# CS 7800: Advanced Algorithms

Class 1: Intoduction + Stable Matching

Jonathan Ullman September 5, 2025

#### Me

#### Jonathan Ullman

- Feel free to call me Jon
- Research: Foundations of Trustworthy Al and Statistics
- Office: 177 Huntington 616
- Office Hours:
  - Tricky because I'm in 177
  - Will poll for a good time
  - Always available by appt





#### The TA Team

#### John Abascal

- Will help us part-time
- He has an adorable sausage dog
- Office: 177 Huntington 6<sup>th</sup> Floor



## Algorithms

#### What is an algorithm?

An explicit, precise, unambiguous, mechanicallyexecutable sequence of elementary instructions for solving a computational problem. -Jeff Erickson

## Algorithms

What is algorithms (the subfield of CS)?

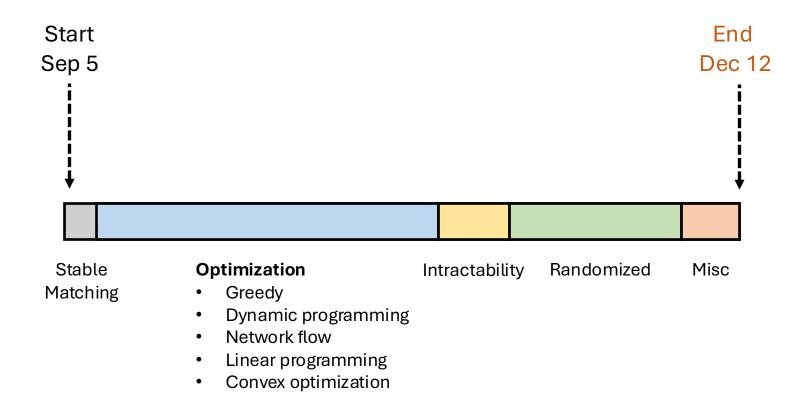
The rigorous mathematical study of computational problems and the algorithms for solving them.

## Algorithms

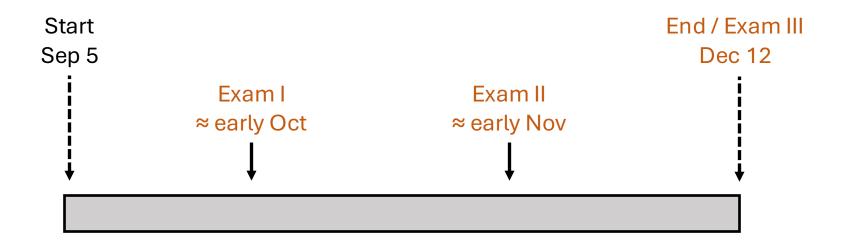
#### What is CS 7800: Advanced Algorithms?

- (1) An overview of the most fundamental algorithms and techniques that we believe every PhD computer scientist should know.
- (2) A mental workout to help you develop analytic and mathematical reasoning and communication skills for computer science research.

#### Course Structure



#### Course Structure



#### **Evaluation:**

- 3x exams = 75%
  - Not cumulative but the material builds on itself
- 6x assignments = 25%
  - Drop the lowest score

#### Grading:

- Standard scale (e.g. A/A- is 90%+)
- Generously curved as needed
- Typical distribution:
  - 50% get A/A-, 50% get B+/B
  - I'm more generous with small classes

#### Course Website

http://jonathan-ullman.github.io/cs7800-f25

Home Course Info Schedule

# CS 7800: Advanced Algorithms Fall 2025

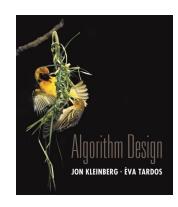
#### Course Schedule

This schedule will be updated continuously throughout the term.

Date	Торіс	Reading	Notes
Fri 09/05/25	<ul><li>Class 1: Introduction</li><li>Course Overview</li><li>Stable Matching</li></ul>	_	HW0 Out: [pdf] [tex]
	[slides after]		
Tue 09/09/25	Class 2: Greedy Algorithms  • Interval Scheduling  • Minimizing Lateness	KT 4.1-4.2	_
	[slides before] [slides after]		

#### Recommended Resources

- Algorithm Design by Kleinberg and Tardos
  - We'll follow this closely in the 1<sup>st</sup> half
  - Can easily find copies



- Algorithms by Jeff Erickson
  - Useful for review, alternate perspective, and some advanced topics
  - Will use this more in the 2<sup>nd</sup> half
  - Free on the web

#### **Algorithms**



Jeff Erickson

## Assignments

- 6 HW Assignments (probably)
  - Approximately every two weeks
  - Late days: total of 5, max of 2 per assignment
  - Further extensions granted for special circumstances
- All questions are algorithms and related mathematics, no programming
- Review HW0 out now, due Friday 9/12 at 11:59pm!
  - No late days—I want to quickly test your background

## Assignment Philosophy/Policies

- This course has two related-yet-different goals
  - #1: give a working knowledge of algorithms (everyone has to)
  - #2: exercise and stretch your brain (you get out what you put in)
- Exams are for #1 and are most of the evaluation
- Homework is to prepare you for exams and for #2
  - A few assigned/graded problems so you get feedback
  - More optional/ungraded problems so you can get exercise
- Al/Honestly Policy: You're adults and scholars, act like it
  - You can easily ace the assignments using AI, I can't reliably stop you
  - Using AI won't prepare you for exams, which are most of your grade
  - Using AI won't make you a better scholar
  - Using Al wastes my time giving feedback

## **Assignment Logistics**

- Homework must be typeset in LaTeX!
  - You'll have to learn it sometime!
  - Many good resources available
  - Many good editors available (Overleaf, TexStudio)
  - I will provide source to get you started

The Not So Short Introduction to LATEX  $2\varepsilon$ 

Or LATEX 2E in 157 minutes

## **Assignment Logistics**

- I use Gradescope for homework
  - Entry code: D3ERDX



#### **Discussion Forum**

I've used Piazza in the past but I'm open minded!

## Introductions!

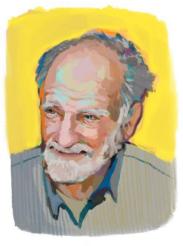
# Stable Matching

## National Residency Matching Program

- National system for matching US medical school graduates to medical residencies
  - Roughly 40,000 doctors per year
  - Assignment is almost entirely algorithmic



David Gale (1921-2008) PROFESSOR, UC BERKELEY



Lloyd Shapley
PROFESSOR EMERITUS, UCLA



Alvin Roth
PROFESSOR, STANFORD

# (Centralized) Labor Markets

# Matchings

	1st	2nd	3rd	4th	5th
MGH	Bob	Alice	Dorit	Ernie	Clara
BW	Dorit	Bob	Alice	Clara	Ernie
BID	Bob	Ernie	Clara	Dorit	Alice
МТА	Alice	Dorit	Clara	Bob	Ernie
СН	Bob	Dorit	Alice	Ernie	Clara

	1st	2nd	3rd	4th	5th
Alice	СН	MGH	BW	MTA	BID
Bob	BID	BW	MTA	MGH	СН
Clara	BW	BID	MTA	СН	MGH
Dorit	MGH	СН	MTA	BID	BW
Ernie	MTA	BW	СН	BID	MGH

# Matchings

# Stable Matchings

#### Ask the Audience

• Either find a stable matching or convince yourself that there is no stable matching

	1st	2nd	3rd
MGH	Alice	Bob	Clara
BW	Bob	Clara	Alice
BID	Alice	Clara	Bob

	1st	2nd	3rd
Alice	BW	BID	MGH
Bob	BW	MGH	BID
Clara	MGH	BID	BW

# Gale-Shapley Algorithm

# Gale-Shapley Demo

	1st	2nd	3rd	4th	5th
MGH	Bob	Alice	Dorit	Ernie	Clara
BW	Dorit	Bob	Alice	Clara	Ernie
BID	Bob	Ernie	Clara	Dorit	Alice
MTA	Alice	Dorit	Clara	Bob	Ernie
СН	Bob	Dorit	Alice	Ernie	Clara

	1st	2nd	3rd	4th	5th
Alice	СН	MGH	BW	MTA	BID
Bob	BID	BW	MTA	MGH	СН
Clara	BW	BID	MTA	СН	MGH
Dorit	MGH	СН	MTA	BID	BW
Ernie	MTA	BW	СН	BID	MGH

#### Observations

# Gale-Shapley Algorithm: Analysis

# Gale-Shapley Algorithm: Analysis

# Gale-Shapley Algorithm: Analysis

## Real World Impact

TABLE I
STABLE AND UNSTABLE (CENTRALIZED) MECHANISMS

Market	Stable	Still in use (halted unraveling)
American medical markets		
NRMP	yes	yes (new design in '98)
Medical Specialties	yes	yes (about 30 markets)
British Regional Medical Markets		
Edinburgh ('69)	yes	yes
Cardiff	yes	yes
Birmingham	no	no
Edinburgh ('67)	no	no
Newcastle	no	no
Sheffield	no	no
Cambridge	no	yes
London Hospital	no	yes
Other healthcare markets		
Dental Residencies	yes	yes
Osteopaths (<'94)	no	no
Osteopaths (≥'94)	yes	yes
Pharmacists	yes	yes
Other markets and matching process	ses	
Canadian Lawyers	yes	yes (except in British Columbia since 1996)
Sororities	yes (at equilibrium)	yes

Table 1. Reproduced from Roth (2002, Table 1).

## Real World Challenges

- Doctors ↔ Hospitals
  - Have to deal with two-body problems
  - Have to make sure doctors do not game the system
- Kidneys ↔ Patients
  - Not all matches are feasible (blood types, immunity)
  - Certain pairs must be matched
- Students ↔ Public Schools
  - Siblings, walking zones, diversity
- Rabbis ↔ Synagogues
  - No idea why, just a fun example

