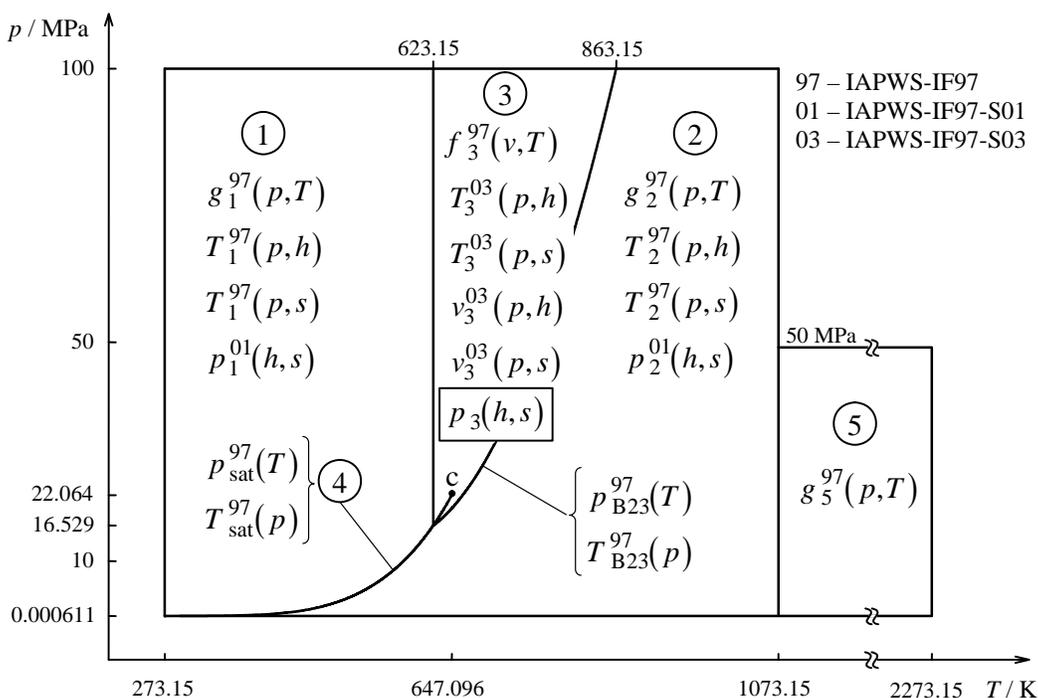
Figure 1. Regions and equations of IAPWS-IF97, IAPWS-IF97-S01, IAPWS-IF97-S03, and equations $p_3(h,s)$ of this release

In modeling steam power cycles, thermodynamic properties as a function of the variables enthalpy and entropy (h,s) are also required in region 3. With IAPWS-IF97, these calculations require two-dimensional iteration of specific volume v and temperature T using the functions $h(v,T)$ and $s(v,T)$ that can be explicitly calculated from the basic equation of region 3 $f(v,T)$. With specific volume and temperature, the other properties in region 3 can be calculated. While these calculations are not frequently required in

region 3, the relatively large computing time required for two-dimensional iteration can be significant.

In order to avoid such iterations, this release provides an explicit calculation of the functions $v(h,s)$ and $T(h,s)$ in region 3.

However, extensive test calculations have shown that is it not possible to set up explicit backward equations for the functions $T_3(h,s)$ and $v_3(h,s)$ that meet the IAPWS requirements for numerical consistency. Therefore, equations for the backward function $p_3(h,s)$ were developed, see Figure 1. With the pressure p, the temperature can be calculated from the IAPWS-IF97-S03 backward equation $T_3^{03}(p,h)$[1], and the specific volume can be calculated from the IAPWS-IF97-S03 backward equation $v_3^{03}(p,s)$[2].

Since IAPWS provides explicit equations for functions with independent variables (h,s) in liquid region 1, vapor region 2 and region 3 (this supplementary release), these functions can be calculated without iteration. However, iteration is necessary to find the region for the given variables (h,s). In order to avoid such iterations, this release provides equations as functions of h and s for the region boundaries. Section 4 contains the comprehensive description of these equations.

In modeling power cycles and steam turbines, thermodynamic properties as a function of the variables (h,s) are also required for region 4, the two-phase vapor/liquid (wet steam) region. These calculations require a one-dimensional iteration using the saturation-pressure equation and the basic equations. In order to avoid such iterations, this release provides an equation $T_{sat}(h,s)$ for the two-phase region. With the temperature T, the pressure p can be calculated by using the IAPWS-IF97 saturation-pressure equation $p_{sat}^{97}(T)$, and the vapor fraction x can be calculated explicitly by using the IAPWS-IF97 basic equations. The comprehensive description of the equation $T_{sat}(h,s)$ can be found in Section 5.

The numerical consistencies of all backward equations presented in Sections 3, 4, and 5 with the IAPWS-IF97 basic equations are sufficient for most applications in heat cycle and steam turbine calculations. For applications where the demands on numerical consistency are extremely high, iterations using the IAPWS-IF97 basic equations may be necessary. In these cases, the backward equations can be used for calculating very accurate starting values. The time required to reach the convergence criteria of the iteration will be significantly reduced.

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[1] The alternative use of the IAPWS-IF97-S03 backward equation $T_3^{03}(p,s)$ leads to worse numerical consistency.

[2] The alternative use of the IAPWS-IF97-S03 backward equation $v_3^{03}(p,h)$ leads to worse numerical consistency.

The backward equations presented here can only be used in their range of validity described in Sections 3.2, 4.3, 4.4, 4.5, 4.6, and 5.3. They should not be used for determining any thermodynamic derivatives.

In any case, depending on the application, a conscious decision is required whether to use the backward equations or to calculate the corresponding values by iteration from the basic equation of IAPWS-IF97.

3 Backward Functions $p(h,s)$, $T(h,s)$, and $v(h,s)$ for Region 3

3.1 Numerical Consistency Requirements

For numerical consistency, the maximum permissible temperature difference $\left|\Delta T\right|_{\text{tol}} = 25\,\text{mK}$ between the backward function $T_3(h,s)$ and the basic equation $f_3^{97}(v,T)$ was determined by IAPWS [8] for the backward functions $T_3^{03}(p,h)$ and $T_3^{03}(p,s)$.

The tolerance $\left|\Delta v/v\right|_{\text{tol}} = 0.01\,\%$ of the backward function $v_3(h,s)$ is the IAPWS requirement for the backward functions $v_3^{03}(p,h)$ and $v_3^{03}(p,s)$ [6, 7].

In order to fulfill the required numerical consistency of the functions $T_3(h,s)$ and $v_3(h,s)$, the relative maximum deviation between the pressure calculated from the backward equation $p_3(h,s)$ and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ must be smaller than $0.01\,\%$.

At the critical point $\left[T_\text{c} = 647.096\,\text{K},\ v_\text{c} = 1/\left(322\text{ kg m}^{-3}\right),\ p_\text{c} = 22.064\,\text{MPa}\right]$ [10], more stringent consistency requirements were arbitrarily set. These were $\left|\Delta T\right|_{\text{tol}} = 0.49\,\text{mK}$, $\left|\Delta v/v\right|_{\text{tol}} = 0.0001\,\%$, and $\left|\Delta p/p\right|_{\text{tol}} = 0.002\,\%$.

Table 1 shows the resulting permissible values for the numerical consistency of the backward functions $T_3(h,s)$, $v_3(h,s)$, and $p_3(h,s)$ with the basic equation $f_3^{97}(v,T)$.

Table 1. Numerical consistency values $\left|\Delta T\right|_{\text{tol}}$ of [8] required for $T_3(h,s)$, tolerances $\left|\Delta v/v\right|_{\text{tol}}$ of [6, 7] required for $v_3(h,s)$, and tolerances $\left|\Delta p/p\right|_{\text{tol}}$ required for $p_3(h,s)$

	$\left|\Delta T\right|_{\text{tol}}$	$\left|\Delta v/v\right|_{\text{tol}}$	$\left|\Delta p/p\right|_{\text{tol}}$
Region 3	25 mK	0.01 %	0.01 %
Critical Point	0.49 mK	0.0001 %	0.002 %

3.2 Structure of the Equation Set

The equation set consists of backward equations $p(h,s)$ for region 3. Region 3 is defined by:

$$623.15 \text{ K} \leq T \leq 863.15 \text{ K and } p_{B23}^{97}(T) \leq p \leq 100 \text{ MPa},$$

where p_{B23}^{97} represents the B23-equation of IAPWS-IF97. The division of region 3 into subregions for the backward equations $p_3(h,s)$ is identical to the division for the backward equations $T_3^{03}(p,h)$, $v_3^{03}(p,h)$ and $T_3^{03}(p,s)$, $v_3^{03}(p,s)$, see Figure 2.
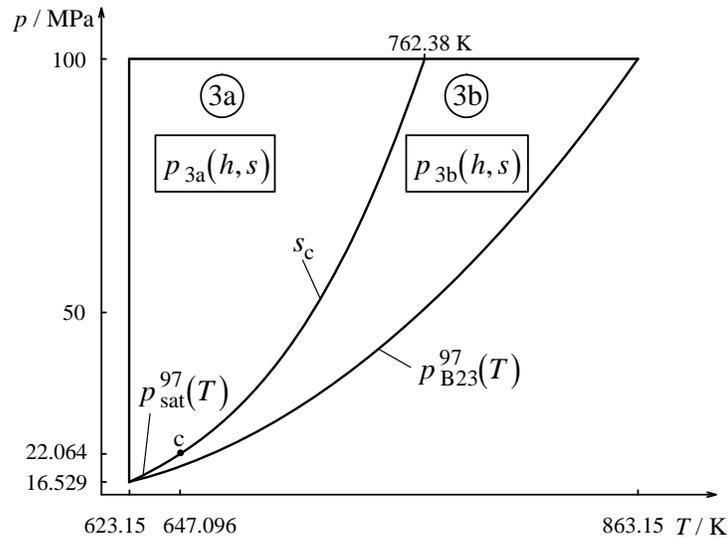
Figure 2. Division of region 3 into two subregions 3a and 3b for the backward equations $p(h,s)$

Table 2 shows the decisions that have to be made in order to find the correct subregion for the function $p(h,s)$.

Table 2. Criteria for finding the correct subregion, 3a or 3b, for the backward function $p(h,s)$

Backward Function $p(h,s)$		
	Subregion	
	3a	3b
for $p < p_c$:	$h \leq h_{3a}'(s)$	$h \geq h_{2c3b}''(s)$
for $p \geq p_c$:	$s \leq s_c$	$s > s_c$

For pressures greater than or equal to p_c, the boundary between the subregions 3a and 3b corresponds to the critical isentropic line

$$s = s_c = 4.412\ 021\ 482\ 234\ 76 \text{ kJ kg}^{-1} \text{ K}^{-1},$$

see Figure 2. For the functions $p(h,s)$, input points can be tested directly to identify the subregion since the specific entropy is an independent variable.

3.3 Backward Equations $p(h,s)$

The Equations. The backward equation $p_{3a}(h,s)$ for subregion 3a has the following dimensionless form:

$$\frac{p_{3a}(h,s)}{p^*} = \pi_{3a}(\eta,\sigma) = \sum_{i=1}^{33} n_i (\eta - 1.01)^{I_i} (\sigma - 0.750)^{J_i} \quad , \tag{1}$$

where $\pi = p/p^*$, $\eta = h/h^*$, and $\sigma = s/s^*$, with $p^* = 99 \text{ MPa}$, $h^* = 2300 \text{ kJ kg}^{-1}$, and $s^* = 4.4 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (1) are listed in Table 3.

The backward equation $p_{3b}(h,s)$ for subregion 3b has the dimensionless form

$$\frac{p^*}{p_{3b}(h,s)} = \frac{1}{\pi_{3b}}(\eta,\sigma) = \sum_{i=1}^{35} n_i (\eta - 0.681)^{I_i} (\sigma - 0.792)^{J_i} \quad , \tag{2}$$

where $\pi = p/p^*$, $\eta = h/h^*$, and $\sigma = s/s^*$, with $p^* = 16.6 \text{ MPa}$, $h^* = 2800 \text{ kJ kg}^{-1}$, and $s^* = 5.3 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (2) are listed in Table 4.

Computer-program verification. To assist the user in computer-program verification of Eqs. (1) and (2), Table 5 contains test values for calculated pressures.

Table 3. Coefficients and exponents of the backward equation $p_{3a}(h,s)$ for subregion 3a in its dimensionless form, Eq. (1)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	$0.770\ 889\ 828\ 326\ 934 \times 10^1$	18	5	28	$0.538\ 069\ 315\ 091\ 534 \times 10^{20}$
2	0	1	$-0.260\ 835\ 009\ 128\ 688 \times 10^2$	19	6	28	$0.143\ 619\ 827\ 291\ 346 \times 10^{22}$
3	0	5	$0.267\ 416\ 218\ 930\ 389 \times 10^3$	20	7	24	$0.364\ 985\ 866\ 165\ 994 \times 10^{20}$
4	1	0	$0.172\ 221\ 089\ 496\ 844 \times 10^2$	21	8	1	$-0.254\ 741\ 561\ 156\ 775 \times 10^4$
5	1	3	$-0.293\ 542\ 332\ 145\ 970 \times 10^3$	22	10	32	$0.240\ 120\ 197\ 096\ 563 \times 10^{28}$
6	1	4	$0.614\ 135\ 601\ 882\ 478 \times 10^3$	23	10	36	$-0.393\ 847\ 464\ 679\ 496 \times 10^{30}$
7	1	8	$-0.610\ 562\ 757\ 725\ 674 \times 10^5$	24	14	22	$0.147\ 073\ 407\ 024\ 852 \times 10^{25}$
8	1	14	$-0.651\ 272\ 251\ 118\ 219 \times 10^8$	25	18	28	$-0.426\ 391\ 250\ 432\ 059 \times 10^{32}$
9	2	6	$0.735\ 919\ 313\ 521\ 937 \times 10^5$	26	20	36	$0.194\ 509\ 340\ 621\ 077 \times 10^{39}$
10	2	16	$-0.116\ 646\ 505\ 914\ 191 \times 10^{11}$	27	22	16	$0.666\ 212\ 132\ 114\ 896 \times 10^{24}$
11	3	0	$0.355\ 267\ 086\ 434\ 461 \times 10^2$	28	22	28	$0.706\ 777\ 016\ 552\ 858 \times 10^{34}$
12	3	2	$-0.596\ 144\ 543\ 825\ 955 \times 10^3$	29	24	36	$0.175\ 563\ 621\ 975\ 576 \times 10^{42}$
13	3	3	$-0.475\ 842\ 430\ 145\ 708 \times 10^3$	30	28	16	$0.108\ 408\ 607\ 429\ 124 \times 10^{29}$
14	4	0	$0.696\ 781\ 965\ 359\ 503 \times 10^2$	31	28	36	$0.730\ 872\ 705\ 175\ 151 \times 10^{44}$
15	4	1	$0.335\ 674\ 250\ 377\ 312 \times 10^3$	32	32	10	$0.159\ 145\ 847\ 398\ 870 \times 10^{25}$
16	4	4	$0.250\ 526\ 809\ 130\ 882 \times 10^5$	33	32	28	$0.377\ 121\ 605\ 943\ 324 \times 10^{41}$
17	4	5	$0.146\ 997\ 380\ 630\ 766 \times 10^6$				

Table 4. Coefficients and exponents of the backward equation $p_{3b}(h,s)$ for subregion 3b in its dimensionless form, Eq. (2)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	2	$0.125\ 244\ 360\ 717\ 979 \times 10^{-12}$	19	−4	8	$0.355\ 073\ 647\ 696\ 481 \times 10^4$
2	−12	10	$-0.126\ 599\ 322\ 553\ 713 \times 10^{-1}$	20	−3	1	$-0.115\ 303\ 107\ 290\ 162 \times 10^{-3}$
3	−12	12	$0.506\ 878\ 030\ 140\ 626 \times 10^1$	21	−3	3	$-0.175\ 092\ 403\ 171\ 802 \times 10^1$
4	−12	14	$0.317\ 847\ 171\ 154\ 202 \times 10^2$	22	−3	5	$0.257\ 981\ 687\ 748\ 160 \times 10^3$
5	−12	20	$-0.391\ 041\ 161\ 399\ 932 \times 10^6$	23	−3	6	$-0.727\ 048\ 374\ 179\ 467 \times 10^3$
6	−10	2	$-0.975\ 733\ 406\ 392\ 044 \times 10^{-10}$	24	−2	0	$0.121\ 644\ 822\ 609\ 198 \times 10^{-3}$
7	−10	10	$-0.186\ 312\ 419\ 488\ 279 \times 10^2$	25	−2	1	$0.393\ 137\ 871\ 762\ 692 \times 10^{-1}$
8	−10	14	$0.510\ 973\ 543\ 414\ 101 \times 10^3$	26	−1	0	$0.704\ 181\ 005\ 909\ 296 \times 10^{-2}$
9	−10	18	$0.373\ 847\ 005\ 822\ 362 \times 10^6$	27	0	3	$-0.829\ 108\ 200\ 698\ 110 \times 10^2$
10	−8	2	$0.299\ 804\ 024\ 666\ 572 \times 10^{-7}$	28	2	0	$-0.265\ 178\ 818\ 131\ 250$
11	−8	8	$0.200\ 544\ 393\ 820\ 342 \times 10^2$	29	2	1	$0.137\ 531\ 682\ 453\ 991 \times 10^2$
12	−6	2	$-0.498\ 030\ 487\ 662\ 829 \times 10^{-5}$	30	5	0	$-0.522\ 394\ 090\ 753\ 046 \times 10^2$
13	−6	6	$-0.102\ 301\ 806\ 360\ 030 \times 10^2$	31	6	1	$0.240\ 556\ 298\ 941\ 048 \times 10^4$
14	−6	7	$0.552\ 819\ 126\ 990\ 325 \times 10^2$	32	8	1	$-0.227\ 361\ 631\ 268\ 929 \times 10^5$
15	−6	8	$-0.206\ 211\ 367\ 510\ 878 \times 10^3$	33	10	1	$0.890\ 746\ 343\ 932\ 567 \times 10^5$
16	−5	10	$-0.794\ 012\ 232\ 324\ 823 \times 10^4$	34	14	3	$-0.239\ 234\ 565\ 822\ 486 \times 10^8$
17	−4	4	$0.782\ 248\ 472\ 028\ 153 \times 10^1$	35	14	7	$0.568\ 795\ 808\ 129\ 714 \times 10^{10}$
18	−4	5	$-0.586\ 544\ 326\ 902\ 468 \times 10^2$				

Table 5. Selected pressure values calculated from Eqs. (1) and (2)[a]

Equation	h / kJ kg^{-1}	s / kJ kg^{-1} K^{-1}	p / MPa
	1700	3.8	$2.555\ 703\ 246 \times 10^1$
$p_{3a}(h,s)$, Eq. (1)	2000	4.2	$4.540\ 873\ 468 \times 10^1$
	2100	4.3	$6.078\ 123\ 340 \times 10^1$
	2600	5.1	$3.434\ 999\ 263 \times 10^1$
$p_{3b}(h,s)$, Eq. (2)	2400	4.7	$6.363\ 924\ 887 \times 10^1$
	2700	5.0	$8.839\ 043\ 281 \times 10^1$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

Numerical Consistency with the Basic Equation of IAPWS-IF97. The maximum relative deviations and root-mean-square relative deviations between the calculated pressure, Eqs. (1) and (2), and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ together with the permissible differences are listed in Table 6. The calculation of the root-mean-square value is described in Section 1.

The critical pressure is met exactly by the equations $p_3(h,s)$.

Table 6. Maximum differences and root-mean-square differences between the pressure calculated from Eqs. (1) and (2) and from the IAPWS-IF97 basic equation $f_3^{97}(v,T)$, and related permissible values

Subregion	Equation	$\left|\Delta p/p\right|_{\text{tol}}$	$\left|\Delta p/p\right|_{\text{max}}$	$\left|\Delta p/p\right|_{\text{RMS}}$
3a	(1)	0.01 %	0.0070 %	0.0030 %
3b	(2)	0.01 %	0.0084 %	0.0036 %

Consistency at Boundary Between Subregions. The maximum relative pressure deviation between the two backward equations, Eq. (1) and Eq. (2), along the boundary s_c, has the following value

$$\left|\frac{\Delta p}{p}\right|_{\text{max}} = \left|\frac{p_{3a}(h,s_c) - p_{3b}(h,s_c)}{p_{3b}(h,s_c)}\right|_{\text{max}} = 0.00074\ \% \ .$$

Thus, the relative pressure differences between the two backward equations $p(h,s)$ of the adjacent subregions 3a and 3b are smaller than the numerical consistencies of these equations with the IAPWS-IF97 basic equation.

3.4 Backward Function *T(h,s)*

Calculation of the Backward Function T(h,s).   The $p_3(h,s)$ equations described in Section 3.3 together with the IAPWS-IF97-S03 backward equation $T_3^{03}(p,h)^{3)}$ make it possible to determine temperature *T* from enthalpy *h* and entropy *s* without iteration.

For calculating *T* from a given *h* and *s* in region 3, the following steps should be made:

- Use the entropy line s_c (see Figure 2) to identify the subregion (3a or 3b) for the given values of *h* and *s.* Then, calculate the pressure *p* using the equation $p_{3a}(h,s)$, Eq. (1), or $p_{3b}(h,s)$, Eq. (2).

- Use the IAPWS-IF97-S03 boundary equation $h_{3ab}^{03}(p)$ to identify the subregion (3a or 3b) for the given value of *h* and the previously calculated value of *p*. Then, calculate temperature *T* for the subregion using the backward equation $T_{3a}^{03}(p,h)$ or $T_{3b}^{03}(p,h)$.

Numerical Consistency with the Basic Equation of IAPWS-IF97.   The maximum temperature differences and related root-mean-square differences between the calculated temperature and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ together with the permissible differences are listed in Table 7. The temperature differences were calculated as follows for subregions 3a and 3b:

$$\Delta T = T_3^{03}\left[p_3\left(h_3^{97}, s_3^{97} \right), h_3^{97} \right] - T .$$

The function T_3^{03} represents the calculation of $T(p,h)$ using the backward equations of region 3 including the determination of which subregion (3a or 3b) contains the point.

Table 7. Maximum differences and root-mean-square differences between calculated temperatures and IAPWS-IF97 basic equation $f_3^{97}(v,T)$ compared with the permissible differences

Subregion	$\left|\Delta T\right|_{tol}$	$\left|\Delta T\right|_{max}$	$\left|\Delta T\right|_{RMS}$
3a	25 mK	23.7 mK	10.5 mK
3b	25 mK	22.4 mK	9.9 mK

The maximum temperature deviations are less than the permissible value in subregions 3a and 3b. Therefore, the accuracy of pressure calculated by the equations $p_{3a}(h,s)$ and $p_{3b}(h,s)$, Eqs. (1) and (2), is sufficient for calculating temperature using the backward equations $T_3^{03}(p,h)$.

[3)] The alternative use of the IAPWS-IF97-S03 backward equation $T_3^{03}(p,s)$ leads to worse numerical consistency.

Consistency at Boundary Between Subregions. The subregion boundary between the backward equations $T_{3a}^{03}(p,h)$ and $T_{3b}^{03}(p,h)$ is the boundary equation $h_{3ab}^{03}(p)$. The subregion boundary between the backward equations $p_{3a}(h,s)$ and $p_{3b}(h,s)$ is the critical isentropic line $s = s_c$. These two boundary lines are independent but cross each other at 4 points in the pressure range from p_c to 100 MPa. The discontinuity between $T_{3a}^{03}(p,h)$ and $T_{3b}^{03}(p,h)$ overlaps that between $p_{3a}(h,s)$ and $p_{3b}(h,s)$ at these 4 points. The greatest temperature difference was determined as:

$$\left|\Delta T\right|_{\max} = \left|T_{3a}^{03}\left(p_{3a}(h,s),h\right) - T_{3b}^{03}\left(p_{3b}(h,s),h\right)\right|_{\max} = 0.68 \text{ mK} ,$$

where $h = 2.128\,526\,988 \times 10^3 \text{ kJ kg}^{-1}$ and $s = s_c$.

3.5 Backward Function *v(h,s)*

Calculation of the Backward Function v(h,s). The $p_3(h,s)$ equations described in Section 3.3 together with the IAPWS-IF97-S03 backward equation $v_3^{03}(p,s)$ [4] make it possible to determine specific volume *v* from enthalpy *h* and entropy *s* without iteration.

For calculating *v* from given *h* and *s* in region 3, the following steps should be made:

- Use the entropy line s_c (see Figure 2) to identify the subregion (3a or 3b) for the given values of *h* and *s*. Then, calculate the pressure *p* using the equation $p_{3a}(h,s)$, Eq. (1), or $p_{3b}(h,s)$, Eq. (2).

- Since the entropy line s_c is the subregion boundary for the backward equations $v_3^{03}(p,s)$, calculate the specific volume *v* for the subregion identified previously using the backward equation $v_{3a}^{03}(p,s)$ or $v_{3b}^{03}(p,s)$.

Numerical Consistency with the Basic Equation of IAPWS-IF97. The maximum relative deviations and related root-mean-square relative deviations between the calculated specific volume and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ together with the permissible differences are listed in Table 8. The specific volume relative deviations for subregions 3a and 3b were calculated using the relation

$$\Delta v = \left(v_3^{03}\left[p_3(h_3^{97},s_3^{97}),s_3^{97}\right] - v\right)\Big/ v .$$

The function v_3^{03} represents the calculation of $v(p,s)$ using the backward equations of region 3 including the determination of which subregion (3a or 3b) contains the point.

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[4] The alternative use of the IAPWS-IF97-S03 backward equation $v_3^{03}(p,h)$ leads to worse numerical consistency.

Table 8. Maximum relative differences and root-mean-square relative differences between calculated specific volume and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$, and related permissible values

Subregion	$\left\|\Delta v/v\right\|_{\text{tol}}$	$\left\|\Delta v/v\right\|_{\text{max}}$	$\left\|\Delta v/v\right\|_{\text{RMS}}$
3a	0.01 %	0.0097 %	0.0053 %
3b	0.01 %	0.0095 %	0.0043 %

The maximum relative deviations of specific volume are less than the permissible value in subregions 3a and 3b. Therefore, the accuracy of pressure calculated by the equations $p_{3a}(h,s)$ and $p_{3b}(h,s)$, Eqs. (1) and (2), is sufficient for calculating specific volume using the backward equations $v_3^{03}(p,s)$.

Consistency at Boundary Between Subregions. The maximum relative specific volume difference between the two backward equations of the adjacent subregions 3a and 3b along the boundary line s_{c} has the following value:

$$\left\|(\Delta v/v)\right\|_{\text{max}} = \left.\left|\frac{v_{3a}^{03}\!\left[p_{3a}(h,s_{\text{c}}),\ s_{\text{c}}\right] - v_{3b}^{03}\!\left[p_{3b}(h,s_{\text{c}}),\ s_{\text{c}}\right]}{v_{3b}^{03}\!\left[p_{3b}(h,s_{\text{c}}),\ s_{\text{c}}\right]}\right|\right|_{\text{max}} = 0.00028 \ \%.$$

3.6 Computing Time in Relation to IAPWS-IF97

A very important motivation for the development of the backward equations $p(h,s)$ for region 3 was reducing the computing time to obtain thermodynamic properties and differential quotients from the given variables (h,s). In IAPWS-IF97, time-consuming iterations, *e.g.*, the two-dimensional Newton method, are required. Using the $p_3(h,s)$ equation combined with the backward equations $T_3^{03}(p,h)$ and $v_3^{03}(p,s)$ of IAPWS-IF97-S03, the calculation to obtain p, T and v as a function of h and s is about 11 times faster than that of the two-dimensional Newton method.

4 Equations for Region Boundaries Given Enthalpy and Entropy

4.1 Determination of Region Boundaries

IAPWS provides explicit equations for functions of the variables (h,s) in liquid region 1, vapor region 2 and region 3 (this release). The region boundaries must be determined for the given enthalpy and entropy to decide where the given state point is located.

The boundaries between single-phase regions 1, 2, and 3 and two-phase region 4 are the saturated liquid line $x = 0$ and the saturated vapor line $x = 1$ (see Figure 3).

Figure 3. Regions of IAPWS-IF97 and boundary equations $h_1'(s)$, $h_{3a}'(s)$, $h_{2ab}''(s)$, $h_{2c3b}''(s)$, $h_{B13}(s)$, and $T_{B23}(h,s)$ of this release

A one-dimensional iteration using the IAPWS-IF97 basic equation $g_1^{97}(p,T)$ and the saturation-pressure equation $p_{\text{sat}}^{97}(T)$ is required to calculate the enthalpy from a given entropy on the saturated liquid line of region 1. For the calculation of the enthalpy from a given entropy on the saturated vapor line of region 2, a one-dimensional iteration using the IAPWS-IF97 basic equation $g_2^{97}(p,T)$ and the saturation-pressure equation $p_{\text{sat}}^{97}(T)$ is required. On the saturated liquid and saturated vapor lines of region 3, a two-dimensional iteration using the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ and the saturation-pressure equation $p_{\text{sat}}^{97}(T)$ is required to calculate the enthalpy from a given entropy. The boundary equations $h_1'(s)$, $h_{3a}'(s)$, $h_{2ab}''(s)$, and $h_{2c3b}''(s)$ provided in this release make it possible

to determine without iteration whether the given state point is located in the single-phase regions 1, 2, and 3 or in the two-phase region 4.

The boundary between regions 1 and 3 is the isotherm $T = 623.15\,\text{K}$ (see Figure 1). A one-dimensional iteration using the IAPWS-IF97 basic equation $g_1^{97}(p,T)$ is necessary to calculate the isotherm from a given enthalpy and entropy. In order to avoid this iteration, the boundary equation $h_{\text{B13}}(s)$ has been developed (see Figure 3).

The boundary between regions 2 and 3 is defined by the B23-equation of IAPWS-IF97 (see Figure 1). Figure 3 shows that the B23-equation has more than one value of enthalpy for a given entropy. In order to determine the region of the given state point, the boundary line must be divided into at least three subregions. In each subregion, a one-dimensional iteration using the IAPWS-IF97 basic equation $g_2^{97}(p,T)$ and the B23-equation $p_{\text{B23}}^{97}(T)$ is necessary to calculate the B23-boundary from a given enthalpy and entropy. The equation $T_{\text{B23}}(h,s)$ and the IAPWS-IF97-S01 backward equation $p_{2c}^{01}(h,s)$ allow the determination of the boundary between regions 2 and 3 without iteration.

4.2 Numerical Consistency Requirements

The numerical consistency requirements on the equations for region boundaries result from IAPWS requirements on backward functions. That means, backward functions of the variables enthalpy and entropy have to meet their corresponding requirements when using the equations for region boundaries.

4.3 Equations $h_1'(s)$ and $h_{3a}'(s)$ for the Saturated Liquid Line

In order to meet the requirements for numerical consistency, the saturated liquid line was divided into two sections. Section 1 adjoins region 1 of IAPWS-IF97. Section 3a adjoins subregion 3a of this release.

The equation $h_1'(s)$ of Section 1 describes the saturated liquid line from the triple-point temperature 273.16 K to 623.15 K and can be simply extrapolated to 273.15 K so that it covers the entropy range (see Figure 3)

$$s'(273.15\,\text{K}) \leq s \leq s'(623.15\,\text{K}),$$

$$\text{where } s'(273.15\,\text{K}) = -1.545\,495\,919 \times 10^{-4}\ \text{kJ kg}^{-1}\,\text{K}^{-1}$$

$$\text{and } s'(623.15\,\text{K}) = 3.778\,281\,340\ \text{kJ kg}^{-1}\,\text{K}^{-1}.$$

The equation $h_{3a}'(s)$ of Section 3a describes the saturated liquid line in the entropy range (see Figure 3)

$$s'(623.15\,\text{K}) < s \leq s_c,$$

$$\text{where } s'(623.15\,\text{K}) = 3.778\,281\,340\ \text{kJ kg}^{-1}\,\text{K}^{-1}$$

$$\text{and } s_c = 4.412\,021\,482\,234\,76\ \text{kJ kg}^{-1}\,\text{K}^{-1}.$$

The Equations. The boundary equation $h_1'(s)$ has the following dimensionless form:

$$\frac{h_1'(s)}{h^*} = \eta_1'(\sigma) = \sum_{i=1}^{27} n_i \left(\sigma - 1.09\right)^{I_i} \left(\sigma + 0.366 \times 10^{-4}\right)^{J_i} , \tag{3}$$

where $\eta = h/h^*$ and $\sigma = s/s^*$, with $h^* = 1700 \text{ kJ kg}^{-1}$ and $s^* = 3.8 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (3) are listed in Table 9.

The boundary equation $h_{3a}'(s)$ has the following dimensionless form:

$$\frac{h_{3a}'(s)}{h^*} = \eta_{3a}'(\sigma) = \sum_{i=1}^{19} n_i \left(\sigma - 1.09\right)^{I_i} \left(\sigma + 0.366 \times 10^{-4}\right)^{J_i} , \tag{4}$$

where $\eta = h/h^*$ and $\sigma = s/s^*$, with $h^* = 1700 \text{ kJ kg}^{-1}$ and $s^* = 3.8 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (4) are listed in Table 10.

Table 9. Coefficients and exponents of the boundary equation $h_1'(s)$ in its dimensionless form, Eq. (3)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	14	0.332 171 191 705 237	15	8	4	0.194 486 637 751 291 × 10^2
2	0	36	0.611 217 706 323 496 × 10^{-3}	16	12	2	−0.357 915 139 457 043 × 10^1
3	1	3	−0.882 092 478 906 822 × 10^1	17	12	4	−0.335 369 414 148 819 × 10^1
4	1	16	−0.455 628 192 543 250	18	14	1	−0.664 426 796 332 460
5	2	0	−0.263 483 840 850 452 × 10^{-4}	19	14	22	0.323 321 885 383 934 × 10^5
6	2	5	−0.223 949 661 148 062 × 10^2	20	16	10	0.331 766 744 667 084 × 10^4
7	3	4	−0.428 398 660 164 013 × 10^1	21	20	12	−0.223 501 257 931 087 × 10^5
8	3	36	−0.616 679 338 856 916	22	20	28	0.573 953 875 852 936 × 10^7
9	4	4	−0.146 823 031 104 040 × 10^2	23	22	8	0.173 226 193 407 919 × 10^3
10	4	16	0.284 523 138 727 299 × 10^3	24	24	3	−0.363 968 822 121 321 × 10^{-1}
11	4	24	−0.113 398 503 195 444 × 10^3	25	28	0	0.834 596 332 878 346 × 10^{-6}
12	5	18	0.115 671 380 760 859 × 10^4	26	32	6	0.503 611 916 682 674 × 10^1
13	5	24	0.395 551 267 359 325 × 10^3	27	32	8	0.655 444 787 064 505 × 10^2
14	7	1	−0.154 891 257 229 285 × 10^1				

Table 10. Coefficients and exponents of the boundary equation $h'_{3a}(s)$ in its dimensionless
form, Eq. (4)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	1	0.822 673 364 673 336	11	6	4	0.191 413 958 471 069
2	0	4	0.181 977 213 534 479	12	7	2	0.581 062 241 093 136 × 10⁻¹
3	0	10	−0.112 000 260 313 624 × 10⁻¹	13	7	28	−0.165 505 498 701 029 × 10⁴
4	0	16	−0.746 778 287 048 033 × 10⁻³	14	7	32	0.158 870 443 421 201 × 10⁴
5	2	1	−0.179 046 263 257 381	15	10	14	−0.850 623 535 172 818 × 10²
6	3	36	0.424 220 110 836 657 × 10⁻¹	16	10	32	−0.317 714 386 511 207 × 10⁵
7	4	3	−0.341 355 823 438 768	17	10	36	−0.945 890 406 632 871 × 10⁵
8	4	16	−0.209 881 740 853 565 × 10¹	18	32	0	−0.139 273 847 088 690 × 10⁻⁵
9	5	20	−0.822 477 343 323 596 × 10¹	19	32	6	0.631 052 532 240 980
10	5	36	−0.499 684 082 076 008 × 10¹				

Computer-program verification. To assist the user in computer-program verification of
Eqs. (3) and (4), Table 11 contains test values for calculated enthalpies.

Table 11. Selected enthalpy values calculated from
Eqs. (3) and (4)[a]

Equation	s / kJ kg⁻¹ K⁻¹	h / kJ kg⁻¹
$h'_1(s)$, Eq. (3)	1.0	3.085 509 647 × 10²
	2.0	7.006 304 472 × 10²
	3.0	1.198 359 754 × 10³
$h'_{3a}(s)$, Eq. (4)	3.8	1.685 025 565 × 10³
	4.0	1.816 891 476 × 10³
	4.2	1.949 352 563 × 10³

[a] It is recommended that programmed functions be verified using
8 byte real values for all variables.

Numerical Consistency with the Basic Equations of IAPWS-IF97. The maximum
deviations between the calculated enthalpy, Eqs. (3) and (4), and the IAPWS-IF97 basic
equations $g_1^{97}(p,T)$ and $f_3^{97}(v,T)$ are listed in Table 12.

The enthalpy value $h'(273.15\,\text{K})$ is met exactly by the equation $h'_1(s)$. The enthalpy
value of the critical point $h_c = 2.087\,546\,845 \times 10^3$ kJ kg⁻¹ is met exactly by the equation
$h'_{3a}(s)$.

Table 12. Maximum differences between the enthalpy calculated
from Eqs. (3) and (4) and from the IAPWS-IF97 basic
equations $g_1^{97}(p,T)$ and $f_3^{97}(v,T)$

Region	$|\Delta h|_{\text{max}}$
1	0.0034 kJ kg⁻¹
3	0.0045 kJ kg⁻¹

Consistency of the Backward Equations with the Basic Equations of IAPWS-IF97 along $h'_1(s)$ and $h'_{3a}(s)$. The maximum pressure differences between the IAPWS-IF97-S01 backward equation $p_1^{01}(h,s)$ and the IAPWS-IF97 basic equation $g_1^{97}(p,T)$ along the boundary equation $h'_1(s)$, Eq. (3), are listed in Table 13. Table 13 also shows the relative maximum pressure difference between the backward equation $p_{3a}(h,s)$, Eq. (1), and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ along the boundary equation $h'_{3a}(s)$, Eq. (4).

Table 13. Maximum differences between the pressure calculated from $p_1^{01}(h,s)$ and $p_{3a}(h,s)$, Eq. (1), and from the IAPWS-IF97 basic equations $g_1^{97}(p,T)$ and $f_3^{97}(v,T)$

Region		Calculation of $\left\|\Delta p\right\|_{\max}$	$\left\|\Delta p\right\|_{\mathrm{tol}}$	$\left\|\Delta p\right\|_{\max}$
1	$p \le 2.5$ MPa	$\left\|\dfrac{\Delta p}{p}\right\|_{\max} = \left\|\dfrac{p_1^{01}\big(h'_1(s),s\big) - p}{p}\right\|_{\max}$	0.6 %	0.59 %
	$p > 2.5$ MPa	$\left\|\Delta p\right\|_{\max} = \left\|p_1^{01}\big(h'_1(s),s\big) - p\right\|_{\max}$	15 kPa	12.9 kPa
3		$\left\|\dfrac{\Delta p}{p}\right\|_{\max} = \left\|\dfrac{p_{3a}\big(h'_{3a}(s),s\big) - p}{p}\right\|_{\max}$	0.01 %	0.0065 %

The absolute maximum temperature difference between the IAPWS-IF97 backward equation $T_1^{97}(p,h)$[5] and the IAPWS-IF97 basic equation $g_1^{97}(p,T)$ along the boundary equation $h'_1(s)$, Eq. (3), is listed in Table 14. It also shows the absolute maximum temperature difference between the IAPWS-IF97-S03 backward equation $T_{3a}^{03}(p,h)$[5] and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ along the boundary equation $h'_{3a}(s)$, Eq. (4).

Table 14. Maximum differences between the temperature calculated from $T_1^{97}(p,h)$ and $T_{3a}^{03}(p,h)$ and from the IAPWS-IF97 basic equations $g_1^{97}(p,T)$ and $f_3^{97}(v,T)$

Region	Calculation of $\left\|\Delta T\right\|_{\max}$	$\left\|\Delta T\right\|_{\mathrm{tol}}$	$\left\|\Delta T\right\|_{\max}$
1	$\left\|\Delta T\right\|_{\max} = \left\|T_1^{97}\Big[p_1^{01}\big(h'_1(s),s\big),h'_1(s)\Big] - T\right\|_{\max}$	25 mK	23.8 mK
3	$\left\|\Delta T\right\|_{\max} = \left\|T_{3a}^{03}\Big[p_{3a}\big(h'_{3a}(s),s\big),h'_{3a}(s)\Big] - T\right\|_{\max}$	25 mK	4.8 mK

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[5] The alternative calculation of the temperature on the saturated-liquid line using the IAPWS-IF97 saturation-temperature equation $T_{\mathrm{sat}}^{97}\Big(p_1^{01}\big(h'_1(s),s\big)\Big)$ or $T_{\mathrm{sat}}^{97}\Big(p_{3a}\big(h'_{3a}(s),s\big)\Big)$ leads to worse numerical consistency.

The maximum relative difference of specific volume between the IAPWS-IF97-S03 backward equation $v_{3a}^{03}(p,s)$ and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ along the boundary equation $h_{3a}'(s)$, Eq. (4), is listed in Table 15.

Table 15. Maximum relative difference between the specific volume calculated from $v_{3a}^{03}(p,s)$ and from the IAPWS-IF97 basic equation $f_3^{97}(v,T)$

Region	Calculation of $\left\vert \Delta v/v \right\vert_{\max}$	$\left\vert \Delta v/v \right\vert_{\text{tol}}$	$\left\vert \Delta v/v \right\vert_{\max}$
3	$\left\vert \dfrac{\Delta v}{v} \right\vert_{\max} = \left\vert \dfrac{v_{3a}^{03}\left[p_{3a}\left(h_{3a}'(s),s\right),s \right] - v}{v} \right\vert_{\max}$	0.01 %	0.0058 %

The maximum pressure differences, the maximum temperature differences and the maximum relative difference of specific volume are smaller than the permissible values. Therefore, the accuracy of enthalpy calculated by the equations $h_1'(s)$ and $h_{3a}'(s)$, Eqs. (3) and (4), is sufficient.

4.4 Equations $h_{2ab}''(s)$ and $h_{2c3b}''(s)$ for the Saturated Vapor Line

In order to meet the requirements for numerical consistency, the saturated vapor line was divided into two sections. Section 2ab adjoins subregions 2a and 2b of IAPWS-IF97. Section 2c3b adjoins subregion 2c of IAPWS-IF97 and subregion 3b of this release.

The equation $h_{2ab}''(s)$ of section 2ab describes the saturated vapor line from the triple point to $5.85\ \text{kJ kg}^{-1}\ \text{K}^{-1}$ and can be simply extrapolated to 273.15 K so that it covers the entropy range (see Figure 3)

$$5.85\ \text{kJ kg}^{-1}\ \text{K}^{-1} \leq s \leq s''(273.15\ \text{K}),$$

where $s''(273.15\ \text{K}) = 9.155\ 759\ 395\ \text{kJ kg}^{-1}\ \text{K}^{-1}$.

The equation $h_{2c3b}''(s)$ of section 2c3b describes the saturated vapor line in the entropy range (see Figure 3)

$$s_\text{c} \leq s < 5.85\ \text{kJ kg}^{-1}\ \text{K}^{-1},$$

where $s_\text{c} = 4.412\ 021\ 482\ 234\ 76\ \text{kJ kg}^{-1}\ \text{K}^{-1}$.

The Equations. The boundary equation $h_{2ab}''(s)$ has the following dimensionless form:

$$\frac{h_{2ab}''(s)}{h^*} = \eta_{2ab}''(\sigma) = \exp\left[\sum_{i=1}^{30} n_i \left(\sigma_1^{-1} - 0.513\right)^{I_i} \left(\sigma_2 - 0.524\right)^{J_i} \right], \qquad (5)$$

where $\eta = h/h^*$, $\sigma_1 = s/s_1^*$, and $\sigma_2 = s/s_2^*$, with $h^* = 2800\ \text{kJ kg}^{-1}$, $s_1^* = 5.21\ \text{kJ kg}^{-1}\ \text{K}^{-1}$, and $s_2^* = 9.2\ \text{kJ kg}^{-1}\ \text{K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (5) are listed in Table 16.

The boundary equation $h''_{2c3b}(s)$ has the following dimensionless form:

$$\frac{h''_{2c3b}(s)}{h^*} = \eta''_{2c3b}(\sigma) = \left[\sum_{i=1}^{16} n_i\,(\sigma-1.02)^{I_i}\,(\sigma-0.726)^{J_i}\right]^4,\qquad(6)$$

where $\eta = h/h^*$ and $\sigma = s/s^*$, with $h^* = 2800\ \text{kJ kg}^{-1}$ and $s^* = 5.9\ \text{kJ kg}^{-1}\ \text{K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (6) are listed in Table 17.

Table 16. Coefficients and exponents of the boundary equation $h''_{2ab}(s)$ in its dimensionless form, Eq. (5)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	1	8	$-0.524\ 581\ 170\ 928\ 788 \times 10^3$	16	28	8	$0.660\ 788\ 766\ 938\ 091 \times 10^{16}$
2	1	24	$-0.926\ 947\ 218\ 142\ 218 \times 10^7$	17	28	12	$0.166\ 320\ 055\ 886\ 021 \times 10^{23}$
3	2	4	$-0.237\ 385\ 107\ 491\ 666 \times 10^3$	18	28	20	$-0.218\ 003\ 784\ 381\ 501 \times 10^{30}$
4	2	32	$0.210\ 770\ 155\ 812\ 776 \times 10^{11}$	19	28	22	$-0.787\ 276\ 140\ 295\ 618 \times 10^{30}$
5	4	1	$-0.239\ 494\ 562\ 010\ 986 \times 10^2$	20	28	24	$0.151\ 062\ 329\ 700\ 346 \times 10^{32}$
6	4	2	$0.221\ 802\ 480\ 294\ 197 \times 10^3$	21	32	2	$0.795\ 732\ 170\ 300\ 541 \times 10^7$
7	7	7	$-0.510\ 472\ 533\ 393\ 438 \times 10^7$	22	32	7	$0.131\ 957\ 647\ 355\ 347 \times 10^{16}$
8	8	5	$0.124\ 981\ 396\ 109\ 147 \times 10^7$	23	32	12	$-0.325\ 097\ 068\ 299\ 140 \times 10^{24}$
9	8	12	$0.200\ 008\ 436\ 996\ 201 \times 10^{10}$	24	32	14	$-0.418\ 600\ 611\ 419\ 248 \times 10^{26}$
10	10	1	$-0.815\ 158\ 509\ 791\ 035 \times 10^3$	25	32	24	$0.297\ 478\ 906\ 557\ 467 \times 10^{35}$
11	12	0	$-0.157\ 612\ 685\ 637\ 523 \times 10^3$	26	36	10	$-0.953\ 588\ 761\ 745\ 473 \times 10^{20}$
12	12	7	$-0.114\ 200\ 422\ 332\ 791 \times 10^{11}$	27	36	12	$0.166\ 957\ 699\ 620\ 939 \times 10^{25}$
13	18	10	$0.662\ 364\ 680\ 776\ 872 \times 10^{16}$	28	36	20	$-0.175\ 407\ 764\ 869\ 978 \times 10^{33}$
14	20	12	$-0.227\ 622\ 818\ 296\ 144 \times 10^{19}$	29	36	22	$0.347\ 581\ 490\ 626\ 396 \times 10^{35}$
15	24	32	$-0.171\ 048\ 081\ 348\ 406 \times 10^{32}$	30	36	28	$-0.710\ 971\ 318\ 427\ 851 \times 10^{39}$

Table 17. Coefficients and exponents of the boundary equation $h''_{2c3b}(s)$ in its dimensionless form, Eq. (6)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	$0.104\ 351\ 280\ 732\ 769 \times 10^1$	9	8	2	$0.743\ 957\ 464\ 645\ 363 \times 10^4$
2	0	3	$-0.227\ 807\ 912\ 708\ 513 \times 10^1$	10	8	20	$-0.356\ 896\ 445\ 355\ 761 \times 10^{20}$
3	0	4	$0.180\ 535\ 256\ 723\ 202 \times 10^1$	11	12	32	$0.167\ 590\ 585\ 186\ 801 \times 10^{32}$
4	1	0	$0.420\ 440\ 834\ 792\ 042$	12	16	36	$-0.355\ 028\ 625\ 419\ 105 \times 10^{38}$
5	1	12	$-0.105\ 721\ 244\ 834\ 660 \times 10^6$	13	22	2	$0.396\ 611\ 982\ 166\ 538 \times 10^{12}$
6	5	36	$0.436\ 911\ 607\ 493\ 884 \times 10^{25}$	14	22	32	$-0.414\ 716\ 268\ 484\ 468 \times 10^{41}$
7	6	12	$-0.328\ 032\ 702\ 839\ 753 \times 10^{12}$	15	24	7	$0.359\ 080\ 103\ 867\ 382 \times 10^{19}$
8	7	16	$-0.678\ 686\ 760\ 804\ 270 \times 10^{16}$	16	36	20	$-0.116\ 994\ 334\ 851\ 995 \times 10^{41}$

Computer-program verification. To assist the user in computer-program verification of Eqs. (5) and (6), Table 18 contains test values for calculated enthalpies.

Table 18. Selected enthalpy values calculated from Eqs. (5) and (6)[a]

Equation	s / kJ kg^{-1} K^{-1}	h / kJ kg^{-1}
$h''_{2ab}(s)$, Eq. (5)	7.0	$2.723\,729\,985 \times 10^3$
	8.0	$2.599\,047\,210 \times 10^3$
	9.0	$2.511\,861\,477 \times 10^3$
$h''_{2c3b}(s)$, Eq. (6)	5.5	$2.687\,693\,850 \times 10^3$
	5.0	$2.451\,623\,609 \times 10^3$
	4.5	$2.144\,360\,448 \times 10^3$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

Numerical Consistency with the Basic Equations of IAPWS-IF97. The maximum deviations between the calculated enthalpy, Eqs. (5) and (6), and the IAPWS-IF97 basic equations $g_2^{97}(p,T)$ and $f_3^{97}(v,T)$ are listed in Table 19.

The enthalpy value $h''(273.15\,\text{K})$ is met exactly by the equation $h''_{2ab}(s)$. The enthalpy value at the critical point $h_c = 2.087\,546\,845 \times 10^3$ kJ kg^{-1} is met exactly by the equation $h''_{2c3b}(s)$.

Table 19. Maximum differences between the enthalpy calculated from Eqs. (5) and (6) and from the IAPWS-IF97 basic equations $g_2^{97}(p,T)$ and $f_3^{97}(v,T)$

Region		Equation	$\left|\Delta h\right|_{\text{max}}$
2	$s \geq 5.85$ kJ kg^{-1} K^{-1}	$h''_{2ab}(s)$, Eq. (5)	0.0012 kJ kg^{-1}
	$s < 5.85$ kJ kg^{-1} K^{-1}	$h''_{2c3b}(s)$, Eq. (6)	0.0058 kJ kg^{-1}
3		$h''_{2c3b}(s)$, Eq. (6)	0.0073 kJ kg^{-1}

Consistency of the Backward Equations with the Basic Equations of IAPWS-IF97 along $h''_{2ab}(s)$ and $h''_{2c3b}(s)$. The maximum relative pressure differences between the IAPWS-IF97-S01 backward equations $p_2^{01}(h,s)$ and the IAPWS-IF97 basic equation $g_2^{97}(p,T)$ and between the backward equation $p_{3b}(h,s)$, Eq. (2), and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ along the boundary equations $h''_{2ab}(s)$ and $h''_{2c3b}(s)$, Eqs. (5) and (6), are listed in Table 20. The function p_2^{01} represents the calculation of $p(h,s)$ from the IAPWS-IF97-S01 backward equations of region 2 including the determination of the subregion (2a, 2b or 2c).

Table 20. Maximum differences between the pressure calculated from $p_2^{01}(h,s)$ and $p_{3b}(h,s)$, Eq. (2), and from the IAPWS-IF97 basic equations $g_2^{97}(p,T)$ and $f_3^{97}(v,T)$

Region		Calculation of $\left|\Delta p\right|_{max}$	$\left|\Delta p\right|_{tol}$	$\left|\Delta p\right|_{max}$
2	$s \geq 5.85 \text{ kJ kg}^{-1}\text{ K}^{-1}$	$\left.\dfrac{\Delta p}{p}\right|_{max} = \left.\left|\dfrac{p_2^{01}\left(h_{2ab}^{"}(s),s\right)-p}{p}\right|\right._{max}$	0.0035 %	0.0027 %
	$s < 5.85 \text{ kJ kg}^{-1}\text{ K}^{-1}$	$\left.\dfrac{\Delta p}{p}\right|_{max} = \left.\left|\dfrac{p_2^{01}\left(h_{2c3b}^{"}(s),s\right)-p}{p}\right|\right._{max}$	0.0088 %	0.0066 %
3		$\left.\dfrac{\Delta p}{p}\right|_{max} = \left.\left|\dfrac{p_{3b}\left(h_{2c3b}^{"}(s),s\right)-p}{p}\right|\right._{max}$	0.01 %	0.0050 %

The absolute maximum temperature differences between the IAPWS-IF97 backward equations $T_2^{97}(p,h)^{[6]}$ and the IAPWS-IF97 basic equation $g_2^{97}(p,T)$ and between the IAPWS-IF97-S03 backward equation $T_{3b}^{03}(p,h)^{[6]}$ and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ along the boundary equations $h_{2ab}^{"}(s)$ and $h_{2c3b}^{"}(s)$, Eqs. (5) and (6), are listed in Table 21. The function T_2^{97} represents the calculation of $T(p,h)$ from the IAPWS-IF97 backward equations of region 2 including the determination of the subregion (2a, 2b or 2c).

Table 21. Maximum differences between the temperature calculated from $T_2^{97}(p,h)$ and $T_{3b}^{03}(p,h)$ and from the IAPWS-IF97 basic equations $g_2^{97}(p,T)$ and $f_3^{97}(v,T)$

Region		Calculation of $\left|\Delta T\right|_{max}$	$\left|\Delta T\right|_{tol}$	$\left|\Delta T\right|_{max}$
2	$s \geq 5.85 \text{ kJ kg}^{-1}\text{ K}^{-1}$	$\left|\Delta T\right|_{max} = \left|T_2^{97}\left[p_2^{01}\left(h_{2ab}^{"}(s),s\right),h_{2ab}^{"}(s)\right]-T\right|_{max}$	10 mK	9.5 mK
	$s < 5.85 \text{ kJ kg}^{-1}\text{ K}^{-1}$	$\left|\Delta T\right|_{max} = \left|T_2^{97}\left[p_2^{01}\left(h_{2c3b}^{"}(s),s\right),h_{2c3b}^{"}(s)\right]-T\right|_{max}$	25 mK	24.1 mK
3		$\left|\Delta T\right|_{max} = \left|T_{3b}^{03}\left[p_{3b}\left(h_{2c3b}^{"}(s),s\right),h_{2c3b}^{"}(s)\right]-T\right|_{max}$	25 mK	6.2 mK

The maximum relative difference of specific volume between the IAPWS-IF97-S03 backward equation $v_{3b}^{03}(p,s)$ and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ along the boundary equation $h_{2c3b}^{"}(s)$, Eq. (6), is listed in Table 22.

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[6] The alternative calculation of the temperature on the saturated vapor line using the saturation-temperature equation $T_{sat}^{97}\left(p_2^{01}\left(h_{2ab}^{"}(s),s\right)\right)$, $T_{sat}^{97}\left(p_2^{01}\left(h_{2c3b}^{"}(s),s\right)\right)$ or $T_{sat}^{97}\left(p_{3b}\left(h_{2c3b}^{"}(s),s\right)\right)$ of IAPWS-IF97 leads to worse numerical consistency.

Table 22. Maximum relative difference between the specific volume calculated from
$v_{3b}^{03}(p,s)$ and from the IAPWS-IF97 basic equation $f_3^{97}(v,T)$

Region	Calculation of $\left|\Delta v / v\right|_{max}$	$\left|\Delta v / v\right|_{tol}$	$\left|\Delta v / v\right|_{max}$
3	$\left\|\dfrac{\Delta v}{v}\right\|_{max} = \left\|\dfrac{v_{3b}^{03}\left(h_{2c3b}^{"}(s),s\right)-v}{v}\right\|_{max}$	0.01 %	0.0041 %

The maximum pressure differences, the maximum temperature differences and the maximum relative difference of specific volume are smaller than the permissible values. Therefore, the accuracy of enthalpy calculated by the equations $h_{2ab}^{"}(s)$ and $h_{2c3b}^{"}(s)$, Eqs. (5) and (6), is sufficient.

4.5 Equation $h_{B13}(s)$ for Boundary between Regions 1 and 3

The equation $h_{B13}(s)$ describes the enthalpy as a function of entropy for the isotherm $T = 623.15\ \text{K}$ from the saturated liquid line up to 100 MPa. The corresponding entropy range is (see Figure 3):

$$s\left(100\ \text{MPa}, 623.15\ \text{K}\right) \leq s \leq s'\left(623.15\ \text{K}\right),$$

$$\text{where}\ \ s'\left(623.15\ \text{K}\right) = 3.778\,281\,340\ \text{kJ kg}^{-1}\,\text{K}^{-1}$$
$$\text{and}\ \ s\left(100\ \text{MPa}, 623.15\ \text{K}\right) = 3.397\,782\,955\ \text{kJ kg}^{-1}\,\text{K}^{-1}$$

calculated from the IAPWS-IF97 basic equation $g_1^{97}(p,T)$.

The Equation. The boundary equation $h_{B13}(s)$ has the following dimensionless form:

$$\frac{h_{B13}(s)}{h^*} = \eta_{B13}(\sigma) = \sum_{i=1}^{6} n_i\,(\sigma - 0.884)^{I_i}\,(\sigma - 0.864)^{J_i}\ , \tag{7}$$

where $\eta = h/h^*$ and $\sigma = s/s^*$, with $h^* = 1700\ \text{kJ kg}^{-1}$ and $s^* = 3.8\ \text{kJ kg}^{-1}\,\text{K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (7) are listed in Table 23.

Table 23. Coefficients and exponents of the equation
$h_{B13}(s)$ in its dimensionless form, Eq. (7)

i	I_i	J_i	n_i
1	0	0	0.913 965 547 600 543
2	1	−2	−0.430 944 856 041 991 × 10^{-4}
3	1	2	0.603 235 694 765 419 × 10^{2}
4	3	−12	0.117 518 273 082 168 × 10^{-17}
5	5	−4	0.220 000 904 781 292
6	6	−3	−0.690 815 545 851 641 × 10^{2}

Computer-program verification. To assist the user in computer-program verification of Eq. (7), Table 24 contains test values for calculated enthalpies.

Table 24. Selected enthalpy values calculated from Eq. (7)[a]

Equation	s / kJ kg^{-1} K^{-1}	h / kJ kg^{-1}
	3.7	$1.632\ 525\ 047 \times 10^3$
$h_{B13}(s)$, Eq. (7)	3.6	$1.593\ 027\ 214 \times 10^3$
	3.5	$1.566\ 104\ 611 \times 10^3$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

Numerical Consistency with the Basic Equations of IAPWS-IF97. Equation (7) does not exactly describe the isotherm $T = 623.15\ \text{K}$. The maximum temperature deviation was determined as:

$$\left|\Delta T\right|_{\text{max}} = \left|T_{\text{it}}\left(p, h_{B13}(s)\right) - T\right|_{\text{max}} = 3.20\ \text{mK}$$

where T_{it} was obtained by iteration using the IAPWS-IF97 basic equation $g_1^{97}(p,T)$. Therefore, the boundary equation $h_{B13}(s)$, Eq. (7), reproduces the isotherm $T = 623.15\ \text{K}$ within the tolerance of 25 mK.

Consistency of the Backward Equations with the Basic Equations of IAPWS-IF97 along $h_{B13}(s)$. The maximum pressure difference between the IAPWS-IF97-S01 backward equation $p_1^{01}(h,s)$ and the IAPWS-IF97 basic equation $g_1^{97}(p,T)$ along the boundary equation $h_{B13}(s)$, Eq. (7), is:

$$\left|\Delta p\right|_{\text{max}} = \left|p_1^{01}\left(h_{B13}(s), s\right) - p\right|_{\text{max}} = 14.4\ \text{kPa}.$$

The maximum temperature difference between the IAPWS-IF97 backward equation $T_1^{97}(p,h)$ and the IAPWS-IF97 basic equation $g_1^{97}(p,T)$ along the boundary equation $h_{B13}(s)$, Eq. (7), was determined as:

$$\left|\Delta T\right|_{\text{max}} = \left|T_1^{97}\left[p_1^{01}\left(h_{B13}(s), s\right), h_{B13}(s)\right] - T\right|_{\text{max}} = 24.2\ \text{mK}.$$

The maximum pressure difference is smaller than the permissible value of 15 kPa and the maximum temperature difference is smaller than the permissible value of 25 mK. Therefore, the accuracy of the boundary equation $h_{B13}(s)$, Eq. (7), is sufficient.

4.6   Equation $T_{B23}(h,s)$ for the Boundary between Regions 2 and 3

The boundary between regions 2 and 3 is defined by the B23-equation $p_{B23}^{97}(T)$ of IAPWS-IF97 (Figure 1). Since $p_{B23}^{97}(T)$ has an S-shape in the *h-s* plane, see Figure 4, it is not possible to develop equations $h(s)$ or $s(h)$ that meet the numerical consistency requirements. Therefore, an equation $T_{B23}(h,s)$ for the boundary between regions 2 and 3 has been developed. The equation $T_{B23}(h,s)$ and the IAPWS-IF97-S01 backward equation $p_{2c}^{01}(h,s)$ simplify the determination of the region for given variables *h* and *s*.
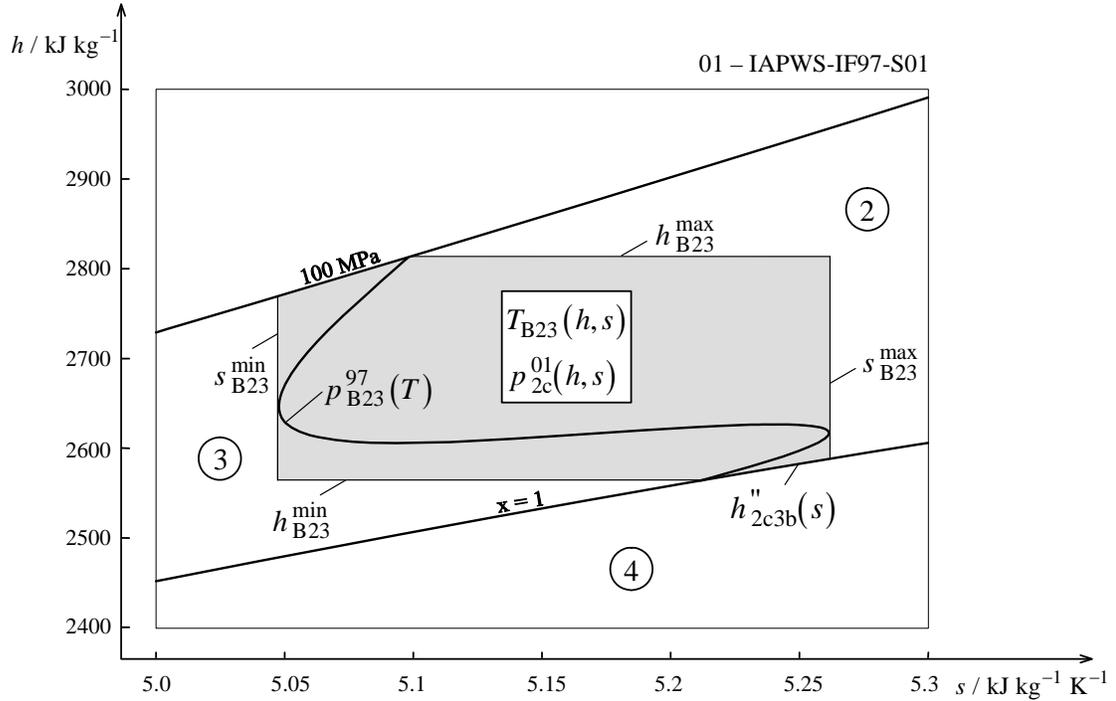
Figure 4.   Illustration of the B23-equation $p_{B23}^{97}(T)$ of IAPWS-IF97 and the range of validity of the boundary equation $T_{B23}(h,s)$ in an *h-s* diagram

The range of validity of the equation $T_{B23}(h,s)$ is from the saturated vapor line $x=1$ up to 100 MPa in the entropy range

$$s_{B23}^{\min} \le s \le s_{B23}^{\max},$$

where $s_{B23}^{\min}= 5.048\,096\,828 \text{ kJ kg}^{-1}\text{ K}^{-1}$

and $s_{B23}^{\max}= 5.260\,578\,707 \text{ kJ kg}^{-1}\text{ K}^{-1}$,

and in the enthalpy range

$$h_{B23}^{\min} \le h \le h_{B23}^{\max},$$

where $h_{B23}^{\min}= h''(623.15\text{ K}) = 2.563\,592\,004 \times 10^3 \text{ kJ kg}^{-1}$

and $h_{B23}^{\max}= h(100\text{ MPa}, 863.15\text{ K}) = 2.812\,942\,061 \times 10^3 \text{ kJ kg}^{-1}$,

see Figure 4. The saturated vapor line can be calculated by the equation $h''_{2c3b}(s)$, Eq. (5).

The Equation.   The equation $T_{B23}(h,s)$ has the following dimensionless form:

$$\frac{T_{B23}(h,s)}{T^*} = \theta_{B23}(\eta,\sigma) = \sum_{i=1}^{25} n_i (\eta - 0.727)^{I_i} (\sigma - 0.864)^{J_i} \ , \tag{8}$$

where $\theta = T/T^*$, $\eta = h/h^*$, and $\sigma = s/s^*$, with $T^* = 900$ K, $h^* = 3000$ kJ kg^{-1}, and $s^* = 5.3$ kJ kg^{-1} K^{-1}. The coefficients n_i and exponents I_i and J_i of Eq. (8) are listed in Table 25.

Table 25. Coefficients and exponents of the boundary equation $T_{B23}(h,s)$ in its dimensionless form, Eq. (8)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	10	$0.629\ 096\ 260\ 829\ 810 \times 10^{-3}$	14	3	−2	$0.149\ 276\ 502\ 463\ 272$
2	−10	8	$-0.823\ 453\ 502\ 583\ 165 \times 10^{-3}$	15	3	−1	$0.698\ 733\ 471\ 798\ 484$
3	−8	3	$0.515\ 446\ 951\ 519\ 474 \times 10^{-7}$	16	5	−5	$-0.252\ 207\ 040\ 114\ 321 \times 10^{-1}$
4	−4	4	$-0.117\ 565\ 945\ 784\ 945 \times 10^{1}$	17	6	−6	$0.147\ 151\ 930\ 985\ 213 \times 10^{-1}$
5	−3	3	$0.348\ 519\ 684\ 726\ 192 \times 10^{1}$	18	6	−3	$-0.108\ 618\ 917\ 681\ 849 \times 10^{1}$
6	−2	−6	$-0.507\ 837\ 382\ 408\ 313 \times 10^{-11}$	19	8	−8	$-0.936\ 875\ 039\ 816\ 322 \times 10^{-3}$
7	−2	2	$-0.284\ 637\ 670\ 005\ 479 \times 10^{1}$	20	8	−2	$0.819\ 877\ 897\ 570\ 217 \times 10^{2}$
8	−2	3	$-0.236\ 092\ 263\ 939\ 673 \times 10^{1}$	21	8	−1	$-0.182\ 041\ 861\ 521\ 835 \times 10^{3}$
9	−2	4	$0.601\ 492\ 324\ 973\ 779 \times 10^{1}$	22	12	−12	$0.261\ 907\ 376\ 402\ 688 \times 10^{-5}$
10	0	0	$0.148\ 039\ 650\ 824\ 546 \times 10^{1}$	23	12	−1	$-0.291\ 626\ 417\ 025\ 961 \times 10^{5}$
11	1	−3	$0.360\ 075\ 182\ 221\ 907 \times 10^{-3}$	24	14	−12	$0.140\ 660\ 774\ 926\ 165 \times 10^{-4}$
12	1	−2	$-0.126\ 700\ 045\ 009\ 952 \times 10^{-1}$	25	14	1	$0.783\ 237\ 062\ 349\ 385 \times 10^{7}$
13	1	10	$-0.122\ 184\ 332\ 521\ 413 \times 10^{7}$				

Computer-program verification.   To assist the user in computer-program verification of Eq. (8), Table 26 contains test values for calculated temperatures.

Table 26. Selected temperature values calculated from Eq. (8)[a]

Equations	h / kJ kg^{-1}	s / kJ kg^{-1} K^{-1}	T / K
	2600	5.1	$7.135\ 259\ 364 \times 10^{2}$
$T_{B23}(h,s)$, Eq. (8)	2700	5.15	$7.685\ 345\ 532 \times 10^{2}$
	2800	5.2	$8.176\ 202\ 120 \times 10^{2}$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

Use of the Equation.   The equation $T_{B23}(h,s)$ and the IAPWS-IF97-S01 backward equation $p_{2c}^{01}(h,s)$ can be used to determine whether a given state point (h,s) is located in region 2 or in region 3.

For entropies $s_{B23}^{min} < s < s_{B23}^{max}$ and enthalpies $h_{B23}^{min} < h < h_{B23}^{max}$, the following steps should be performed:

- Calculate temperature T_{B23} from the given h and s using equation $T_{B23}(h, s)$, Eq. (8).

- Calculate pressure p_{B23} from the given h and s using the IAPWS-IF97-S01 backward equation $p_{2c}^{01}(h, s)$.

- Calculate pressure p_{B23}^{97} using the B23-equation $p_{B23}^{97}(T)$ of IAPWS-IF97 where $T = T_{B23}$ was previously calculated.
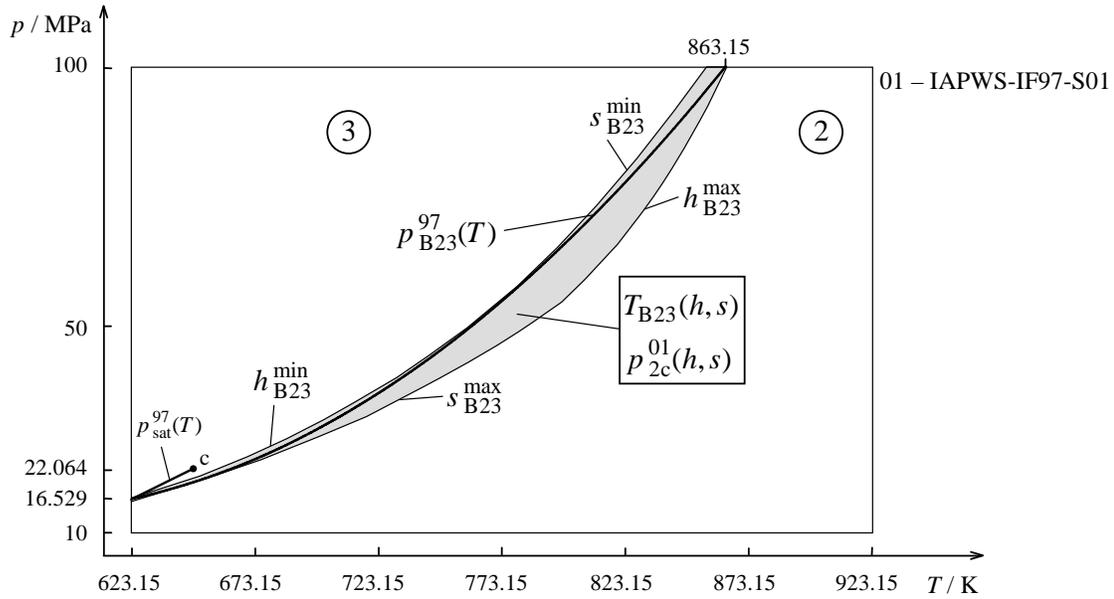
Figure 5. Illustration of the B23-equation $p_{B23}^{97}(T)$ of IAPWS-IF97 and the range of validity of the boundary equation $T_{B23}(h, s)$ in a p-T diagram

Equation $T_{B23}(h, s)$ should only be used to determine the region for a given state point at the boundary between regions 2 and 3.

Numerical Consistency with the B23-Equation of IAPWS-IF97. The maximum relative deviation between the calculated pressure $p_{2c}^{01}(h, s)$ and the IAPWS-IF97 B23-equation $p_{B23}^{97}(T)$, where $T = T_{B23}(h, s)$, has the following value

$$\left| \frac{\Delta p}{p} \right|_{max} = \left| \frac{p_{2c}^{01}(h, s) - p_{B23}^{97}\left(T_{B23}(h, s)\right)}{p_{B23}^{97}\left(T_{B23}(h, s)\right)} \right|_{max} = 0.0045\ \% .$$

Thus, the combination of the equations $T_{B23}(h, s)$ and $p_{2c}^{01}(h, s)$ reproduces the B23-boundary of IAPWS-IF97 within the maximum allowed value 0.0088 %.

Consistency of the Backward Equations with the Basic Equations of IAPWS-IF97 along the B23-Boundary. The maximum relative pressure difference between the IAPWS-IF97-S01 backward equation $p_{2c}^{01}(h,s)$ and the IAPWS-IF97 basic equation $g_2^{97}(p,T)$ along the boundary calculated as explained in *Use of the Equation* was determined as:

$$\left|\frac{\Delta p}{p}\right|_{\max} = \left|\frac{p_{2c}^{01}(h,s)-p}{p}\right|_{\max} = 0.0029\ \%\ .$$

The absolute maximum temperature difference between the IAPWS-IF97 backward equation $T_{2c}^{97}(p,h)$ and the IAPWS-IF97 basic equation $g_2^{97}(p,T)$ along the boundary has the value

$$|\Delta T|_{\max} = \left|T_{2c}^{97}\left(p_{2c}^{01}(h,s),h\right)-T\right|_{\max} = 24.1\ \text{mK}\ .$$

The maximum relative pressure difference is smaller than the permissible value of 0.0088 % and the maximum temperature difference is smaller than the permissible value of 25 mK. Therefore, the combination of the equations $T_{B23}(h,s)$, Eq. (8), and $p_{2c}^{01}(h,s)$ of IAPWS-IF97-S01 is sufficiently accurate to determine the region for a given state point (h,s).

4.7 Computing Time in Relation to IAPWS-IF97

A very important motivation for the development of the equations for region boundaries was reducing the computing time to determine the region for a given state point (h,s). By using backward equations, Eqs. (3) to (8), users can determine the region without time-consuming iterations, which require repeated evaluation of forward functions of IAPWS-IF97.

5 Backward Functions $T_{\text{sat}}(h,s)$, $p_{\text{sat}}(h,s)$, and $x(h,s)$ for the Two-Phase Region

5.1 Calculation of Saturation Properties from a Given Enthalpy and Entropy

In modeling power cycles and steam turbines, thermodynamic properties as a function of the variables (h,s) are required in the two-phase (wet steam) region. The important region for steam turbine calculations is the range $s \geq s''(623.15\ \text{K})$, see Figure 6, where the saturation temperature is less than 623.15 K and the two-phase region is located between regions 1 and 2. In this region, the calculation of saturation properties from a given h and s requires iteration using the IAPWS-IF97 basic equations $g_1^{97}(p,T)$ and $g_2^{97}(p,T)$ and the saturation-pressure equation $p_{\text{sat}}^{97}(T)$. In order to avoid such iterations, this supplementary release provides an equation $T_{\text{sat}}(h,s)$ for the two-phase region.
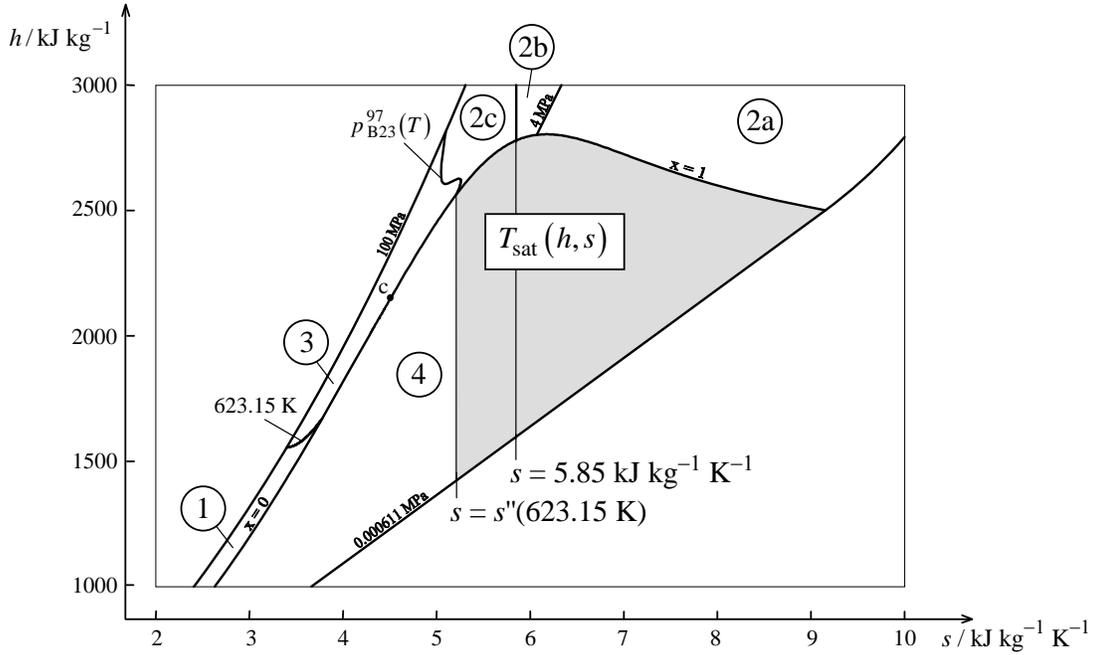
Figure 6. Regions and subregions of IAPWS-IF97 and the backward equation $T_{\mathrm{sat}}(h,s)$ of this release

5.2 Numerical Consistency Requirements

For the backward function $T_{\mathrm{sat}}(h,s)$, the IAPWS requirements for the IAPWS-IF97 backward equations $T_2^{97}(p,h)$ and $T_2^{97}(p,s)$ in the subregions 2a, 2b, and 2c [9] were taken as acceptable values for the numerical consistency. Table 27 contains these values.

The permissible values $|\Delta p/p|_{\mathrm{tol}}$ for the numerical consistency of the backward function $p_{\mathrm{sat}}(h,s)$ result from the requirements on the IAPWS-IF97-S01 backward equation $p_2^{01}(h,s)$ in the subregions 2a, 2b, and 2c [4, 5]. Table 27 shows the relative tolerances for the backward function $p_{\mathrm{sat}}(h,s)$.

Table 27. Permissible values $|\Delta T|_{\mathrm{tol}}$ of [9] for the function $T_{\mathrm{sat}}(h,s)$ and relative tolerances $|\Delta p/p|_{\mathrm{tol}}$ of [4, 5] for the function $p_{\mathrm{sat}}(h,s)$

Entropy Range (Fig. 6)	$|\Delta T|_{\mathrm{tol}}$	$|\Delta p/p|_{\mathrm{tol}}$
$s \geq 5.85\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$	10 mK	0.0035 %
$s < 5.85\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$	25 mK	0.0088 %

5.3 Backward Equation $T_{sat}(h,s)$

Range of Validity. The range of validity of the backward equation $T_{sat}(h,s)$ is the two-phase region for $s \geq s''(623.15\,\text{K})$ with $s''(623.15\,\text{K}) = 5.210\,887\,825\,\text{kJ kg}^{-1}\,\text{K}^{-1}$ (see Fig. 6). The corresponding temperature range is $273.15\,\text{K} \leq T \leq 623.15\,\text{K}$.

The Equation. The backward equation $T_{sat}(h,s)$ for the two-phase region has the following dimensionless form:

$$\frac{T_{sat}(h,s)}{T^*} = \theta_{sat}(\eta,\sigma) = \sum_{i=1}^{36} n_i \left(\eta - 0.119\right)^{I_i} \left(\sigma - 1.07\right)^{J_i}, \tag{9}$$

where $\theta = T/T^*$, $\eta = h/h^*$, and $\sigma = s/s^*$, with $T^* = 550\,\text{K}$, $h^* = 2800\,\text{kJ kg}^{-1}$, and $s^* = 9.2\,\text{kJ kg}^{-1}\,\text{K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (9) are listed in Table 28.

Table 28. Coefficients and exponents of the equation $T_{sat}(h,s)$ in its dimensionless form, Eq. (9)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	0.179 882 673 606 601	19	5	4	0.125 028 363 714 877 × 10^1
2	0	3	−0.267 507 455 199 603	20	5	16	0.101 316 840 309 509 × 10^4
3	0	12	0.116 276 722 612 600 × 10^1	21	6	6	−0.151 791 558 000 712 × 10^1
4	1	0	0.147 545 428 713 616	22	6	8	0.524 277 865 990 866 × 10^2
5	1	1	−0.512 871 635 973 248	23	6	22	0.230 495 545 563 912 × 10^5
6	1	2	0.421 333 567 697 984	24	8	1	0.249 459 806 365 456 × 10^{-1}
7	1	5	0.563 749 522 189 870	25	10	20	0.210 796 467 412 137 × 10^7
8	2	0	0.429 274 443 819 153	26	10	36	0.366 836 848 613 065 × 10^9
9	2	5	−0.335 704 552 142 140 × 10^1	27	12	24	−0.144 814 105 365 163 × 10^9
10	2	8	0.108 890 916 499 278 × 10^2	28	14	1	−0.179 276 373 003 590 × 10^{-2}
11	3	0	−0.248 483 390 456 012	29	14	28	0.489 955 602 100 459 × 10^{10}
12	3	2	0.304 153 221 906 390	30	16	12	0.471 262 212 070 518 × 10^3
13	3	3	−0.494 819 763 939 905	31	16	32	−0.829 294 390 198 652 × 10^{11}
14	3	4	0.107 551 674 933 261 × 10^1	32	18	14	−0.171 545 662 263 191 × 10^4
15	4	0	0.733 888 415 457 688 × 10^{-1}	33	18	22	0.355 777 682 973 575 × 10^7
16	4	1	0.140 170 545 411 085 × 10^{-1}	34	18	36	0.586 062 760 258 436 × 10^{12}
17	5	1	−0.106 110 975 998 808	35	20	24	−0.129 887 635 078 195 × 10^8
18	5	2	0.168 324 361 811 875 × 10^{-1}	36	28	36	0.317 247 449 371 057 × 10^{11}

Computer-program verification. To assist the user in computer-program verification of Eq. (9), Table 29 contains test values for calculated temperatures.

Table 29. Selected temperature values calculated from Eq. (9)[a]

Equation	h / kJ kg^{-1}	s / kJ kg^{-1} K^{-1}	T / K
	1800	5.3	$3.468\ 475\ 498 \times 10^2$
$T_{\text{sat}}(h,s)$, Eq. (9)	2400	6.0	$4.251\ 373\ 305 \times 10^2$
	2500	5.5	$5.225\ 579\ 013 \times 10^2$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

Numerical Consistency with the Saturation-Temperature Equation of IAPWS-IF97. The maximum deviations and related root-mean-square differences between the calculated temperature, Eq. (9), and the IAPWS-IF97 saturation-temperature equation $T_{\text{sat}}^{97}(p)$ were calculated as follows:

$$\left|\Delta T_{\text{sat}}\right|_{\text{max}} = \left|T_{\text{sat}}(h,s) - T_{\text{sat}}^{97}(p)\right|_{\text{max}}.$$

The maximum differences are listed in Table 30. The calculation of the root-mean-square value is described in Section 1.

Table 30. Maximum differences and related root-mean-square differences between the temperature calculated from Eq. (9) and from the IAPWS-IF97 saturation-temperature equation $T_{\text{sat}}^{97}(p)$

Entropy Range	$\left|\Delta T\right|_{\text{tol}}$	$\left|\Delta T\right|_{\text{max}}$	$\left|\Delta T\right|_{\text{RMS}}$
$s \geq 5.85$ kJ kg^{-1} K^{-1}	10 mK	0.67 mK	0.33 mK
$s < 5.85$ kJ kg^{-1} K^{-1}	25 mK	0.86 mK	0.45 mK

5.4 Backward Function $p_{\text{sat}}(h,s)$

Calculation of the Backward Function $p_{sat}(h,s)$.   To calculate the saturation pressure as a function of enthalpy and entropy, the following steps should be performed.

- Calculate temperature T_{sat} from the given h and s using the backward equation $T_{\text{sat}}(h,s)$, Eq. (9), for the two-phase region.

- Calculate saturation pressure p_{sat} using the IAPWS-IF97 saturation-pressure equation $p_{\text{sat}}^{97}(T)$, where $T = T_{\text{sat}}$ was previously calculated.

Numerical Consistency with the Saturation-Pressure Equation of IAPWS-IF97.   The maximum relative deviations and root-mean-square relative deviations between the calculated pressure $p_{\text{sat}}(h,s)$ and the IAPWS-IF97 saturation-pressure equation $p_{\text{sat}}^{97}(T)$ were calculated as follows:

$$\left|\frac{\Delta p_{\text{sat}}}{p_{\text{sat}}}\right|_{\text{max}} = \left|\frac{p_{\text{sat}}^{97}(T_{\text{sat}}(h,s)) - p_{\text{sat}}^{97}(T)}{p_{\text{sat}}^{97}(T)}\right|_{\text{max}}.$$

The resulting maximum relative deviations and root-mean-square relative deviations are listed in Table 31.

Table 31. Maximum relative deviations and root-mean-square relative deviations between the pressure $p_{\text{sat}}(h,s)$ and the IAPWS-IF97 saturation-pressure equation $p_{\text{sat}}^{97}(T)$

Entropy Range	$\left\lvert\dfrac{\Delta p}{p}\right\rvert_{\text{tol}}$	$\left\lvert\dfrac{\Delta p}{p}\right\rvert_{\text{max}}$	$\left\lvert\dfrac{\Delta p}{p}\right\rvert_{\text{RMS}}$
$s \geq 5.85\ \text{kJ kg}^{-1}\ \text{K}^{-1}$	0.0035 %	0.0029 %	0.0012 %
$s < 5.85\ \text{kJ kg}^{-1}\ \text{K}^{-1}$	0.0088 %	0.0034 %	0.0013 %

Table 31 shows that the maximum relative deviations are smaller than the tolerances. Therefore, the accuracy of the temperature calculated by the equation $T_{\text{sat}}(h,s)$, Eq. (9), is sufficient for calculating pressure as a function of h and s.

5.5 Backward Function $x(h,s)$

Calculation of the Backward Function x(h,s). To calculate the vapor fraction as a function of enthalpy and entropy, the following steps should be performed.

- Calculate temperature T_{sat} from the given h and s using the backward equation $T_{\text{sat}}(h,s)$, Eq. (9), for the two-phase region.

- Calculate saturation pressure p_{sat} using the IAPWS-IF97 saturation-pressure equation $p_{\text{sat}}^{97}(T)$, where $T = T_{\text{sat}}$ was previously calculated.

- Calculate the vapor fraction x using the relation $x = (h - h')/(h'' - h')$[7], where $h' = h_1^{97}(p_{\text{sat}}, T_{\text{sat}})$ and $h'' = h_2^{97}(p_{\text{sat}}, T_{\text{sat}})$.

Numerical Consistency with the Basic Equations of IAPWS-IF97. The maximum deviations and related root-mean-square deviations between the calculated vapor fraction $x(h,s)$ and vapor fractions calculated from the IAPWS-IF97 basic equations $g_1^{97}(p,T)$ and $g_2^{97}(p,T)$ were calculated as follows:

$$|\Delta x|_{\text{max}} = |x(h,s) - x|_{\text{max}} \quad \text{with } x = \frac{h - h_1^{97}\left(p_{\text{sat}}^{97}(T), T\right)}{h_2^{97}\left(p_{\text{sat}}^{97}(T), T\right) - h_1^{97}\left(p_{\text{sat}}^{97}(T), T\right)}.$$

The maximum deviations and root-mean-square deviations are listed in Table 32.

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[7] The calculation of the vapor fraction using the saturated liquid entropy s' and the saturated vapor entropy s'' leads to worse numerical consistency.

Table 32. Maximum differences of vapor fraction $x(h,s)$ from the IAPWS-IF97 basic equations $g_1^{97}(p,T)$ and $g_2^{97}(p,T)$.

Entropy Range	$\|\Delta x\|_{\mathrm{max}}$	$\|\Delta x\|_{\mathrm{RMS}}$
$s \geq 5.85\,\mathrm{kJ\ kg^{-1}\ K^{-1}}$	0.64×10^{-6}	0.25×10^{-6}
$s < 5.85\,\mathrm{kJ\ kg^{-1}\ K^{-1}}$	4.40×10^{-6}	0.57×10^{-6}

Table 32 shows that the vapor fraction $x(h,s)$ can be calculated to 5 digits of accuracy. Therefore, the accuracy of temperature calculated by the equation $T_{\mathrm{sat}}(h,s)$, Eq. (9), is sufficient for calculating vapor fraction as a function of h and s.

5.6 Computing Time in Relation to IAPWS-IF97

A very important motivation for the development of the backward equation $T_{\mathrm{sat}}(h,s)$ was reducing the computing time to obtain thermodynamic properties from given variables (h,s) in the two-phase region. In IAPWS-IF97, time-consuming iterations are required. Using the $T_{\mathrm{sat}}(h,s)$ equation combined with the IAPWS-IF97 saturation-pressure equation $p_{\mathrm{sat}}^{97}(T)$, and the IAPWS-IF97 basic equations $g_1^{97}(p,T)$ and $g_2^{97}(p,T)$, the calculation to obtain p, T and x as a function of h and s is about 11 times faster than the iterative method.

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