Modern Artificial Intelligence: Homework 2 *

Deadline: 2021/04/14 00:00am

March 23, 2021

Compress all of your code and pdf files into a zip file. Then upload it to LMS system on time. Please cite references (e.g., websites or books) if you learn something from them.

NOTICE: DO NOT use any toolbox for this homework. If you use any library, you should make sure it is included correctly. If your code cannot independently run on Prof. Tseng's Matlab. You only got partial points.

1. Bayesian inference (40%):

There is a Hidden Markov Model in Figure 1. Assume the hidden variables X_i to be boolean ($X_i \in \{1, 0\}$), where i = 0 $\sim$ 10. Assume the measurement variables Z_i to be boolean ($z_i \in \{1, 0\}$),where i = 1 $\sim$ 10. Let $P(X_0 = 1) = P(X_0 = 0) = 0.5$. Let the transition matrix $P(X_{t+1}|X_t)$ and sensor matrix $P(Z_t|Xt)$ be given by

$$T = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}, Z = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$$

, where in the T matrix,

$T_{11} = P(X_{t+1} = 1|X_t = 1); T_{12} = P(X_{t+1} = 0|X_t = 1);$

$T_{21} = P(X_{t+1} = 1|X_t = 0); T_{22} = P(X_{t+1} = 0|X_t = 0);$

and in the Z matrix,

$Z_{11} = P(Z_t = 1|X_t = 1); Z_{12} = P(Z_t = 0|X_t = 1);$

$Z_{21} = P(Z_t = 1|X_t = 0); Z_{22} = P(Z_t = 0|X_t = 0);$

Consider the sequences of measurements $Z_{1:10} = \{0, 0, 0, 1, 1, 1, 1, 0, 0, 0\}$

Please answer the following questions.

1) Given $Z_{1:10}$, compute the smoothed estimates of X_t, where t=1$\sim$10. (20%)

2) Find the most likely sequence of states $X_{1:10}$ given evidence $Z_{1:10}$. (20%)

Code Delivery:

Matlab code (.m file). The two files should be *smooth_seq.m* and *viterbi_seq.m*. Compress all of your code into a zip file. This code should display: The probability or output sequence of each algorithm.

*The homework and the programming assignment must NOT be the result of cooperative work. Each student must work individually in order to understand the material in depth. You CANNOT copy the homework or the programming assignment of somebody else.

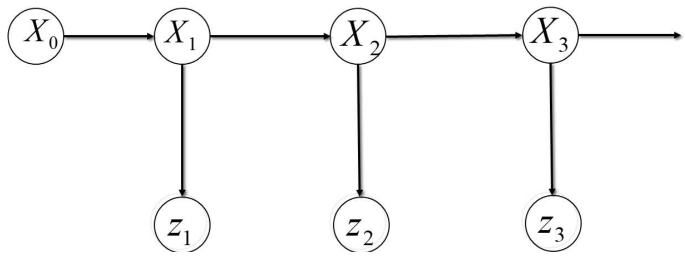

Figure 1: Illustration of Hidden Markov Model(HMM).

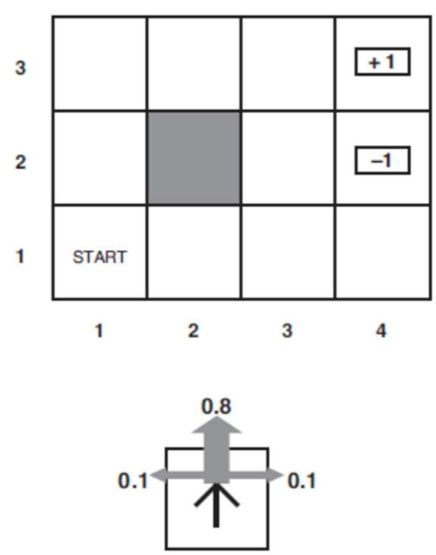

Figure 2: Illustration of 4×3 world.

2. MDP (40%):

As Fig.2 shows, there is a 4×3 world. Its goal is to move to the cell {4,3}. The environment is defined as follows:

$state : s \in S = \{1..4, 1..3\}$

$initial \quad state : s = \{1, 1\}$

$action : a \in A = \{left, up, right, down\}$

$transition\ model : P(s'|s, a)$ as the bottom figure of Fig.2 shows.

$cost : R(s) = -0.04$ when $s' \neq (4, 3)$ or $s' \neq (4, 2)$

$R(s) = +1$ when $s = (4, 3)$

$R(s) = -1$ when $s = (4, 2)$

$discount : \gamma = 0.9$

$terminal : s = \{4, 3\}$

If the robot hits the walls, it will be bounced.

Please answer the following questions:

1) Write a program to solve this MDP using value iteration. Display the optimal action and utility

value at each state for each iteration until converge. (20%)
2) Write a program to solve this MDP using policy iteration. Display the optimal action and utility value at each state for each iteration until converge. (20%)

Code Delivery:

Matlab code (.m file). The two files should be $MDP\_VI.m$ and $MDP\_PI.m$. Compress all of your code into a zip file. This code should display: The optimal action and utility value in each state at each iteration until converge. For example, you can display the action of each state as follows:
iteration=3
action=
rrr
u u
urul

3. MDP in real life problems (20%):

Define a Markov decision processing (MDP) for a real problem. Please answer the following questions:
1) Define the state, action, transitional probability, rewards, and discount terms. (5%)
2) Plot a picture (state machine) for this MDP. (5%)
3) Explain your formulation and the possible optimal solutions. (10%)