Seminar 12:

Chapter 7: 1, 2, 4, 6, 7, 8

$Q: a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{10}x + 2a_{01}y + a_{00} = 0$

$\xrightarrow{\text{isometries}}$ (rotation + translation)

$\lambda_1 x^2 + \lambda_2 y^2 = k$ or $\lambda_1 x^2 + v_2 y = k \xrightarrow{\text{rescaling}} \lambda_1, \lambda_2 \in \{\pm 1\}$

n = rank Q	$(p, n-p)$	equation	
2	$(0,2)$ or $(2,0)$	$x^2 + y^2 + 1 = 0$	imaginary ellipse
2	$(1,1)$	$x^2 - y^2 - 1 = 0$	hyperbola ♥
2	$(0,2)$ or $(2,0)$	$x^2 + y^2 - 1 = 0$	ellipse (circle) ♥
2	$(0,2)$ or $(2,0)$	$x^2 + y^2 = 0$	point / two complex lines
2	$(1,1)$	$x^2 - y^2 = 0$	two real lines
1	$(0,1)$ or $(1,0)$	$x^2 + 1 = 0$	two complex lines
? 1	$(1,1)$	$x^2 - 1 = 0$	two real lines
? 1	$(1,1)$	$x^2 = 0$	double line
1	$(0,1)$ or $(1,0)$	$x^2 = y$ $x^2 - y = 0$	parabola ♥

7.2. Write down a quadratic eqn. with associated matrix A and find $M \in SO(2)$ with diagonalizes A.

a) $A = \begin{pmatrix} 6 & 2 \\ 2 & 9 \end{pmatrix}$

$P_A(x) = \det(A - xI_4) = \begin{vmatrix} 6-x & 2 \\ 2 & 9-x \end{vmatrix} = (6-x)(9-x) - 4 =$

$= x^2 - 15x + 50$; $x_{1,2} = \frac{15 \pm \sqrt{225 - 200}}{2} = \begin{cases} 10 \\ 5 \end{cases} \Rightarrow \begin{cases} \lambda_1 = 10 \\ \lambda_2 = 5 \end{cases}$

$S(\lambda_1) = S(10) = \{(x, y) \mid A \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \lambda_1 \cdot \begin{pmatrix} x \\ y \end{pmatrix}\} = \{(x, y) \mid (A - \lambda_1 I_2) \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\}$

$$\begin{pmatrix} -4 & 2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -4x + 2y = 0 \\ 2x - y = 0 \end{cases} \Leftrightarrow y = 2x \Rightarrow S(\lambda_1) = \langle (1, 2) \rangle$$

We choose $v_1 = \frac{1}{\sqrt{5}}(1, 2)$

$$S(\lambda_2) = \left\{ (x, y) \,\middle|\, \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\} =$$

$$= \left\{ (x, y) \,\middle|\, \begin{cases} x + 2y = 0 \\ 2x + 4y = 0 \end{cases} \right\} = \{(-2y, y) \mid y \in \mathbb{R}\} = \langle (-2, 1) \rangle$$

We choose $v_2 = \frac{1}{\sqrt{5}}(2, -1)$. This gives us:

$$M_{B', B} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \end{pmatrix} \qquad \det(M_{B', B}) = -1$$

We wand det $M = 1$. So instead of choosing $v_2 = \frac{1}{\sqrt{5}}(2, -1)$ we choose $v_2 = \frac{1}{\sqrt{5}}(-2, 1) \Rightarrow M = M_{B', B} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

Regarding the quadratic equation, let us take:

$$Q: (x \;\; y) \cdot \begin{pmatrix} 6 & 2 \\ 2 & 9 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + (2 \;\; 3) \begin{pmatrix} x \\ y \end{pmatrix} + 5 = 0$$

$$Q: (6x + 2y \;\;\; 2x + 9y) \begin{pmatrix} x \\ y \end{pmatrix} + 2x + 3y + 5 = 0$$

$$Q: 6x^2 + 2xy + 2xy + 9y^2 + 2x + 3y + 5 = 0$$

$$Q: 6x^2 + 4xy + 9y^2 + 2x + 3y + 5 = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = M \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = M_{B', B} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} x' - 2 \\ 2x' + y \end{pmatrix}$$

$$Q: \underbrace{(x' \;\; y') \cdot M^T}_{(x \;\; y)} \cdot \begin{pmatrix} 6 & 2 \\ 2 & 9 \end{pmatrix} \cdot \underbrace{M \cdot \begin{pmatrix} x' \\ y' \end{pmatrix}}_{\begin{pmatrix} x \\ y \end{pmatrix}} + (2 \;\; 3) \cdot M \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} + 5 = 0$$

~~$Q: 10x'^2 + 5y'^2$~~

$Q: (x' \ y') \begin{pmatrix} 10 & 0 \\ 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} + \frac{1}{\sqrt{5}} \cdot (2 \ 3) \cdot \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} + 5 = 0$

$Q: 10x'^2 + 5y'^2 + \frac{8}{\sqrt{5}} x' - \frac{1}{\sqrt{5}} y' + 5 = 0$

$Q: (10x'^2 + \frac{8}{\sqrt{5}} x') + (5y'^2 - \frac{1}{\sqrt{5}} y') + 5 = 0$

$10 \cdot (x'^2 + 2 \cdot \frac{2}{5\sqrt{5}} x' + \frac{4}{125}) + 5 \cdot (y'^2 - 2 \cdot \frac{1}{10\sqrt{5}} y' + \frac{1}{500}) + 5 - \frac{8}{25} - \frac{1}{100} = 0$

$10 \cdot (x' + \frac{2}{5\sqrt{5}})^2 + 5 \cdot (y - \frac{1}{10\sqrt{5}})^2 + \frac{467}{100} = 0$

$x'' = x' + \frac{2}{5\sqrt{5}}$; $y'' = y' - \frac{1}{10\sqrt{5}}$

$\Rightarrow Q: 10x''^2 + 5y''^2 + \frac{467}{100} = 0 \Rightarrow$ imaginary ellipse

7.4. Write down the associated matrix and bring the equation to the canonical form.

So far the transformations have been isometries, but if we want, we can do: $\begin{cases} x''' = \sqrt{\frac{1000}{467}} x'' \\ y''' = \sqrt{\frac{500}{467}} y'' \end{cases} \Rightarrow Q: x'''^2 + y'''^2 + 1 = 0$

a) $Q: -x^2 + xy + y^2 = 0$

$M_Q = \begin{pmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$; $P_{M_Q}(x) = \begin{vmatrix} -1-x & \frac{1}{2} \\ \frac{1}{2} & 1-x \end{vmatrix} = x^2 + 2x + \frac{3}{4} \Rightarrow \begin{cases} -\frac{1}{2} \\ -\frac{3}{2} \end{cases}$

$= -(1+x)(1-x) - \frac{1}{4} = -(1-x^2) - \frac{1}{4} = x^2 - \frac{5}{4} \Rightarrow \lambda_{1,2} = \pm \frac{\sqrt{5}}{2}$

$M_Q \begin{pmatrix} x \\ y \end{pmatrix} = \lambda_1 \begin{pmatrix} x \\ y \end{pmatrix} \Leftrightarrow \begin{pmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{cases} -x + \frac{1}{2} y = \frac{\sqrt{5}}{2} x \\ \frac{1}{2} x + y = \frac{\sqrt{5}}{2} y \end{cases} \Rightarrow \begin{cases} -\frac{1}{2} x + \frac{1}{2} y = 0 \\ \frac{1}{2} x - \frac{1}{2} y = 0 \end{cases} \Rightarrow S(\lambda_1) = \langle (1,1) \rangle$

$v_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$M_Q \begin{pmatrix} x \\ y \end{pmatrix} = \lambda_2 \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{cases} -x + \frac{1}{2}y = -\frac{3}{2}x \\ \frac{1}{2}x - y = -\frac{3}{2}x \end{cases} \Rightarrow \begin{cases} \frac{1}{2}x + \frac{1}{2}y = 0 \\ \frac{1}{2}x + \frac{1}{2}y = 0 \end{cases}$

$\Rightarrow S(\lambda_2) = \langle (1, 1) \rangle \Rightarrow v_2 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}; \ |M| = \frac{1}{2}(-1 \ -1) \Rightarrow v_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\Rightarrow M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$(x \ y) \, M_Q \begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow (x' \ y') \underbrace{M^T \cdot M_Q \cdot M}_{= \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{3}{2} \end{pmatrix}} \begin{pmatrix} x' \\ y' \end{pmatrix} = 0$

$M^T \cdot M_Q \cdot M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} =$

$= \frac{1}{2} \begin{pmatrix} -1 + \frac{1}{2} & \frac{1}{2} - 1 \\ 1 + \frac{1}{2} & -\frac{1}{2} - 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} =$

$= \frac{1}{2} \begin{pmatrix} -\frac{1}{2} - \frac{1}{2} & \frac{1}{2} - \frac{1}{2} \\ \frac{3}{2} - \frac{3}{2} & -\frac{3}{2} - \frac{3}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{3}{2} \end{pmatrix}$

$\left(-\frac{1}{2}x' \ \ -\frac{3}{2}y'\right) \begin{pmatrix} x' \\ y' \end{pmatrix} = 0$

$-\frac{1}{2}x'^2 - \frac{3}{2}y'^2 = 0 \ \Big| \cdot (-2)$

$x'^2 + 3y'^2 = 0$

$\begin{cases} x'' = x' \\ y'' = \sqrt{3}\,y' \end{cases} \Rightarrow x''^2 + y''^2 = 0$

Lagrange method:

$-x^2 + xy - y^2 = 0$

$-(x^2 - xy) - y^2 = 0$

$-\left(x^2 - 2\cdot x\cdot \frac{y}{2} + \frac{y^2}{4}\right) + \frac{y^2}{4} + y^2 = 0$

$-\left(x - \frac{y}{2}\right)^2 - \frac{3y^2}{4} = 0$

$\left(x - \frac{y}{2}\right)^2 + \frac{3}{4}y^2 = 0$

$\begin{cases} x = x - \frac{y}{2} \\ y = \frac{\sqrt{3}}{2} y \end{cases} \Rightarrow x'^2 + y'^2 = 0$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

7.7. Find the canonical equation:

a) $Q: 5x^2 + 4xy + 8y^2 - 32x - 56y + 80 = 0$

$Q: (5x^2 + 4xy) + 8y^2 - 32x - 56y + 80 = 0$

$Q: \left(5x^2 + 2\sqrt{5}x\cdot\frac{2}{\sqrt{5}}y + \frac{4}{5}y^2\right) + 8y^2 - \frac{4}{5}y^2 - 32x - 56y + 80 = 0$

$Q: \left(\sqrt{5}x + \frac{2}{\sqrt{5}}y\right)^2 + \frac{36}{5}y^2 - 32x - 56y + 80 = 0$

$x' = \sqrt{5}x + \frac{2}{\sqrt{5}}y \Rightarrow \sqrt{5}x = x' - \frac{2}{\sqrt{5}}y$

$x = \frac{1}{\sqrt{5}}x' - \frac{2}{5}y$

$Q: x'^2 + \frac{36}{5}y^2 - \frac{32}{\sqrt{5}}x' + \frac{32\cdot 2}{5}y - 56y + 80 = 0$

$Q: x'^2 - \frac{32}{\sqrt{5}}x' + \frac{36}{5}y^2 + \frac{64}{5}y - 56y + 80 = 0$

$Q: \left(x'^2 - \frac{32}{\sqrt{5}}x'\right) + \left(\frac{36}{5}y^2 - \frac{216}{5}y\right) + 80 = 0$

$Q: \left(x'^2 - 2x'\cdot\frac{16}{\sqrt{5}} + \frac{256}{5}\right) + \left(\frac{36}{5}y - 2\cdot\frac{6}{\sqrt{5}}y\cdot\frac{18}{\sqrt{5}} + \frac{18^2}{5}\right) -$

$-\frac{256}{5} + 18^2 \quad + 80 = 0$

$$x'' = x' - \frac{16}{\sqrt{5}}$$

$$y'' = \frac{6}{\sqrt{5}} y - \frac{18}{\sqrt{5}}$$

$$Q: x''^2 + y''^2 = \frac{180}{5}$$

$$x''' = \sqrt{\frac{5}{180}} \cdot x''$$

$$y''' = \sqrt{\frac{5}{180}} \cdot y''$$

$$\Rightarrow Q: x'''^2 + y'''^2 = 1$$

Seminar 13:

Ch7: 9, 10 a)

Ch8: 3, 4, 5, 10, 14

$$Q: a_{11} x^2 + 2a_{12} xy + a_{22} y^2 + 2a_{10} x + 2a_{01} y + a_{00} = 0$$

$$M_Q = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \qquad \hat{M}_Q = \left(\begin{array}{cc|c} a_{11} & a_{12} & a_{10} \\ a_{12} & a_{22} & a_{01} \\ \hline a_{10} & a_{01} & a_{00} \end{array}\right)$$

$\hat{\Delta} := \det \hat{M}_Q$

$\delta := \det M_Q$

$T := \operatorname{Tr} M_Q$

$\hat{\Delta}$	δ	T	Curve Q
	$\delta > 0$		a point
$\hat{\Delta} = 0$	$\delta = 0$		two lines or the empty set
	$\delta < 0$		two lines
	$\delta > 0$	$T < 0$	an ellipse
$\hat{\Delta} \neq 0$	$\delta > 0$	$T > 0$	the empty set
	$\delta = 0$	-	a parabola
	$\delta < 0$	-	a hyperbola

7.9. Discuss the type of the curve $x^2 + \lambda xy + y^2 - 6x - 16 = 0$ in terms of $\lambda \in \mathbb{R}$. (Homework: solve this using the techniques from last time: diagonalization and Lagrange)

$$M_Q = \begin{pmatrix} 1 & \frac{\lambda}{2} \\ \frac{\lambda}{2} & 1 \end{pmatrix} \Rightarrow \delta = 1 - \frac{\lambda^2}{4}$$

$$\hat{M}_Q = \begin{pmatrix} 1 & \frac{\lambda}{2} & -3 \\ \frac{\lambda}{2} & 1 & 0 \\ -4 & 0 & -16 \end{pmatrix} \Rightarrow \hat{\delta} = 4\lambda^2 - 25$$

I. $\lambda = \frac{5}{2} \Rightarrow \left.\begin{matrix} \hat{\delta} = 0 \\ \delta < 0 \end{matrix}\right\} \Rightarrow$ two lines

II. $\lambda = -\frac{5}{2} \Rightarrow \left.\begin{matrix} \hat{\delta} = 0 \\ \delta < 0 \end{matrix}\right\} \Rightarrow$ two lines

III. $|\lambda| < 2$ and $\lambda \neq \pm\frac{5}{2} \Rightarrow \left.\begin{matrix} \hat{\delta} \neq 0 \\ \delta < 0 \end{matrix}\right\} \Rightarrow$ hyperbola

IV. $\lambda = \pm 2 \Rightarrow \left.\begin{matrix} \hat{\delta} \neq 0 \\ \delta = 0 \end{matrix}\right\} \Rightarrow$ parabola

V. $\lambda \in (-2, 2)$ and $\lambda \neq \pm\frac{5}{2} \Rightarrow \left.\begin{matrix} \hat{\delta} \neq 0 \\ \delta > 0 \\ T > 0 \end{matrix}\right\} \Rightarrow$ empty set

7.10. Decide (using the Lagrange method) what surfaces are described by the equations:

a) $x^2 + 2y^2 + z^2 + xy + yz + zx = 1$

b) $xy + yz + zx = 1$

a) $x^2 + \frac{y^2}{4} + \frac{z^2}{4} + 2xy \cdot \frac{1}{2} + 2yz \cdot \frac{1}{4} + 2zx \cdot \frac{1}{2} + \frac{7y^2}{4} + \frac{3z^2}{4} + \frac{1}{2}yz = 1$

$$\left(x + \frac{y}{2} + \frac{z}{2}\right)^2 + \frac{7}{4}y^2 + \frac{3z^2}{4} + \frac{1}{2}yz = 1$$

$$x' := x + \frac{y}{2} + \frac{z}{2}$$

$$(x')^2 + \frac{7}{4}y^2 + 2\frac{\sqrt{7}}{2}yz \cdot \frac{1}{2\sqrt{7}} + \frac{1}{28}z^2 + \frac{20}{28}z^2 = 1$$

$$(x')^2 + \left(\frac{\sqrt{7}}{2} y + \frac{1}{2\sqrt{7}} z\right)^2 + \frac{5}{7} z^2 = 1$$

$$y' := \frac{\sqrt{7}}{2} y + \frac{1}{2\sqrt{7}} z$$

$$(x')^2 + (y')^2 + \frac{5}{7} z^2 = 1$$

$$z' := \frac{\sqrt{5}}{\sqrt{7}} z \quad \Rightarrow \quad (x')^2 + (y')^2 + (z')^2 = 1$$

$\Rightarrow$ ellypsoid

e) We don't have squares, so we make them: $x = x' + y$

$$Q: (x'+y)y + yz + z(x'+y) = 1$$

$$x'y + y^2 + yz + zx' + zy = 1$$

$$\left(y^2 + 2\cdot\frac{1}{2} yx' + 2\cdot\frac{1}{2} yz + \frac{(x')^2}{4} + \frac{z^2}{4} + \frac{1}{2}x'z\right) + \frac{1}{2}x'z - \frac{(x')^2}{4} - \frac{z^2}{4}$$

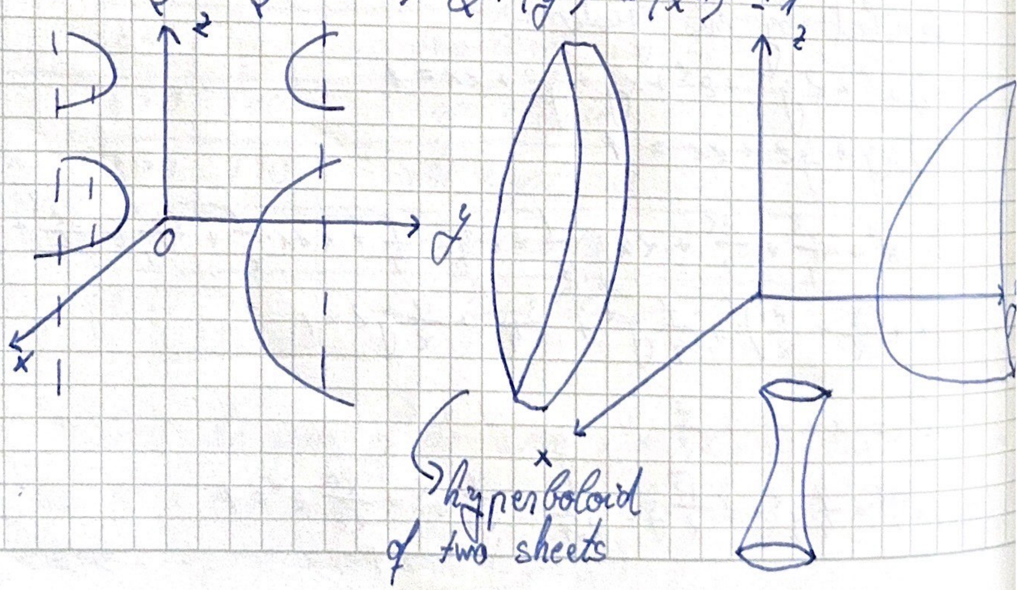
$$\left(y + \frac{x'}{2} + \frac{z}{2}\right)^2 - \frac{(x')^2}{4} - \frac{z^2}{4} + \frac{1}{2}x'z = 1$$

$$y' := y + \frac{x'}{2} + \frac{z}{2}$$

$$(y')^2 - \left(\frac{(x')^2}{4} - 2\cdot\frac{x'}{2}\cdot\frac{z}{2} + \frac{z^2}{4}\right) = 1$$

$$(y')^2 - \left(\frac{x'}{2} - \frac{z}{2}\right)^2 = 1$$

$$x'' := \frac{x'}{2} - \frac{z}{2} \quad \Rightarrow \quad Q: (y')^2 - (x'')^2 = 1$$



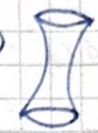
$$\left(y+\frac{x'}{2}+z\right)^2-\frac{(x')^2}{4}-z^2=1$$

$$y' := y+\frac{x'}{2}+z$$

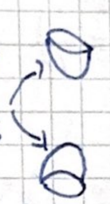
$$(y')^2-\frac{(x')^2}{4}-z^2=1 \Rightarrow \frac{(x')^2}{4}-(y')^2+z^2=-1$$

$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1} \to$ ellypsoid

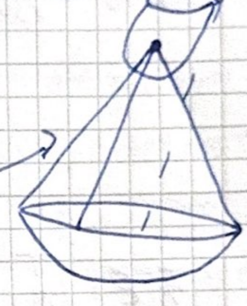
$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1} \to$ hyperboloid



$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1} \to$ hyperboloid of two sheets



$\boxed{\frac{x^2}{p}+\frac{y^2}{q}=2z} \to$ elliptic paraboloid



$\boxed{\frac{x^2}{p}-\frac{y^2}{q}=2z} \to$ hyperbolic paraboloid

Chapter 8:

$\mathcal{S}: f(x,y,z)=0$

$T_{(x_0,y_0,z_0)}\mathcal{S}: \nabla f \cdot (x-x_0, y-y_0, z-z_0)=0$

$$\frac{\partial f}{\partial x}(x_0,y_0,z_0)\cdot(x-x_0)+\frac{\partial f}{\partial y}(x_0,y_0,z_0)(y-y_0)+\frac{\partial f}{\partial z}(x_0,y_0,z_0)(z-z_0)=0$$

8.4. Determine the tangent planes to the ellipsoid $\mathcal{E}_{2,3,2\sqrt{2}}: \frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{8}=1$ which are parallel to the plane $\pi: 3x-2y+5z+1=0$.

$T_{P(x_0,y_0,z_0)}\mathcal{E}: \frac{x_0 x}{4}+\frac{y_0 y}{9}+\frac{z_0 z}{8}=1$

$\overrightarrow{n_\pi}=(3,-2,5)$; $\overrightarrow{n_{T_P\mathcal{E}}}=\left(\frac{x_0}{4},\frac{y_0}{9},\frac{z_0}{8}\right)$; $\pi \parallel T_P\mathcal{E} \Rightarrow$

$$\Rightarrow \frac{x_0}{\frac{4}{3}} = \frac{y_0}{\frac{9}{-2}} = \frac{z_0}{\frac{8}{5}} \Leftrightarrow \frac{x_0}{12} = \frac{y_0}{-18} = \frac{z_0}{40}$$

$$\Leftrightarrow \begin{cases} y_0 = \frac{-18x_0}{12} = \frac{-3x_0}{2} \\ z_0 = \frac{40x_0}{12} = \frac{10x_0}{3} \end{cases}$$

$$\frac{{}^{18/}x_0^2}{4} + \frac{{}^{2/}9x_0^2}{36} + \frac{100x_0^2}{72} = 1 \;/\cdot 72$$

$$\Rightarrow 18x_0^2 + 18x_0^2 + 100x_0^2 = 72 \Rightarrow 136x_0^2 = 72$$

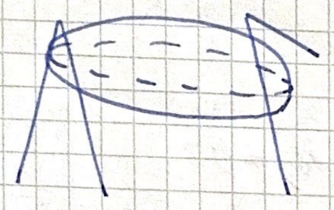
$$\Rightarrow x_0^2 = \frac{72^{18}}{136} = \frac{9}{17} \Rightarrow x_0 = \pm\sqrt{\frac{9}{17}}$$

$$y_0 = \mp \frac{3}{2}\sqrt{\frac{9}{17}}$$

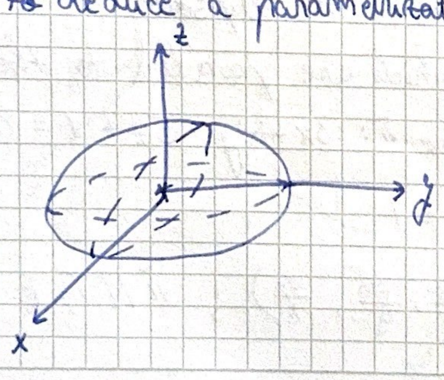
$$z_0 = \pm \frac{10}{3}\sqrt{\frac{9}{17}}$$

$$T_{P_1}\mathcal{E}: \sqrt{\frac{9}{17}} \cdot \frac{x}{4} - \frac{3\sqrt{9}}{2\sqrt{17}} \cdot \frac{y}{9} + \frac{10\sqrt{9}}{3\sqrt{17}} \cdot \frac{z}{8} = 1$$

$$T_{P_2}\mathcal{E}: -\sqrt{\frac{9}{17}} \cdot \frac{x}{4} + \frac{3\sqrt{9}}{18\sqrt{17}} \cdot y + \frac{10\sqrt{9}}{24\sqrt{17}} \cdot z = 1$$



8.10. Use a parametrization of an ellipse and a rotation matrix to deduce a parametrization of an ellipsoid of revolution.



The equation of an ellipse in the (xOy) plane is: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which we cane parametrize as:

$$\begin{cases} x = a\cos\theta \\ y = b\sin\theta \end{cases}, \quad \theta \in [0, 2\pi)$$

In 3D this is: $\begin{cases} x = a\cos\theta \\ y = b\sin\theta \\ z = 0 \end{cases}$

$$[Rot_{Oy}] = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$[Rot_{Oz}] = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow$ So the surface of revolution (   ) has the equation:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos t & 0 & -\sin t \\ 0 & 1 & 0 \\ \sin t & 0 & \cos t \end{pmatrix} \cdot \begin{pmatrix} a\cos\theta \\ b\sin\theta \\ 0 \end{pmatrix} = \begin{pmatrix} a\cos t\cdot\cos\theta \\ b\sin\theta \\ a\sin t\cos\theta \end{pmatrix}$$

$$\Rightarrow \mathcal{J}: \frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{a^2} = 1$$