PARTIAL II

PART I

P1.

$\vec{BC} = C - B = (0,1)$

$\vec{i} = (1,0)$

$\langle \vec{BC}, \vec{i} \rangle = 0 \cdot 1 + 1 \cdot 0 = 0 \Rightarrow \vec{BC} \perp \vec{i}$

P2.

$$AC = \frac{x - x_A}{x_C - x_A} = \frac{y - y_A}{y_C - y_A}$$

$$\frac{x-2}{\cancel{2}} = \frac{-y+4}{\cancel{-2}}$$

$y_{AC} = -x + 6 \Leftrightarrow x + y - 6 = 0$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}} = \frac{|4+1-6|}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

P3. $CM \perp AB \Rightarrow$

$$m_{CM} = \frac{-1}{m_{AB}}, \quad m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{-3}{2}$$

$$m_{CM} = \frac{2}{3}$$

$$CM: \quad y - y_C = m_{CM} \cdot (x - x_C)$$

$$y - 2 = \frac{2}{3}(x-4)$$

$$y = \frac{2}{3}(x-4) + 2 \quad (1)$$

$BN \perp AC \Rightarrow$

$$m_{BN} = \frac{-1}{m_{AC}} = \frac{-1}{-1} = 1$$

$$BN: \quad y - y_B = m_{BN}(x - x_B)$$

$$y - 1 = 1(x-4)$$

$$y = x - 3 \quad (2)$$

$$(1) + (2) \Rightarrow x - 3 = \frac{2}{3}(x-4) + 2$$

$$\Rightarrow O(7, 4)$$

P4. $\cancel{A_{K'} = 2\begin{bmatrix}-1\\1\end{bmatrix} + 4\begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}2\\ \end{bmatrix}}$

$$[A]_{K'} = \begin{bmatrix}-1 & 0\\ 1 & 1\end{bmatrix}^{-1} \cdot \left(\begin{bmatrix}2\\4\end{bmatrix} - \begin{bmatrix}4\\0\end{bmatrix} \right)$$

$$= -\begin{pmatrix}1 & 0\\ -1 & -1\end{pmatrix}\begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$$

$$\cancel{[C]_{K'} = 4\begin{pmatrix}-1\\1\end{pmatrix} + 2\begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}4\\6\end{pmatrix} = \begin{pmatrix}0\\ \end{pmatrix}}$$

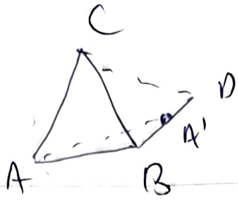
$$= -\begin{pmatrix}1 & 0\\ -1 & -1\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}0\\2\end{pmatrix}$$

AC: $\dfrac{y-y_A}{y_C-y_A} = \dfrac{x-x_A}{x_C-x_A} \iff \dfrac{y-2}{0} = \dfrac{x-2}{0-2}$

$\iff -2y+4 = x-2$

$y = \dfrac{-x}{2} + 3$

Part IV



P5

$\vec{AB} = (0,0,1)$

$\vec{BC} = (-1,1,1)$

$\vec{AC} = (-1,1,2)$

$dist(A, BD) = ?$

$\vec{BD} = (0,-1,0)$

$\vec{BA} = (0,0,-1)$

$\vec{DA} = (0,1,-1)$

$$AA' = \frac{|\vec{BA} \times \cancel{\vec{BD}}\ \vec{DA}|}{|\vec{BD}|} = \frac{1}{1} = 1$$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = i$$

P6

$\vec{BC} = (-1,1,1)$

$\vec{BD} = (0,-1,0)$

$$n = |\vec{BC} \times \vec{BD}| = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 0 & -1 & 0 \end{vmatrix} = i + k$$

$= (+1, 0, 1)$

$\Rightarrow (BCD): x + z = d$

$B(1,2,1) \Rightarrow B \in (BCD)$

$\Rightarrow d = -2, \quad x + z - 2 = 0$

$$L: \begin{cases} x = 1 + t \\ y = 2 \\ z = 0 + t \end{cases} \Rightarrow 1 + t + t = 2 \quad t = \frac{1}{2} \Rightarrow P\left(\frac{3}{2}, 2, \frac{1}{2}\right)$$

P7 $(\vec{AB} \times \vec{AC}) \cdot k$

$\vec{AB} = (0,0,1)$; $\vec{AC} = (-1,1,2)$

$(\vec{AB} \cdot \vec{AC}) = -i - j$

$$\begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = -i + j = (-1,1,0)$$

P8

$\vec{BA} = (0,0,-1)$ $\quad \vec{DC} = (-1,2,1)$

$\vec{DA} = (0,1,-1)$

$$\begin{vmatrix} 0 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = 1 > 0 \Rightarrow \text{right oriented}$$

P9 $[A]_{K'} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 2 \\ -1 & -2 & 1 \end{pmatrix}^{-1} \Big((1,2,0) - (1,1,1) \Big)$

$$= \begin{pmatrix} -3 & -1 & -1 \\ 2 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} (0,1,-1) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$[B]_{K'} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \quad [C]_{K'} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$(\vec{BA} \cdot \vec{BC}) = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & -1 & 1 \end{vmatrix} = -j - k \sim \langle 0,-1,-1 \rangle$$

$B = (-1, 1, 0)$

$-y - z = d$

$-1 = d \Rightarrow (ABC) = -y - z + 1 = 0$