Saturation Problem Formalization

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1 System Model

- Macrocell (MC)
  - MC Antennas: $M' = \{m'_1, m'_2, ..., m'_N\}, N \in \mathbb{N}, |M'| = n_{m'}|\forall m'_n \in M' : n \in \mathbb{N}$
  - MC Micro Data Centers: $S' = \{s'_1, s'_2, ..., s'_{n_{s'}}\}, |S'| = n_{s'}|n_{s'} \in \mathbb{N}$
  - Link Budget: L' (mbps), $L' \in \mathbb{N}$
- Smallcell (SC)
  - SC Antennas: $M = \{m_1, m_2, ..., m_N\}, N \in \mathbb{N}, |M| = n_m|\forall m_n \in M : n \in \mathbb{N}$
  - SC Cluster: set of $n_c|n_c \leq n_m$ antennas
  - MC Micro Data Centers: one MDC $s|s \in S = \{s_1, s_2, ..., s_{n_s}\}, |S| = n_s|n_s \in \mathbb{N}$ for each SC Cluster
  - Link Budget: L (mbps), $L \in \mathbb{N}$
- Micro Data Center (MDC)
  - $s|s \in S \cup S'$
- Covered Area
  - Distance, in kilometers (km), between MC antennas
  - Distance, in km, between SC antennas and MC antennas
- User Equipment (UE)
  - Static or Dynamic (moving)
- Number of allocated vBBU's (Virtual Base Band Unit) in time t (hour)
  - $a_{is}(t) \in \mathbb{N}$: vector indicating the number of allocated vBBU's (of each machine class $i \in I$ for each MDC $s \in S \cup S'$) in time t – decision variable

- Machine classes and specifications
  - Classes $I = \{1, 2, ..., c\} | c \in \mathbb{N}, |I| = c$ machine classes
  - Computational power $P_{is} = \{P_{s1}, P_{s2}, ..., P_{sc}, P_{s'1}, P_{s'2}, ..., P_{s'c}\} | \forall p \in P_{is} : p \in \mathbb{N}$: vector of positive integers in Million Instructions Per Second (MIPS) for each machine class $i \in I$ and each type of MDC $s \in S \cup S'$
  - Number of cores $N_{is} = \{N_{s1}, N_{s2}, ..., N_{sc}, N_{s'1}, N_{s'2}, ..., N_{s'c}\} | \forall n \in N_{is} : n \in \mathbb{N}$: vector of positive integers indicating the number of cores for each machine class $i \in I$ and each type of MDC $s \in S \cup S'$
  - Pricing $A_{is} = \{A_{s1}, A_{s2}, ..., A_{sc}, A_{s'1}, A_{s'2}, ..., A_{s'c}\} | \forall a \in A_{is} : a \in \mathbb{R}^+$: vector of positive real numbers indicating the pricing in United States Dollar (USD) for each machine class $i \in I$ and each type of MDC $s \in S \cup S'$
- Association
  - $b_{ism}(t)$: vector of zeros (not associated) and ones (associated) to represent the association between a machine class $i \in I$ in an MDC $s \in S \cup S'$ and an antenna $m \in M \cup M'$
- Workload
  - $\Gamma_m(t) \in \mathbb{R}^+$: vector of real numbers between 0 and 1 indicating the percentage of usage of each antenna $m \in M \cup M'$ in time (hour)
  - W: channel decoding process (number of instructions), $W \in \mathbb{N}$
- Migration
  - $c_{sm}(t)$: binary variable to indicate if a migration occurred in an antenna $m \in M \cup M'$ from an MDC $s \in S \cup S'$ in time t ($c_{sm}(t) = 1$) or not ($c_{sm}(t) = 0$) with $c_{sm}(t) \in \mathbb{Z} | 0 \leq c_{sm}(t) \leq 1$
  - K: migration cost (USD), $K \in \mathbb{R}^+$

2 Problem Formalization

2.1 Objective Function

Minimize the cost and the number of allocated vBBU's:

$$min \left(\sum_{t}^{T} \sum_{i}^{I} \sum_{s}^{S} a_{is}(t) A_{is} + \sum_{t}^{T} \sum_{s}^{S} \sum_{m}^{M} c_{sm}(t) K \right) \tag{1}$$

2.2 Decision Variables

$$b_{ism}(t) \tag{2}$$

$$a_{is}(t) \tag{3}$$

2.3 Constraints

2.3.1 Horizontal Allocation

$$a_{is}(t)P_{is}N_{is} - \sum_{m}^{M} b_{ism}(t)W\Gamma_m(t) \geq 0 \qquad \forall t \in T; \forall i \in I; \forall s \in S \cup S' \qquad (4)$$

Ef_is

2.3.2 Vertical Allocation

$$b_{ism}(t)\frac{P_{sm}}{P_{is}} \leq 1 \qquad \forall t \in T; \forall i \in I; \forall s \in S \cup S'; m \in M \cup M' \qquad (5)$$

2.3.3 Migration Cost

$$c_{sm} + b_{sm}(t) - \sum_{s'|s' \neq s} b_{s'm}(t-1) = 1 \qquad \forall t \in T; \forall s \in S \cup S'; \forall m \in M \qquad (6)$$

3 Trade-offs

- Centralize (minimizing the number of MDC's in use) or distribute the workload?
- Association decision: why associate with a specific MDC and not with another?
- Workload transfer decision: keep the workload in one MDC or send to another?

4 Scenario Components Definition

4.1 Parameters

- Macrocell (MC)
  - MC MDC's: $S' = \{s'_1, s'_2, s'_3, s'_4, s'_5, s'_6, s'_7\}$
  - MC Antennas: $M' = \{m'_1, m'_2, m'_3, m'_4, m'_5, m'_6, m'_7\}$, one MC for each MC MDC
  - Link Budget: 900 mbps
- Smallcell (SC)
  - MC MDC's: $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$
  - SC Antennas: $M = \{m_1, m_2, m_3, m_4, ..., m_25, m_26, m_27, m_28\}$, four SC for each SC MDC
  - Link Budget: 300 mbps
- Micro Data Center (MDC)
  - $s | s \in S \cup S'$

- Covered Area (3 scenarios)
  1. 1 km (urban)
  2. 25 km (urban-countryside)
  3. 50 km (countryside)
- Number of allocated vBBU's in time t (hour)
  - $a_{is}(t)$: vector indicating the number of allocated vBBU's (of each type for each MDC) in time t (decision variable)
- Machine classes and specifications
  - $I = \{1, 2, 3\}$
  - First column for machine specification values of MDC's $s \in S$ and the second column for the power values for MDC's $s' \in S'$

    $P_{is} = [0.5, 1.0, 1.5, 1.0, 2.0, 3.0] = \begin{bmatrix} 0.5 & 1.0 \\ 1.0 & 2.0 \\ 1.5 & 3.0 \end{bmatrix}$, $N_{is} = [4, 8, 16, 16, 32, 64] = \begin{bmatrix} 4 & 16 \\ 8 & 32 \\ 16 & 64 \end{bmatrix}$,

    $A_{is} = [20, 30, 40, 30, 50, 90] = \begin{bmatrix} 20 & 30 \\ 30 & 50 \\ 40 & 90 \end{bmatrix}$
- Association
  - $b_{ism}(t)$: vector of zeros (not associated) and ones (associated) to represent the association between a machine class $i \in I$ in an MDC $s \in S \cup S'$ and an antenna $m \in M \cup M'$
- Workload
  - $\Gamma_m(t)$: vector of real numbers between 0 and 1 indicating the percentage of usage of each antenna $m \in M \cup M'$ in time t following a **normal distribution**
  - W: $200.7 = 1400$ instructions
- Migration
  - c_{sm}: one if a migration occurred in an antenna $m \in M \cup M'$ from an MDC $s \in S \cup S'$ in time t or zero (no migration in time t)
  - K: migration cost