Neoclassical Growth Theory

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Introduction and Motivation

Neoclassical Growth Theory

- The starting point for any study of economic growth is the neoclassical growth model.
  - Benchmark model despite limiting assumptions
  - Emphasizes the role of capital accumulation
- The model shows how economic policy can raise an economy's transitional growth rate by inducing people to save more.
- However, in the long-run the growth rate will revert to the rate of technological progress
- Policy shifts levels, not growth.
  - Neoclassical theory takes technological progress as being independent of economic forces
- Underlying this result is the principle of diminishing marginal productivity

The Basic Solow-Swan Model

Model Setup

Aggregate Production Function: $Y = F(K)$

Key Assumptions:

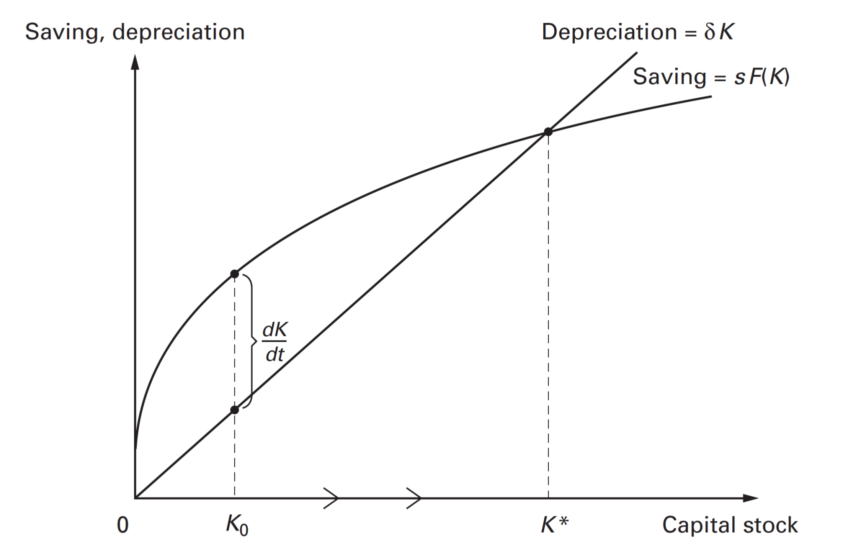
- Labor supply and technology initially constant
- Diminishing returns: $F'(K) > 0$, $F''(K) < 0$
- Inada conditions: $\lim_{K \to 0} F'(K) = \infty$, $\lim_{K \to \infty} F'(K) = 0$

Capital Accumulation: $\dot{K} = sF(K) - \delta K$

- People save fraction s of income
- Capital depreciates at rate δ

Key insight: Given the initial stock of capital K0 we can determine the entire future time path of capital and output.

Graphical Analysis



Why does growth stop?

Adding Population Growth and Technology

With Population Growth:

- Per capita dynamics: $\dot{k} = sf(k) - (n + \delta)k$
- Population "dilutes" capital per person
- Still no per capita growth in steady state

With Technological Progress:

- Production: $Y = A(t)L^{1-\alpha}K^{\alpha}$ where $A(t) = A_0 e^{gt}$
- Growth in "efficiency units": AL grows at rate $n + g$
- Per efficiency unit: $\dot{\kappa} = s\kappa^{\alpha} - (n + g + \delta)\kappa$, where $\kappa = K/AL$

Take-home: In the long run, the only parameter affecting the growth rate is the exogenous rate of technological progress g.

Overview

- In the short-run the economy can grow for a while by accumulating capital…
- … But without technological change, decreasing returns to capital choke off growth.
- With technological change, growth can be sustained…
- … But, the model provides no account as to what drives technological change.
- Provides no economic explanation for persistent differences in growth.

The Ramsey-Cass-Koopmans Model

Endogenizing the Saving Rate

Limitation of Fixed Saving Rate:

- Ignores consumption smoothing motives
- No role for interest rates in saving decisions

Ramsey Approach: Representative household maximizes:

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1$$

subject to: $K_{t+1} = K_t + F(K_t) - c_t - \delta K_t$

Key Result: Same long-run conclusions (with no explanation), but richer dynamics

- Provides microfoundations for saving behavior
- Links growth theory to standard consumer theory
- Enables welfare analysis of growth policies

The Euler Equation[1]

Optimal Consumption Path:

$$\frac{\dot{c}}{c} = \frac{(r - \rho)}{\varepsilon}$$

where:

- $r = F'(K) - \delta$ (net return to capital)
- $\rho > 0$ = rate of time preference (where $\beta = \frac{1}{1+\rho}$).
- ε = elasticity of marginal utility

Intuition:

- If $r > \rho$: Saving more attractive $\Rightarrow$ consumption grows
- If $r < \rho$: Consumption now preferred $\Rightarrow$ consumption falls

[1] Continuous time, infinite horizon in which we assume isoelastic utility, s.t., individuals have the same elasticity of substitution $1/\varepsilon$ between present and future consumption no matter the level of consumption. $u(c) = \frac{c^{1-\varepsilon}-1}{1-\varepsilon}$ where $u'(c) = c^{-\varepsilon}$; $u''(c) = -\varepsilon c^{-(\varepsilon} + 1)$.

Empirical Predictions and Evidence

Conditional Convergence

Key Question: Do poor countries catch up with rich ones?

Neoclassical Prediction: "Conditional convergence"

- Countries converge in levels if they share same fundamentals:
  - Same technology (argued/assumed to be true in the empirical literature)
  - Same saving rate s, depreciation δ, population growth n
- Poor countries grow faster when further from their steady state
- Different fundamentals $\Rightarrow$ different steady states $\Rightarrow$ no convergence

Empirical Evidence on Convergence

- Empirical tests focus on whether poor countries grow faster than rich countries, after controlling for the fundamentals (s, n, and δ),

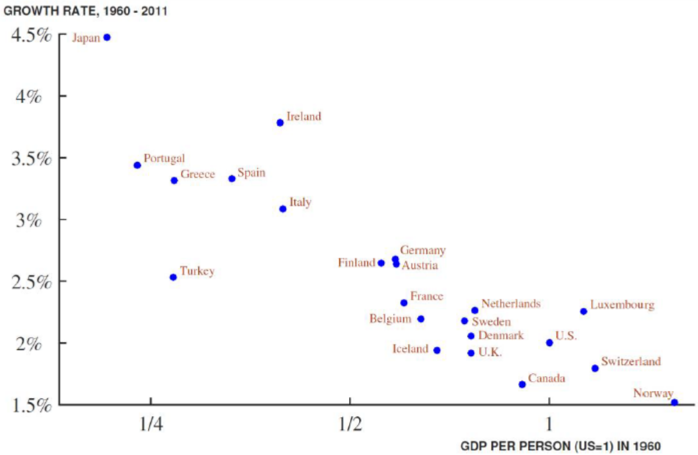
$$\frac{1}{T}\log\frac{y_{i,t+T}}{y_{i,t}} = \beta_0 + \beta_1 \log y_{i,t} + \beta_2 X_{i,t} + \epsilon_{i,t}$$

- growth rates can vary either because of differences in the parameters determining their steady-states ($\beta_2 X_{i,t}$) or because of differences in initial positions ($\beta_1 \log y_{i,t}$).

- An estimated value of $\beta_1 < 0$ is taken as evidence of conditional convergence.

Convergence in the OECD

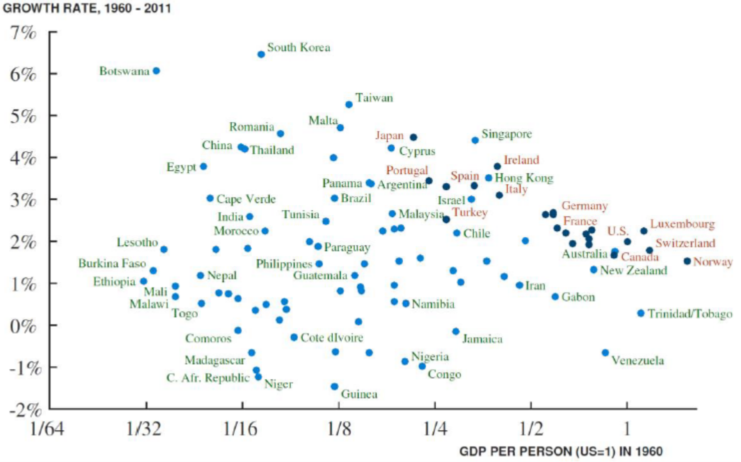
Figure 25: Convergence in the OECD



Source: The Penn World Tables 8.0. Countries in the OECD as of 1970 are shown.

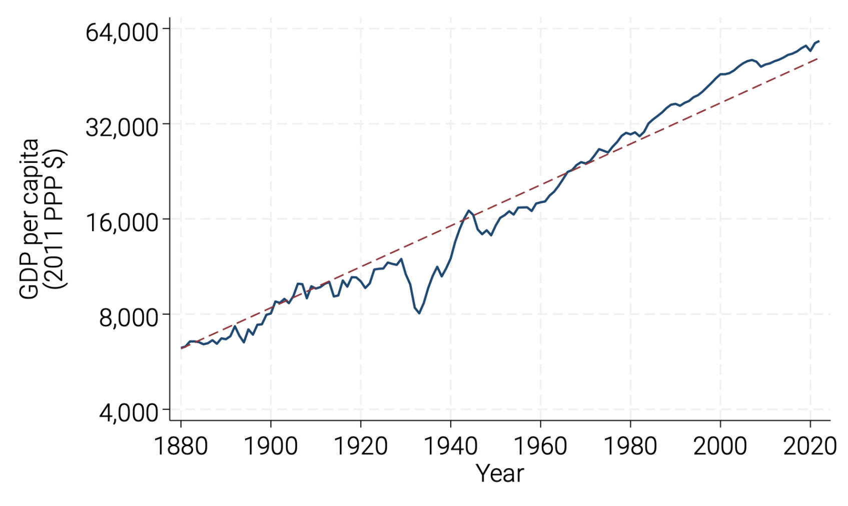
Limited Convergence World Wide

Figure 26: The Lack of Convergence Worldwide

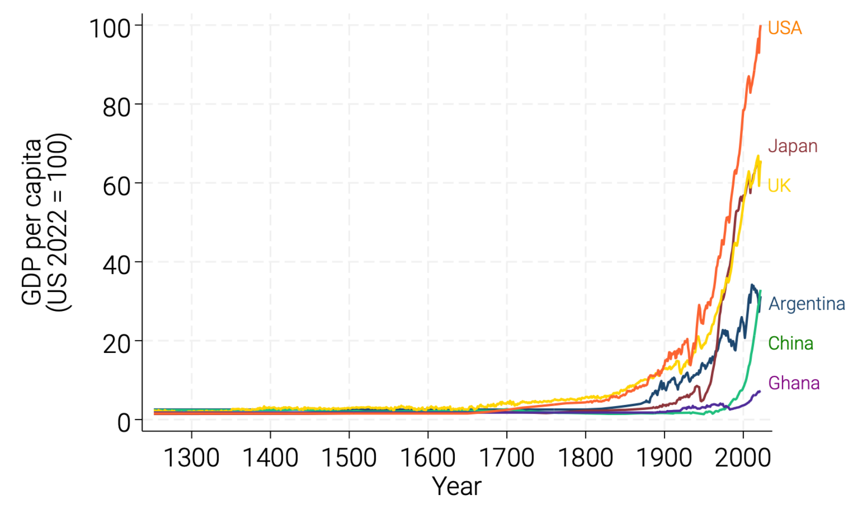


Source: The Penn World Tables 8.0.

Diminishing Returns?



Divergence



GDP per capita
(US 2022 = 100)
100
80
60
40
20
0
1300
1400
1500
1600
1700
1800
1900
2000
Year
USA
Japan
UK
Argentina
China
Ghana

Frontier Growth vs. Catchup Growth

- Limited support for conditional convergence overall — "Some countries just have faster technological progress"?

$$A_{it} = \tilde{A}_{it}\hat{A}_{it}$$

- $\hat{A}_{it}$ – the world technology frontier, changes slowly
- $\tilde{A}_{it}$ – changes rapidly due to events, changes in policies (miracles and disasters)
    - For countries close to the frontier, growth is only possible by moving the frontier.
    - What forces determine the evolution of $\hat{A}_{it}$? (Endogenous growth theory)
- For countries far away from the frontier, reasonable to think that the frontier is exogenous.
    - What forces determine $\tilde{A}_{it}$? (Development Economics)