Mogoran Jonathan - 915b

①

$$x^2+y^2+z^2=1 \Rightarrow f(x,y,z)=x^2+y^2+z^2-1$$

The eq. of the tg. plane is given by:

$$f_x(x_0,y_0,z_0)\cdot(x-x_0)+f_y(x_0,y_0,z_0)\cdot(y-y_0)$$

$$+f_z(x_0,y_0,z_0)\cdot(z-z_0)=0,$$

where $\cdot$ is the dot product

$$f_x(x_0,y_0,z_0)=2x_0$$

$$f_y(x_0,y_0,z_0)=2y_0$$

$$f_z(x_0,y_0,z_0)=2z_0$$

$$2x_0(x-x_0)+2y_0(y-y_0)+2z_0(z-z_0)=0$$

Using Desmos 3D, we can check if the equation is good. For $(1,0,0)$, we see that

$2\cdot 1(x-1)=0$ is indeed the tangent plane to the unit sphere (see attachment).

(2) $f:\mathbb{R}^2\to\mathbb{R}$

$$f(x,y)=\frac{1}{2}(x^2+by^2),\ b>0$$

$$\nabla f(x,y)=(x,by)$$

$$(x_{k+1},y_{k+1})=(x_k,y_k)-s_k\nabla f(x_k,y_k)$$

that is

$$x_{k+1} = (1 - s_k)\, x_k$$

$$y_{k+1} = (1 - b s_k)\, y_k$$

$$\varphi(s_k) = f(x_{k+1}, y_{k+1}) =$$

$$= \frac{1}{2}\left[\left((1-s_k)\,x_k\right)^2 + b\left((1-b s_k)\,y_k\right)^2\right]$$

$$= \frac{1}{2}\left(\frac{d}{ds}\left[b\, y_k^2 \cdot (1-b s_k)^2\right] + \frac{d}{ds}\left[x_k^2 (1-s_k)^2\right]\right)$$

$$= -2b^2\, y_k^2\, (1-b s_k) - 2x^2 (1-s_k) -$$

$$2$$

$$= \left(b^3 y_k^2 + x_k^2 \right) s - b^2 y_k^2 - x_k^2$$

We want

$$\varphi'(s) = 0$$

$$\left(b^3 y_k^2 + x_k^2 \right) s - b^2 y_k^2 - x_k^2 = 0$$

$$\Leftrightarrow \quad s = \frac{b^2 y_k^2 + x_k^2}{b^3 y_k^2 + x_k^2} \text{ — the optimal}$$

step size.