$\boxed{PB1}$ $\vec{AB} = (-1, 7)$ $\vec{AC} = (-9, 3)$, $\vec{BC} = (-8, 4)$

① $\vec{AB} \cdot \vec{AC} = (-1 \cdot -9) + (7 \cdot 3) = 30$

$\vec{AB} \cdot \vec{BC} = -20$

$\vec{AC} \cdot \vec{BC} = 60$

$\Rightarrow$ not right angled

② $M_{AB} = \left(\frac{7}{2}, \frac{5}{2}\right)$, $M_{AC} = \left(\frac{-1}{2}, \frac{1}{2}\right)$

$$m_{AB} = \frac{y_B - y_A}{x_A - x_B} = 7 \Rightarrow \perp = \frac{-1}{7}$$

$$m_{AC} = \frac{y_C - y_A}{x_C - x_A} = \frac{-1}{3} \Rightarrow \perp = 3$$

$$\left.\begin{aligned} y - \frac{5}{2} &= \frac{1}{7}\left(x - \frac{7}{2}\right) \\ y - \frac{1}{2} &= 3\left(x + \frac{1}{2}\right) \end{aligned}\right\} \Rightarrow (0, 2) \text{ circumcenter}$$

④ $A = \frac{1}{2} \begin{vmatrix} 4 & -1 & 1 \\ 3 & 6 & 1 \\ -5 & 2 & 1 \end{vmatrix} = 30$

$$\frac{\quad}{5} \quad \frac{\quad}{5}$$

$$H\left(\frac{1}{5};\frac{8}{5}\right)$$

c) $[\vec{AB}]_K = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

$$[\vec{AC}]_K = \begin{bmatrix} -9 \\ 3 \end{bmatrix}$$

$$M_{K,K'} = \begin{pmatrix} -1 & -9 \\ 7 & 3 \end{pmatrix} \Rightarrow \det = -3+63 = 60$$

$$^{t}M_{K,K'} = \begin{pmatrix} -1 & 7 \\ -9 & 3 \end{pmatrix} \Rightarrow M_{K',K} = \begin{pmatrix} \frac{1}{20} & \frac{3}{20} \\ -\frac{7}{60} & -\frac{1}{60} \end{pmatrix}$$

$$[A]_{k'} = \cancel{[\vec{OA}]_{k'} = (\vec{O'O} + \vec{OA}) \; [\vec{A'A}]_{k'} = [}$$

$$= [\vec{AA}]_{k'} = [\vec{AO}]_{k'} + [\vec{OA}]_{k'} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$[B]_{k'} = [\vec{AB}]_{k'} = [\vec{AO} + \vec{OB}]_{k'} = [\vec{OB}]_{k'} - [\vec{OA}]_{k'} =$$

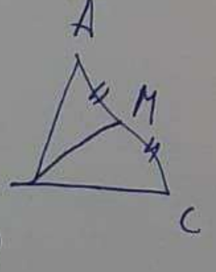
$$= M_{k'k}\left([\vec{OB}]_k - [\vec{OA}]_k\right) = \begin{pmatrix} \frac{3}{60} & \frac{9}{60} \\ -\frac{7}{60} & -\frac{1}{60} \end{pmatrix}\begin{pmatrix}-1\\7\end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$[C]_{k'} = [\vec{OC}]_{k'} - [\vec{OA}]_{k'} = M_{k'k}\begin{pmatrix}-9\\3\end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{60} & \frac{9}{60} \\ -\frac{7}{60} & -\frac{1}{60} \end{pmatrix}\begin{pmatrix}-9\\3\end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



M mij AC $\Rightarrow M\left(-\frac{1}{2}; \frac{1}{2}\right)$

$$BM: \frac{y - y_B}{y_M - y_B} = \frac{x - x_B}{x_M - x_B} \Leftrightarrow \frac{y-6}{\frac{1}{2}-6} = \frac{x-3}{-\frac{1}{2}-3}$$

$$\frac{y-6}{-\frac{11}{2}} = \frac{x-3}{-\frac{7}{2}} \Leftrightarrow -\frac{7}{2}(y-6) = -\frac{11}{2}(x-3)$$

$$-7y + 42 = -11x + 33 \Leftrightarrow BM: 11x - 7y + 9 = 0$$

Pb 2.

a) $\vec{AB} = (0, 0, 1)$
$\vec{AC} = (-1, 1, 2)$ $\Rightarrow$ $\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 2 \\ 0 & -1 & 1 \end{vmatrix} = 1 \Rightarrow 1$
$\vec{AD} = (0, -1, 1)$

> 0 right-oriented

b) $v_1 = u_1 = (0, 0, 1) - \vec{AB}$

$v_2 = u_2 - pr_{v_1} u_2 = u_2 - \frac{u_2 v_1}{v_1 \cdot v_1} \cdot v_1$ $\Bigg\} \Rightarrow$

$u_2 \cdot v_1 = 2, \quad v_1 \cdot v_1 = 1$

$\Rightarrow v_2 = (-1, 1, 2) - (0, 0, 2) = (-1, 1, 0)$

$v_3 = u_3 - \frac{v_3 \cdot v_1}{v_1 \cdot v_1} \cdot v_1 - \frac{u_3 \cdot v_2}{v_2 \cdot v_2} \cdot v_2$

$\left.\begin{array}{l} u_3 \cdot v_1 = 1 \\ u_3 \cdot v_2 = -1 \\ v_2 \cdot v_2 = 2 \end{array}\right| \Rightarrow v_3 = (0, -1, 1) - (0, 0, 1) - \left(\frac{1}{2}, \frac{-1}{2}, 0\right) =$

$= \left(\frac{1}{2}, \frac{-1}{2}, 0\right)$

PB2

c) $AB = (0, 0, 1)$
$AC = (-1, 1, 2)$

$n = AB \times AC = (-1, -1, 0)$

$$\left.\begin{array}{l} -x - y = d \\ A \in (ABC) \end{array}\right\} \Rightarrow -x - y + 3 = 0$$

$$P = D - \frac{n \cdot D + d}{n \cdot n} \cdot n$$

n – normal vector

$n \cdot D = (-1, -1, 0) \cdot (1, 1, 1) = -2$

$n \cdot n = 2$

$$P = (1, 1, 1) - \frac{-2 + (-3)}{2} \cdot n = \begin{pmatrix} -\frac{2}{3} \\ -\frac{2}{3} \\ 1 \end{pmatrix}$$

d) $\vec{BC} = a\vec{AB} + b\vec{AC} + c\vec{AD}$

$\vec{BD} = a'\vec{AB} + b'\vec{AC} + c'\vec{AD}$

$$n = \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\Rightarrow x' + y' + z' = 0$

PB3

using Grassman $x \times y = (x \cdot y_2)y_1 - (x \cdot y_1)y_2$

$(a \times b) \times (c \times d) = [(a \times b) \cdot d]c -$

$- [(a \times b) \cdot c]d = [a, b, d]c - [a, b, c]d$

P4. $d(P, H) = |PH|$

$$d(P, H) = \left| \frac{\langle n, \vec{QP} \rangle}{|n|^2} n \right| = \frac{|\langle n, QP \rangle|}{|n|}$$

$$= \frac{|a_1 p_1 + a_2 p_2 + \dots a_n p_n - b|}{\sqrt{a_1^2 + a_2^2 + \dots + a_n^2}}$$