Semi-Endogenous Growth Theory

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Does population growth affect long-run growth?

Scale Effects Revisited

All endogenous growth models so far predict scale effects

The Prediction:

- More researchers $\Rightarrow$ faster innovation $\Rightarrow$ higher growth
- Larger countries should grow faster
- Population growth should accelerate productivity growth

The Evidence Against Scale Effects

Jones' Critique:

- U.S. R&D employment increased 9-fold since 1950
- R&D intensity (as % of GDP) increased substantially
- But: Productivity growth has been roughly constant (or declining)

Cross-Country Evidence:

- No robust relationship between country size and growth
- China and India don't grow faster than small countries
- OECD countries have similar long-run growth despite size differences

Leads to overprediction of growth effects.

Two Solutions

Solution 1: Young (1998) - Endogenous Product Variety

- More researchers, but also more products to improve
- Research effort spreads across expanding variety space
- Dilutes scale effects

Solution 2: Jones (1995) - Semi-Endogenous Growth

- Distinguish growth in *levels* from growth in *growth rates*
- Idea fishing: Each innovation harder than the last
- Requires more researchers just to maintain constant growth

The Semi-Endogenous Growth Model

Key Modification: Idea Fishing

Schumpeterian: $\dot{A}_t = R_t A_t^{\phi}$

- If $\phi = 1$: Standing on shoulders (knowledge spillovers)
- Leads to scale effects

Semi-Endogenous: $\dot{A}_t = R_t A_t^{\phi}$ with $\phi < 1$

- $\phi < 1$: "Fishing out" – each innovation harder than last
- As frontier advances, innovation becomes more difficult
- Offsetting effects: Knowledge helps but frontier harder to push

Key Insight:

- Need *more* researchers just to maintain constant growth
- Policy changes affect *levels*, not long-run growth rates

Population and Growth

Balanced Growth Path:

- Population growth $L_t = L_0 e^{nt}$,
- Research effort grows with population: $R_t = \bar{s} L_t$
- Technology growth: $\frac{\dot{A}_t}{A_t} = \bar{s} L_t A_t^{\beta}$, where $\beta \equiv 1 - \phi > 0$

Long-Run Growth Rate:

$$g = \frac{\sigma n}{\beta} = \gamma n$$

where $\gamma = \frac{\sigma}{\beta}$ is the degree of IRS.

- the more important ideas are to production ($\uparrow \sigma$), and the faster is population growth ($\uparrow n$), the higher the growth rate.
- the harder it is to find new ideas ($\uparrow \beta$), the lower the growth rate.
- Growth rate depends on population growth, not population level

Transitional Dynamics

What if R&D investment increases?

- **Short run**: Productivity grows faster (level effect kicks in)
- **Long run**: Growth rate returns to γn
- **Net effect**: Higher level of technology, same growth rate

Intuition:

- More R&D temporarily accelerates innovation
- But as frontier advances, innovation gets harder
- Eventually returns to balanced growth path
- Similar to Solow, but for technology not capital

Policy Implications

Semi-Endogenous vs. Fully Endogenous

	Fully Endogenous	**Semi-Endogenous**
R&D subsidy	$\uparrow$ growth rate	$\uparrow$ level only
Patent protection	$\uparrow$ growth rate	$\uparrow$ level only
Population size	Matters for g	Doesn't matter
Population growth	Matters for g	Matters for g

Key Difference:

- Fully endogenous: Policies have permanent growth effects
- Semi-endogenous: Policies have level effects

Are Level Effects Meaningful?

Suppose R&D subsidy raises technology level by 20%

- GDP permanently 20% higher
- Consumption permanently 20% higher
- Welfare gain is enormous even without growth effect.

Implications:

- Don't dismiss policies just because they don't affect long-run g
- Level effects may be more important than growth rate effects

Population and Development

Demographic Transition:

- Population growth varies across development stages
- Fertility transitions affect long-run growth
- Semi-endogenous model: $g = \gamma n$ directly links demographics to growth

Very Long-Run Questions:

- "How did population growth affect pre-industrial stagnation?"
- "Will aging populations reduce global growth?"
- "Does the demographic transition explain growth accelerations?"

Convergence and Divergence

Growth-Rate Convergence:

- Long-run $g = \gamma n$, so similar n delivers common growth rates
- Policy and R&D effort shift technology levels without changing steady-state g

Level Divergence:

- Different innovation effort or initial A_0 leaves lasting output gaps
- Slower population growth pushes economies onto permanently lower paths

Implications:

- Demography becomes the bottleneck
- Convergence requires closing level gaps before population growth stalls

Semi-Endogenous Growth: Key Lessons

- Scale effects problem: Fully endogenous models overpredict
    - More researchers but constant growth $\Rightarrow$ ideas getting harder
    - Need modification to fit long-run data
- Idea fishing: $\phi < 1$ eliminates scale effects
    - Population growth matters, population level doesn't
    - $g = \gamma n$ in long run
- Policies affect levels, not long-run growth rates
    - But level effects can be huge!
    - Don't dismiss policies without growth effects
- Key model for questions about population and very long-run growth

The Endogenous Growth Toolkit

You Now Have Four Models:

- **AK**: Simplest, for broad investment climate questions
- **Product Variety**: R&D incentives, market size, trade
- **Schumpeterian**: Creative destruction, firm turnover, frontier innovation
- **Semi-Endogenous**: Population, demographics, very long run

Key Skill: Match model to question

- What mechanisms matter?
- What can we abstract from?
- Start from the simplest model that captures key forces

Remember: Models are tools, not truths