# The Product-Variety Model

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How do we pay for innovation?

# From AK to Innovation-Based Growth

- AK models can deliver sustained growth through constant returns, but the mechanism is reduced-form capital deepening with no explicit innovation choice.

- Motivated a second-wave of endogenous growth models:

  - **Horizontal innovation**: new varieties expand the menu of inputs and raise productivity via specialization (Romer, 1990).

  - **Vertical innovation**: quality improvements replace old products and trigger creative destruction (Aghion & Howitt, 1992).

# Core Ingredients of Product Variety

- Creating a new product requires a sunk R&D outlay that yields a nonrival but excludable blueprint.
- This is added to the stock of knowledge.
- Monopololistic competition over products creates profits which create a reward for innovation.
- Expanding variety of specialized intermediate products leads to induced and sustained growth (A.A. Young, 1928).

# Simple Product-Variety Model

## Households and Preferences

- Fixed population $L$ of identical households supplying one unit of labor inelastically.

$$u(c_t) = \frac{c_t^{1-\varepsilon} - 1}{1 - \varepsilon}, \varepsilon > 0$$

- In steady state, the growth rate $g$ and interest rate $r$ must obey the Euler equation, leading to

$$g = \frac{r - \rho}{\varepsilon}$$

## Final-Good Production

- Final output is produced under perfect competitions, combining labor and intermediates:

$$Y_t = L^{1-\alpha} \int_0^{M_t} x_i^{\alpha} \, di, \qquad 0 < \alpha < 1.$$

  where $M_t$ is the measure of product varieties and $x_i$ is the usage of variety $i$.

- Labor supply is fixed, so expanding $M_t$ raises productivity by deepening specialization.

## Variety Raises Output

- In equilibrium, each variety is produced in the same amount, $x = X_t/M_t$.
- Substituting back into the production function yields

$$Y_t = L^{1-\alpha} M_t^{1-\alpha} X_t^{\alpha}.$$

  which is increasing in $M_t$, given the factor inputs $L$ and $X_t$.

- Final good is used for consumption, investment (producing blueprints), and producing intermediate products, so $\text{GDP}_t = Y_t - X_t$.

# Intermediate Production

- A blueprint grants monopoly power over variety $i$.
- Monopolist maximized profit,

$$\Pi_i = p_i x_i - x_i$$

- Demand inherited from the final-good firm:

$$p_i = \frac{\partial Y_t}{\partial x_i} = \alpha L^{1-\alpha} x_i^{\alpha-1}.$$

- Flow profit is therefore $\Pi_i = \alpha L^{1-\alpha} x_i^{\alpha} - x_i$.

## Equilibrium Quantity, Profits, and GDP

- Profit maximization delivers symmetric quantity

$$x^* = L\alpha^{\frac{2}{1-\alpha}}$$

- Equilibrium profit per variety is,

$$\Pi = \frac{1-\alpha}{\alpha} L\alpha^{\frac{2}{1-\alpha}}$$

- Final-good output and GDP are both proportional to the degree of product variety:

$$Y_t = M_t L^{1-\alpha} x^{\alpha}, \qquad \text{GDP}_t = M_t(L^{1-\alpha} x^{\alpha} - x)$$

- Thus, growth in GDP will be proportional to growth in variety, $M_t$.

$$g = \frac{1}{M_t} \frac{dM_t}{dt}$$

## Research

- How much product variety grows depends on the amount $R$ of final output that is used in research,

$$dM_t/dt = \lambda(R_t)$$

where $\lambda$ is the productivity of R&D.

- With free entry value of a blueprint is $\Pi/r$ (present value of perpetual monopoly profit discounted at market interest rate $r$) $\Rightarrow r = \lambda\Pi$

# Balanced Growth with Final-Good R&D

- Combining Euler and arbitrage conditions:

$$g = \frac{\lambda \Pi - \rho}{\varepsilon}.$$

- Higher $L$ or $\lambda$ raise $g$ through larger monopoly rents.

- Provides the scale-effect prediction we can confront with data.

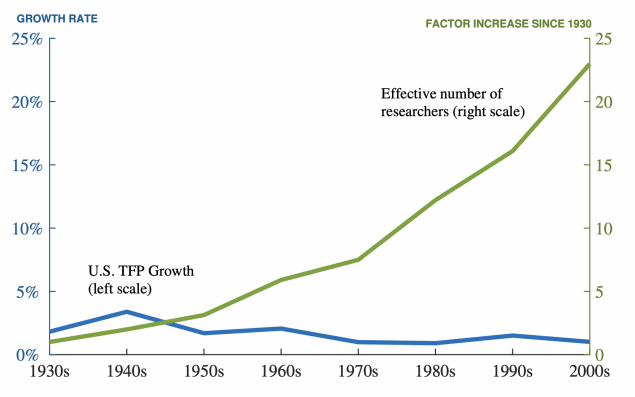
# Theory Meets Evidence

# Core Empirical Predictions

- **Scale effects**: larger $L$ or integration that expands market size should raise $g$.
- **R&D incentives**: monopoly rents motivate innovation; policies altering markups or R&D costs shift growth.
- **Exit dynamics**: varieties never obsolete $\Rightarrow$ exit reduces productivity.

# Evidence on Scale Effects

- Jones (1995) shows U.S. growth stayed near 2% while researchers increased sharply.



# Turnover and Creative Destruction

- Variety model prediction: exit is bad news because it reduces the number of varieties.

- In practice, entry *and* exit are tied to productivity growth:

    - Early evidence from Comin & Mulani (2007) and Fogel, Morck & Yeung, (2008).

## Convergence Predictions

- Balanced growth: $g = (\lambda\Pi - \rho)/\varepsilon$ depends on research productivity, discounting, and market size $L$.
- No diminishing returns $\Rightarrow$ no built-in transitional convergence; countries with larger markets or higher $\lambda$ sustain faster long-run growth.
- Initial differences in variety $M_0$ persist without knowledge diffusion, so income levels diverge even if parameters coincide.
- Trade integration matters: accessing world markets raises effective market size and R&D rents. With ideas shared globally, small economies match frontier $g$; without diffusion, gaps in $M_t$ and income levels persist.
- Partial openness generates convergence clubs: economies sharing markets and ideas converge together, while autarkic countries remain in a slower-growth group.

## Key Lessons

- Product variety links monopoly rents to innovation and explains why policy levers affecting R&D matter for growth.
- Offers clear predictions for market-size shocks, IP policy, and R&D subsidies.
- Can rationalize convergence and divergence patterns through trade and knowledge diffusion.
- But, overstates scale effects and cannot accommodate productivity-enhancing exit.
- To formalize obsolescence, we need to move away from horizontal models of product development into vertical models of quality improvements.