CM146, Fall 2018
Problem Set 02: Jonathan Chu

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1 (on CCLE)

2 (on CCLE)

3 Understanding Linear Separability

(a) **Solution:** If $\delta = 0$, we have:

$$y_i(\boldsymbol{w^T x_i} + \theta) \geq 1$$

Trivially, this inequality holds when $sgn(y_i) = sgn(\boldsymbol{w^T x_i} + \theta)$ and $|\boldsymbol{w^T x_i} + \theta| \geq 1$.

The matching signs of y_i and $\boldsymbol{w^T x_i}$ indicate these values of $\boldsymbol{w}$ and θ satisfy equation (1). Therefore D is linearly separable.

(b) **Solution:** For $0 < \delta < 1$, it still holds that $sgn(y_i) = sgn(\boldsymbol{w^T x_i} + \theta)$, meaning that D remains linearly separable, and $|\boldsymbol{w^T x_i} + \theta| < 1$ for some i, which is not of any concern.

However, if the minimum $\delta \geq 1$, we have

$$y_i(\boldsymbol{w^T x_i} + \theta) \geq c,\ c \leq 0$$

and D is not linearly separable.

(c) **Solution:** The minimum value we can achieve for δ is 0. If this is the case, we seek $\boldsymbol{w}$ and $\boldsymbol{x}$ that satisfy the following:

$$y_i(\boldsymbol{w^T x_i} + \theta) \geq 0$$

Trivially, $\boldsymbol{w} = \boldsymbol{0}$, $\theta = 0$ satisfy the above inequality yet clearly will not separate D.

(d) **Solution:** Trivially, we see that $\boldsymbol{w} = [1, 1, 1]$, $\theta = 0$, $\delta = 0$ is an optimal solution.

In fact, any solution with

$$\delta = 0$$

$$|\boldsymbol{w^T x_i} + \theta| = |w_1x_1 + w_2x_2 + w_3x_3 + \theta| \geq 1$$

$$\implies |w_1 + w_2 + w_3 + \theta| \geq 1 \text{ and } |-w_1 + -w_2 + -w_3 + \theta| \geq 1$$

is optimal.