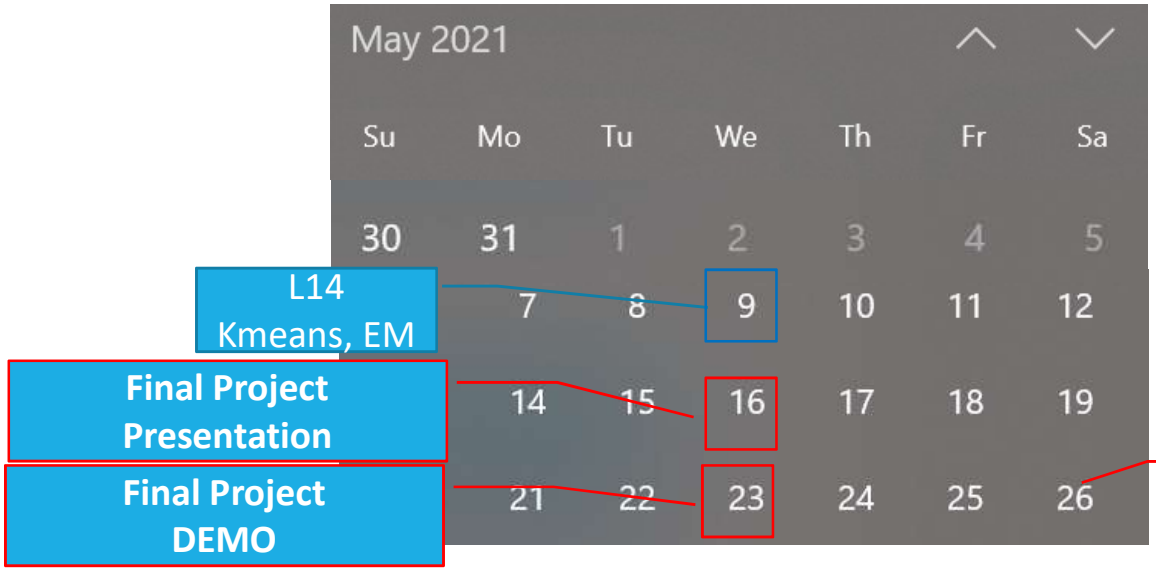
K-means and EM

KUO-SHIH TSENG
DEPARTMENT OF MATHEMATICS
NATIONAL CENTRAL UNIVERSITY, TAIWAN

2021/06/09

Course Announcement



Please take your time on your project!
There are only 2 weeks!
We will have presentation and demo via google meet.

Final Project report

Course Announcement

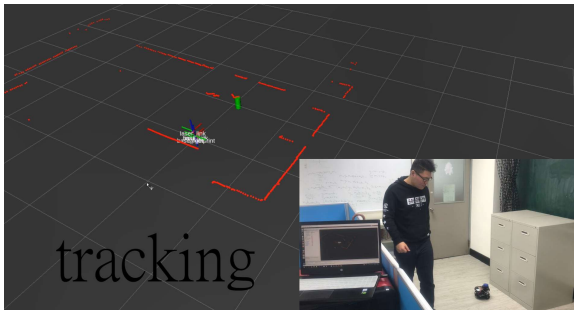
- Project meeting: (6/9, 6/16)
  - 13:30~13:45: 陳宇揚
  - 13:45~14:00: 李家妤
  - 14:00~14:15: 陳羽暉 蔡沐霖 高文顯
  - 14:15~14:30: 邱韋翔
  - 14:30~14:45: 林寶德
  - 14:45~15:00: 張軒旗

Course Announcement

- Project presentation(6/18) and demo(6/25)
  - Presentation/Demo (20 mins) + Q&A (~5mins)
  - 03:00～03:30: 陳宇揚
  - 03:30～04:00: 李家妤
  - 04:00～04:30: 陳羽暉 蔡沐霖 高文顯
  - 04:30～05:00: 邱韋翔
  - 05:00～05:30: 林寶德
  - 05:30～06:00: 張軒旗
- Google meet: meet.google.com/doq-cwbn-ocv
- Presentation: share your desktop with ppt presentation
- Demo: play a video and explain your project.

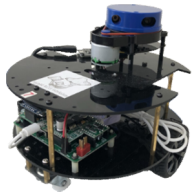
Course Announcement

- You should have a video demo in the presentation and a robot demo in Demo. **Video demo** displays the learning processing. **Robot demo** displays the learned optimal decisions.
- **Video demo (in your presentation)**



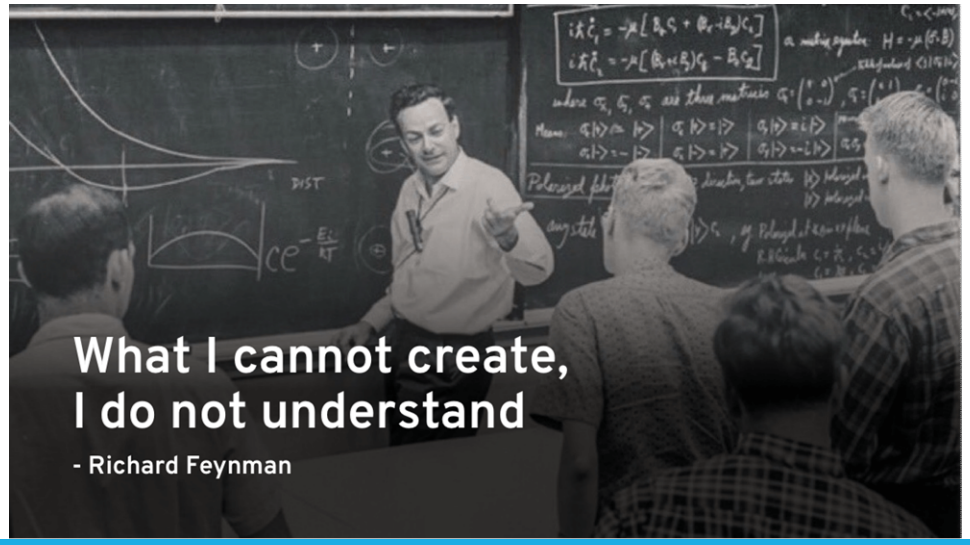
You can use Camtasia to record your video.
https://www.techsmith.com/video-editor.html
The video should shows how the robot learn to its task.
For example, episode #1 to #20

- **Robot demo/simulation (15 mins)**



Course Announcement

- This video will be uploaded on the website of AI Maker Lab.
- http://w2.math.ncu.edu.tw/about/aimakerlab



Course Announcement

- Final report DL: **06/26(Sat.)** 11:59pm
- Check "Final Project Report sample.pdf" on LMS.
- You must write a final report (4-8 pages), which includes
  - Introduction
  - Methods/Approaches
  - Experiments
  - Conclusion and Discussion
  - Reference
- Upload your report to LMS on time!
- **DON'T** send me your code. That's your intellectual property (IP) even if I am your teacher. I trust what you have done. If you use somebody's code without citations, you got 0 in the final project.

Course Announcement

- Final project poster
- If your project is excellent, you can make a poster in front of M-213.

Your poster here!

Outline

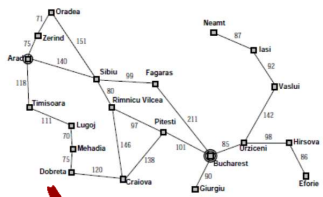
- Need
- K-means
- Expectation Maximization (EM)
- EX: subgoal clustering
- EX: Gaze patterns
- Review of MAI

Outline

[Problem solving]

Search problems

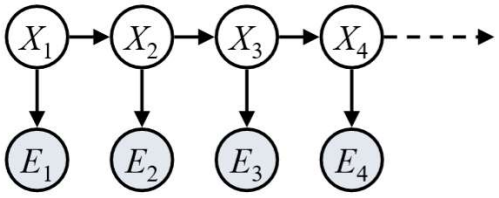
Adversarial Search



[Perception and Uncertainty]

Bayes Theorem

Bayes Filter and Smoothing

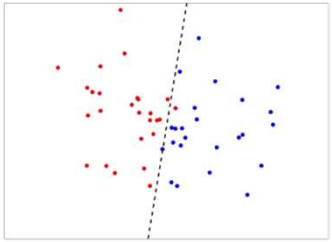


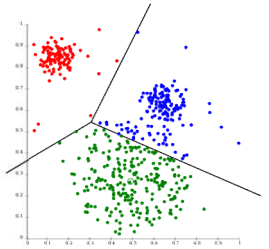
[Learning and Decision-making]

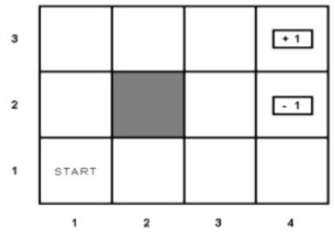
Supervised learning

Unsupervised learning

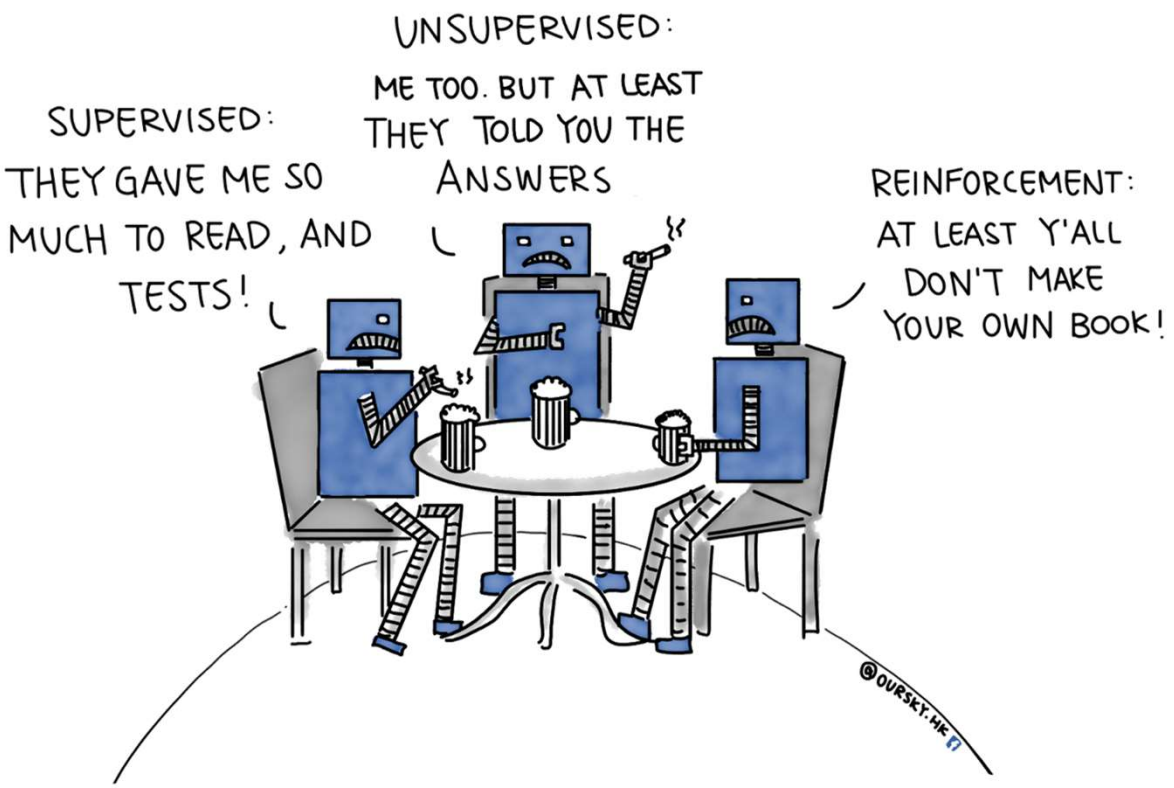
Reinforcement learning







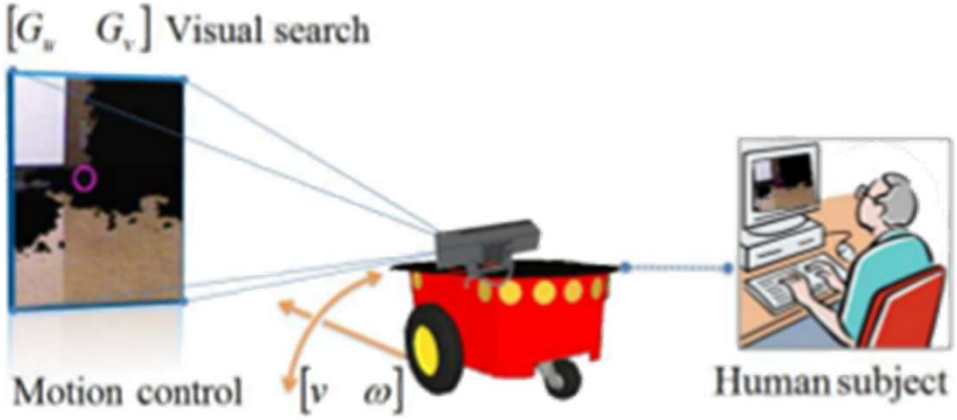
Need



From: Facebook OURSKY

Need

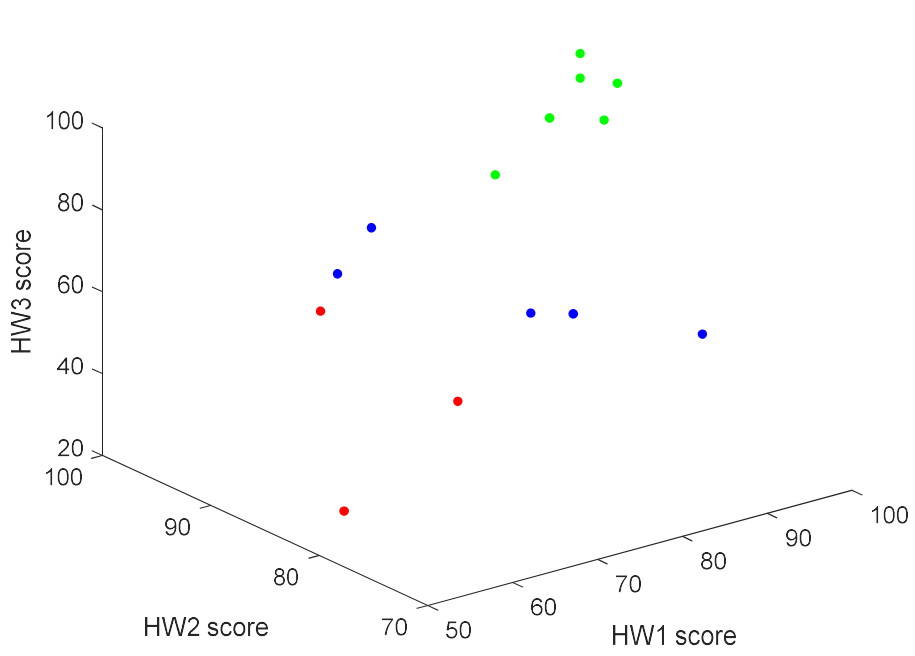
- For some applications, we got some data without labels. We have to divided then into **meaningful** and/or **useful** ***K*** groups for analysis. Since there are no labels, it is called unsupervised learning or called clustering analysis.
- For example, a human remotely controls a robot moving in a 2D plane. How could we extract the robot motion behavior?

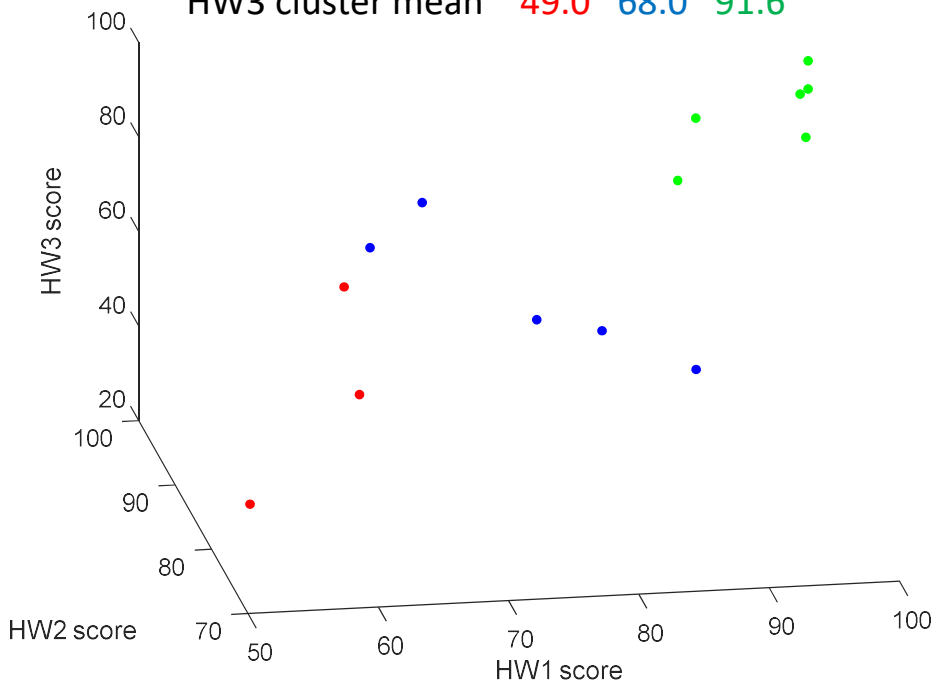


Need

- Clustering MAI students into 3 group according to 3 homework scores.

HW1 cluster mean	58.6	77.3	97.4
HW2 cluster mean	81.6	84.5	92.4
HW3 cluster mean	49.0	68.0	91.6





Need

- K-means for image segmentation

Christopher M. Bishop, "Pattern Recognition and Machine Learning," Springer, 2013.

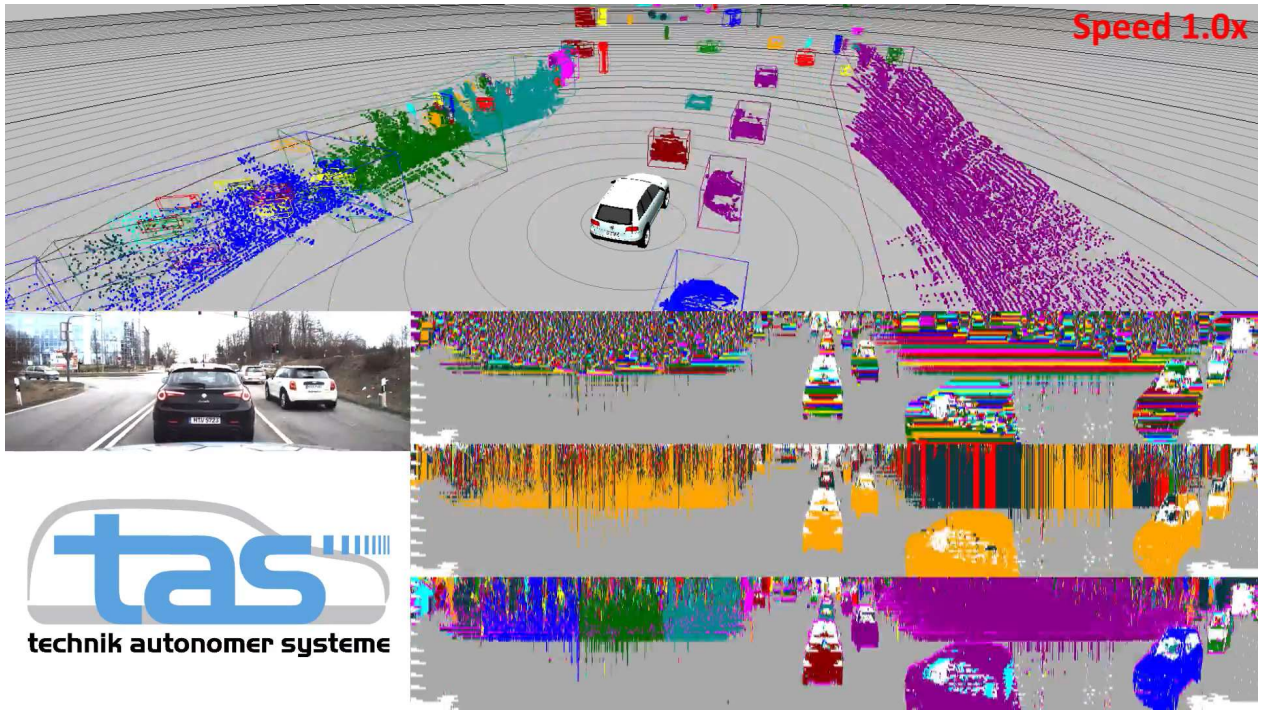
Need

- Semantic Segmentation for Autonomous Driving

https://www.youtube.com/watch?v=HbPhvct5kvs

Need

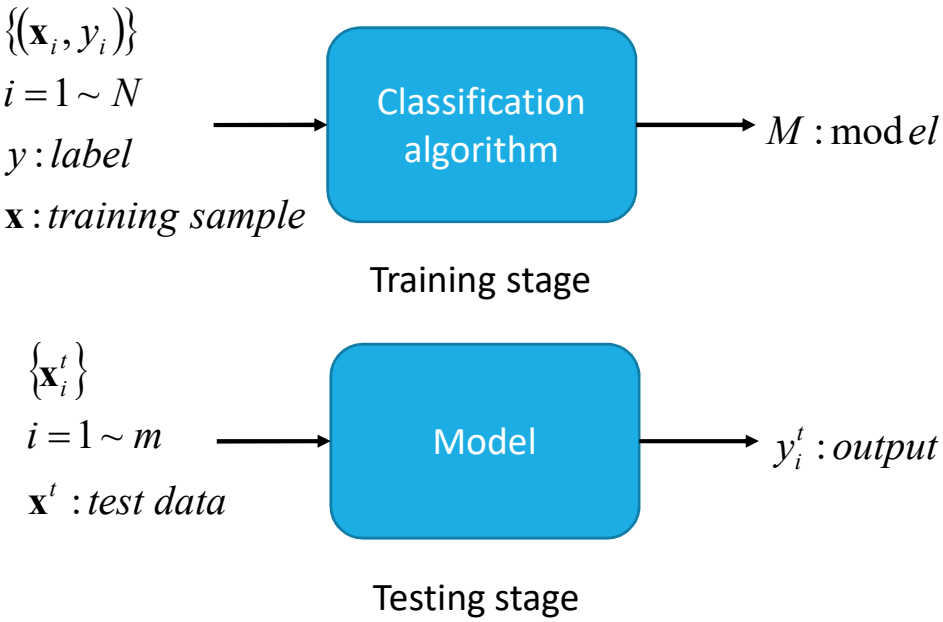
- Clustering for point clouds



https://www.youtube.com/watch?v=DdbvfwdRZqM

Need

- Supervised learning v.s. Unsupervised learning



There are algorithms to decide "***K***" according to data.

Need

- There are many clustering algorithms for different purposes. If you want to learn more, please read [1] for details.
  - **K-means**
  - **Expectation Maximization (EM)**
  - DBSCAN
  - Self-Organizing Maps (SOM)
  - etc.

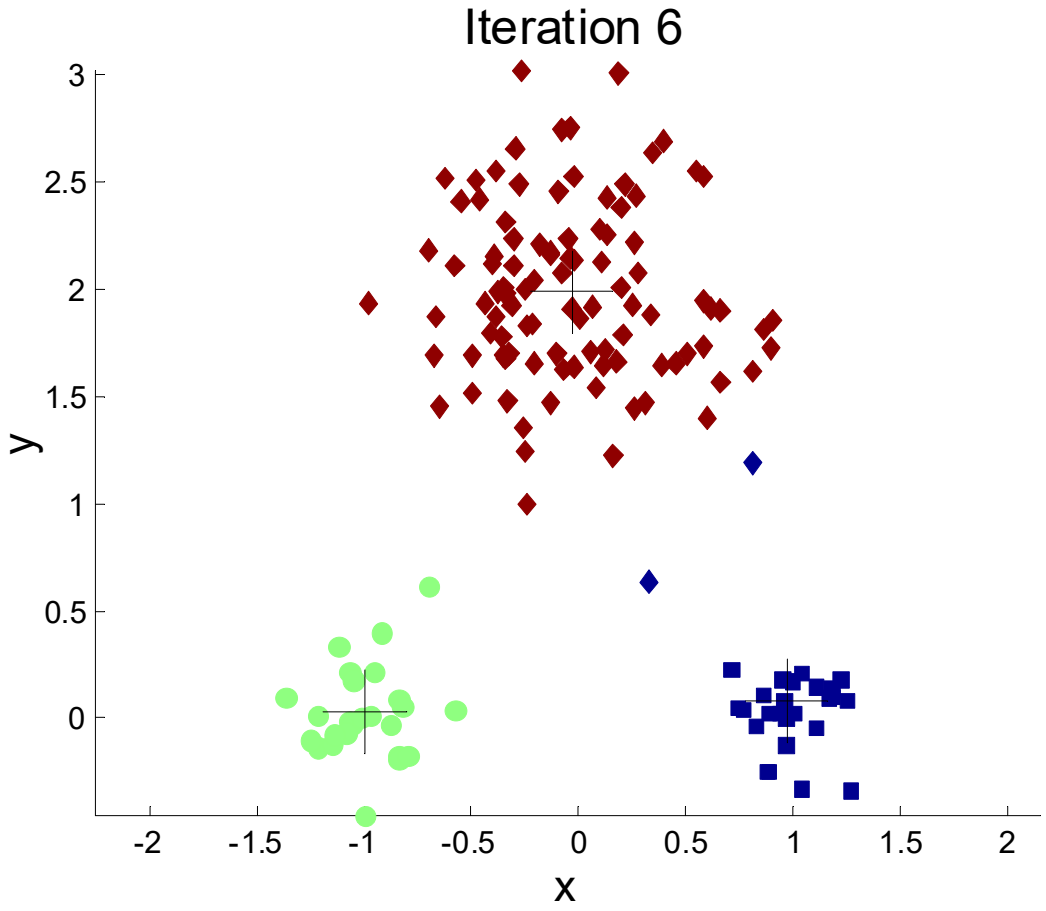
[1] Pang-Ning Tan, Michael Steinbach, Anuj Karpatne and Vipin Kumar, "Introduction to Data Mining", Pearson, 2nd Edition, 2018

K-means

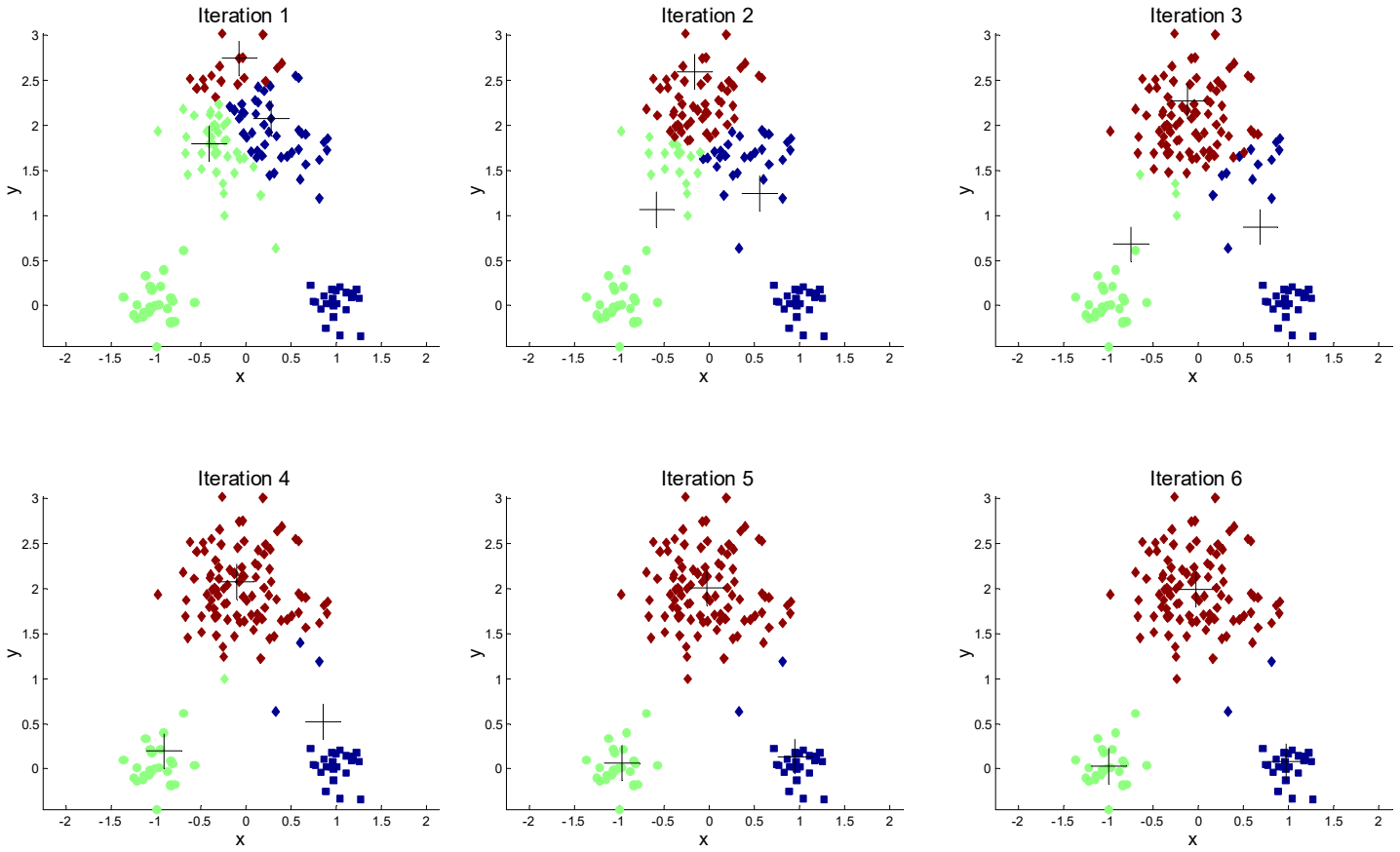
- The major concept of K-means is to update the K centroids iteratively.
- The complexity is O(nKID)
  - n: # of data points
  - I: # of iterations
  - K: # of clusters
  - d: # of feature dimensions

1: Select K points as the initial centroids.
2: **repeat**
3: Form K clusters by assigning all points to the closest centroid.
4: Recompute the centroid of each cluster.
5: **until** The centroids don't change

K-means

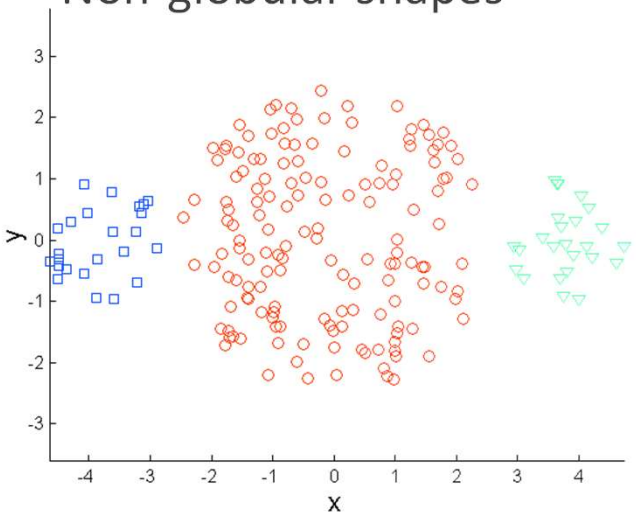


K-means

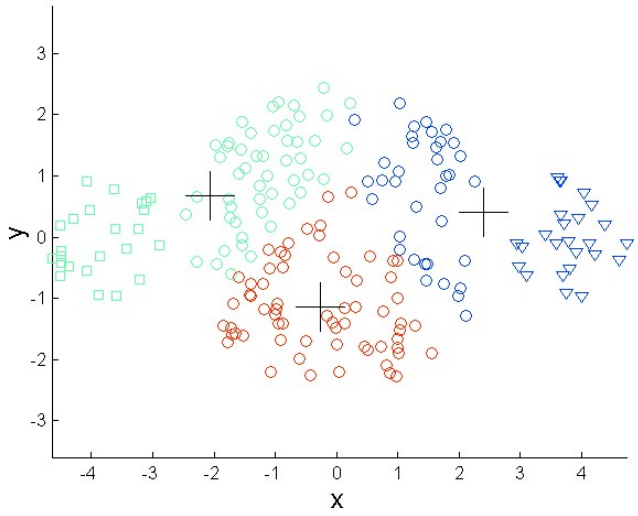

Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
x
y

K-means

- ☹: K-means has problems when clusters are of differing
  - Sizes
  - Densities
  - Non-globular shapes



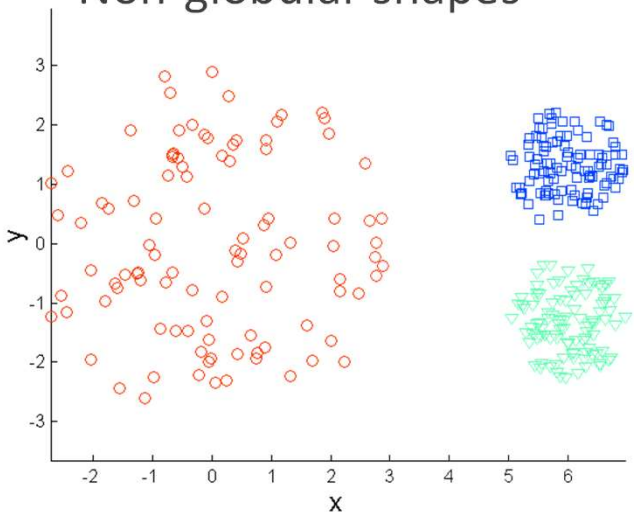
Original Points



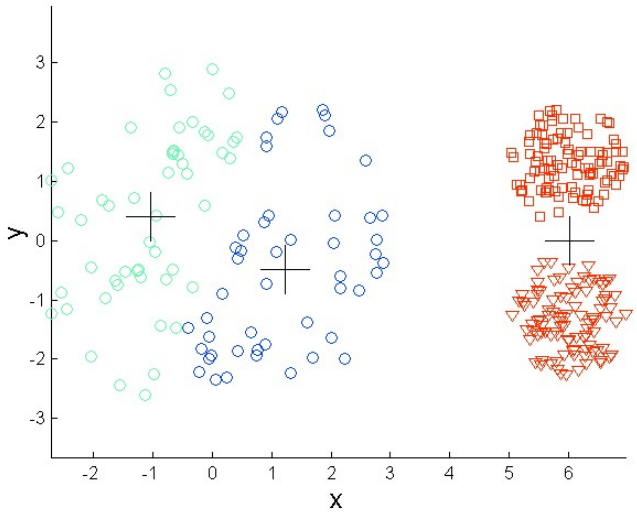
K-means (3 Clusters)

K-means

- ☹: K-means has problems when clusters are of differing
  - Sizes
  - Densities
  - Non-globular shapes



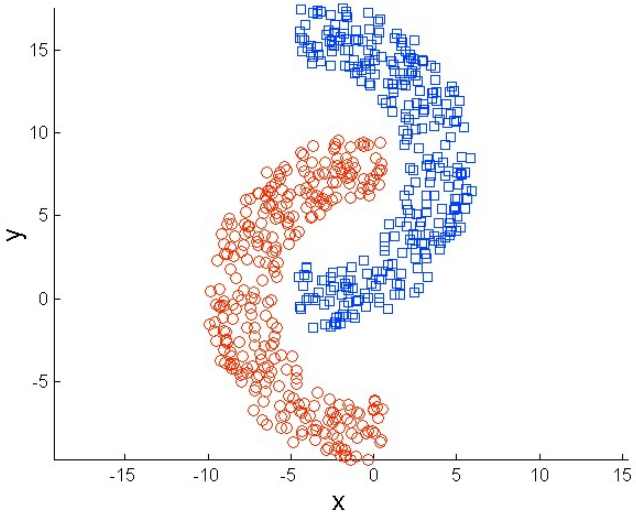
Original Points



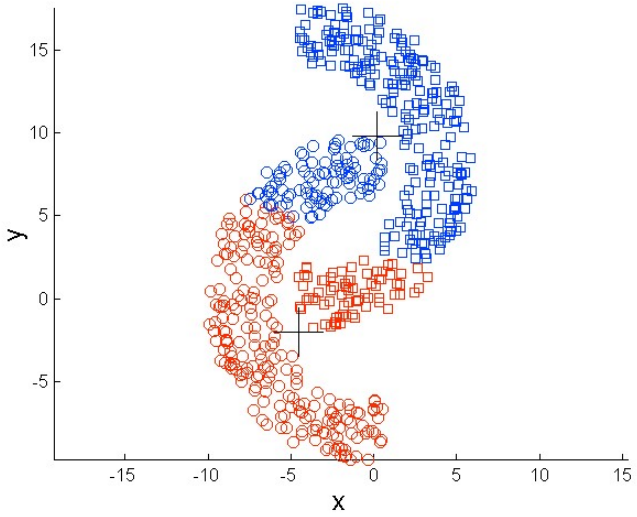
K-means (3 Clusters)

K-means

- ☹: K-means has problems when clusters are of differing
  - Sizes
  - Densities
  - Non-globular shapes



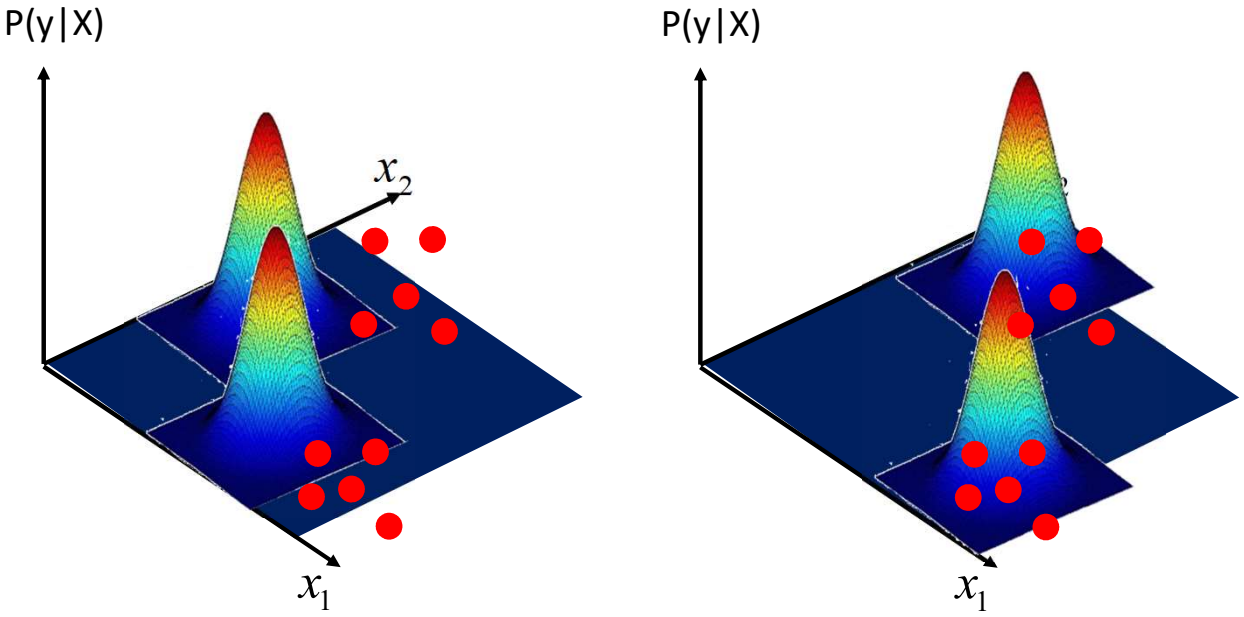
Original Points



K-means (2 Clusters)

Expectation Maximization (EM)

- The major concept of EM is to
  1. **Expectation**: Compute the probability of j-th data belonging to i-th group based on given distribution
  2. **Maximization**: Compute new distribution parameters



Expectation Maximization (EM)

- The major concept of EM is to
  1. **Expectation**: Compute the probability of j-th data belonging to i-th group based on given distribution
  2. **Maximization**: Compute new distribution parameters

$[Input]\ K, \mathbf{x}_j, j = 1 \sim N$

$[Output]\ labels$

$0.\ \text{Initialize probability distriubtion}$

$1.\ \text{E-step}:$

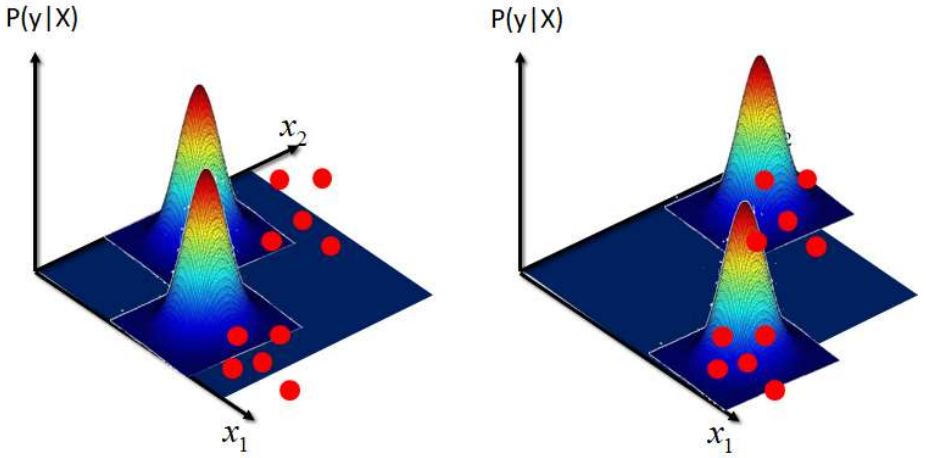
$p_{ij} = P(C = i \mid \mathbf{x}_j) = \alpha P(\mathbf{x}_j \mid C = i) P(C = i)$

$2.\ \text{M-step}:$

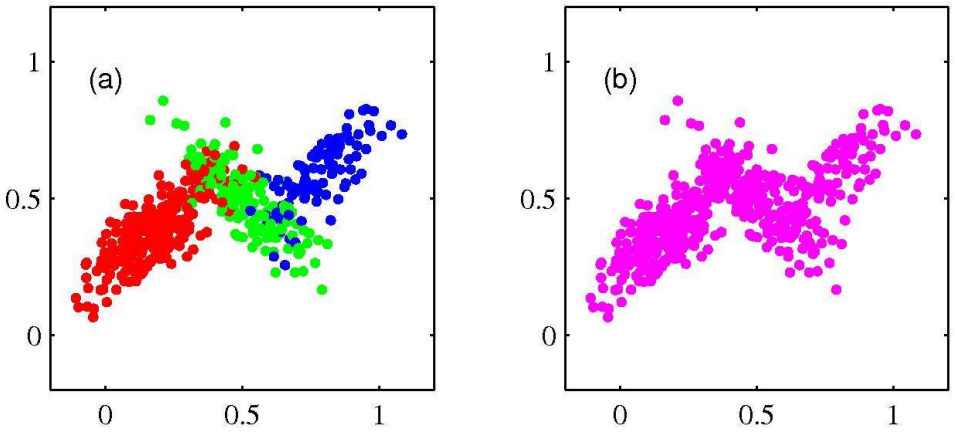
$$\boldsymbol{\mu}_i \leftarrow \frac{1}{n_i} \sum_j p_{ij} \mathbf{x}_j$$

$$\Sigma_i \leftarrow \frac{1}{n_i} \sum_j p_{ij} (\mathbf{x}_j - \boldsymbol{\mu}_i)(\mathbf{x}_j - \boldsymbol{\mu}_i)^T$$

$$P(C = i) \leftarrow n_i / N$$

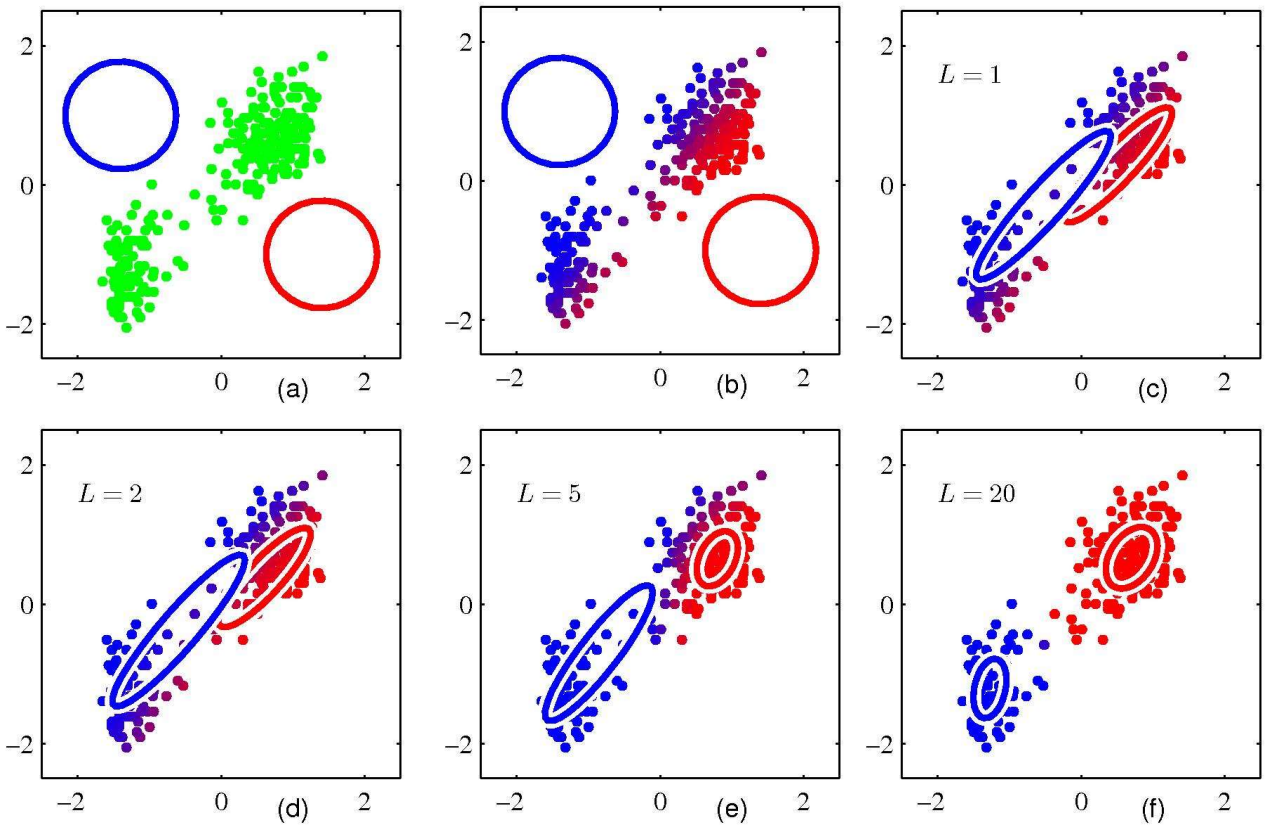


Expectation Maximization (EM)



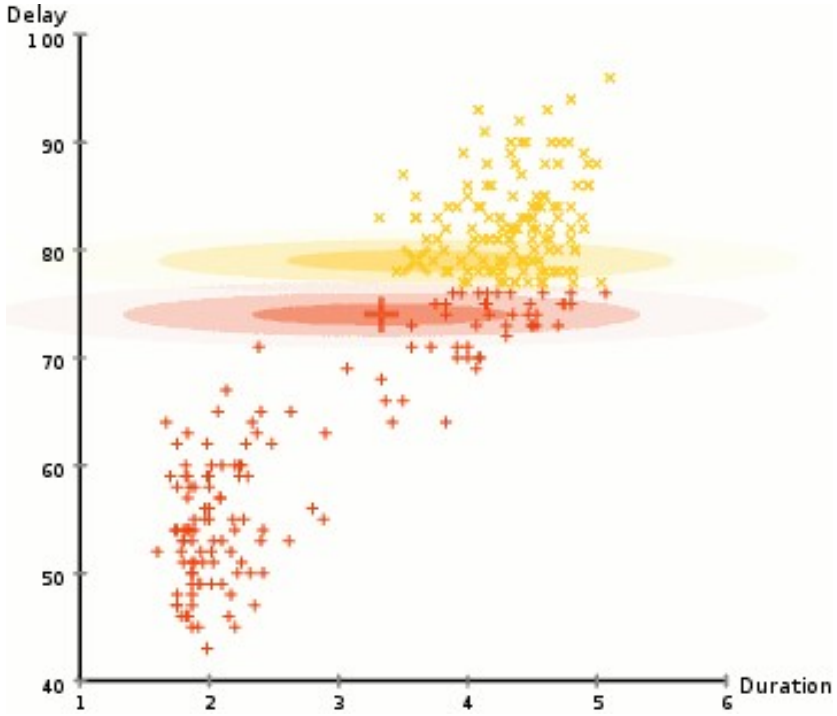
Christopher Bishop, "Pattern Recognition and Machine Learning", Springer, 2006.

Expectation Maximization (EM)



Expectation Maximization (EM)

- Animation of EM



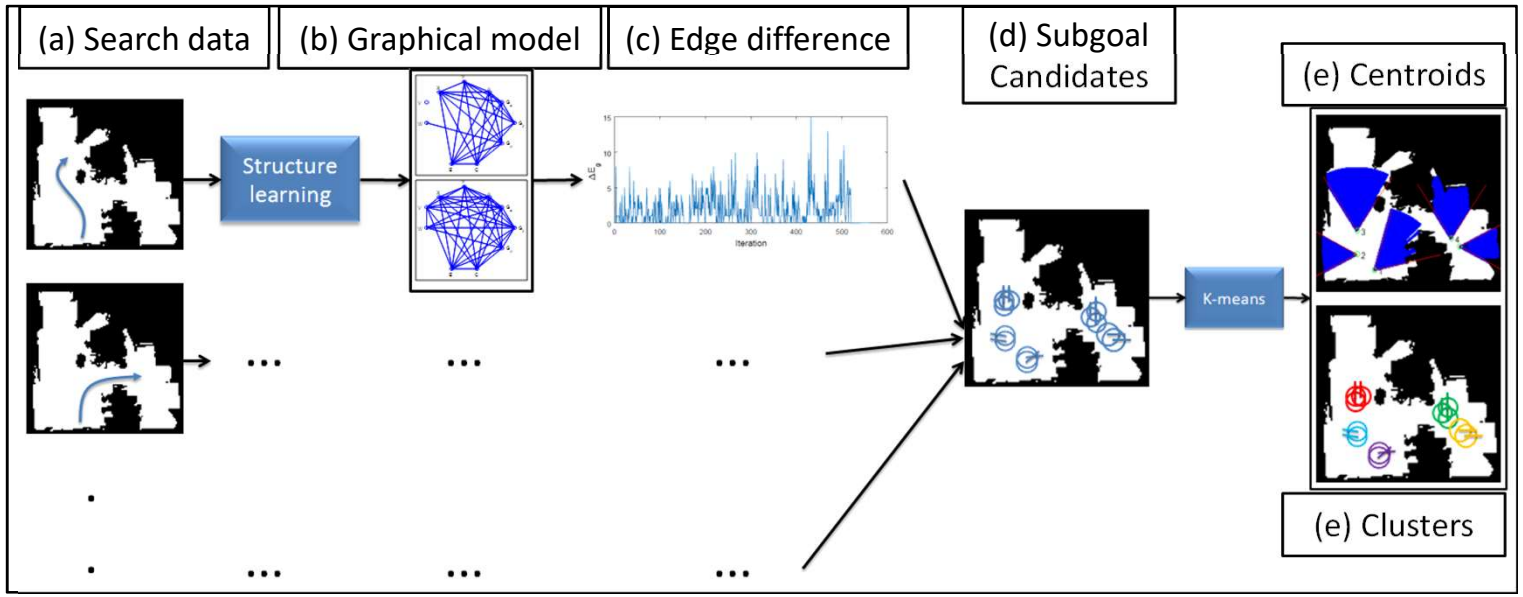
https://en.wikipedia.org/wiki/Expectation–maximization_algorithm

Expectation Maximization (EM)

- ☺
  - K-means is a special case of EM. EM can deal with different sizes and elliptical shapes.
  - EM can handle data with random processes since it is based on statistical assumptions.
- ☹
  - Slow. Impractical for large numbers of data.
  - Cannot work well with a few data.

EX: Subgoal clustering

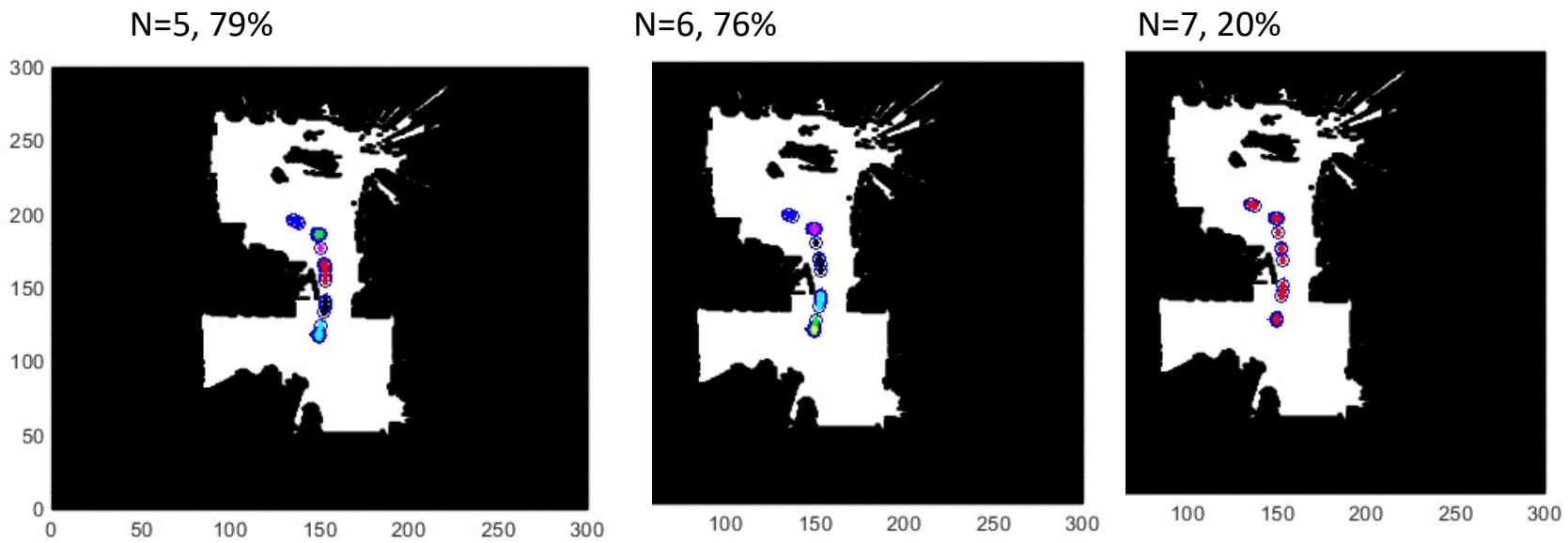
- Find the subgoal from subgoal candidates



Kuo-Shih Tseng and Bérénice Mettler, "Human Planning and Coordination in Spatial Search Problems", 1st IFAC Conference on Cyber-Physical and Human-Systems, 2016.

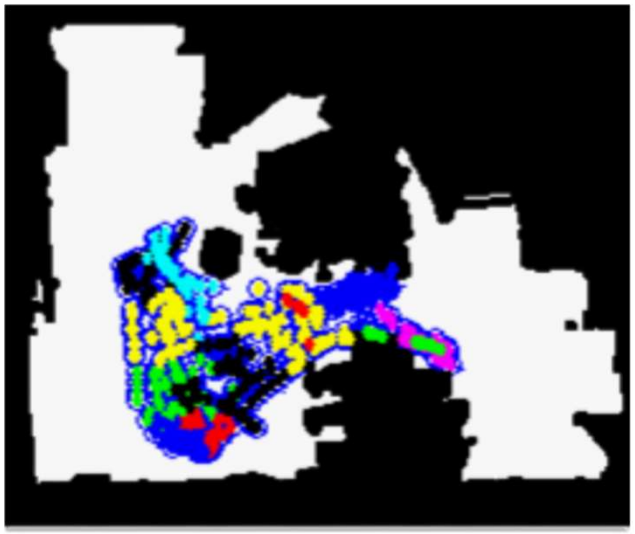
EX: Subgoal clustering

- Extract robot positions from trajectories

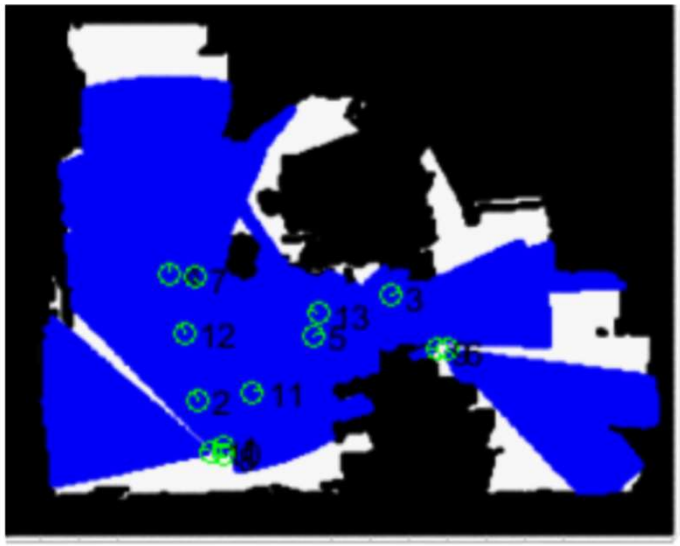


EX: Subgoal clustering

- Cluster robot position into K groups.



(d) Subgoal candidates



(e) Clustered subgoals

EX: Gaze patterns

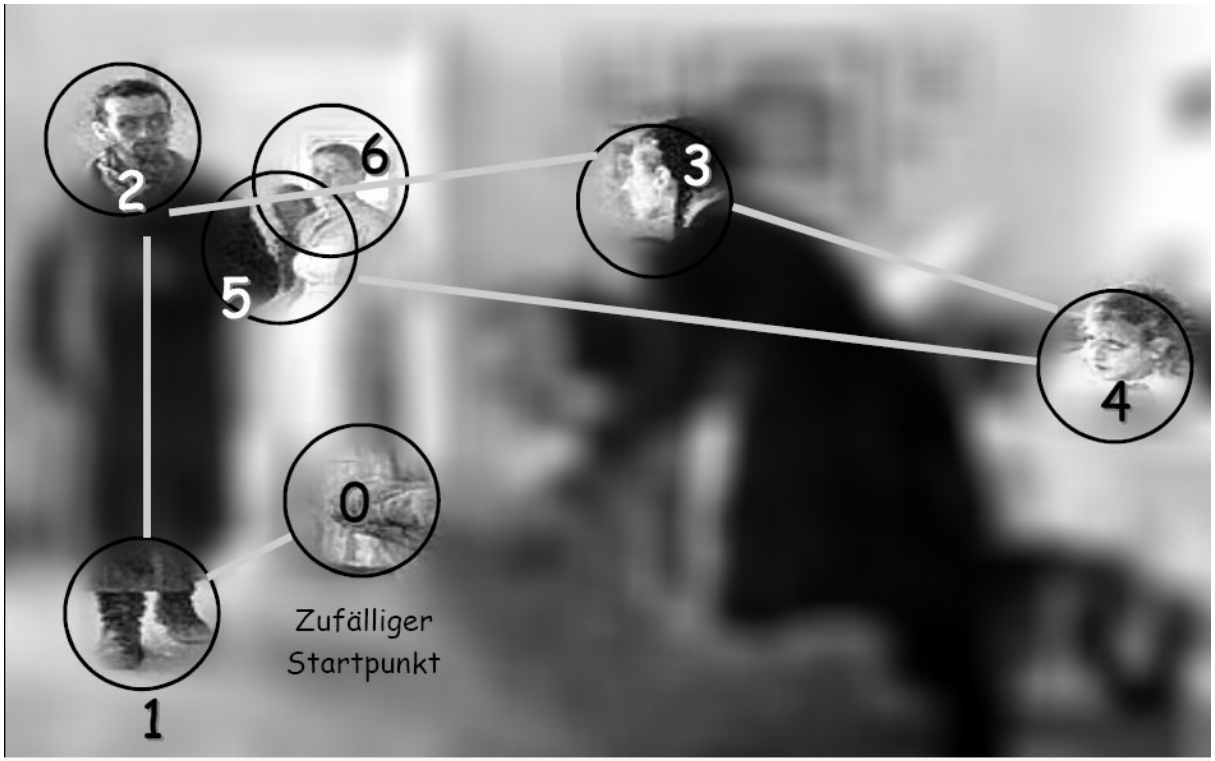


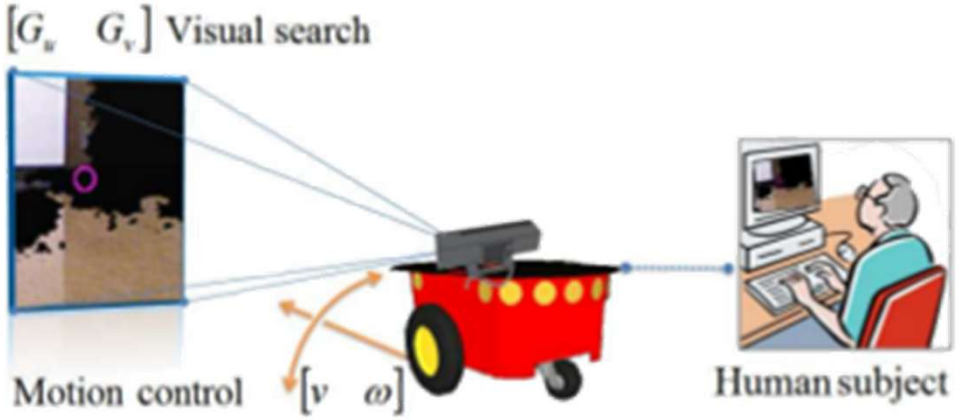
Bild 11: Foveale Ergänzung durch die ersten 6 Fixationen (nach Daten von *Yarbus,1967*)

1. Fixation
2. Smooth pursuit
3. Saccades

https://en.wikipedia.org/wiki/Eye_movement

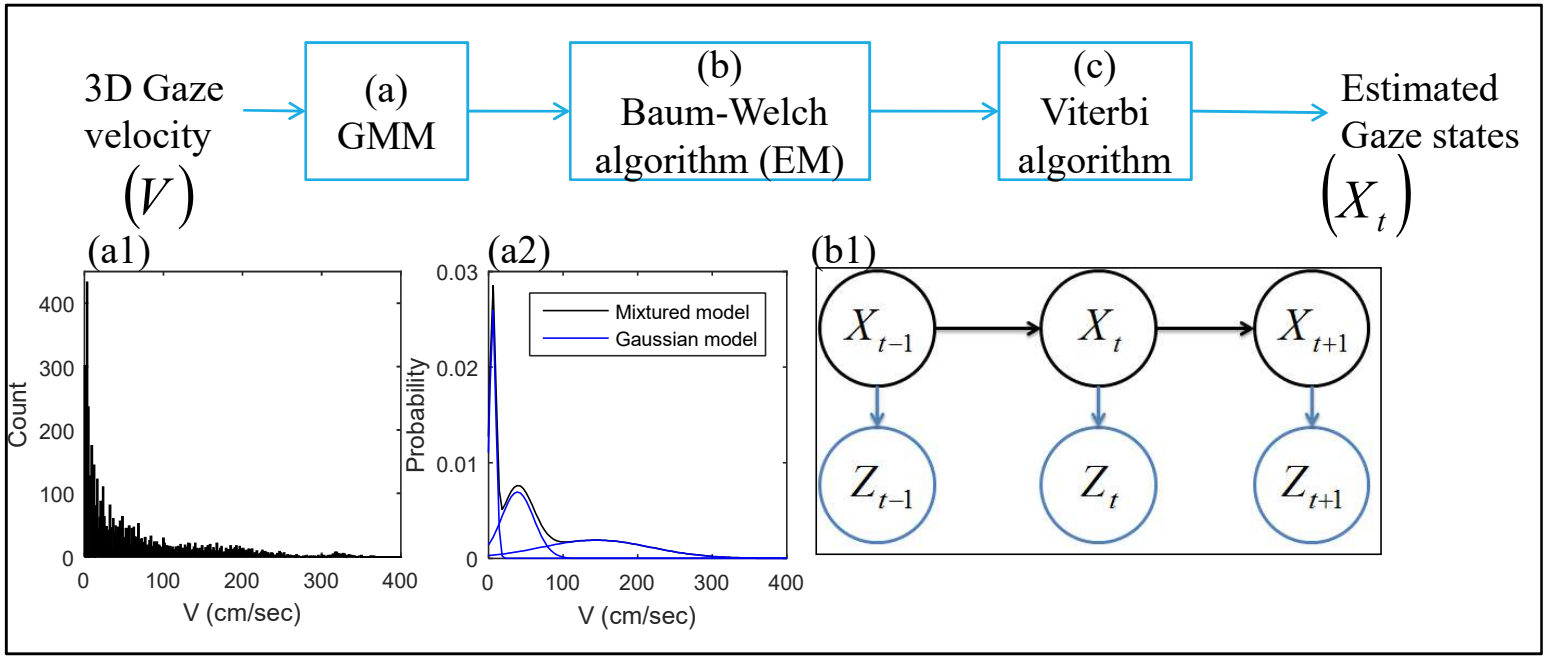
EX: Gaze patterns

- A human subject remotely control a robot to search for objects. The gaze tracker can detect where the subject is looking.
- We want to know the gaze patterns from noisy data.



[1] Kuo-Shih Tseng and Bérénice Mettler, "Analysis of Coordination Patterns between Gaze and Control in Human Spatial Search", 2nd IFAC Conference on Cyber-Physical and Human-Systems, 2018.

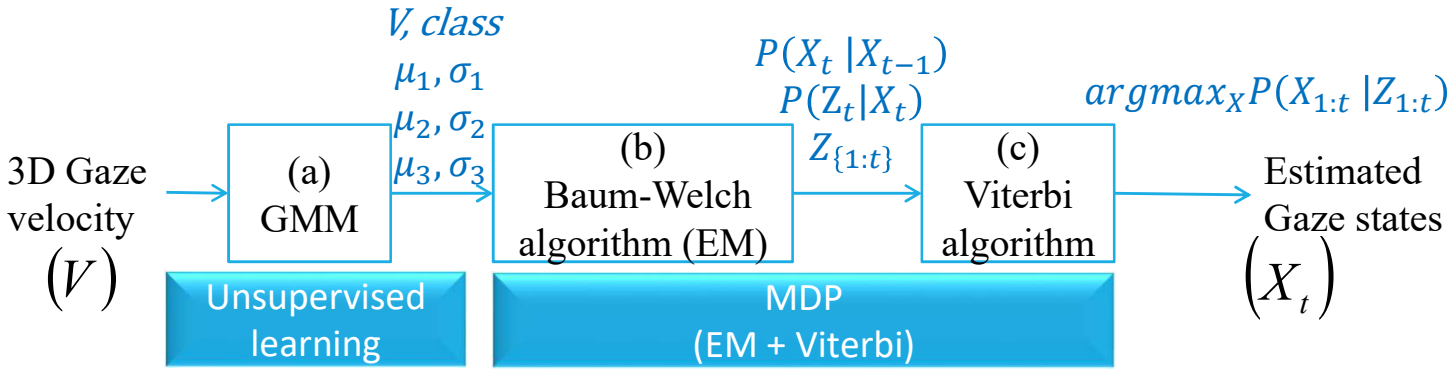
EX: Gaze patterns

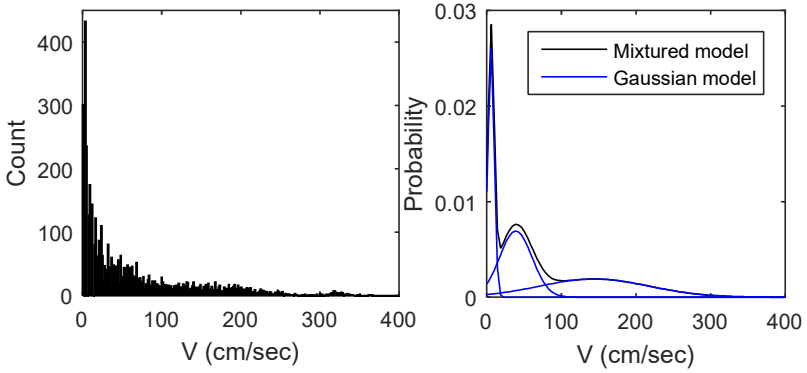


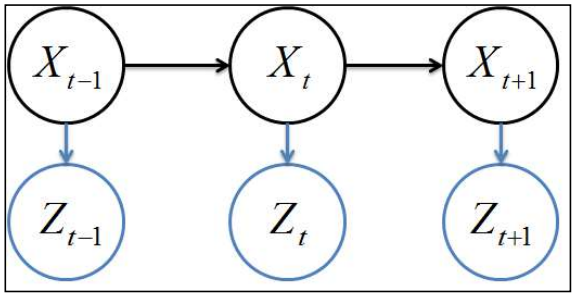
(a-d) The blocks represent GMM, Baum-Welch, Viterbi and SICE algorithms.
(a1) The histogram represents the distribution of 3D gaze velocity.
(a2) The blue and black curves represent clustered Gaussian distributions and mixture model, respectively.
(b1) The black and blue circles represent hidden states (X) and observations (Z), respectively.
(c) The Viterbi algorithm computes the most likely gaze state sequence (X).

EX: Gaze patterns

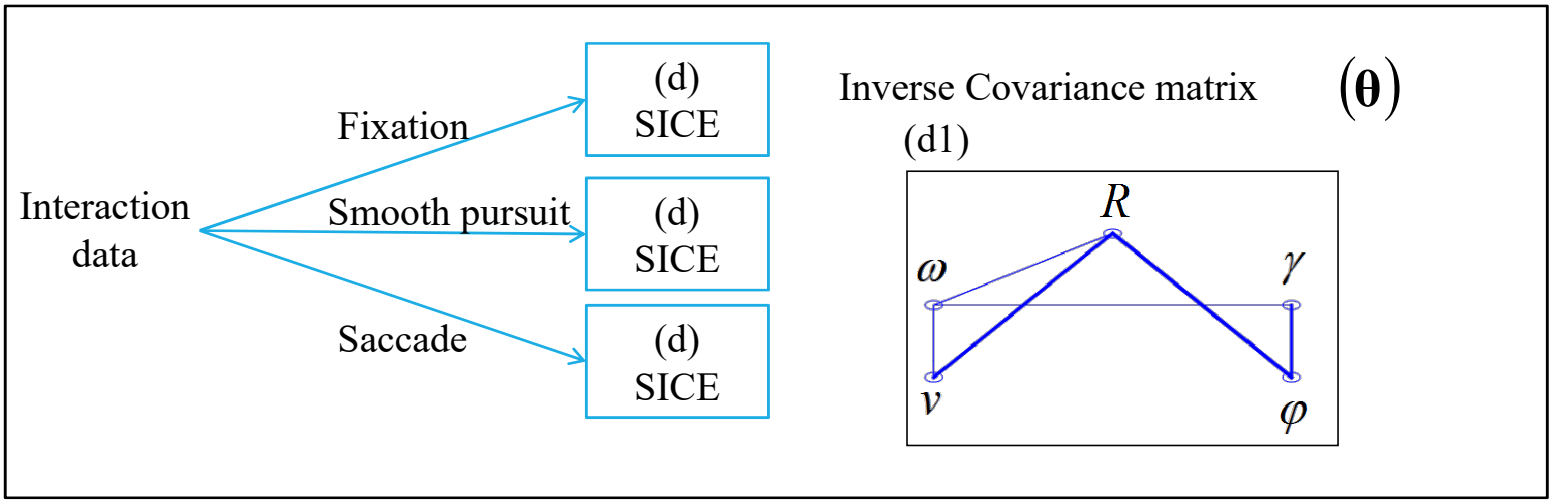
- Utilizing AI algorithms, we can explore human data!







EX: Gaze patterns

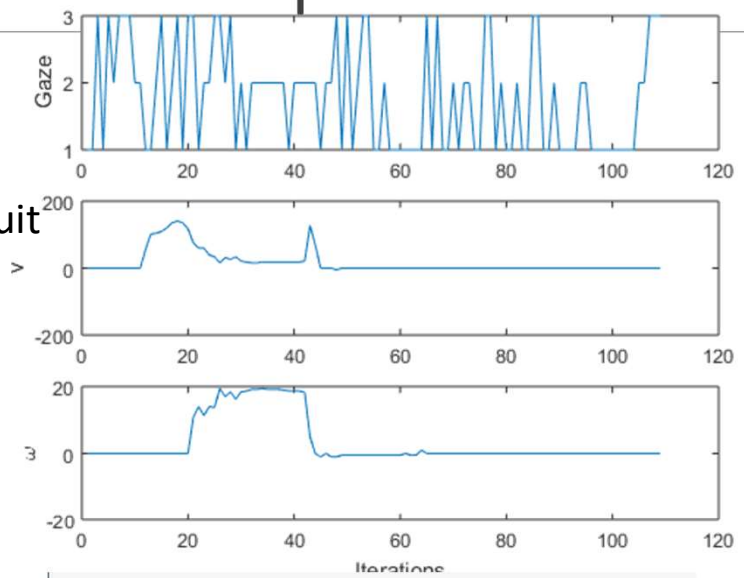


(d1) The Nodes represent the variables and edges represent the links between two variables. For example, if there is a link between v and w, it means that the two variables are conditionally correlated.

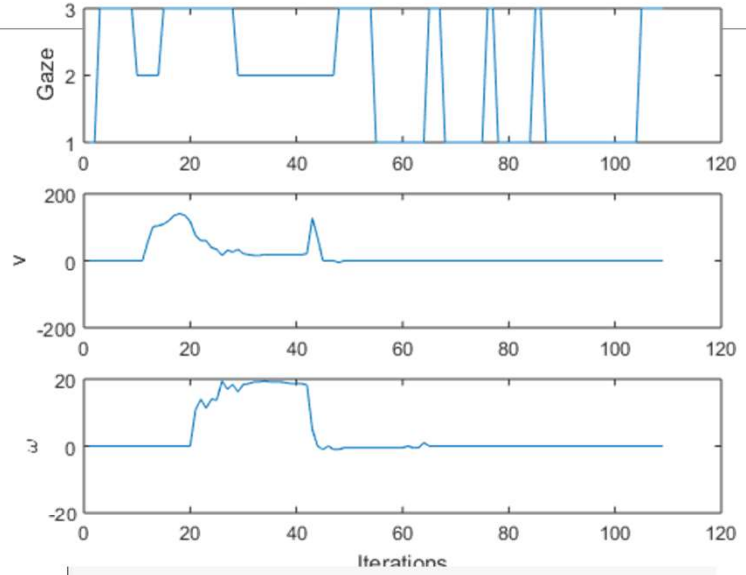
EX: Gaze patterns

Zt={1,2,3}

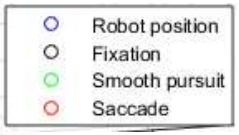
1. Fixation
2. Smooth pursuit
3. Saccades

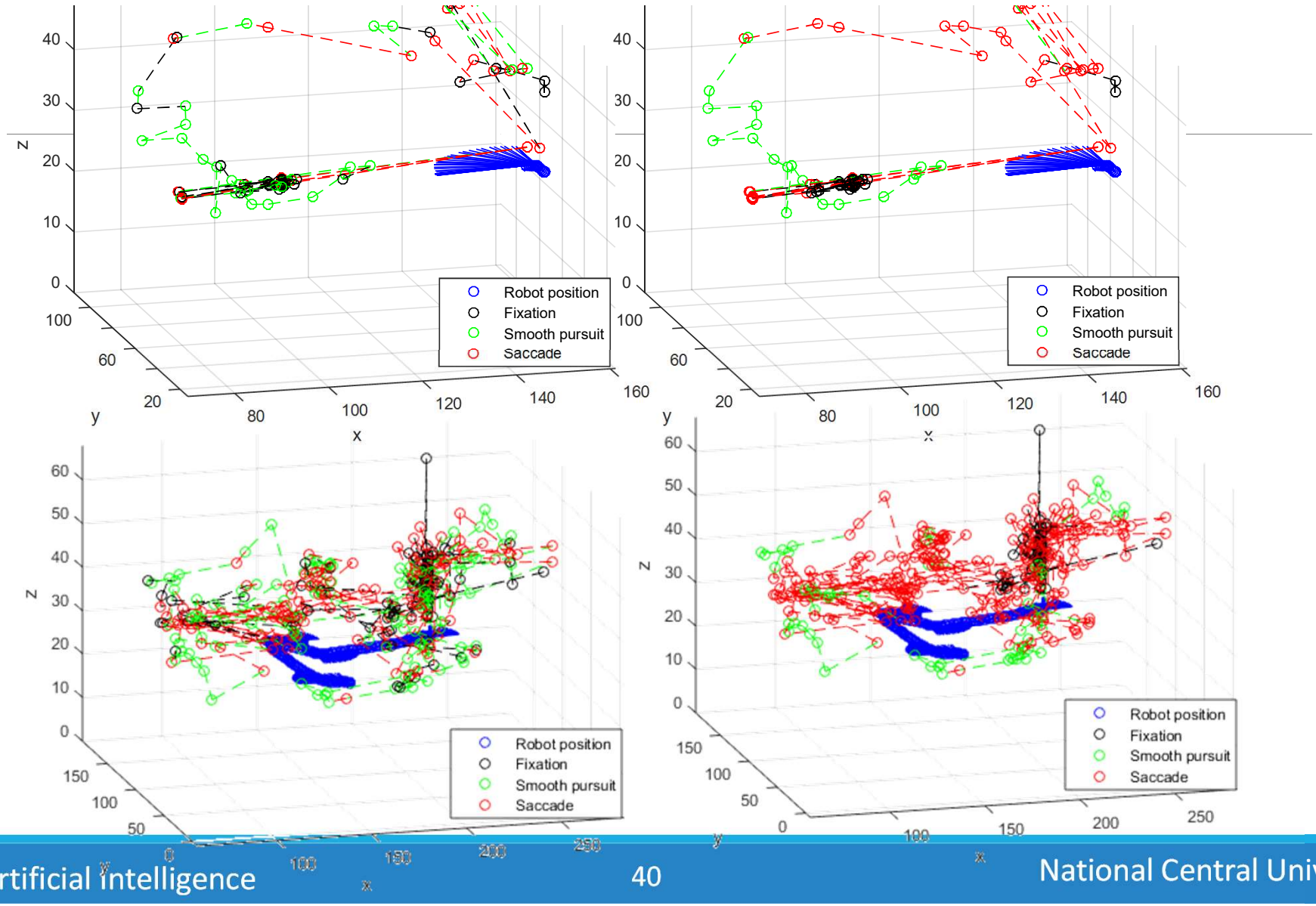


HMM+Viterbi



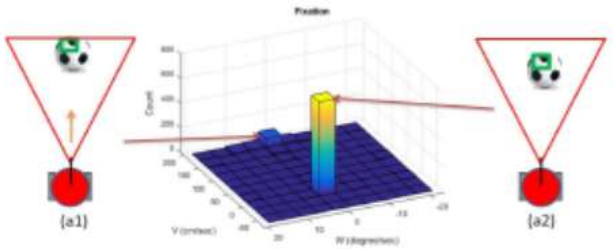
Xt={1,2,3}
Via
HMM+Viterbi



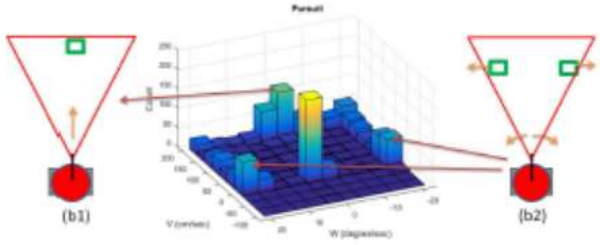

Robot position
Fixation
Smooth pursuit
Saccade
x
y
z

EX: Gaze patterns

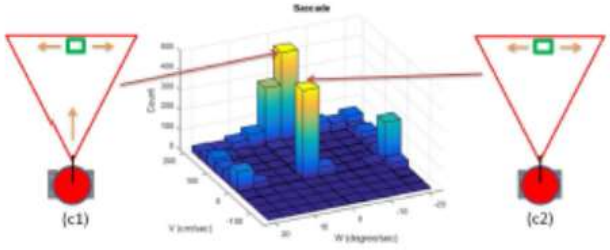
1. Fixation is used primarily to look at the target;
2. Smooth pursuit is coupled to robot rotation and used to search for new explored area;
3. Saccade is coupled with forward motion and used to search for new explored area.
4. The analysis also shows that interaction patterns of pursuit mode predict the operators' search performance.



(a) Fixation



(b) Smooth pursuit



(c) Saccade

Review of MAI

$$1.\ f = \min(g + h)$$

$$2.\ P(A \mid B) = \eta P(B \mid A) P(A)$$

$$3.\ U(s) = R(s) + \gamma \max_a \left[\sum_{s'} U(s') P(s' \mid s, a) \right]$$

$$4.\ w_i \leftarrow w_i + \alpha \left[R(s) + \gamma \max_{a'} Q(s', a') - Q(s, a) \right] \frac{\partial Q(s, a)}{\partial w_i}$$

$$5.\ y = WX \Rightarrow y = W\Phi(X) \Rightarrow y = \Phi(WX) \Rightarrow y = \Phi(WY)$$

$$\Rightarrow y = \Phi_n(WX)$$

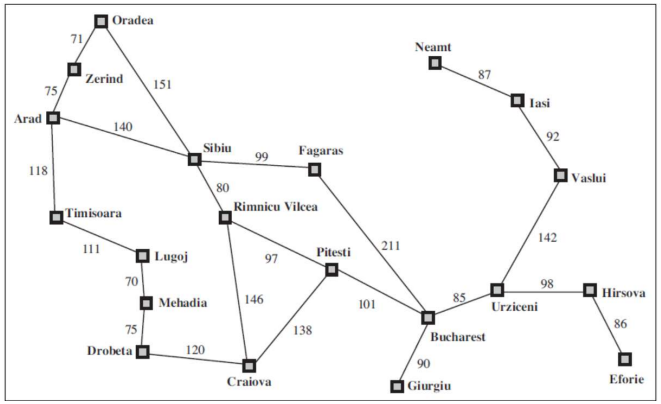
The essence of AI –
Learning from the past experience, predicting the future and making optimal decisions.

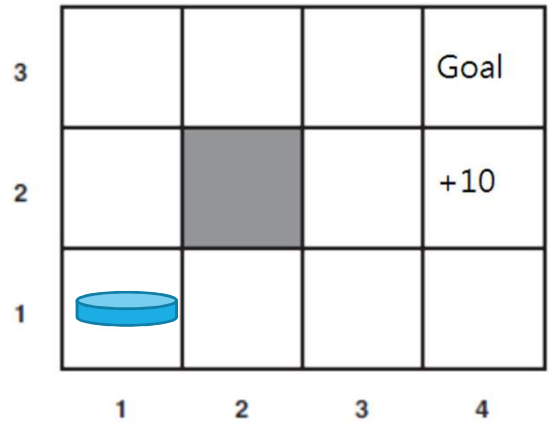
Review of MAI

1. $f = \min(g + h)$

$$f(i) = \underbrace{g(i)}_{past} + \underbrace{h(i)}_{future}$$

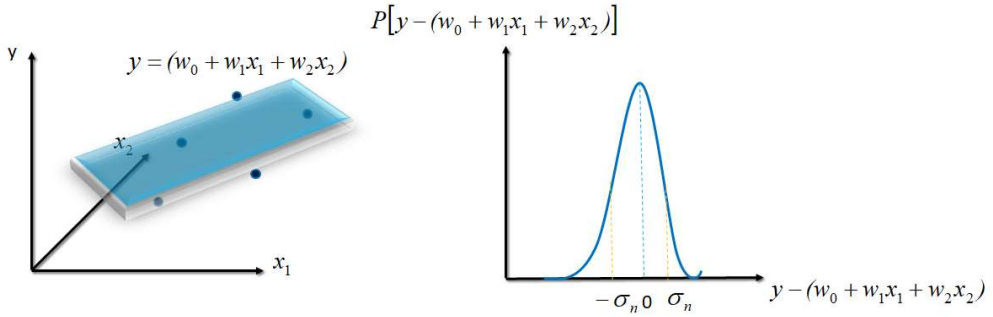
$$\mathrm{H(s)} = \underbrace{\mathrm{c(s,a,s')}}_{past} + \underbrace{h(s')}_{future}$$



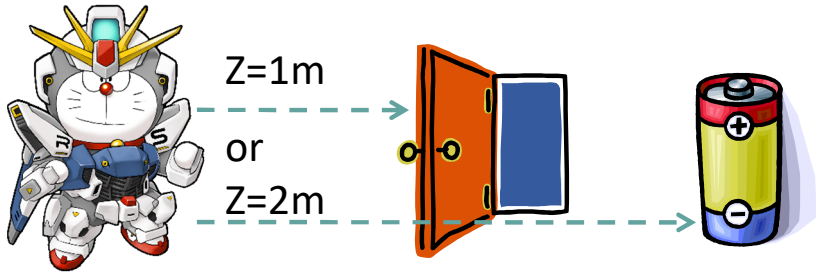


Review of MAI

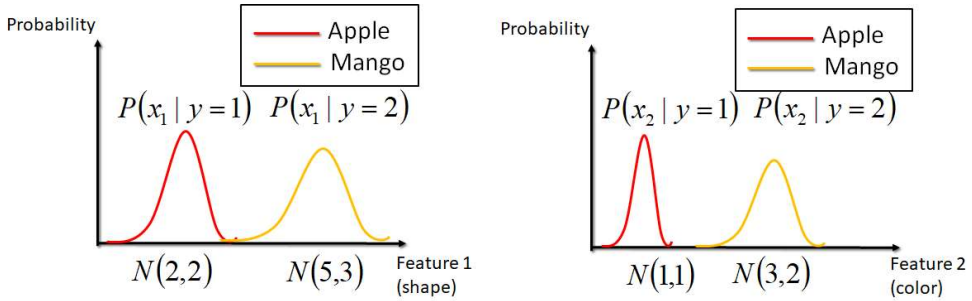
$$2.\ P(A \mid B) = \eta P(B \mid A) P(A)$$



GP for function approximation



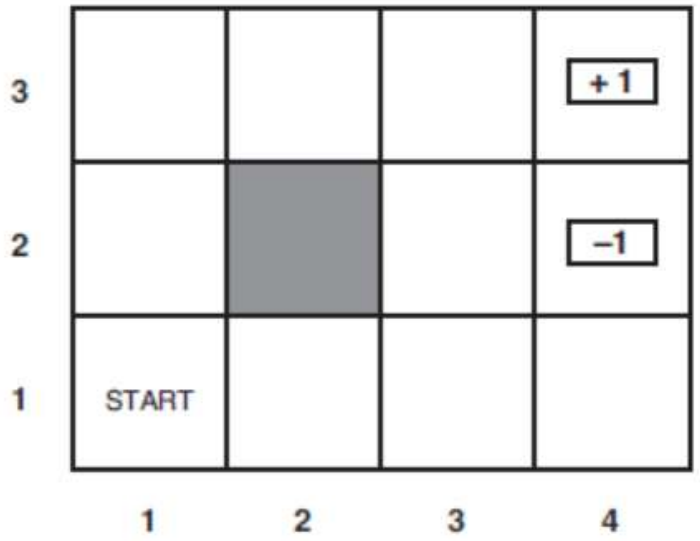
Sensing estimation



Naïve Bayes for classification

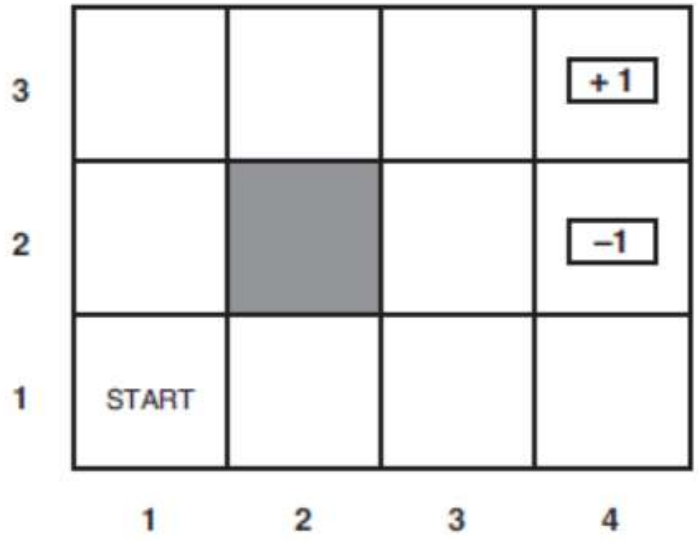
Review of MAI

$$3.\ U(s) = R(s) + \gamma \max_a \left[\sum_{s'} U(s')P(s'|s,a) \right]$$



Review of MAI

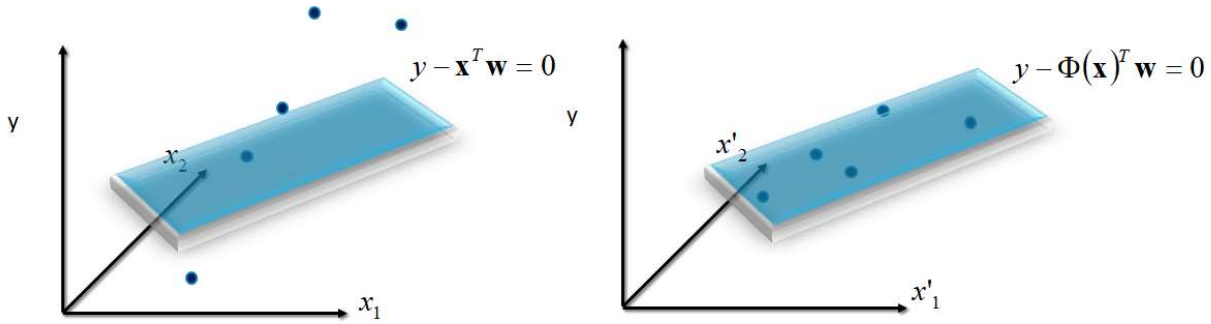
$$4.\ w_i \leftarrow w_i + \alpha\left[R(s) + \gamma \max_{a'} Q(s', a') - Q(s, a)\right]\frac{\partial Q(s, a)}{\partial w_i}$$

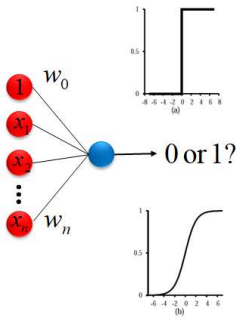


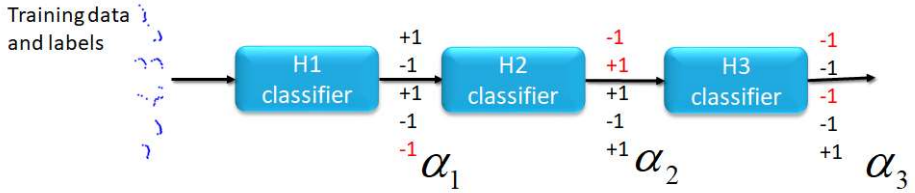
Review of MAI

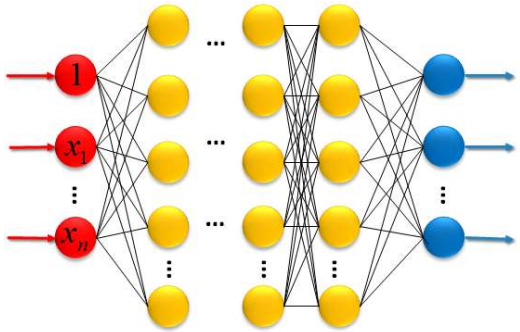
$$5.\ y = WX \Rightarrow y = W\Phi(X) \Rightarrow y = \Phi(WX) \Rightarrow y = \Phi(WY)$$

$$\Rightarrow y = \Phi_n(WX)$$









Review of MAI

- Modern Artificial Intelligence

$$2.\ P(A \mid B) = \eta P(B \mid A) P(A)$$

$$4.\ w_i \leftarrow w_i + \alpha \left[R(s) + \gamma \max_{a'} Q(s', a') - Q(s, a) \right] \frac{\partial Q(s, a)}{\partial w_i}$$

$$5.\ y = WX \Rightarrow y = W\Phi(X) \Rightarrow y = \Phi(WX) \Rightarrow y = \Phi(WY) \Rightarrow y = \Phi_n(WX)$$

The essence of AI –
Learning from the past experience, predicting the future and making optimal decisions.

Review of MAI

"They don't appear to want to take over. They just want to dance."

AI is not your enemy!

Humans will cooperate with AI. It can help you achieve your dream!

Q&A