Forecasting and Risk (BANA 4090)

ARIMA models (Part IV)

Zhaohu(Jonathan) Fan

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- Main topics:
  - Seasonal ARIMA models

Seasonal ARIMA models

ARIMA models

- The general **idea** of ARIMA models is to **capture autocorrelation**.
  - **Autocorrelation**: measure linear relationship between **lagged values** of a time series y.
  - For example, we measure the relationship between:
    - y_t and y_{t-1}
    - y_t and y_{t-2}
    - etc.
- **Major assumption**: stationarity.
- **Advantages**: strong underlying theory, flexible, etc.
- **Key concepts**: order, differencing.
  - Autoregressive Integrated Moving Average models (ARIMA (p, d, q)models):
    - AR: $p =$ order of the autoregressive part.
    - I: $d =$ degree of first differencing involved.
    - MA: $q =$ order of the moving average part.

Stationarity

- First-order (Lag-1) differencing (used for removing trend)
  - The differenced series is the **change** between each observation in the original series:
  - $y_t^{'} = y_t - y_{t-1}$. In R, `diff(data, lag=1)` or `diff(data)`.
- Seasonal (Lag-m) differencing (used for removing seasonality)
  - is the difference between an observation and the corresponding observation from the previous year.
  - $y_t^{'} = y_t - y_{t-m}$, where m= number of seasons.
  - For monthly data m=12. In R, `diff(data, lag=12)`.
  - For quarterly data m=4. In R, `diff(data, lag=4)`.

Stationarity (cont'd)

- Augmented Dickey-Fuller Test (ADF)

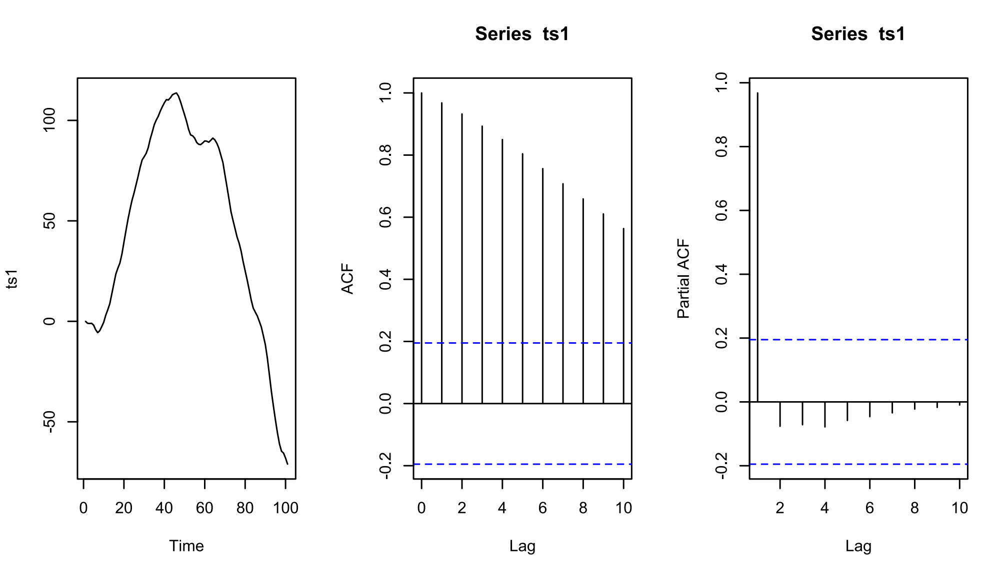
    - In general, the Augmented Dickey-Fuller test is defined as:
        - H_0 (null hypothesis): the data series is not stationary
        - H_a (alternative hypothesis): the data series is stationary.

- The `adf.test()` from the R package `tseries` will do a Augmented Dickey-Fuller test.

Identifying ARIMA (**p, d, q**) models

Model	ACF	PACF
AR(p)	exponentially decay	cut off at lag p
MA(q)	cut off at lag q	exponentially decay
ARMA(p,q)	exponentially decay after lag q	exponentially decay
ARIMA(p,d,q)	slowly decrease	exponentially decay

- ARIMA (1,1,1)
- Note: **ACF** starts at lag 0 and **PACF** starts at lag 1.



Seasonal ARIMA models

- (Non-seasonal) ARIMA models: non-seasonal data.
- ARIMA models are also capable of modelling a wide range of seasonal data by including additional seasonal terms.
- Seasonal ARIMA models:

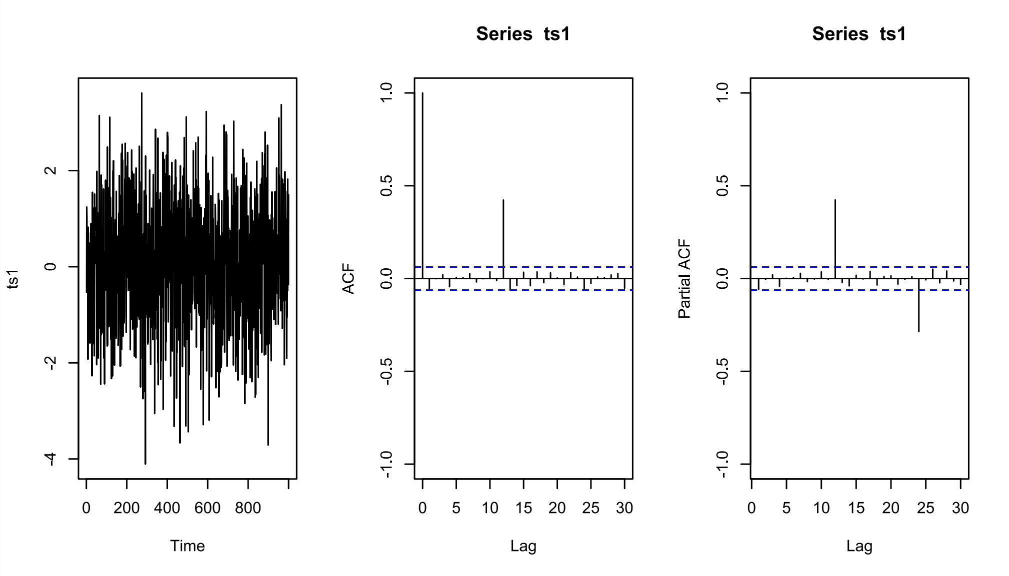
ARIMA	$\underbrace{(p, d, q)}$	$\underbrace{(P, D, Q)_m}$
	$\uparrow$	$\uparrow$
	Non-seasonal part	Seasonal part of
	of the model	of the model

where $m =$ number of seasons/number of observations per year.

```
- For monthly data m=12.
- For quarterly data m=4.
```

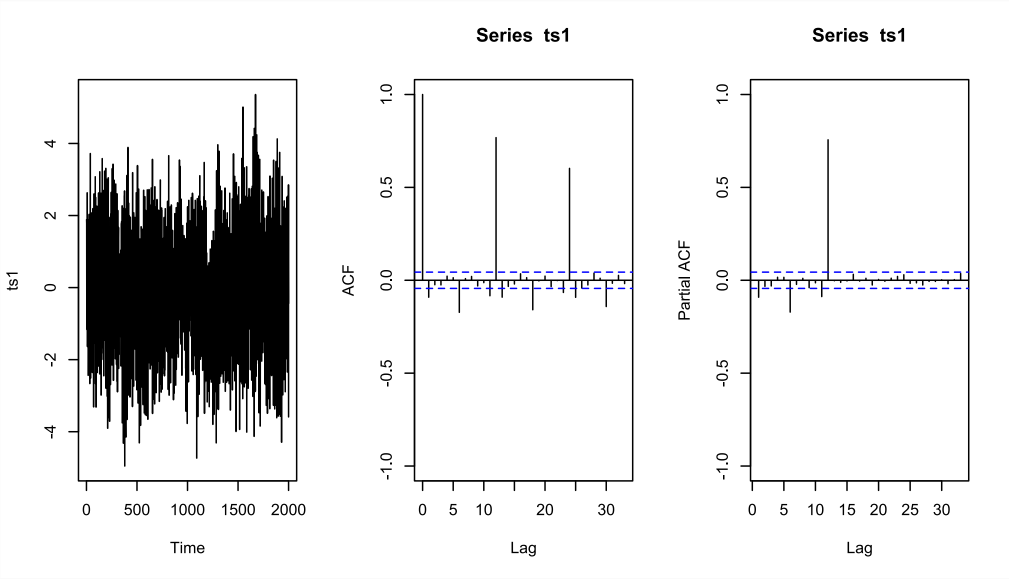
Seasonal ARIMA models(cont'd)

- The seasonal part of an AR or MA model will be seen in the seasonal lags of the PACF and ACF.
- **Seasonal ARIMA (0,0,0) (0,0,1)** $_{12}$ will show:
  - a spike at lag 12 in the **ACF** but no other significant spikes.
  - The **PACF** will show exponential decay in the seasonal lags (lags 12, 24, …).
  - Note: **ACF** starts at lag 0 and **PACF** starts at lag 1.



Seasonal ARIMA models(cont'd)

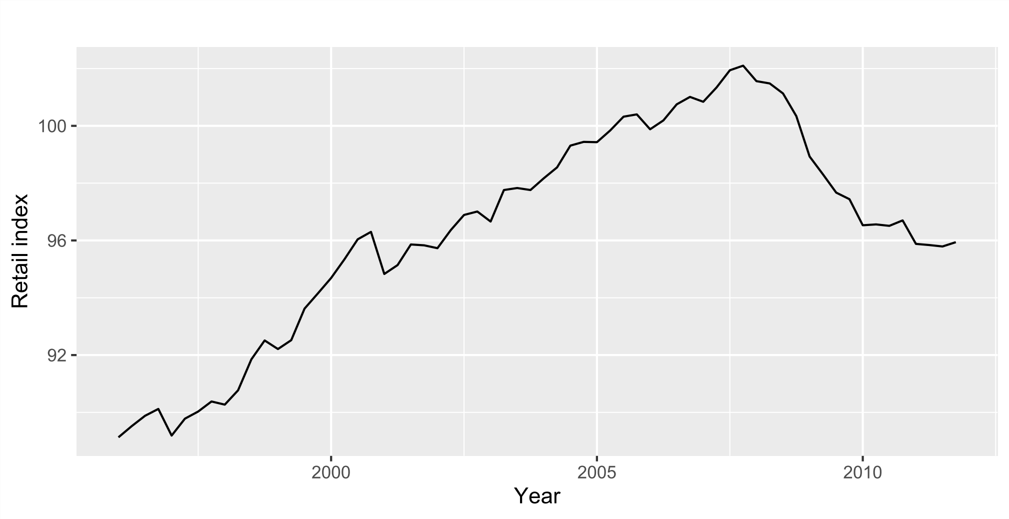
- **Seasonal ARIMA (0,0,0)(1,0,0)** $_{12}$ will show:
  - exponential decay in the seasonal lags of the **ACF**.
  - a single significant spike at lag 12 in the **PACF**.
- Note: **ACF** starts at lag 0 and **PACF** starts at lag 1



Data Example: European quarterly retail

- Quarterly retail trade index in the Euro area (17 countries), 1996-2011.

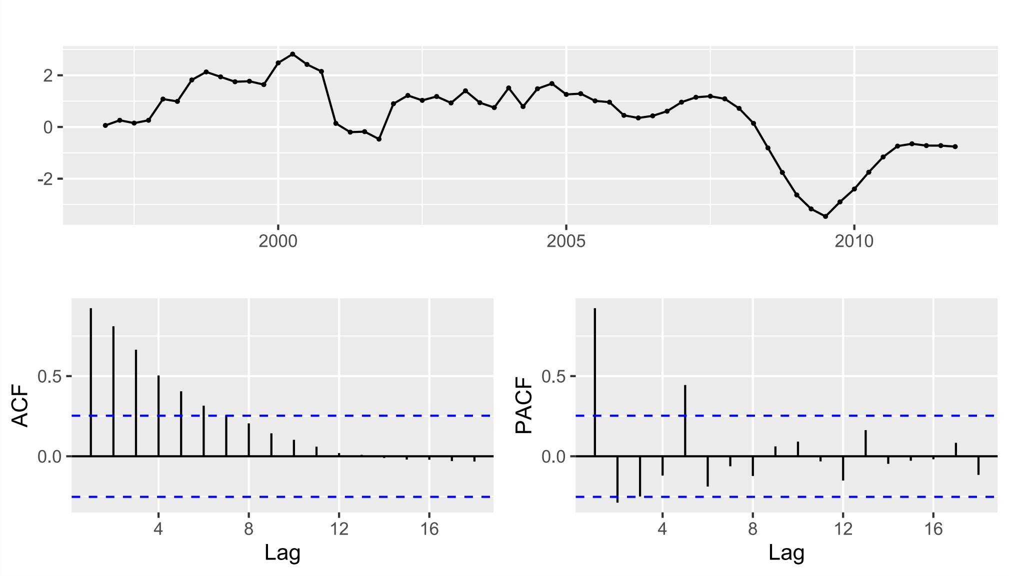
```
library(fpp2)
autoplot(euretail) +
  xlab("Year") + ylab("Retail index")
```



Data Example (cont'd)

- Taking the seasonal difference

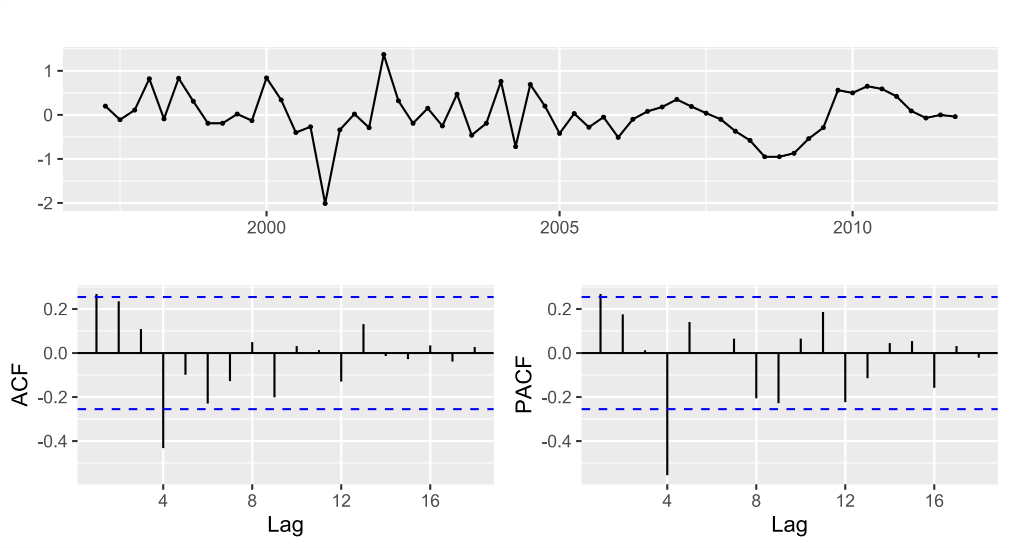
```
euretail %>% diff(lag=4) %>% ggtsdisplay()
```



Data Example (cont'd)

- Both seasonal and first differences are applied:

```
euretail %>% diff(lag=4) %>% diff() %>%
  ggtsdisplay()
```



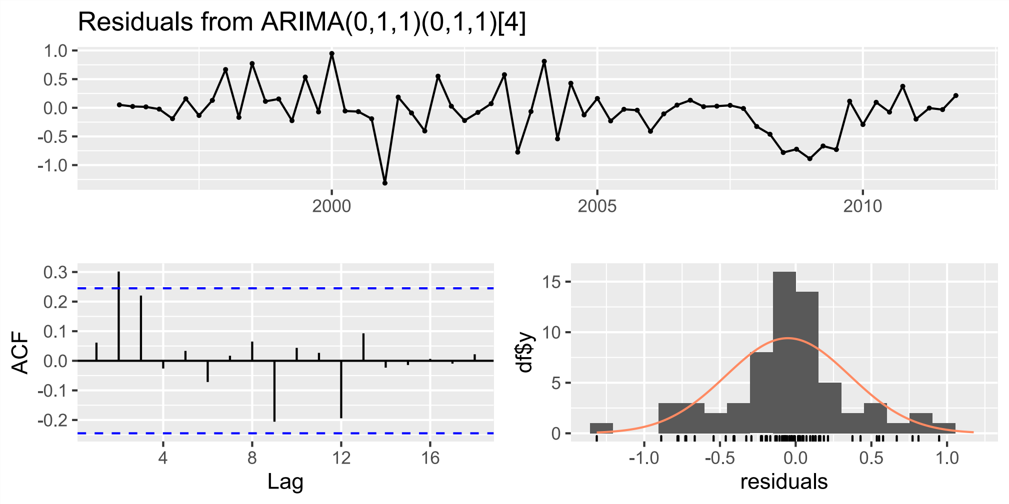
Data Example (cont'd)

- $d = 1$ and $D = 1$ seems necessary.
- Significant spike at lag 1 in ACF suggests non-seasonal MA(1) component.
- Significant spike at lag 4 in ACF suggests seasonal MA(1) component.
- Initial candidate model: ARIMA(0,1,1)(0,1,1) $_4$.
- We could also have started with ARIMA(1,1,0)(1,1,0) $_4$.

Data Example (cont'd)

- ARIMA(0,1,1)(0,1,1) $_4$

```
fit ← Arima(euretail, order=c(0,1,1),
  seasonal=c(0,1,1))
checkresiduals(fit)
```




```
##
##      Ljung-Box test
##
## data:  Residuals from ARIMA(0,1,1)(0,1,1)[4]
## Q* = 10.654, df = 6, p-value = 0.09968
##
```

Data Example (cont'd)

- ACF and PACF of residuals show significant spikes at lag 2, and maybe lag 3.

- BIC of ARIMA(0,1, 3)(0,1,1) $_4$ model is 77.64592.

```
fit <- Arima(euretail, order=c(0,1,3),
  seasonal=c(0,1,1))
BIC(fit)
```

```
## [1] 77.64592
```

Data Example (cont'd)

- BIC of ARIMA(0,1, 2)(0,1,1) $_4$ model is 81.83933.

```
(fit2 <- Arima(euretail, order=c(0,1,2),
  seasonal=c(0,1,1)))
```

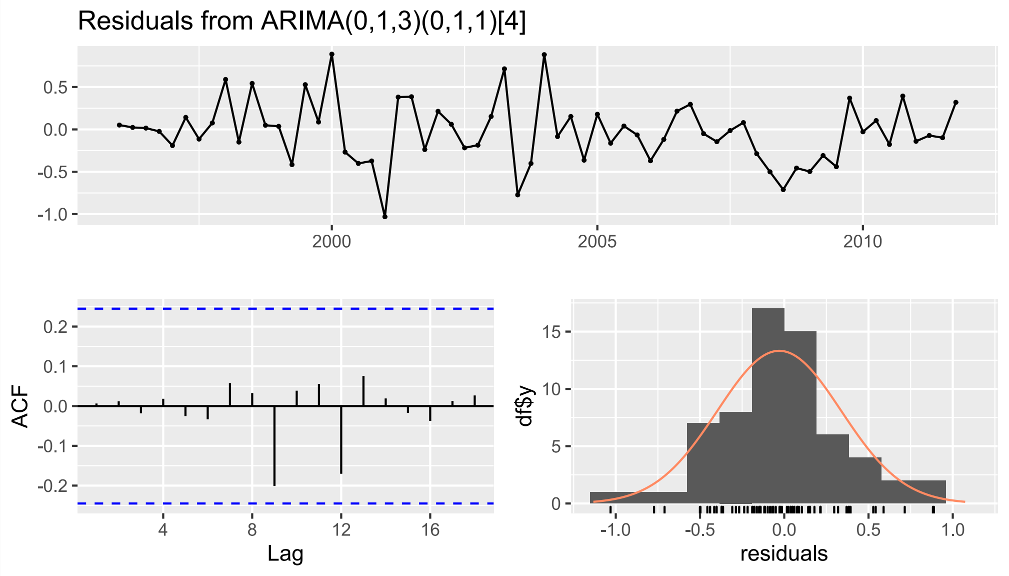
```
## Series: euretail
## ARIMA(0,1,2)(0,1,1)[4]
##
## Coefficients:
##          ma1     ma2     sma1
##       0.2303  0.2502  -0.6991
## s.e.  0.1484  0.1188   0.1284
##
## sigma^2 estimated as 0.1789:  log likelihood=-32.76
## AIC=73.53   AICc=74.27   BIC=81.84
```

```
BIC(fit2)
```

```
## [1] 81.83933
```

Data Example (cont'd)

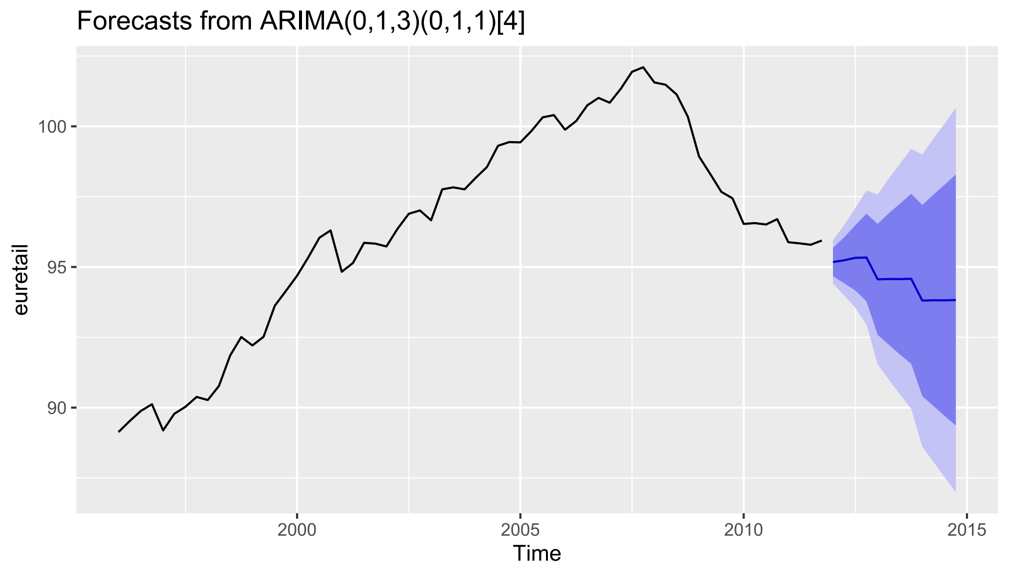
```
checkresiduals(fit)
```




```
##
##      Ljung-Box test
##
## data:  Residuals from ARIMA(0,1,3)(0,1,1)[4]
## Q* = 0.51128, df = 4, p-value = 0.9724
##
## Model df: 4.   Total lags used: 8
```

Data Example (cont'd)

```
autoplot(forecast(fit, h=12))
```



Data Example (cont'd)

```
auto.arima(euretail)
```

```
## Series: euretail
## ARIMA(0,1,3)(0,1,1)[4]
##
## Coefficients:
##          ma1     ma2     ma3     sma1
##       0.2630  0.3694  0.4200  -0.6636
## s.e.  0.1237  0.1255  0.1294   0.1545
##
## sigma^2 estimated as 0.156:  log likelihood=-28.63
## AIC=67.26   AICc=68.39   BIC=77.65
```

Data Example (cont'd)

```r
auto.arima(euretail, stepwise=FALSE, approximation=FALSE)
```

```
## Series: euretail
## ARIMA(0,1,3)(0,1,1)[4]
##
## Coefficients:
##          ma1     ma2     ma3     sma1
##       0.2630  0.3694  0.4200  -0.6636
## s.e.  0.1237  0.1255  0.1294   0.1545
##
## sigma^2 estimated as 0.156:  log likelihood=-28.63
## AIC=67.26   AICc=68.39   BIC=77.65
```