# Growth Accounting

Jonathan Colmer

November 10, 2025

## Growth Accounting: Basics

- Start from an aggregate technology $Y = F(K, AL)$
- How much of growth is "due to":
  - Growth in inputs (capital, labor, etc.)
  - Growth in technology (A)
- An exercise is accounting, not causal inference.
- Doesn't attempt to explain why growth in inputs differ, but provides direction.
- Developed by Abramovitz (1956) and Solow (1957).

## From Production to Growth Rates

$$Y(t) = F[K(t), A(t)L(t)]$$

If we differentiate with respect to time and divide by $Y$, we get:

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{K(t)}{Y(t)} \frac{\partial Y(t)}{\partial K(t)} \frac{\dot{K}(t)}{K(t)} + \frac{L(t)}{Y(t)} \frac{\partial Y(t)}{\partial L(t)} \frac{\dot{L}(t)}{L(t)} + \frac{A(t)}{Y(t)} \frac{\partial Y(t)}{\partial A(t)} \frac{\dot{A}(t)}{A(t)}$$

# Elasticities and the Solow Residual

$$g_Y = \alpha_K g_K + \alpha_L g_L + g_A$$

- Much is measurable:
  - Growth in output ($\frac{\dot{Y}(t)}{Y(t)}$)
  - Growth in capital ($\frac{\dot{K}(t)}{K(t)}$)
  - Growth in labor ($\frac{\dot{L}(t)}{L(t)}$)
  - Output elasticities w.r.t. capital ($\alpha_k$) and labor ($\alpha_L$)
- we're left with the residual $g_A$, which captures everything else (technology, utilization, misallocation, measurement error, etc.)

# Measurement: Capital

- Ideally, we want the flow of services from capital.
- In practice, we measure the stock and assume that the flow is proportional.
- Perpetual-inventory method:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

    - Start with some initial stock $K_0$
    - Measure investment series $I_t$ from national income and product accounts
    - Calibrate $\delta$
- Errors in $K_t$ accumulate and mechanically spill into the residual

# Measurement: Labor

- Simplest measure is hours worked.
- But misses "The knowledge, skills, health or values embodied in people that make them productive." – Gary Becker.
- Increases in output may come from higher-quality labor, not just more hours.
- Jorgensen and Grilliches (1967) disaggregate labor by schooling and weight by relative wages
- Equivalent adjustments can be made for capital

# Measurement: Output Elasticities

- If capital and labor earn their marginal products,

$$r_t = \frac{\partial Y}{\partial K}, \qquad w_t = \frac{\partial Y}{\partial L}$$

- Then, output elasticities are factor shares,

$$\alpha_K = \frac{r_t K_t}{Y_t}, \qquad \alpha_L = \frac{w_t L_t}{Y_t}$$

- Data on factor shares are widely used.

- But, only valid under idealized assumptions.

# Why Not Estimate?

- One could regress $\dot{Y}/Y$ on $\dot{K}/K$ and $\dot{L}/L$.
- This would allow us to recover $\alpha_K$ and $\alpha_L$, as well as the residual $g_A$.
- Problems?

# Why Not Estimate?

- One could regress $\dot{Y}/Y$ on $\dot{K}/K$ and $\dot{L}/L$.
- This would allow us to recover $\alpha_K$ and $\alpha_L$, as well as the residual $g_A$.
- Problems?
  - Endogeneity: productivity affects both output and inputs
  - Measurement error: noisy capital and labor series would bias estimates of elasticities
- Using factor shares circumvents both problems but imposes the strong price-equals-marginal-product assumption

# Growth Accounting and Sources of Growth

- Growth accounting is not causal analysis
- Example:

$$Y = AK^{\alpha}(Le^{x_t})^{1-\alpha}$$

- Suppose $A$ and $L$ are constant
- $x$ is labor-augmenting growth in technology
- Taking logs and differentiating w.r.t time gives,

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + (1-\alpha)x$$

# Growth Accounting and Sources of Growth

$$\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + (1 - \alpha)x$$

- Balanced growth path in Solow/Ramsey implies $\dot{K}/K = \dot{Y}/Y = x$
- Capital accumulation mechanically picks up $\alpha x$ even if technology drives everything
- A small residual does not imply technology is unimportant; it may simply reflect endogenous factors
- To attribute both direct and induced effects to technology, divide measured TFP growth by $(1 - \alpha)$

# Alternative Per-Worker Decomposition

- Start with Cobb-Douglas production function,

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

- Divide both sides by $Y_t^{\alpha}$ and raise to power $1/(1-\alpha)$,

$$Y_t = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} H_t \underbrace{Z_t}_{A_t^{\frac{1}{1-\alpha}}}$$

- Divide by $L_t$,

$$\frac{Y_t}{L_t} = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_t}{L_t} Z_t$$

# Alternative Per-Worker Decomposition

$$\frac{Y_t}{L_t} = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_t}{L_t} Z_t$$

- Decomposes per capita (or per hour) growth into three components:
  - Capital deepening: $K/Y$
  - Growth in human capital per hour: $H/L$
  - TFP: $Z$
- Importantly Solow and Ramsey model imply that $K/Y$ is constant along the balanced growth path
- Take logs and differentiae w.r.t. time, to get the growth accounting equation.

# U.S. Growth Experience (Jones, 2016)

| Period | $g_{Y/L}$ | $K/Y$ | Labor comp. | TFP |
|---|---|---|---|---|
| **1948–2013** | **2.5** | **0.1** | **0.3** | **2.0** |
| 1948–1973 | 3.3 | -0.2 | 0.3 | 3.2 |
| 1973–1990 | 1.6 | 0.5 | 0.3 | 0.8 |
| 1990–1995 | 1.6 | 0.2 | 0.7 | 0.7 |
| 1995–2000 | 3.0 | 0.3 | 0.3 | 2.3 |
| 2000–2007 | 2.7 | 0.2 | 0.3 | 2.2 |
| 2007–2013 | 1.7 | 0.1 | 0.5 | 1.1 |

- TFP growth accounts for most growth in the U.S.

- By contrast, input growth accounts for most growth in many fast-growing countries.

# Philippon (2023): Additive Growth

- Standard view: $A_{t+\tau} = A_t(1+g)^\tau$ so $\log A$ drifts linearly
- Alternative view: $A_{t+\tau} = A_t + b\tau$ so technology follows a linear trend in levels
- Matters for forecasting and for how we interpret growth slowdowns.

# Philippon (2023): Exponential or Linear?

$$\text{Exponential:} \quad \log A_{t+\tau} \approx \log A_t + g\tau$$
$$\text{Linear:} \quad A_{t+\tau} = A_t + b\tau$$

- Linear specification implies constant expected increments in TFP
- Exponential specification implies constant proportional growth

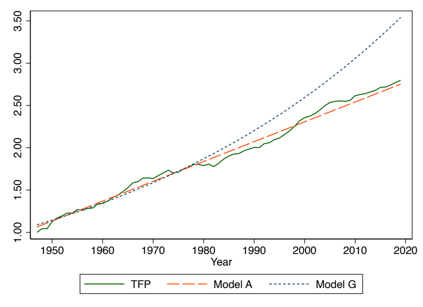
# Philippon (2023): Additive Growth?

- Consider the U.S. post-WWII sample
- Use the first half of sample to predict $A(t)$ for the second half
- Compare prediction of:
  - Person who believes in exponential growth (Model G)
  - Person who believes in linear growth (Model A)

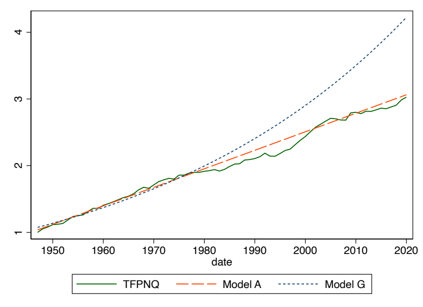
# Linear Growth Fits Better Out of Sample

Figure 1: Out-of-Sample TFP Forecasts

(a) BCL Data




(b) Fernald Data

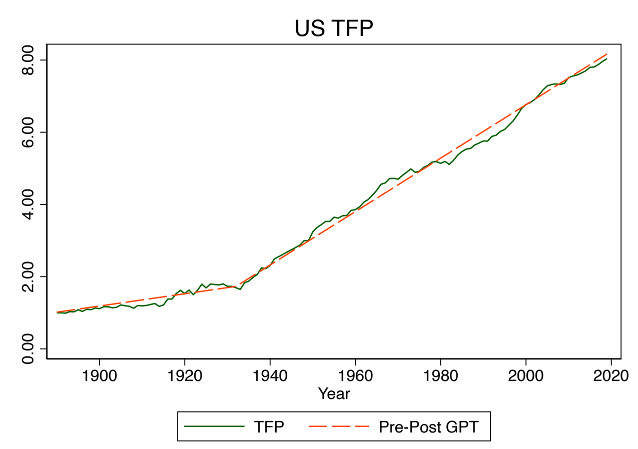


## Long-Run Perspective

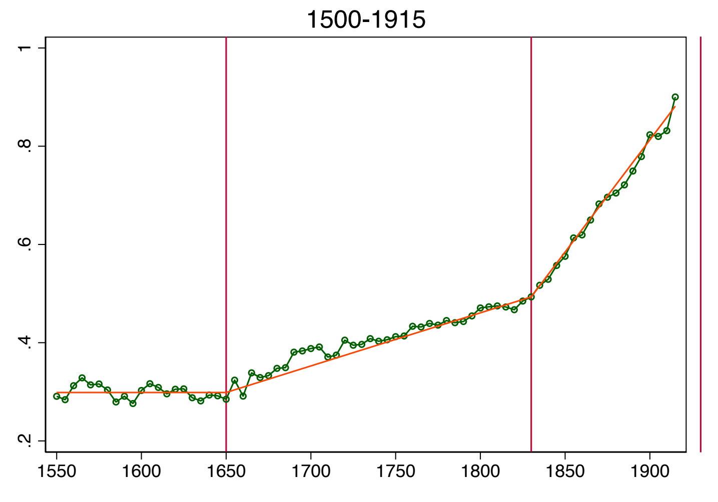
- Growth cannot have been linear forever — $A(t)$ would have been negative at some point.
- Philippon proposes that "General Purpose Technologies" cause breaks
  - Enlightenment/glorious revolution
  - Steam engine/industrial revolution
  - Electrification/second industrial revolution

# Occasional Breaks over Longer Sample

Figure 10: Linear US TFP with One Break



# Occasional Breaks over Longer Sample




Source: U.K. Pseudo-TFP (GDP per capita to the power 2/3).

## Takeaways

- Additive growth pushes back against the ideas that ideas multiply ideas.
- Instead, ideas add.
- Implies secular stagnation is natural without major innovations — growth rates trend down as income levels rise.
- Points towards increasing the slope of the TFP trend through GPT diffusion and adoption, rather than scaling R&D.
- Maybe ideas multiply ideas, but there is a constant adjustment capacity, i.e., only so many ideas can be adopted at a given point in time. This would look like additive TFP growth.