HW4

0313405_呂京祐_資財 07

Prove that $MD_{floating} = MD_{Fix} - \sum_{i=j+1}^{n} \frac{1}{(1+y)^{i-1}}$

where the bond is priced at par, and the principal be \$1 for simplicity.

$$P = \frac{c}{(1+y)} + \frac{c}{(1+y)^2} + \cdots + \frac{c}{(1+y)^j} + \frac{y}{(1+y)^{(j+1)}} + \cdots + \frac{y}{(1+y)^n} + \frac{P}{(1+y)^n}$$

$$\frac{\partial P}{\partial y} = \frac{-c}{(1+y)^2} + \frac{-2c}{(1+y)^3} + \cdots + \frac{-jc}{(1+y)^{(j+1)}} + \left[\frac{1}{(1+y)^{(j+1)}} + \cdots + \frac{1}{(1+y)^n}\right] - y\left[\frac{j+1}{(1+y)^{(j+2)}} + \cdots + \frac{n-1}{(1+y)^{(n+1)}}\right] + \frac{-nP}{(1+y)^{(n+1)}}$$

$$\frac{-(1+y)}{P}\frac{\partial P}{\partial y} = \frac{1}{P}\left\{\left[\frac{c}{(1+y)} + \frac{2c}{(1+y)^2} + \cdots + \frac{jc}{(1+y)^j}\right] - \left[\frac{1}{(1+y)^j} + \cdots + \frac{1}{(1+y)^{(n-1)}}\right] + y\left[\frac{j+1}{(1+y)^{(j+1)}} + \cdots + \frac{n-1}{(1+y)^n}\right] + \frac{nP}{(1+y)^n}\right\}$$

where the bond is priced at par, and the principal be \$1, then $c = y$, $P = 1$

$$MD_{floating} = \sum_{i=1}^{j} \frac{ic}{(1+y)^i} + \sum_{i=j+1}^{n} \frac{ic}{(1+y)^i} + \frac{n}{(1+y)^n} - \sum_{i=j+1}^{n} \frac{1}{(1+y)^{(i-1)}}$$

$$MD_{floating} = \sum_{i=1}^{n} \frac{ic}{(1+y)^i} + \sum_{i=j+1}^{n} \frac{ic}{(1+y)^i} + \frac{n}{(1+y)^n} - \sum_{i=j+1}^{n} \frac{1}{(1+y)^{(i-1)}}$$

$$MD_{floating} = \sum_{i=1}^{n} \frac{ic}{(1+y)^i} + \frac{n}{(1+y)^n} - \sum_{i=j+1}^{n} \frac{1}{(1+y)^{(i-1)}}$$

$$MD_{floating} = MD_{Fix} - \sum_{i=j+1}^{n} \frac{1}{(1+y)^{(i-1)}}$$