Option 1

Solve 5 exercises chosen from the list below using predicate logic

1 Definition of deduction

$\boxed{1}$

D (universe of all people and animals)

hounds, cat, mice, John are const.

unary: HN, LS ; binary: owns

$HN : D \to \{T, F\}$, $hn(x) = T$ if x howls at night

$owns : D \times D \to \{T, F\}$, $owns(x, y) = T$ if x owns y

$LS : D \to \{T, F\}$, $ls(x) = T$ if x is a light sleeper

H_1 : HN(hounds)

H_2 : $(\forall x)(\text{owns}(x, \text{cat}) \rightarrow \neg\, \text{owns}(x, \text{mice}))$

H_3 : $(\forall x)(\text{LS}(x) \rightarrow (\forall y)(\text{howl}(y) \rightarrow \neg\,\text{owns}(x,y)))$

H_4 : owns(John, cat) $\vee$ owns(John, hound)

H_5 : $\text{LS}(\text{John}) \rightarrow \neg\,\text{owns}(\text{John}, \text{mice})$

$H_3 \vdash_{\text{Universal instantiation, hounds}}$

H_5 (?)

$(\forall x)(\text{LS}(x) \rightarrow \text{howls}(\text{hounds}) \rightarrow \neg\,\text{owns}(x, \text{hounds}))$

$H_{?3}, H_1 \vdash_{\text{MP}} \forall(x)(\text{ls}(x) \rightarrow \neg\,\text{own}(x, \text{hnd})) : H_6$

$H \vdash$

2: universal inst. $\quad owns(John, cat) \to \neg owns(John, mice) = J_7$

$J_8 = H_4 \quad \neg owns(John, bear) \to owns(John, cat)$

$J_6, J_8 \vdash_{mp} b(x) \to owns(John, cat) = J_9$

$J_9, J_7 \vdash_{mp} b(x) \to \neg owns(John, mice) = J_{10}$ = conclusion

2. Semantic Tableaux

$\boxed{13.}$

Domain (D): universe of things

Constants: Train, doll, lamp

unary: Ch, b, g, good

binary: gets

Ch: $D \to \{T, F\}$ $\quad Ch(x) = T$ if x is a child

$ch: D \to \{T, F\}, \; ch(x) = T$ if x is a child

$b: D \to \{T, F\}, \; b(x) = T$ if x is a boy

$g: D \to \{T, F\}, \; g(x) = T$ if x is a girl

$gets: D \times D \to \{T, F\}, \; gets(x, y) = T$ if x gets y

$good: D \to \{T, F\}, \; good(x) = T$ if x is good

$H_1: \forall x \, (boy(x) \vee girl(x) \to child(x))$

$H_2: \forall x \, (child(x) \to (gets(x, train) \vee get(x, doll)$

$\vee \; gets(x, lump)))$

$H_3: \forall x \, (boy(x) \to \neg gets(x, doll))$

$H_4: \forall x \, (child(x) \wedge good(x) \to \neg gets(x, lump)))$

$C: \neg \exists x (\text{child}(x) \wedge \text{gets}(x, \text{train})) \rightarrow$

$\rightarrow \exists x (\text{boy}(x) \wedge \text{good}(x))$

$\neg C: \exists x (\text{child}(x) \wedge \text{gets}(x, \text{train})) \wedge$

$\exists x (\text{Boy}(x) \wedge \text{good}(x))$

$H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge C$ (1) alpha rule for (1)

$\forall x (\text{boy}(x) \vee \text{girl}(x) \rightarrow \text{child}(x))$ (2)

$\forall x (\text{child}(x) \rightarrow (\text{gets}(x, \text{train}) \vee \text{get}(x, \text{doll}) \vee \text{gets}(x, \text{lamp})))$ (3)

$\forall x (\text{boy}(x) \rightarrow \neg \text{gets}(x, \text{doll}))$ (4)

$\exists x\,(\mathrm{child}(x) \wedge \mathrm{gets}(x, \mathrm{train})) \wedge \exists x\,(\mathrm{Boy}(x) \wedge \mathrm{good}(x))$

3. General Resolution

$\boxed{5}$

H_1: Anyone whom Mary loves is a football star

H_2: Any student who does not pass, does not play.

H_3: John is a student.

H_4: Any student who does not study does not pass.

H_5: Anyone who does not play is not a football star.

C: If John does not study, then Mary does not love John.

Notations:

Domain (D) - universe of beings

John - constant

All the functions are unary.

$M(x): D \to \{T, F\}$, $M(x) = T$ if Mary loves x

$F(x): D \to \{T, F\}$, $F(x) = T$ if x is a football star.

$S(x): D \to \{T, F\}$, $S(x) = T$ if x is a student.

$P(x): D \to \{T, F\}$, $P(x) = T$ if x passes

$L(x): D \to \{T, F\}$, $L(x) = T$ if x plays

$T(x): D \to \{T,F\}$, $T(x) = T$ if x studies

$H_1: \forall x \, (M(x) \to F(x))$

$H_2: \forall x \, ((S(x) \land \neg P(x)) \to \neg L(x))$

$H_3: S(John)$

$H_4: \forall x \, (S(x) \land \neg T(x)) \to \neg P(x))$

$H_5: \forall x \, (\neg L(x) \to \neg F(x))$

$C: \neg T(John) \to \neg M(John)$

$C: T(John) \lor \neg M(John)$

$\neg C: \neg T(John) \land M(John)$

Rewriting we get:

$H_1^c: \boxed{\neg M(x) \lor F(x) = G_1}$ ✓

$H_2^c: \neg(S(x) \land \neg P(x)) \lor \neg L(x)$ (de Morgan's law)

$: \neg S(x) \vee P(x) \vee \neg L(x) = C_2$

$H_3^C: S(John) = C_3$

$H_4^C: \neg(S(x) \wedge \neg T(x)) \vee \neg P(x)$ (de Morgan's law)

$: \neg S(x) \vee T(x) \vee \neg P(x) = C_4$

$H_5^C: L(x) \vee \neg F(x) = C_5$

$(\neg C)^C: \neg T(John) \wedge M(John) = C_6$

$S = \{C_1, C_2, \dots, C_6\}$, general predicate resolution is applied.

$$C_7 = \underset{x \leftarrow John}{Res^{PR}}(C_2, C_3) = P(John) \vee \neg L(John)$$

$$C_8 = Res^{PR}(C_1, C_5) = L(x) \vee \neg M(x)$$

$C_9 = \underset{x \leftarrow John}{Res}(C_7, C_8) = P(John) \vee \neg R(John)$

$C_{10} = \underset{x \leftarrow John}{Res^{P_1}}(C_6, C_9) = \neg T(John) \vee P(John)$

$C_{11} = \underset{x \leftarrow John}{Res^{P_1}}(C_{10}, C_4) = \neg S(John)$

$C_{12} = Res^{P_1}(C_{11}, C_3) = \square$

$S \vdash^{P_1}_{Res} \square$, therefore S is an inconsistent set and the deduction $H_1, \ldots H_5$ holds.

Hence :

If John does not study, then Mary does not love John

(q.e.d)

4. Linear Resolution

H_1: Every coyote chases some road runner.

H_2: Every road runner who says „beep-beep" is smart.

H_3: No coyote catches any smart roadrunners.

H_4: Any coyote who chase some roadrunner but does not catch it is frustrated.

C: If all roadrunners say „beep-beep", then all coyotes are frustrated.

Rewriting we get:

D - domain of beings

All functions are unary.

No constants.

$coyote(x): D \to \{T, F\}$, $coyote(x) = T$ if x is a coyote

$chases(x, y): D \times D \to \{T, F\}$, $Chases(x, y) = T$ if x chases y

$beep(x): D \to \{T, F\}$, $beep(x) = T$ if x says "beep-beep".

$roadrunner(x): D \to \{T, F\}$, $roadrunner(x) = T$ if x is a road runner

$smart(x): D \to \{T, F\}$, $smart(x) = T$ if x is smart.

$catches(x): D \to \{T, F\}$, $catches(x) = T$ if x catches roadrunner

$frustrated(x): D \to \{T, F\}$, $frustrated(x) = T$ if x is frustrated.

$H_1: \forall x \, (coyote(x) \to \exists y \, (Roadrunner(y) \wedge chases(x, y)))$

$H_2: \forall x \, (Roadrunner(x) \wedge Beep(x) \to Smart(x))$

$H_3: \forall x \forall y \, ((coyote(x) \wedge Roadrunner(y) \wedge smart(y)) \to$

$\neg \, catches(x, y))$

$H_4: \forall x \forall y \, (coyote(x) \wedge roadrunner(y) \wedge chases(x, y) \wedge \neg$

$catches(x, y)) \to frustrated(x))$

$C: \exists x \, (Roadrunner(x) \wedge beep(x)) \wedge \exists y \, (coyote(y) \wedge$

$\neg frustrated(y))$

Rewriting as clauses:

C_1: $\neg coyote(x) \vee chases(x, f(x))$

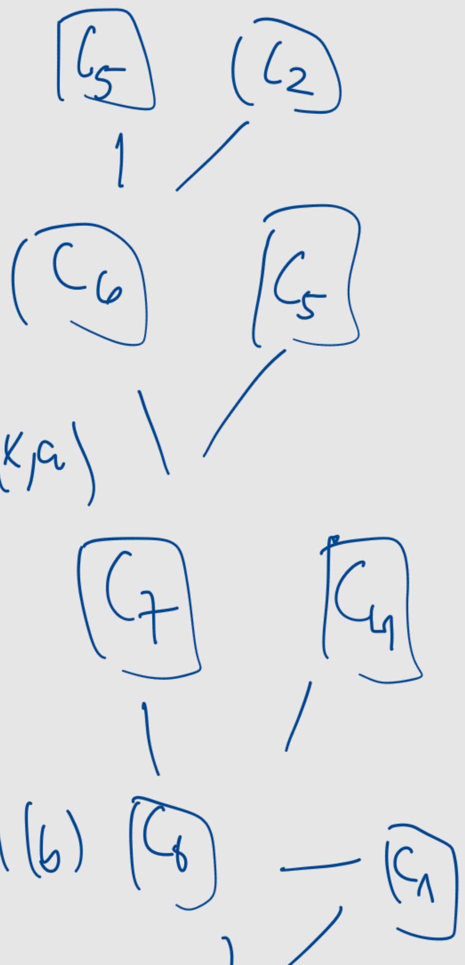
C_2: $\neg roadrunner(x) \vee \neg beep(x) \vee smart(x)$

C_3: $\neg coyote(x) \vee \neg roadrunner(x) \vee \neg smart(y) \vee \neg catches(x,y)$

C_4: $\neg coyote(x) \vee \neg roadrunner(y) \vee \neg chases(x,y) \vee catches(x,y) \vee$

$\vee\ frustrated(x)$

C_5: $roadrunner(a) \wedge beep(a)$ and $coyote(b) \wedge \neg frustrated(b)$



$C_5, C_2 \vdash^{x \leftarrow a}_{res} C_6 = smart(a)$

$C_6, C_3 \vdash^{y \leftarrow a}_{res} C_7 = \neg coyote(x) \vee \neg catches(x,a)$

$C_7, C_5 \vdash^{x \leftarrow b}_{res} C_8 = \neg catches(b,a)$

$C_8, C_4 \vdash_{res} C_9 = \neg chases(b,a) \vee frustrated(b)$

$C_9, C_1 \vdash_{res}^{x \leftarrow b,\ f(b)=a} C_{10} \quad \neg\, Coyote(b)$

$\square$

$C_{10}, C_5 \vdash_{res} \square$

Side clauses C_2, C_5, C_4, C_1.

From the set S we derived the empty clause, therefore S is inconsistent.