Megaran Jonathan
(915)

Seminar 4 - HW

1.

a)

$$\sum_{n\geq 1} \frac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot \ldots \cdot 2n}\cdot \frac{1}{n^2} = ?$$

Ratio test:

$$\lim_{n\to\infty} \frac{x_{n+1}}{x_n} = \lim_{n\to\infty} \frac{\frac{\cancel{1\cdot 3\cdot \ldots \cdot (2n-1)}\cdot(2n+1)}{\cancel{2\cdot 4\cdot \ldots \cdot 2n}\cdot(2n+2)}\cdot\frac{1}{(n+1)^2}}{\frac{\cancel{1\cdot 3\cdot \ldots (2n-1)}}{\cancel{2\cdot 4\cdot \ldots \cdot 2n}}\cdot\frac{1}{n^2}} =$$

$$= \frac{2n+1}{2n+2}\cdot\frac{n^2}{(n+1)^2} \to 1$$

Using R-D:

$$\lim_{n\to\infty} n\cdot\left(\frac{x_n}{x_{n+1}}-1\right)=\lim_{n\to\infty} n\left(\frac{(n+1)^2\cdot(n+1)\cdot 2}{(2n+1)\cdot n^2}-1\right)$$

$$=\lim_{n\to\infty} n\cdot\left(\frac{(n+1)^2\cdot(n+1)\cdot 2-(2n+1)\cdot n^2}{(2n+1)\cdot n^2}\right)=$$

$$=\lim_{n\to\infty} n\cdot\frac{5n^2+6n+2}{2n^3+n^2}=\frac{5}{2}>1\Rightarrow$$

$\Rightarrow x_n$ is convergent.

6. c)

$$\sum_{n\geq 1}\frac{nx^n}{2^n}$$

$$\lim\frac{|x_{n+1}|}{|x_n|}=\lim_{n\to\infty}\frac{\frac{(n+1)x^{\cancel{n+1}}}{2^{\cancel{n+1}}}}{\cdots}=\frac{x}{2}$$

$(n+1)x \quad \left|\frac{x}{n}\right| \quad (n+1)x \quad 2^{n}$

$$\left|\frac{x}{2}\right| < 1 \iff -2 < x < 2$$

si $x \in (-2, 2)$

x_n is convergent

$$R = 2$$