The AK Model

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Why Does Growth Stop in Neoclassical Models?

- Diminishing marginal product of capital
  - As $K \uparrow$, $F'(K) \downarrow$
  - Eventually depreciation catches up with savings
  - Without exogenous technological progress, growth stops
- What if we could eliminate diminishing returns?

The Harrod-Domar Model

$$Y = F(K, L) = \min\{AK, BL\}$$

- Producing one unit needs $1/A$ units of capital and $1/B$ units of labor
- With scarce capital ($AK < BL$): $Y = AK$
- Capital dynamics identical to Solow: $\dot{K} = sY - \delta K$
- On the capital-limited path, $g = \dot{K}/K = sA - \delta$
- Higher saving delivers permanently faster output growth while capital is the bottleneck

Why Harrod-Domar Falls Short

- Cannot account for the sustained growth in output per person.
  - Let ν be population growth rate.
  - growth rate of output per person is $g - \nu$.
  - Eventually, K/L will grow above the limit $B/A \Rightarrow Y = BL$.
  - So $g_{Y/L} \to 0$ as in Solow.
- Sustained growth requires remaining on the capital-limited path forever.

Arrow's (1962) Learning-by-Doing

- Knowledge accumulates as a byproduct of investment
- Individual firms don't internalize spillovers creating an externality.
- At firm level: Diminishing returns to own capital

$$y_j = \bar{A} k_j^{\alpha} L_j^{1-\alpha}, \qquad \text{where } \bar{A} = A_0 \left(\sum_{j=1}^{N} k_j \right)^{\eta}$$

  - Firm takes aggregate capital stock as given
  - Chooses k_j facing diminishing returns ($\alpha < 1$)
  - Perfect competition maintained

- If spillovers strong enough $\alpha + \eta = 1$, we get constant returns in the aggregate,

$$Y = AK^{\alpha+\eta}$$

AK Dynamics

Assuming constant saving, capital accumulation is,

$$\dot{K} = sY - \delta K = sAK^{\alpha+\eta} - \delta K$$

and the growth rate of capital is,

$$g_K = \dot{K}/K = sAK^{\alpha+\eta-1} - \delta$$

Three Cases:

- $\alpha + \eta < 1$: diminishing returns dominate $\Rightarrow K^* = (sA/\delta)^{1/(1-\alpha-\eta)}$, $g \to 0$
- $\alpha + \eta = 1 \Rightarrow Y = AK$ and $g = \dot{K}/K$ (stable permanent growth).
- $\alpha + \eta > 1 \Rightarrow g_K$ rises with K (explosive growth).

Endogenizing Saving

- Endogenizing saving is straightforward here as in the Ramsey version of the neoclassical model (Romer, 1986).
    - Higher discount rate ρ or lower intertemporal elasticity of substitution $1/\varepsilon \Rightarrow$ lower steady-state growth rate g.
    - But, now decentralized growth remains below the social optimum because households and firms ignore how their own k raises the aggregate $\bar{A}$.
    - Investment subsidies to internalize the externality can raise growth permanently.

AK vs. Neoclassical: Key Differences

Can Policy Affect Growth?

- **Neoclassical**: Policies affect levels only
- **AK**: Policies can permanently affect growth rates

Convergence predictions:

- **Neoclassical**: Conditional convergence
- **AK**: Variation in α and ρ lead to permanent differences in growth rates. No convergence.

Empirical evidence:

- More evidence in favor of decreasing returns (neoclassical)
- Still have to grapple with the fact that growth appears to be sustained over time.

The AK Model: Key Lessons

- Simplest model of endogenous growth
    - Eliminates diminishing returns through spillovers
    - Sustains growth without exogenous technological progress
- Policies can permanently affect growth rates
    - Investment subsidies, taxes on capital
    - Anything affecting broad "investment climate"
- But: Technology still a black box
    - No explicit distinction between capital accumulation and technological progress
    - No explicit innovation decisions
    - No role for R&D, patents, competition