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Group 915

Seminar 5

1. b) – We consider the following linear planar systems.

$\dot{x} = -x,$
$\dot{y} = 5y$

(i) Decide the type and stability of the equilibrium point at the origin (0,0).

$$\begin{cases} x' = -x \\ y' = 5y \end{cases} \Rightarrow A = \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$$

We find thei eigenvalues of A, $\lambda_1 = -1$, $\lambda_2 = 5$ (since it is a diagonal matrix no calculation is required, otherwise we would have used the following formula $\lambda_{12} = \frac{Tr(A) \pm \sqrt{Tr(A)^2 - 4det(A)}}{2}$)

$\lambda_1 < 0 < \lambda_2$, $\lambda_{1,2} \in \mathbb{R}$, hence the equilibrium point at the origin is a saddle point. Saddle points are unstable.

(ii) Decide whether it has a global first integral.
(iii) Find a first integral (global or not).

A global first integral is a function $H(x,y)$ that remains constant allong all trajectories of the system and satisfies: $\frac{dH}{dt} = \frac{dH}{dx}\dot{x} + \frac{dH}{dy}\dot{y} = 0$ ①

$$\begin{cases} x' = -x \\ y' = 5y \end{cases} \Leftrightarrow \begin{cases} \dfrac{dx}{dt} = -x \\ \dfrac{dy}{dt} = 5y \end{cases} \Leftrightarrow \int \frac{dy}{y} = -5 \int \frac{dx}{x} \Leftrightarrow$$

$\Leftrightarrow \ln|y| + 5\ln|x| = \ln|C|$ (We decided to express the integral constant in this format) $C \in \mathbb{R}$

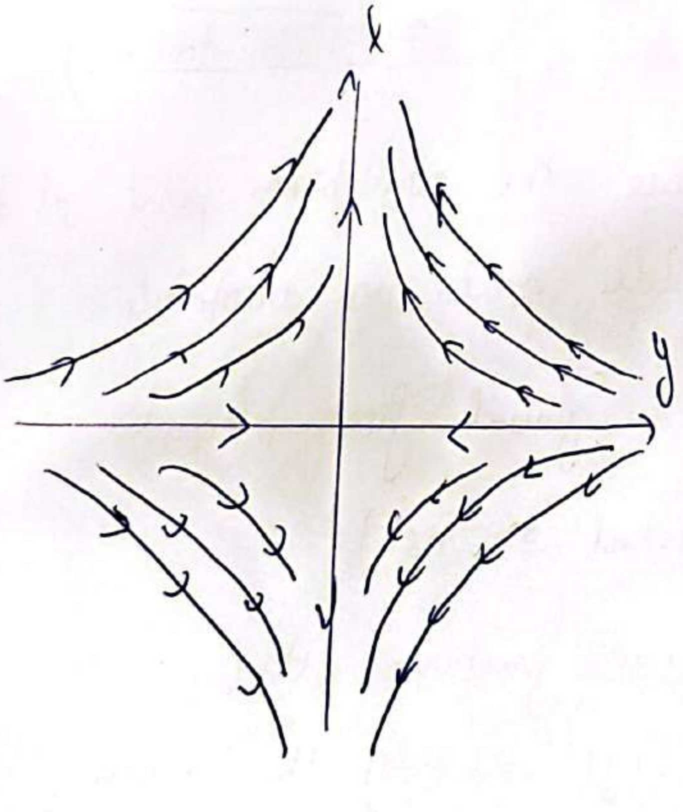
$\Leftrightarrow y x^5 = C$

We define $H(x,y) = yx^5$, $H: \mathbb{R}^2 \to \mathbb{R}$

Checking in 1, we get:

$5x^4 y \cdot (-x) + x^5 \cdot (5y) = 0$, hence H is a global first integral

(iv) Represent the phase portrait.
$$\begin{cases} x' = -x \\ y' = 5y \end{cases}$$



$y = 0 \Rightarrow x > 0,\ x' < 0 \ \leftarrow$
$ x < 0,\ x' > 0 \ \rightarrow$

$x = 0 \Rightarrow y > 0,\ y' > 0 \ \uparrow$
$ y < 0,\ y' < 0 \ \downarrow$