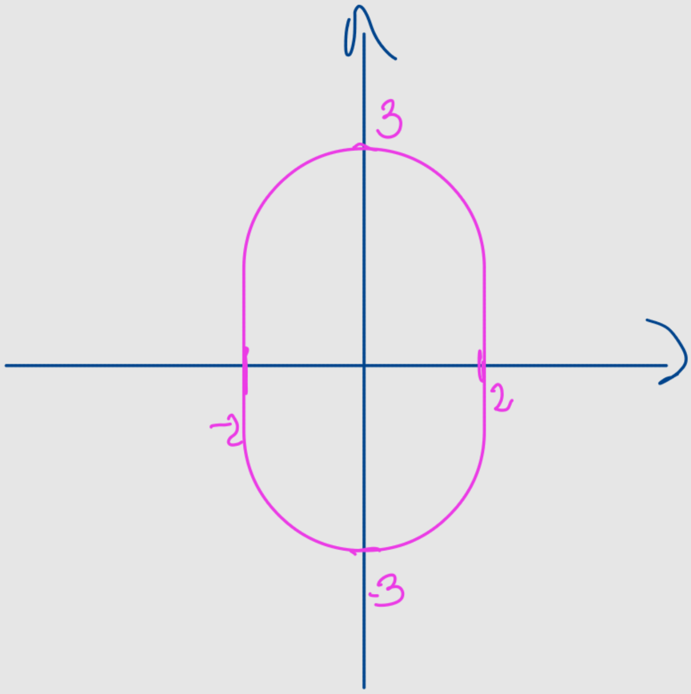
Partial 1

1.

Ellipse equation :



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$(0,3) \in \mathcal{E}_f \Rightarrow$

$$\frac{0}{3} + \frac{9}{b^2} = 1 \Rightarrow b = 3$$

$(2,0) \in \mathcal{G} \Rightarrow$

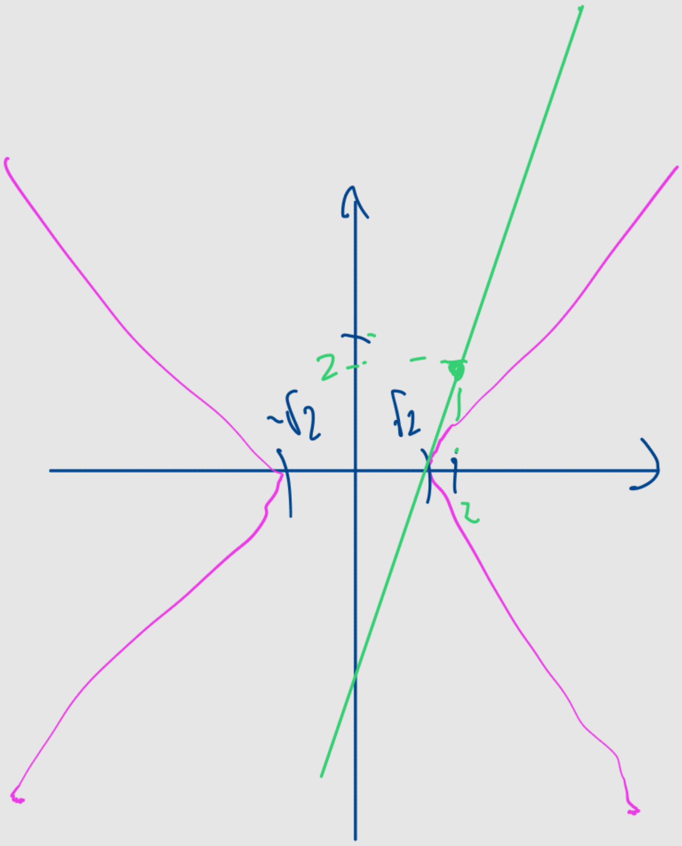
(2)

$$\frac{4}{a^2} = 1 \Rightarrow a = 2$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

2

$$x^2 - y^2 = 2$$



Hyperbola

Equation of the tangent line:

$$x_1 x - y_1 y = 2$$

$M(2,2) \Rightarrow 2x_1 - 2y_1 = 2$

$$\left\{\begin{array}{ll} x_1 - y_1 = 1 & (1) \\ x_1^2 - y_1^2 = 2 & (2) \end{array}\right\} \Rightarrow$$

$(1) \Rightarrow y_1 = x_1 - 1$

$(2) \Rightarrow \cancel{x_1^2} - \cancel{x_1^2} - 1 + 2x_1 = 2$

$x_1 = \frac{3}{2} \Rightarrow y_1 = \frac{1}{2}$

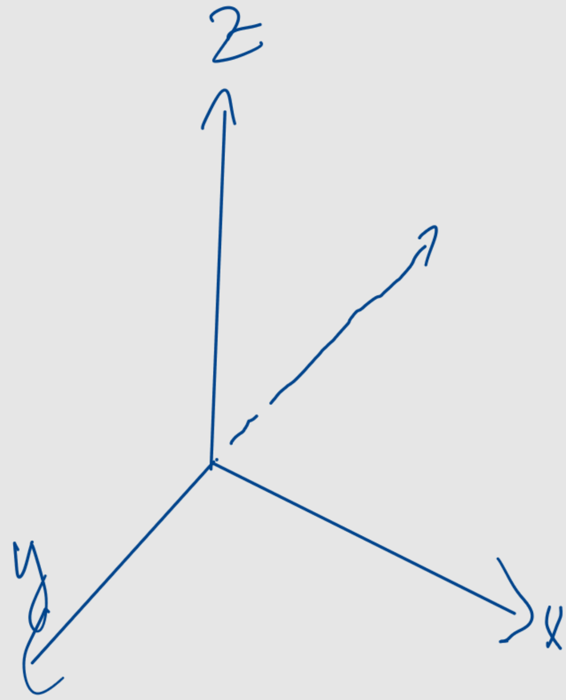
Eq: $\frac{3}{2}x - \frac{1}{2}y = 2$

$y = 3x - 4$

$y = 3x - 1$

3

a)



$S \cap O_y \Rightarrow$

$$\begin{cases} x = 0 \\ z = 0 \end{cases} \Rightarrow y^2 - 2 = 0$$

$$y = \pm\sqrt{2}$$

$$\Rightarrow P \in \{(0, \sqrt{2}, 0), (0, -\sqrt{2}, 0)\}$$

b)

$y^2 + 2xy - 2xz + z - 2 = 0$

$y^2 + 2xy - 2xz + z - 1 = 1$

$(y+x)^2 = y^2 + x^2 + 2yx$

$$\underbrace{(y+x)^2}_{y'} - \underbrace{(x+z)^2}_{z'} + \underbrace{\left(z+\frac{1}{2}\right)^2}_{x'} - \frac{9}{4} = 0$$

$x'^2 + y'^2 - z'^2 = \frac{9}{4}$

$\rightarrow$ hyperboloid with 1 sheet

4

a)

Rotation matrix :

$$\begin{cases} R^T \cdot R = I_n \\ \det(R) = 1 \end{cases}$$

a) $\det(R) \stackrel{?}{=} 1$

$$\begin{vmatrix} \frac{-2}{11} & \frac{6}{11} & \frac{-9}{11} \\ \frac{-6}{11} & \frac{7}{11} & \frac{6}{11} \\ \frac{9}{11} & \frac{6}{11} & \frac{2}{11} \end{vmatrix} \quad (\dots)$$

$= \quad 1 \quad \Rightarrow \det(R) = 1$

1

$$\frac{1}{11}\begin{pmatrix} -2 & 6 & -9 \\ -6 & 7 & 6 \\ 9 & 6 & 2 \end{pmatrix} \frac{1}{11}\begin{pmatrix} -2 & -6 & 9 \\ 6 & 7 & 6 \\ -9 & 6 & 2 \end{pmatrix} =$$

$$= \frac{1}{11}\begin{pmatrix} 121 & 0 & 0 \\ 0 & 121 & 0 \\ 0 & 0 & 121 \end{pmatrix} = I_3 \quad (2)$$

$(1) + (2) \Rightarrow$ R is a rotation matrix

b)

$$tr(R) = 1 + 2\cos(\theta)$$

$$\frac{7}{11} = 1 + 2\cos\theta$$

$$\frac{-2}{11} = \cos(\theta)$$

c)

Finding the eigenvector corresponding to $\lambda_1 = 1$

$$\det(B - \lambda I) = 0$$

$$\lambda = 1$$

$$\Rightarrow B - I = \frac{1}{11}\begin{pmatrix} -13 & 6 & -9 \\ -6 & -4 & 6 \\ 9 & 6 & -9 \end{pmatrix}$$

Solving the system:

$$\begin{pmatrix} 9 & 6 & -9 & 0 \\ -6 & -4 & 6 & 0 \\ 9 & 6 & -9 & 0 \end{pmatrix} \rightsquigarrow \text{Gaussian Elimination} \Rightarrow$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = z \begin{pmatrix} \frac{13}{27} \\ \frac{11}{16} \\ 1 \end{pmatrix} \Leftarrow \text{axis of rotation}$$