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Extra homework 2

1.

$$x_{n+2} = (\alpha-1)x_{n+1} + \beta$$

$$x_{n+2} = \alpha x_{n+1} + (1-\alpha)x_n - x_{n+1}$$

$$x_{n+2} - x_{n+1} = (\alpha-1)x_{n+1} + (1-\alpha)x_n$$

$$= (x_{n+1} - x_n)(\alpha-1)$$

$$x_{n+1} - x_n = (x_n - x_{n-1})(\alpha-1)$$

$$\vdots$$

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$$x_3 - x_2 = (x_2 - x_1)(\alpha-1) \quad +$$

$$x_{n+2} - x_2 = (\alpha-1)(x_{n+1} - x_1)$$

x

$$x_{n+2} = \underbrace{x_{n+1}(\alpha-1)}_{a} + \underbrace{x_2 + (1-\alpha)x_1}_{b}$$

let $\boxed{a = \alpha - 1}$

let $\boxed{b = x_2 + (1-\alpha)x_1}$

$$x_{n+2} = a x_{n+1} + b$$

We can prove that:

$$x_{n+2} = x_2 \cdot a^n + a^{n-1} b + \dots + a \cdot b + b$$

$$x_{n+2} = a x_{n+1} + b$$

$$x_{n+1} = a x_n + b \quad / \cdot a$$

$$x_n = a x_{n-1} + b \quad / \cdot a^2$$

$$\dots\dots$$

$$x_4 = a x_3 + b$$

$$x_3 = a x_2 + b \quad / \cdot a^{n-1}$$

$(+)$

$$x_{n+2} = x_2 \cdot a^n + a^{n-1} b + \dots + ab + b \quad / b$$

$$\frac{x_{n+2}}{a} = \frac{x_2 \cdot a^n}{a} + a^{n+1} + \dots + a + 1 \quad \Big| \cdot (1-a)$$

$$x_{n+2} \cdot \frac{1-a}{a}$$

$$\boxed{\text{let } \Delta = \frac{1-a}{a}}$$

$$\Delta \cdot x_{n+2} = a^n x_2 \cdot \Delta + \cancel{a} 1 - a^n$$

$$x_{n+2} = \frac{a^n(x_2\Delta - 1)}{\Delta} + \frac{1}{\Delta}$$

$$x_{n+2} = \frac{\overbrace{(\alpha-1)^n}^{\in(-1,1)}(x_2\Delta - 1)}{\Delta} + \frac{1}{\Delta}$$

$$\lim_{n\to\infty} x_{n+2} = \frac{1}{\Delta} = \frac{b}{1-a} = \frac{(1-a)x_1 + x_2}{1-(\alpha-1)} =$$

$$= \frac{(1-\alpha)x_1 + x_2}{2-\alpha} = \frac{(1-\alpha)x_1 + x_2}{2-\alpha}$$

We can check the solution:

$$x_{n+2} = (\alpha - 1)x_{n+1} + \beta$$

if $l \in \mathbb{R}$, then $l = \lim x_{n+2} = \lim x_{n+1}$ $(=)$

$(=)$ $l = (\alpha - 1)l + \beta$

$$l - l(\alpha - 1) = \beta$$

$$l(1 - \alpha + 1) = \beta$$

$$l = \frac{\beta}{2-\alpha} = \frac{(1-\alpha)x_1 + x_2}{2-\alpha}$$

So the conclusion stands.