

## Extra homework 5

Weierstrass function is an example of a real function that is continuous everywhere but differentiable nowhere.

$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x),$$

$$0 < a < 1, b > 0 \text{ and } b = 2k+1, k \in \mathbb{N}$$

with the property that

$$ab > 1 + \frac{3}{2} \pi$$

lets' fix  $a=0.5$

$$0.5 \cdot b > 1 + \frac{3}{2} \pi$$

$$b > 11.42$$

Since  $b$  is an odd integer  $\Rightarrow$

$\Rightarrow$  we fix  $a=0.5$ , then  $b$  can

be 13

So the function is:

$$f(x) = \sum_{n=0}^{\infty} (0.5)^n \cos(13^n \pi x)$$

To prove the continuity, we can use Weierstrass M-test.

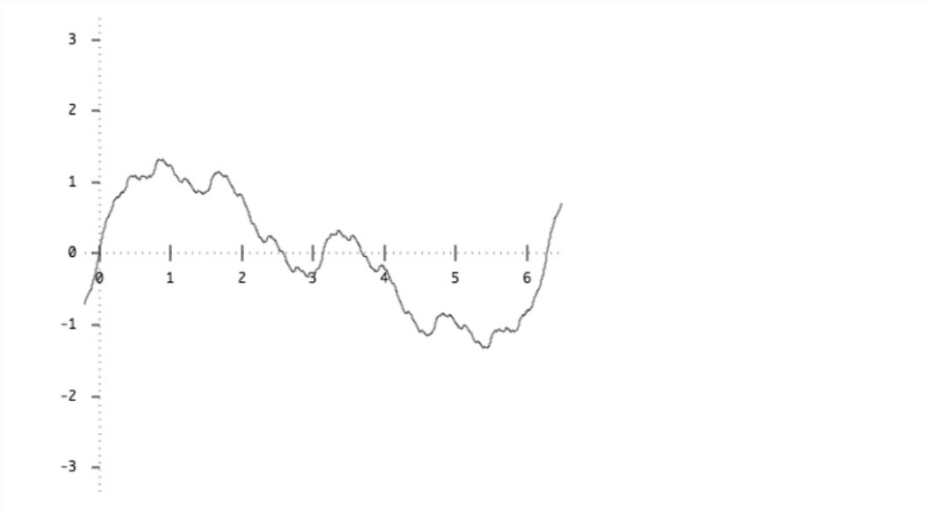
Another example is:

$$w(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \sin(2^n x) =$$

$$= \sin x + \frac{1}{2} \sin 2x + \frac{1}{4} \sin 4x + \dots$$

The graph of the function is also periodic with  $T = 2\pi$

graph of  $w(x)$ :



As the graph suggest, since it is  
bumpy, it is also differentiable nowhere.

