$p \cdot q \mod (X^n + 1) = p \cdot q + w \cdot (X^n + 1)$ for some $w$. We question whether $p(r) \cdot q(r) =? (p \cdot q)(r)$

But we actually have:

$$\begin{aligned}(p \cdot q)(r) \mod (X^n + 1) &= (p \cdot q + w \cdot (X^n + 1))(r) = \\ &= p(r) \cdot q(r) + w(r) \cdot (r^n + 1)\end{aligned}$$

So if we apply modulus on both sides of the equation ($\mod r^n + 1$) we should be able to eliminate the difference: