

— Convolution of Matrices

Convolution

X (the image)

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 1 | 2 | 7 | 4 |
| 1 | 5 | 8 | 9 | 3 | 1 |
| 2 | 7 | 2 | 5 | 1 | 3 |
| 0 | 1 | 3 | 1 | 7 | 8 |
| 4 | 2 | 1 | 6 | 2 | 8 |
| 2 | 4 | 5 | 2 | 3 | 9 |

*

W (the filter)

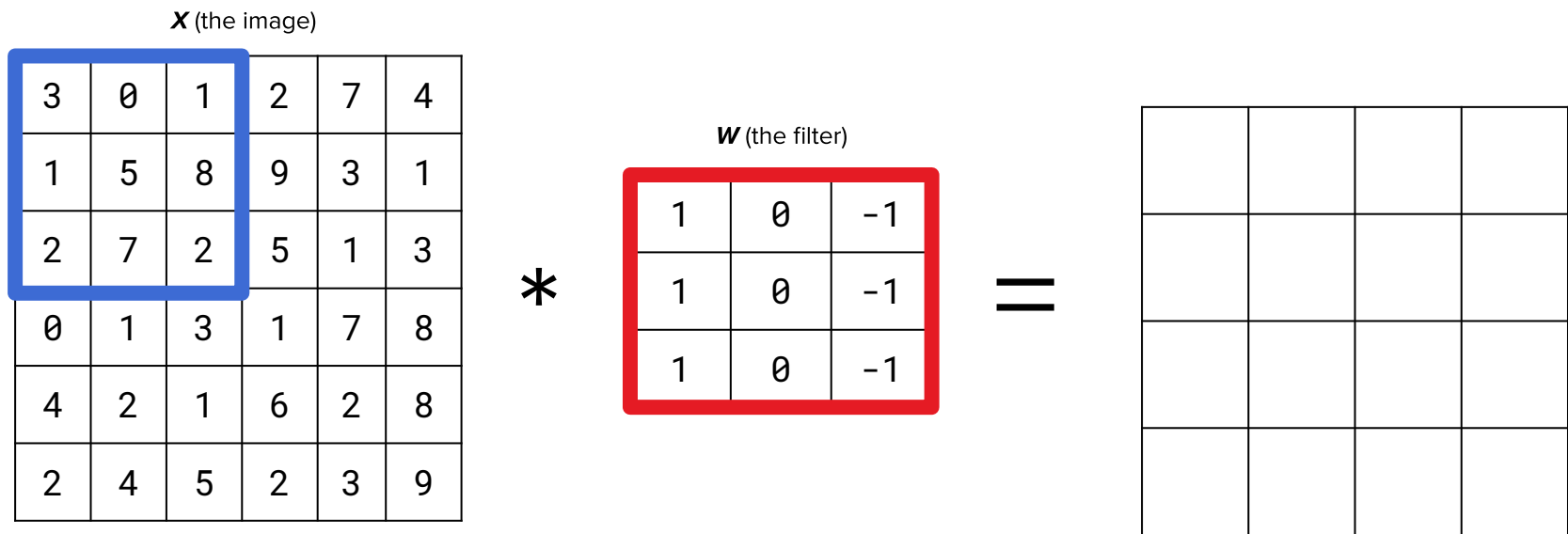
| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

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|--|--|--|--|
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| | | | |
| | | | |

Pass the **filter** over the **image** and compute the element-wise dot product to get the results.

Convolution



Pass the **filter** over the **image** and compute the element-wise dot product to get the results.

Convolution

X (the image)

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 1 | 2 | 7 | 4 |
| 1 | 5 | 8 | 9 | 3 | 1 |
| 2 | 7 | 2 | 5 | 1 | 3 |
| 0 | 1 | 3 | 1 | 7 | 8 |
| 4 | 2 | 1 | 6 | 2 | 8 |
| 2 | 4 | 5 | 2 | 3 | 9 |

W (the filter)

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

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| | | | |
|----|--|--|--|
| -5 | | | |
| | | | |
| | | | |
| | | | |

$$\begin{aligned} & 3*1 + 0*0 + 1*-1 + \\ & 1*1 + 5*0 + 8*-1 + \\ & 2*1 + 7*0 + 2*-1 = -5 \end{aligned}$$

Convolution

X (the image)

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 1 | 2 | 7 | 4 |
| 1 | 5 | 8 | 9 | 3 | 1 |
| 2 | 7 | 2 | 5 | 1 | 3 |
| 0 | 1 | 3 | 1 | 7 | 8 |
| 4 | 2 | 1 | 6 | 2 | 8 |
| 2 | 4 | 5 | 2 | 3 | 9 |

W (the filter)

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

$*$

$=$

| | | | |
|----|--|--|--|
| -5 | | | |
| | | | |
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| | | | |

We then slide the **filter** across the **image** to the next pixel and repeat the process.

Convolution

X (the image)

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 1 | 2 | 7 | 4 |
| 1 | 5 | 8 | 9 | 3 | 1 |
| 2 | 7 | 2 | 5 | 1 | 3 |
| 0 | 1 | 3 | 1 | 7 | 8 |
| 4 | 2 | 1 | 6 | 2 | 8 |
| 2 | 4 | 5 | 2 | 3 | 9 |

W (the filter)

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

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| | | | |
|----|----|--|--|
| -5 | -4 | | |
| | | | |
| | | | |
| | | | |

$$\begin{aligned} &0*1 + 1*0 + 2*-1 + \\ &5*1 + 8*0 + 9*-1 + \\ &7*1 + 2*0 + 5*-1 = -4 \end{aligned}$$

Convolution

X (the image)

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 1 | 2 | 7 | 4 |
| 1 | 5 | 8 | 9 | 3 | 1 |
| 2 | 7 | 2 | 5 | 1 | 3 |
| 0 | 1 | 3 | 1 | 7 | 8 |
| 4 | 2 | 1 | 6 | 2 | 8 |
| 2 | 4 | 5 | 2 | 3 | 9 |

W (the filter)

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

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| | | | |
|----|----|---|--|
| -5 | -4 | 0 | |
| | | | |
| | | | |
| | | | |

$$\begin{aligned} &1*1 + 2*0 + 7*-1 + \\ &8*1 + 9*0 + 3*-1 + \\ &2*1 + 5*0 + 1*-1 = 0 \end{aligned}$$

Convolution

X (the image)

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 1 | 2 | 7 | 4 |
| 1 | 5 | 8 | 9 | 3 | 1 |
| 2 | 7 | 2 | 5 | 1 | 3 |
| 0 | 1 | 3 | 1 | 7 | 8 |
| 4 | 2 | 1 | 6 | 2 | 8 |
| 2 | 4 | 5 | 2 | 3 | 9 |

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W (the filter)

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

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| | | | |
|----|----|---|---|
| -5 | -4 | 0 | 8 |
| | | | |
| | | | |
| | | | |

$$\begin{aligned} &2*1 + 7*0 + 4*-1 + \\ &9*1 + 3*0 + 1*-1 + \\ &5*1 + 1*0 + 3*-1 = 0 \end{aligned}$$

Convolution

X (the image)

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 1 | 2 | 7 | 4 |
| 1 | 5 | 8 | 9 | 3 | 1 |
| 2 | 7 | 2 | 5 | 1 | 3 |
| 0 | 1 | 3 | 1 | 7 | 8 |
| 4 | 2 | 1 | 6 | 2 | 8 |
| 2 | 4 | 5 | 2 | 3 | 9 |

*

W (the filter)

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

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| | | | |
|-----|----|----|-----|
| -5 | -4 | 0 | 8 |
| -10 | -2 | 2 | 3 |
| 0 | -2 | -4 | -7 |
| -3 | -2 | -3 | -16 |

And so on until completion...

What does this actually do for us?

X (the image)

| | | | | | |
|----|----|----|---|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |

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W (the filter)

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

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| | | | |
|--|--|--|--|
| | | | |
| | | | |
| | | | |
| | | | |

What does this actually do for us?

X (the image)

| | | | | | |
|----|----|----|---|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |

*

W (the filter)

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

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| | | | |
|---|----|----|---|
| 0 | 30 | 30 | 0 |
| 0 | 30 | 30 | 0 |
| 0 | 30 | 30 | 0 |
| 0 | 30 | 30 | 0 |

What does this actually do for us?

X (the image)

| | | | | | |
|----|----|----|---|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |

*

W (the filter)

| | | |
|----|----|----|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

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| | | | |
|--|--|--|--|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

What does this actually do for us?

X (the image)

| | | | | | |
|----|----|----|---|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |

*

W (the filter)

| | | |
|----|----|----|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

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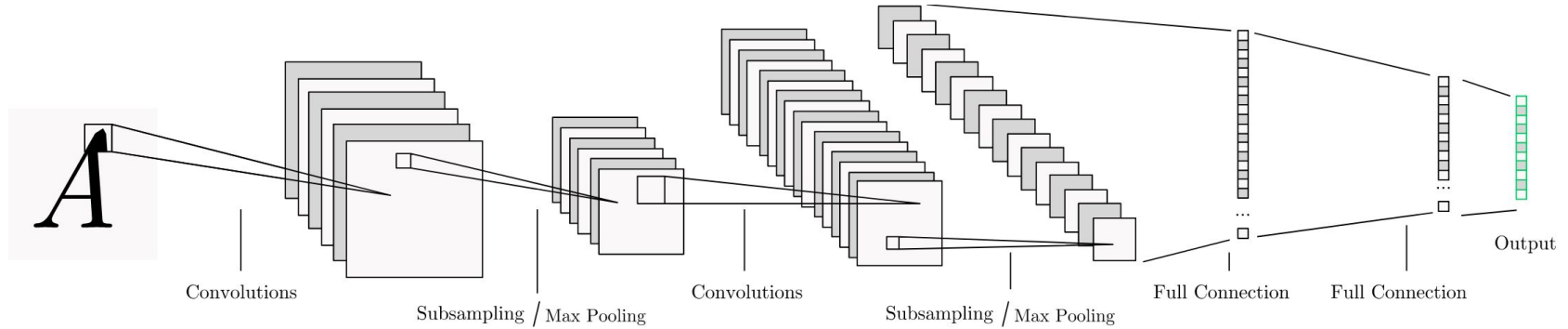
| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |

Edge Detection!



Source: <https://aishack.in/tutorials/image-convolution-examples/>

CNNs in the World



Source: https://hako.github.io/dissertation/figures/figure_5.svg