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A discrete mathematical model for addictive buying: Predicting the affected population evolution

I. García [a], L. Jódar [b], P. Merello [b], F.-J. Santonja [c,*]

[a] Departamento de Comunicación Audiovisual y Publicidad, Universidad del País Vasco, Spain
[b] Instituto Universitario de Matemática Multidisciplinar, Universidad Politécnica de Valencia, Spain
[c] Departamento de Estadística e Investigación Operativa, Universidad de Valencia, Spain

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ABSTRACT

This paper deals with the construction of a discrete mathematical model for addictive buying. Firstly, identifications of consumers buying behavior are performed by using multivariate statistical techniques based on real data bases and sociological approaches. Then the population is divided into appropriate groups according to the level of overbuying and a discrete compartmental model is constructed. The future short term addicted population is computed assuming several future economic scenarios.

1. Introduction

Although addictive buying is not specifically described in DSM-IV [1], uncontrolled problematic buying behavior has been studied by psychologists and psychiatrists since the early 20th century [2]. For some people, shopping is a leisure activity, a way to manage emotions or a means of expressing their self-identity. This however can brings about adverse consequences, including guilt, excessive debt, family conflict, illegal activities, and even suicide attempts.

Some cross-sectional studies have been performed to analyse uncontrolled problematic buying behavior [2]. However, to the best of our knowledge, there are no studies that allow to predict the prevalence of this pathological behavior in the next few years. In this paper, we present a discrete mathematical model that allows us to do it. Taking into account [3] we build the model considering that social behaviors may spread from one person to another.

2. Method

Firstly, in order to know the addictive buying prevalence rate, two surveys have been performed. Secondly, the mathematical model is built. The information contributed by the prevalence analyses allows us to estimate some of the parameters of the model.

2.1. Data and sampling

The prevalence study is based on the database of two surveys performed in Vizcaya (Spain). The first one was in 2001 [4] and the second one in 2010. A total of 350 individuals aged 17 and older (341 individuals for 2001) have been selected using stratified random design to insure representation of the diverse age and gender groups.

* Corresponding author.
E-mail addresses: franciscosantonja@hotmail.com, francisco.santonja@uv.es (F.-J. Santonja).

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2.2. Measurement scales

In both cases (2001 Survey and 2010 Survey), to estimate the prevalence of addictive buying we used the Compulsive Buying Scale proposed in [5]. A cluster analysis (with the K-means algorithm) [6] was performed from the responses to the scale to classify the individuals as rational buyers, excessive buyers and addictive buyers. Descriptive statistic analysis allows us to know the percentage in each case. More details about classification are shown in [4].

2.3. Mathematical model

To build the mathematical model, the population is divided into three subpopulations using the Compulsive Buying Scale proposed in [5]: N, rational buyers, S, excessive buyers and A, addictive buyers such that the total population size (P) at any given time n is given by:

$$P_n = N_n + S_n + A_n. \tag{1}$$

The dynamic of the population can be described by the following system of equations (n, time in months):

$$N_{n+1} - N_n = \mu P_n - \mathrm{d}N_n - \beta_1 \frac{N_n A_n}{P_n} - \beta_2 N_n + \epsilon_2 A_n \tag{2}$$

$$S_{n+1} - S_n = \beta_1 \frac{N_n A_n}{P_n} + \beta_2 N_n - \mathrm{d}S_n - \gamma_1 \frac{S_n A_n}{P_n} - \gamma_2 S_n + \epsilon_1 A_n \tag{3}$$

$$A_{n+1} - A_n = \gamma_1 \frac{S_n A_n}{P_n} + \gamma_2 S_n - \epsilon_2 A_n - \mathrm{d}A_n - \epsilon_1 A_n \tag{4}$$

where the population state vector $PS_n = (N_n, S_n, A_n)$ gives the number of rational buyers, excessive buyers and addictive buyers at time n. See [7] (Section 2.2) for more details about the assumptions to build the mathematical model. In this paper, we model with a discrete model because we consider that buyer's behavior can be abstracted by a succession of steady states intermixed with *jumps* which make their state suddenly change to others.

The parameters of the model are:

- μ, birth rate in Spain. We assume that the birth rate for Vizcaya is the Spanish birth rate.
- d, death rate in Spain. We assume that the death rate for Vizcaya is the Spanish death rate.
- β_1, transmission rate due to social contact with addictive buyers (rational buyers $\rightarrow$ excessive buyers).
- β_2, rate at which a rational buyer transits to the excessive buyer's subpopulation. We admit that this parameter is related to the economic situation.
- γ_1, transmission rate due to social contact with addictive buyers (excessive buyers $\rightarrow$ addictive buyers).
- γ_2, rate at which an excessive buyer transits to the addictive buyers subpopulation. We consider that this parameter is also related to the economic situation.
- ϵ_1, rate at which an addictive buyer reduces his/her addictive behavior himself/herself and becomes an excessive buyer.
- ϵ_2, rate at which an addictive buyer goes to therapy and becomes a rational buyer.

2.3.1. Parameters estimation

The parameter values are:

- $\mu = 0.000833$ month^{-1}. This is the average Spanish birth rate between years 2002–2009 [8].
- $d = 0.000666$ month^{-1}. This is the average Spanish death rate between years 2002–2009 [8].
- $\epsilon_1 = 0.0020$ month^{-1}. We consider that 24.1% of addictive buyers stop his/her addictive behavior himself/herself [9,10] and it is known that an addictive buyer takes 10 years to acknowledge his/her compulsive behavior [11–14] then $\epsilon_1 = 0.241 * 1/(10 * 12)$.
- $\epsilon_2 = 0.000634$ month^{-1}. This parameter is estimated taking into account:
  - the percentage of addictive buyers that begin therapy every year (0.35%). We assume that the information available for Valencian addictive buyers who go into therapy is similar to the information for Vizcaya [15,16].
  - the average percentage of success for therapy programs (50%) [17–20].
  - the median of the therapy duration, 12 weeks (3 months) [17–20].

  Then, we obtained $\epsilon_2 = 0.0035 * 0.5 * 1/3$.
- $\beta_2 = \zeta_n * (0.25 * 0.02)$ month^{-1}. We estimate this parameter admitting that 25% of the population acts rashly with the act of buying (with non-pathological behavior) [21] and 2% of the population have emotional instability (data for the region of Valencia; we assume this proportion for Vizcaya). [22,23].

  ζ_n shows the proportion of the population who have an optimistic opinion related to the economic situation in month n. We consider that this opinion is not constant in time. ζ_n is estimated by fitting the Consumer Confidence Index (CCI) presented by the Ministry of Economic Affairs of Spain [24]. In order to fit, we use Least Square Estimation and the Chow test to analyse the presence of structural breaks in the CCI time serie [25].

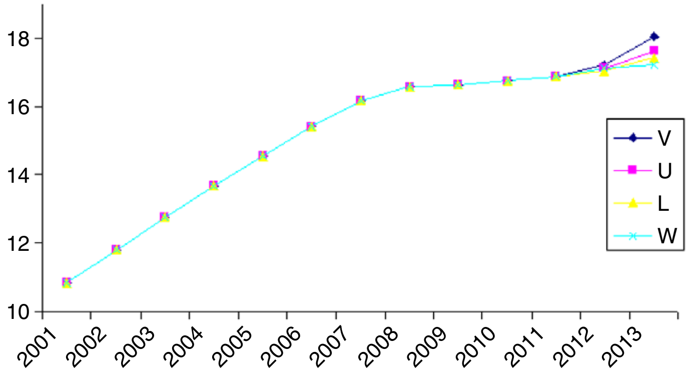

Fig. 1. Percentage of addictive buyers in the next few years according to different economic scenarios.

Table 1
Evolution of proportion of addictive buyers for the next few years. (∗) Predicted by the model.

Year	*V* (%)	*U* (%)	*L* (%)	*W* (%)
2001-January	10.85	10.85	10.85	10.85
2010-May	16.77	16.77	16.77	16.77
2011-December*	**16.88**	**16.88**	**16.88**	**16.88**
2012-December*	**17.22**	**17.11**	**17.04**	**17.05**
2013-December*	**18.04**	**17.61**	**17.42**	**17.21**

The expression for ζ_n is:

$$\zeta_n = \begin{cases} -0.45n + 48.236 & \text{January 2001–December 2005} \\ 0.0199n^2 - 0.2168n + 44.068 & \text{January 2006–July 2007} \\ -7.6962LN(n) + 46.901 & \text{August 2007–December 2008} \\ -0.1369n^2 + 3.0905n + 20.33 & \text{January 2009–} \end{cases}$$

- $\gamma_2 = \zeta_n * (0.25 * 0.02) + k$ month^{-1}. We assume that $\gamma_2 \geq \beta_2$ ($\gamma_2 = \beta_2 + k$), i.e., we admit that the economic situation influences more on excessive buyers than rational buyers.

β_1, γ_1 and k are estimated by fitting the model to real data (surveys performed in Vizcaya, year 2001 and year 2010). Taking as the initial conditions of the model (year 2001-January, i.e., $n = 0$), $N_0 = 0.4897$, $S_0 = 0.4018$ and $A_n = 0.1085$ (Prevalence study for 2001) and the final conditions of the model (year 2010-May, i.e., $n = 112$) $N_{112} = 0.4403$, $S_{112} = 0.3920$ and $A_{112} = 0.1677$ (Prevalence study for 2010), the parameters β_1, γ_1 and k have been estimated by fitting the scaled model. The values of β_1, γ_1 and k that fit the model with real data are $\beta_1 = 0.002453$, $\gamma_1 = 0.0048$ and $k = 0.00013$. More details about this fitting process are shown in [26].

3. Numerical simulations

We use the proposed mathematical model with the parameters listed before to forecast the coming three years (until year 2013). The future short term addicted population is computed assuming several future economic scenarios [27].

- Scenario V. The bad economic situation will finish in one year and half (fast economic recovery).
- Scenario U. The bad economic situation will finish in three years.
- Scenario W. The bad economic situation will experience recoveries and relapses in the next four years.
- Scenario L. The bad economic situation will remain for at least five years (long economic recovery).

Fig. 1 shows the percentage of addictive buyers in the next few years (December of each year). In Table 1, we present some of the numerical values represented in Fig. 1. An increase in addictive buyers can be seen. The mathematical model predicts that around 17%–18% of the population of Vizcaya will present addictive buying behavior at the end of 2013.

4. Conclusions

In this work, we present an approximation to the Spanish addictive buying prevalence for 2010 and some predictions for the next few years. The mathematical model presented allows us to obtain a first approximation of the epidemic evolution in Spain since databases have been designed taking into account Spanish age and gender distribution. According to the numerical simulations described, we can affirm that the addictive buying prevalence rate in Spain is increasing.

We show how mathematical models can be an useful tool to understand social epidemics. Using this type of mathematical approach, we are able to know the dynamics of the epidemic, simulate different situations and analyse the effect of these different scenarios on human behavior.

References

[1] American Psychiatric Association, Diagnostic and Statistical Manual of Mental Disorders, American Psychiatric Publishing, 2004.
[2] D.W. Black, A review of compulsive buying disorder, World Psychiatry 6 (2007) 14–18.
[3] N.A. Christakis, J.H. Fowler, Connected: The Surprising Power of Our Social Networks and How they Shape Our Lives, Hachette Book Group, 2009.
[4] E. Olabarri, I. García, La compra por impulso y la adicción al consumo en el País Vasco, Estudios Sobre Consumo 65 (2003) 86–109 (in Spanish).
[5] G. Valence, A. D'Astous, L. Fortier, Compulsive buying: concept and measurement, Journal of Consumer Policy 11 (1988) 419–433.
[6] J.F. Hair, B. Black, B. Babin, R.E. Anderson, R.L. Tatham, Multivariate Data Analysis, 6th ed., Pearson, 2006.
[7] F.J. Santonja, R.J. Villanueva, L. Jódar, G. González, Mathematical modelling of social obesity epidemic in the region of Valencia, Spain, Mathematical and Computer Modelling of Dynamical Systems 16 (2010) 23–34.
[8] Spanish statistical institute. Available at: http://www.ine.es.
[9] G.D. Walters, Spontaneous remission from alcohol, tobacco, and other drug abuse: seeking quantitative answers to qualitative questions, The American Journal of Drug and Alcohol Abuse 26 (2000) 443–460.
[10] T. Toneatto, J.C. Nett, Natural recovery from problem gambling, in: H. Klingemann, L.C. Sobell (Eds.), Promoting Self-Change from Addictive Behaviors. Practical Implications for Policy, Prevention, and Treatment, Springer, 2007.
[11] S.L. McElroy, P.E. Keck, H.G. Pope, J.M. Smith, S.M. Strakowski, Compulsive buying: a report of 20 cases, Journal of Clinical Psychiatry 55 (1994) 242–248.
[12] S. Schlosser, D.W. Black, S. Repertinger, D. Freet, Compulsive buying: demography, phenomenology and comorbidity in 46 subjects, General Hospital Psychiatry 16 (1994) 205–212.
[13] D. Black, Compulsive buying: a review, Journal of Clinical Psychiatry 57 (1996) 50–55.
[14] D.W. Black, P. Monahan, S. Schlosser, S. Repertinger, Compulsive buying severity: an analysis of compulsive buying scale results in 44 subjects, Journal of Nervous and Mental Disease 189 (2001) 123–126.
[15] Region of Valencia Department of Health, Drugs Addictions Service. Available at: http://www.san.gva.es/cas/prof/homeprof.html.
[16] R. Rodríguez, M. González, A. Fernández, M. Lameiras, Explorando la relación de la adicción a la compra con otros comportamientos excesivos: un estudio piloto, Adicciones 17 (2005) 231–240 (in Spanish).
[17] J.E. Mitchell, M. Burgard, R. Faber, R.D. Crosby, M. de Zwaan, Cognitive behavioral therapy for compulsive buying disorder, Behaviour Research and Therapy 44 (2006) 1859–1865.
[18] A. Mueller, U. Mueller, A. Silbermann, H. Reinecker, S. Bleich, J.E. Mitchell, M. de Zwaan, A. randomized, controlled trial of group cognitive-behavioral therapy for compulsive buying disorder: post treatment and 6-month follow-up results, Journal of Clinical Psychiatry 69 (2008) 1131–1138.
[19] L.M. Koran, R.J. Faber, E. Aboujaoude, M.D. Large, R.T. Serpe, Estimated prevalence of compulsive buying behavior in the United States, American Journal of Psychiatry 163 (2006) 1806–1812.
[20] E. Aboujaoude, N. Gamel, L.M. Koran, A 1-year naturalistic following of patients with compulsive shopping disorder, Journal of Clinical Psychiatry 64 (2003) 946–950.
[21] J. Garcés, Experiencias de trabajo en la prevención y tratamiento de la adicción al consumo, 2000 (in Spanish). Available at: http://webs.uvigo.es/consumoetico/textos/consumo/experiencias.pdf.
[22] Region of Valencia Department of Health, Health Survey, 2000. Available at: http://www.san.gav.es/cas/prof/homeprof.html.
[23] Region of Valencia Department of Health, Health Survey, 2005. Available at: http://www.san.gav.es/cas/prof/homeprof.html.
[24] Ministry of Economic Affairs, Official Credit Institute. Available at: http://www.ico.es/web/contenidos/616/index.
[25] D.N. Gujarati, Econometría, McGraw-Hill, 2004 (in Spanish).
[26] F.J. Santonja, A.C. Tarazona, R.J. Villanueva, A mathematical model of the pressure of an extreme ideology on a society, Computers & Mathematics with Applications 56 (2008) 836–846.
[27] A. Trillas, De la V a la L, el abecedario de la salida de la crisis (in Spanish). Available at: www.elpais.com/articulo/economia/V/L/abecedario/salida/crisis/elpepieco/20090412elpepieco\mathaccent"0250\relax6/Tes?print=1.