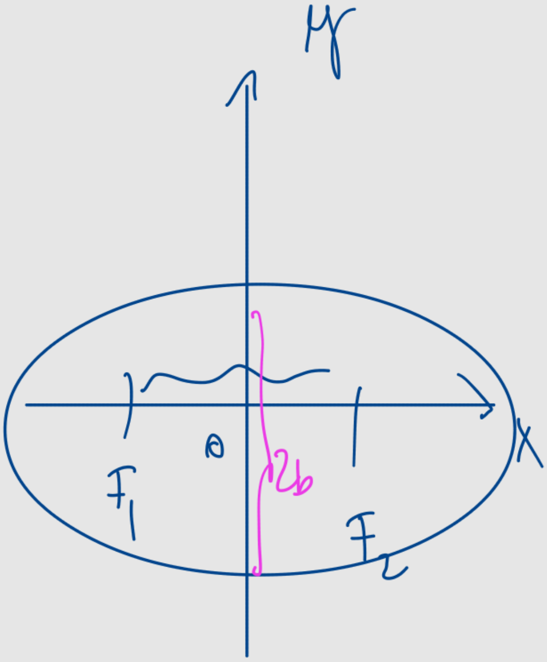
Partial 2

① Focal on X-axis:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$e = \frac{c}{a} = \frac{12}{13} \Rightarrow c = \frac{12a}{13}$$

$$2b = 10 \Rightarrow b = 5$$

$$c^2 = a^2 - b^2$$

$$\frac{144a^2}{169} = a^2 - 25 \Rightarrow a = 13 \Rightarrow c = 12$$

For x axis we get:

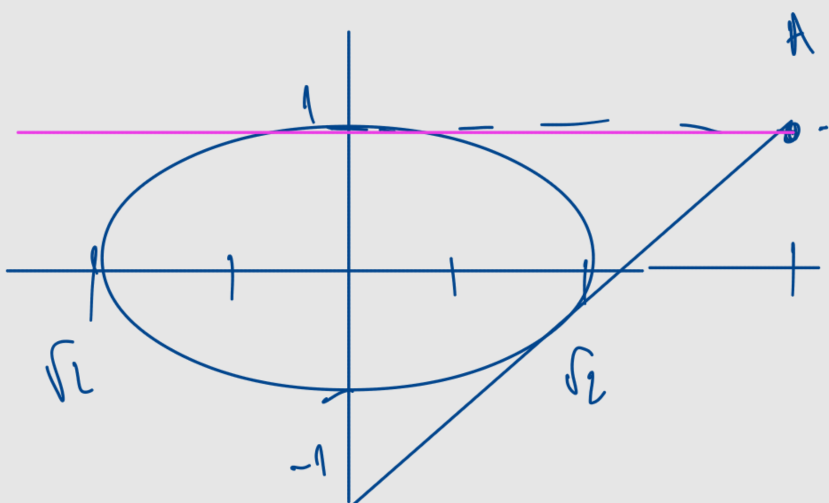
$$\frac{x^2}{169} + \frac{y^2}{25} = 1$$

Since we need to solve on y-axis, rotating means switching a, b:

$$\frac{x^2}{25} + \frac{y^2}{169} = 1$$

2

$$\frac{x^2}{2} + y^2 = 1$$



First tangent is easily proven : $\boxed{y=1}$

Second tangent:

$l_1 : y = ax + b$

① $(4,1) \in l_1 \Rightarrow 4 = a + b \Rightarrow b = 4 - a$

$y = ax + 4 - a$

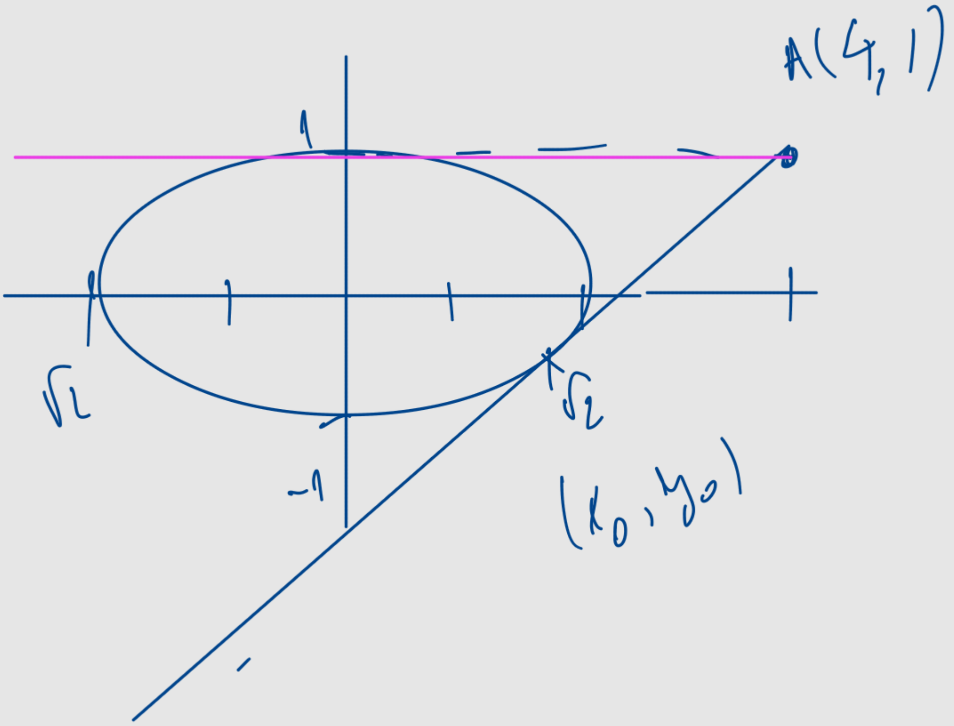
②

$l_1 \in \frac{x^2}{2} + y^2 = 1 \quad \exists \; (x, ax + 4 - a) \in \mathcal{E}$

Substituting back we get a 2nd order equation and we set det $= 0$ and find $a_{1,2}$, hence the solution of the problem.

$\boxed{OR}$ using partial derivatives

$$\frac{x^2}{2} + y^2 = 1$$



$$\frac{d}{dx}\left(\frac{x^2}{2} + y^2\right) = \frac{d}{dx}(1)$$

$$x + 2y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{}$$

dx $\quad 2y$

Slope at $A(4,1)$:

$$\frac{y-1}{x-4} = \frac{-x}{2y} \iff 2y^2 - 2y = -x^2 + 4x \quad (1)$$

$$\frac{x^2}{2} + y^2 = 1$$

$$y^2 = 1 - \frac{x^2}{2} \quad (2)$$

$$y = \pm\sqrt{1 - \frac{x^2}{2}}$$

$$(1)+(2) \Rightarrow 2\left(1 - \frac{x^2}{2}\right) - 2\left(\pm\sqrt{1-\frac{x^2}{2}}\right) = -x^2 + 4x$$

$$2 \cancel{-x^2} \pm 2\sqrt{1-\frac{x^2}{2}} = \cancel{-x^2} + 4x$$

$$1 \pm \sqrt{1-\frac{x^2}{2}} = 2x$$

$\Rightarrow \mid x = \frac{8}{\ldots}$

$$\begin{cases} \quad \\ y_1 = -\sqrt{1 - \frac{\cancel{64}^{32}}{81 \cdot \cancel{2}}} = -\frac{7}{9} \end{cases} \Rightarrow P_1\left(\frac{8}{9}, -\frac{7}{9}\right)$$

$$\begin{cases} x_2 = 0 \\ y_2 = 1 \end{cases} \Rightarrow P_2(0,1)$$

Eq of lines with 2 points

$$\frac{y - y_A}{y_A - y_B} = \frac{x - x_A}{x_A - x_B}$$

P_1A : $y = 1$

P_2A : $y = \frac{9}{7}x - \frac{9}{7}$

(3) $\vec{n} = (1, 1, 1)$ for $P: x + y + z = 0$

???

$\vec{n} \cdot \vec{d} = 0$, $\vec{d} = (a, b, c)$

$$\Rightarrow \vec{d} = (1,-1,0)$$

$P(1,0,0) \in$ hyperboloid $\Bigg\} \Rightarrow$

$$\begin{cases} x = 1+t \\ y = 0-t = -t \\ z = 0+0t = 0 \end{cases}$$

4

$$B = \frac{1}{11}\begin{bmatrix} -2 & 6 & -9 \\ -6 & 7 & 6 \\ 9 & 6 & 2 \end{bmatrix}$$

$\begin{cases} \det(B) = 1 \quad \text{(already checked in partial 1)} \\ B \cdot B^T = I_n \qquad \text{— " —} \end{cases}$

ii) $\mathcal{B} \in \mathfrak{D}(n)$