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Option I – 2. compose exercises

1. Using propositional logic – Truth table

H_1: It is not snowy outside

H_2: We will go hiking only if it is snowy

H_3: If we do not go hiking, we will take a trip.

H_4: If we take a trip, we will come tomorrow.

C: We will come tomorrow.

S - it is snowing outside

H - We will go hiking

T - We will take a trip

Tw - We will come tomorrow

H_1 : $\neg S$

H_2 : $H \to S$

H_3 : $\neg H \to T$

H_4 : $T \to Tw$

S	H	T	Tw	$\neg S$	$\neg H$	$H \to S$	$\neg H \to T$	$T \to Tw$
0	0	0	0	1	1	1	0	1
0	0	0	1	1	1	1	0	1
0	0	1	0	1	1	1	1	0
0	0	1	1	1	1	1	1	1

0	1	0	0	1	0	0	1	1
0	1	0	1	1	0	0	1	1
0	1	1	0	1	0	0	1	0
0	1	1	1	1	0	0	1	1
1	0	0	0	0	1	1	0	1
1	0	0	1	0	1	1	0	1
1	0	1	0	0	1	1	1	0
1	0	1	1	0	1	1	1	1
1	1	0	0	0	0	1	1	1
1	1	0	1	0	0	1	1	1
1	1	1	0	0	0	1	1	0
1	1	1	1	0	0	1	1	1

Conclusions: The conclusion is valid according to the truth table.

2. definition of deduction

A_1: Alice will go shopping if Bella goes and Carla does not go.

H_2: If Dave goes shopping, Bella goes too.

H_3: If Dave is in Toronto, he will go shopping.

H_4: Carl is sick and can't go shopping.

H_5: Dave is in Toronto.

C: Alice goes shopping.

Notations for the propositional variables:

A - Alice will go shopping

B - Bella will go shopping

C - Carl will go shopping

D - Dave will go shopping

Dt - Dave is in Toronto

Propositional formulas:

$H_1: B \wedge \neg C \to A \quad = f_1$

$H_2: D \to B \quad = f_2$

$H_3: Dt \to D \quad = f_3$

$H_4: \neg C \quad = f_4$

$H_5: Dt \quad = f_5$

$C: A \quad = f_4$

$f_5, f_3 \vdash_{mp} D : f_6$

$f_6, f_2 \vdash_{mp} B : f_7$

$f_4, f_7 \vdash B \wedge \neg C : f_8$ (conjunction in conclusion)

$f_8, f_1 \vdash_{mp} A : f_9$

The sequence of formulas $(f_1, f_2, f_3, \ldots, f_9)$ is the deduction of A, based on the hypotheses, Alice will go shopping.

3. Semantic Tableaux

The semantic tableaux method is a refutation proof method, thus we have to negate the conclusion and use theorem of soundness and completeness:

$H_1, H_2, H_3, H_4 \models$ if and only if

$H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C$ has a closed semantic tableaux

Hypothesis:

H_1: It is not windy outside

H_2: We will go surfing only if it is windy

H_3: If we do not go surfing, we will take a bath.

H_4: If we take a bath, we will come tomorrow.

C: We will come tomorrow.

Notations:

S - it is windy outside

H - We will go surfing

T - We will take a bath

Tw - We will come tomorrow

H_1: $\neg S$

H_2: $H \to S$

H_3: $\neg H \to T$

H_4: $T \to Tw$

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C: Tw

$(\neg S) \wedge (H \to S) \wedge (\neg H \to T) \wedge (T \to Tw) \wedge \neg Tw$ (1)

| α rule for (1)

$\neg S$

$H \to S$ (2)

$\neg H \to T$ (3)

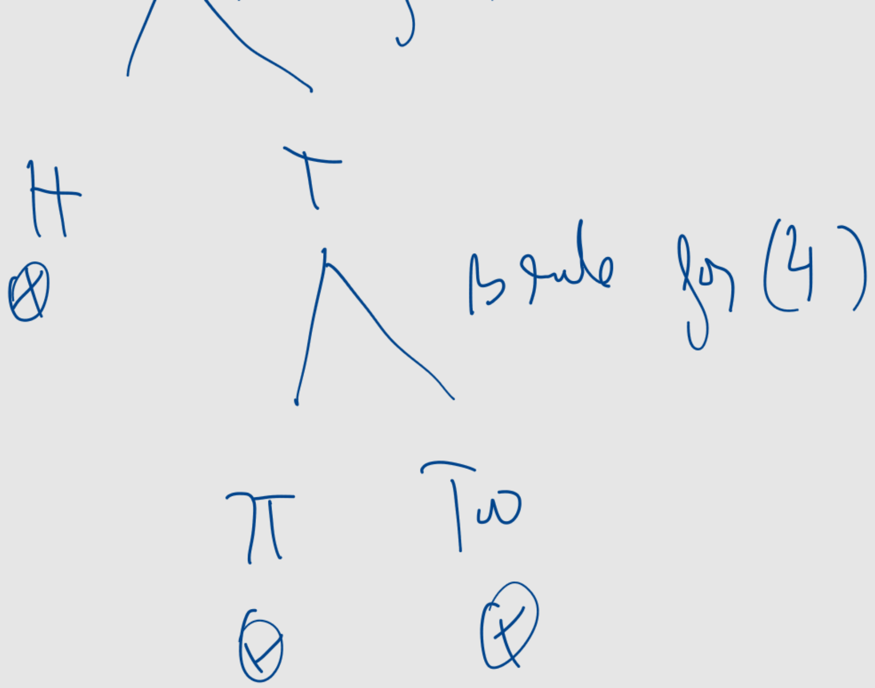
$T \to Tw$ (4)

$\neg Tw$

β rule for (2)

$\neg H$ $\qquad$ S

β rule for (3) $\otimes$



All the branches of the semantic tableau are closed, containing pairs of opposite literals, therefore the conclusion is valid, hence we have solved the problem.

4. General Resolution

Hypothesis:

H_1: It is not beautiful outside

H_2: We will go fishing only if it is beautiful

H_3: If we do not go fishing, we will take a bath.

H_4: If we take a plane, we will come Monday

C: We will come Monday

<u>Notations</u>:

S - it is beautiful outside

H - We will go fishing

T - We will take a plane

Tw - We will come Monday

$H_1: \neg S \quad : C_1$

$H_2: H \to S \equiv \neg H \vee S : C$

$H_3 : \neg H \to T \equiv H \vee T : C_3$

$H_4 : T \to Tw \equiv \neg T \vee Tw : C_4$

$C : Tw$

$\neg C : \neg Tw : C_5$

$X = \{ C_1, C_2, C_3, C_4, C_5 \}$

$CNF(H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C) \vdash_{Res} ?\ \square$

$C_1 : \neg S$

$C_2 : \neg H \vee S$

$C_3 : H \vee T$

$C_4 : T \vee \neg W$

$C_5 : \neg T W$

$C_6 : Res_{TW}(C_4, C_5) = \neg T$

$C_7 : Res_T(C_3, C_6) = H$

$C_8 : Res_H(C_2, C_7) = S$

$C_9 : Res_S(C_1, C_8) = \square$

$CNF(H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C) \vdash_{Res} \square$, so

C is deducible from the hypothesis, therefore „we will come monday" is true.

5. Linear resolution

H_1 : Paul or Quinn or Rache won the game

H_2 : Paul did not win the game.

H_3 : Quinn did not win the game.

H_4 : if rachel won the game, I won it too.

C : I did not win the game.

W – I win the game

P – Paul won the game

Q – Quinn won the game

R – Rachel won the game

$G_1 = p \vee q \vee r$

$C_2 = \neg p$

$C_3 = \neg q$

$C_4 = r \to w \equiv \neg r \vee w$

$C_5 = \neg w$

$\boxed{C_5 = \underline{\neg w}}$ $\qquad$ $\boxed{C_4 = \neg r \vee \underline{w}}$

$\downarrow$

$\boxed{C_6 = \neg \underline{r}}$ $\qquad$ $\boxed{C_1 = p \vee q \vee \underline{r}}$

$\downarrow$

$$
\begin{tikzcd}
\boxed{C_7 = p \vee \underline{q}} \arrow[d] & \boxed{C_3 = \neg q} \arrow[dl] \\
\boxed{C_8 = \underline{p}} \arrow[d] & \boxed{C_2 = \neg p} \arrow[dl] \\
\boxed{C_9 = \square} &
\end{tikzcd}
$$

$S \vdash^{lin}_{Res} \square$, therefore S is inconsistent

and the conclusion is not valid.