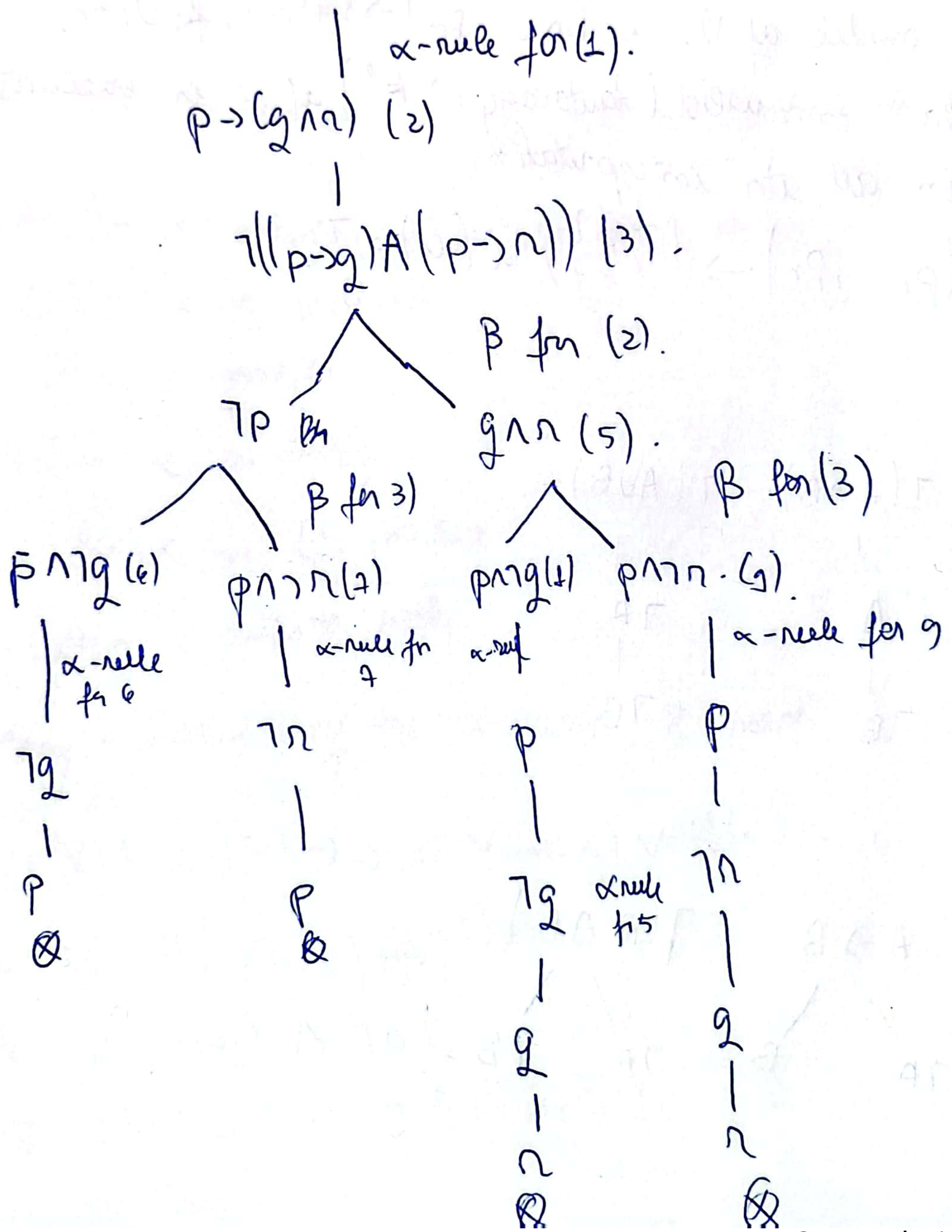
Th a. $U = (p \to (q \wedge r)) \to (p \to q) \wedge (p \to r)$.

semantic tableaux check method.

<u>Theory of soundness and completeness:</u>

U is a valid formula if the semantic tableaux of $\neg U$ has only closed branches and is closed.

$\neg U = \neg((p \to (q \wedge r)) \to (p \to q) \wedge (p \to r))$ (1)



- The semantic tableaux is complete, all the subformulas being explored.
- all the branches are closed making the tableaux closed ⇒

⇒ $\neg U$ is inconsistent ⇒ U is valid ⇒

⇒ $\models (p \to (q \wedge r)) \to (p \to q) \wedge (p \to r)$.

An interpretation which evaluates the formula U as true is called a model of U. $i : (p_1 \dots, p_n) \to \{T, F\}$, $i(U) = T$.

A formula is valid (tautology: " $\models$ ") if it is evaluated as true in all its interpretations.

$\forall i : (p_1 \cdots, p_n) \to \{T, F\}$, $i(U) = T$.

α-rules

$A \wedge B$	$\neg(A \to B)$	$\neg(A \vee B)$
A	A	$\neg A$
B	$\neg B$	$\neg B$

β-rules

$A \vee B$	$A \to B$	$\neg(A \wedge B)$
A / B	$\neg A$ / B	$\neg A$ / $\neg B$.

2. Syntactic proof method ⇒ ~~resolution~~. Deduction.

H_1: Anyone who makes an 'A' at Logic Exam studies or is brilliant or is lucky

H_2: NO CS student is lucky.

H_3: Mary is a CS student and made an 'A' at Logic Exam

H_4: Mary likes to party and does not study.

C: Mary is Brilliant.

- we use the following unary predicate symbols to express propo

mA(x) - x makes an 'A' at Logic Exam studies

b(x) - x is brilliant

s(x) - x studies

l(x) - x is lucky.

CS(x) - x is CS student.

p(x) - x likes to party.

Mary - constant. of the universe. of people.

$H_1: (\forall x)(mA(x) \rightarrow s(x) \vee b(x) \vee l(x)) = f_1$

$H_2: \neg(\exists x)(CS(x) \wedge \neg l(x)) = f_2$

H_4: $p(Mary) \wedge \neg s(Mary) = f_3$

H_3: $CS(Mary) \wedge mA(Mary) = f_4$

C: $b(Mary) = f_5$

$f_1 \vdash_{inst}^{univ} mA(Mary) \to s(Mary) \vee b(Mary) \vee l(Mary) : f_5$.

- the universal variable x was instantiated with the constant Mary.

$f_2 \vdash_{inst}^{univ} CS(Mary) \cancel{AB} \to l(Mary) : f_6$.

- the universal variable x was instantiated with the constant Mary.

$f_3 \vdash^{simpl} \neg s(Mary) : f_7$.

$f_4 \vdash^{simpl} CS(Mary) : f_8$

$f_4 \vdash^{simpl} mA(Mary) : f_9$

$f_1, f_4 \vdash^{mp} s(Mary) \vee b(Mary) \vee l(Mary) \equiv \neg s(Mary) \to (b(Mary) \vee l(Mary)) : f_{10}$

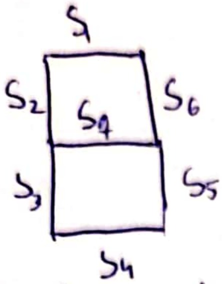
$f_{10}, f_7 \vdash^{mp} b(Mary) \vee l(Mary) : \cancel{f_4} \; \neg l(Mary) \to b(Mary) : f_4$

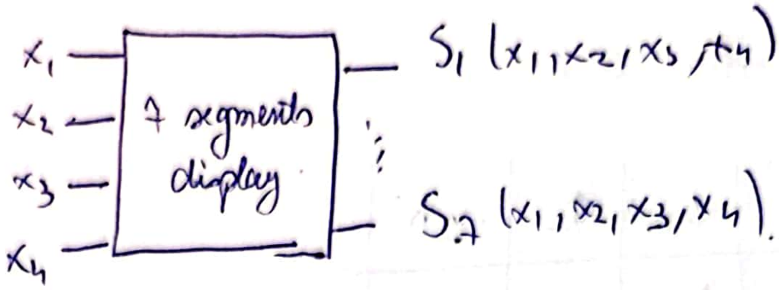
$f_4, f_2 \vdash^{mp} \neg l(Mary) : f_{12}$

$f_{11}, f_{12} \vdash^{mp} b(Mary) : f_{13} = C$.

The sequence of formulas $(f_1, f_2, \ldots, f_{13})$ is a deduction of C from the hypotheses H_1, H_2, H_3, H_4, H_5, therefore based on the hypotheses we conclude: „Mary is brilliant".

3. the combinational circuit has as inputs 4 variables and as outputs 7 functions corresponding to the segment:





	inputs										
Decimal	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	0	0	0	1	1	0
2	0	0	1	0	1	0	1	1	0	1	1
3	0	0	1	1	1	0	0	1	1	1	1
4	0	1	0	0	0	1	0	0	1	1	1
5	0	1	0	1	1	1	0	1	1	0	1
6	0	1	1	0	1	1	1	1	1	0	1
7	0	1	1	1	1	0	0	0	1	1	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	0	1	1	1	1
–	1	0	1	0	0	0	0	0	0	0	1
–	1	0	1	1	0	0	0	0	0	0	1
–	1	1	0	0	0	0	0	0	0	0	1
–	1	1	0	1	0	0	0	0	0	0	1
–	1	1	1	0	0	0	0	0	0	0	1
–	1	1	1	1	0	0	0	0	0	0	1

The simplification of (S_4) follows a dual simplification algorithm. Using the Karnaugh diagram diagram for CCF, the headers of the lines - columns are used to express the indices of the Maxterms.

$S_4 = M_1 \wedge M_4 \wedge M_7 \wedge M_{10} \wedge M_{11} \wedge M_{12} \wedge M_{13} \wedge M_{14} \wedge M_{15}$.

(we look at the 0 values on the S_4 column and create the corresponding CCF).

x_1x_2 \ x_3x_4	00	01	11	10
00		M_1		
01	M_4		M_7	
11	M_{12}	M_{13}	M_{15}	M_{14}
10			M_{11}	M_{10}

<u>Double dual factorizations</u>

- $M_{10} \wedge M_{11} \wedge M_{15} \wedge M_{14} = \bar{x}_1 \vee \bar{x}_3$ = min 1
- $M_{12} \wedge M_{13} \wedge M_{15} \wedge M_{14} = \overset{1}{x}_1 \overset{1}{x}_2 = \bar{x}_1 \vee \bar{x}_2$ = min 2

<u>Single dual factorization:</u>

- $M_{15} \wedge M_7 = \overset{1}{x}_2 \vee \overset{1}{x}_3 \vee \overset{1}{x}_4 = \bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4$ = min 3
- $M_4 \wedge M_{12} = \overset{1}{x}_2 \vee \overset{0}{x}_3 \vee \overset{0}{x}_4 = \bar{x}_2 \vee x_3 \vee x_4$ = min 4

<u>Simple dual factorization</u>

- $M_1: \overset{0}{x}_1 \vee \overset{0}{x}_2 \vee \overset{0}{x}_3 \vee \overset{1}{x}_4 = x_1 \vee x_2 \vee x_3 \vee \bar{x}_4$ = min 5

$C = M(f) \Rightarrow$

$\Rightarrow S_4^{C3} = (\bar{x}_1 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4) \wedge (\bar{x}_2 \vee x_3 \vee x_4) \wedge$

$(x_1 \vee x_2 \vee x_3 \vee \bar{x}_4)$

$$S_4^{Cs} = (\bar{x}_1 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4) \wedge (\bar{x}_2 \vee x_3 \vee x_4) \wedge (x_1 \vee x_2 \vee x_3 \vee \bar{x}_4)$$

$$= \overline{\overline{(\bar{x}_1 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4) \wedge (\bar{x}_2 \vee x_3 \vee x_4) \wedge (x_1 \vee x_2 \vee x_3 \vee \bar{x}_4)}}$$

$$= \overline{\underbrace{\overline{(\overline{x_1 \vee x_1}) \vee (\overline{x_3 \vee x_3})}}_{a} \vee \underbrace{\overline{(\overline{x_1 \vee x_1}) \vee (\overline{x_2 \vee x_2})}}_{b} \vee \underbrace{\overline{(\overline{x_2 \vee x_2}) \vee (\overline{x_3 \vee x_3}) \vee (\overline{x_4 \vee x_4})}}_{c} \vee}$$

$$\overline{\underbrace{\overline{(\overline{x_2 \vee x_2}) \vee x_3 \vee x_4}}_{d} \vee \underbrace{\overline{x_1 \vee x_2 \vee x_3 \vee (\overline{x_4 \vee x_4})}}_{e}}$$

- 4 gates for $\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4$ (NOR).
- 5 gates for : a, b, c, d, e
- a final gate with 5 inputs.