Testing and Sensitivity Analysis for Violations of Parallel Trends

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July 28, 2023

Outline

- What is the parallel trends assumption?
- Why might we be initally skeptical of the parallel trends assumption?
- How can we partially test the validity of parallel trends using pre-treatment info?
- Limitations of pre-trends tests
- Alternative approaches when worried about violations of parallel trends

Set-up

For simplicity, consider the canonical two-period DiD model:

- There are two periods, $t = 0, 1$
- We observe panel data with outcomes Y_{it} for individual i in period t
- Units with $D_i = 1$ are treated beginning period 1; units with $D_i = 0$ are never treated
- Potential outcomes: observed outcome is $Y_{it} = D_i Y_{it}(1) + (1 - D_i) Y_{it}(0)$
- Assume "no anticipation": $Y_{i0}(0) = Y_{i0}(1)$

What is parallel trends?

- The **parallel trends** assumption states that if the treatment hadn't occurred, average outcomes for the treatment and control groups would have evolved in parallel

$$\underbrace{E[Y_{i1}(0) - Y_{i0}(0) \mid D_i = 1]}_{\text{Counterfactual change for treated group}} = \underbrace{E[Y_{i1}(0) - Y_{i0}(0) \mid D_i = 0]}_{\text{Change for untreated group}}$$

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- The parallel trends assumption can also be viewed as a **selection bias stability** assumption:

$$\underbrace{E[Y_{i1}(0) \mid D_i = 1] - E[Y_{i1}(0) \mid D_i = 0]}_{\text{Selection bias in period 1}} = \underbrace{E[Y_{i0}(0) \mid D_i = 1] - E[Y_{i0}(0) \mid D_i = 0]}_{\text{Selection bias in period 0}}$$

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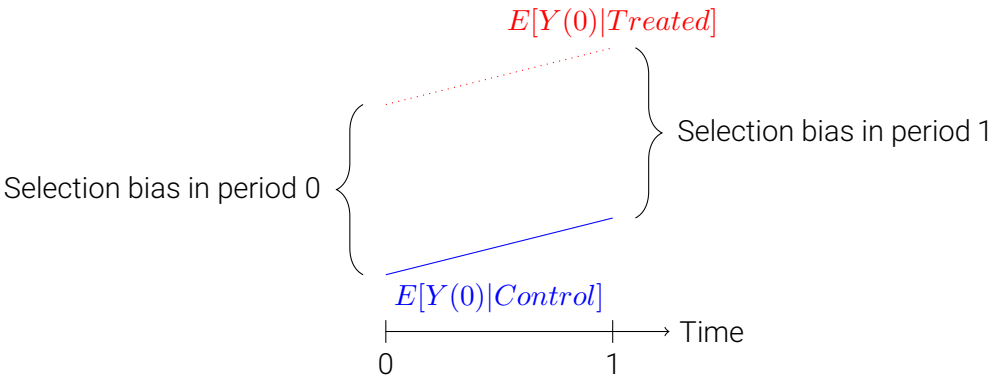
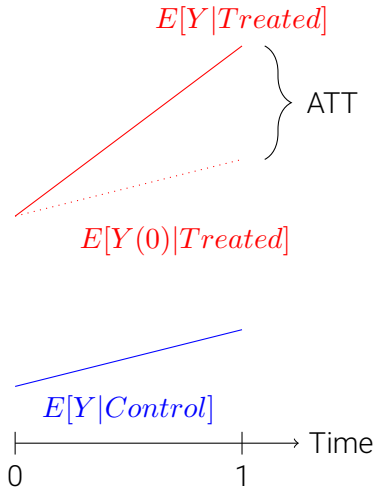
- PT allows for there to be selection bias!
  However, the selection bias has to be the same in both periods

Visualizing PT



Why is PT Useful? It allows us to identify the ATT!



$$\underbrace{E[Y_{i1}(1) - Y_{i1}(0)|D_i = 1]}_{ATT} = (\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00})$$

for $\mu_{td} = E[Y_{it}|D_i = 1]$.

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- The same confounding factors may have different effects on the outcome in different time periods
  - → Suppose people who enroll in a job training program are more motivated to find a job
  - → Motivation might matter more in a bad economy than in a good economy

Why might we be skeptical of PT? Part 2

- Another reason to be skeptical of parallel trends is that its validity will often be **functional form** dependent
- Consider an example:
  - → In period 0, all control units have outcome 10; all treated units have outcome 5.
  - → In period 1, all control units have outcome 15.
  - → If treatment hadn't occurred, would treated units' outcome have increased by 5 also (PT in levels)?
  - → Or would they have increased by 50% ($\sim$ PT in logs)?

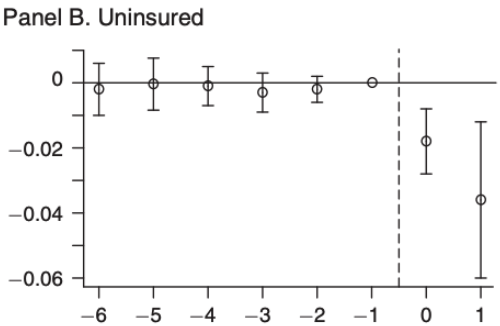
Roth and Sant'Anna (2023) show that PT will depend on functional form unless:

- **Randomization:** treated and control group have same dist. of $Y(0)$ in each period
- **No time effects:** distribution of $Y(0)$ doesn't change over time for either group
- **A hybrid**: θ fraction of the population is as good as randomized; the other $1-\theta$ fraction has no time effects.

Absent these conditions, PT will be violated for at least some functional form; often hard to know if we chose the right one!

Pre-trends to the rescue...

- Luckily, in most DiD applications we have several periods before anyone was treated
- We can test whether the groups were moving in parallel prior to the treatment
  - → If so, then assumption that confounding factors are stable seems more plausible
  - → If not, then it's relatively implausible that would have magically started moving in parallel after treatment date
- Testing for pre-trends provides a natural plausibility check on the parallel trends assumption



- Carey, Miller, and Wherry (2020) do a DiD comparing states who expanded Medicaid in 2014 to states that didn't.

- Report results from "event-study" regression:

$$Y_{its} = \phi_t + \lambda_s + \sum_{r \neq -1} D_i \times 1[t = 2014 + r] \cdot \beta_r + \epsilon_{it}$$

where Y_{its} is insurance for person i in year t in state s, and $D_i = 1$ if in an expansion state.

- Testing for pre-existing trends is a very natural way to assess the plausibility of the PT assumption
- But it also has several *limitations*, highlighted in recent work (Freyaldenhoven et al., 2019; Kahn-Lang and Lang, 2020; Bilinski and Hatfield, 2018b; Roth, 2022)
- Remainder of the talk today will focus on these issues, as well as some solutions.
- Perhaps selfishly, will focus mainly on two of my papers
  - → Roth (2022 AER:I, "Pre-test with Caution: Event-study Estimates After Testing for Parallel Trends")
  - → Rambachan and Roth (Forthcoming RESTUD, "A More Credible Approach to Parallel Trends")

Overview of Limitations

- Parallel pre-trends doesn't necessarily imply parallel (counterfactual) post-treatment trends
  - → If other policies change at the same time as the one of interest — e.g. min wage and UI reform together — can produce parallel pre-trends but non-parallel post-trends
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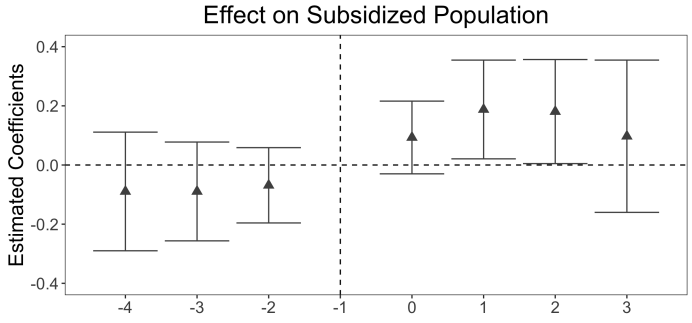
- **Pre-testing issues:** if we only analyze cases without statistically significant pre-trends, this introduces a form of selection bias (which can make things worse)
- If we fail the pre-test, what next? May still want to write a paper (especially if violation is "small")

Issue 1 - Low Power



- He & Wang (2017) study impacts of placing college grads as village officials in China
- Use an "event-study" approach comparing treated and untreated villages

$$Y_{it} = \sum_{k \neq -1} D_{it}^{k} \beta_k + \alpha_i + \phi_t + \epsilon_{it}$$

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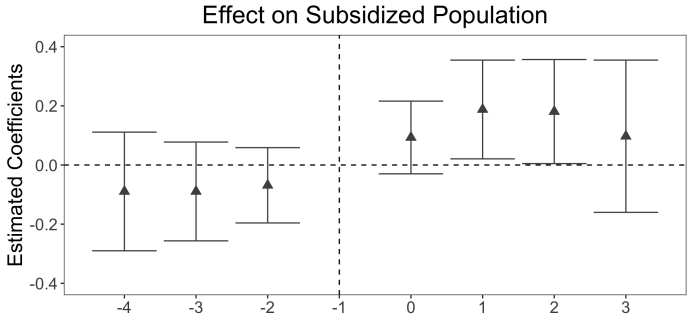

"The estimated coefficients on the leads of treatment ... are statistically indifferent from 0. ... We conclude that the pretreatment trends in the outcomes in both groups of villages are similar, and villages without CGVOs can serve as a suitable control group for villages with CGVOs in the treatment period." (He and Wang, 2017)

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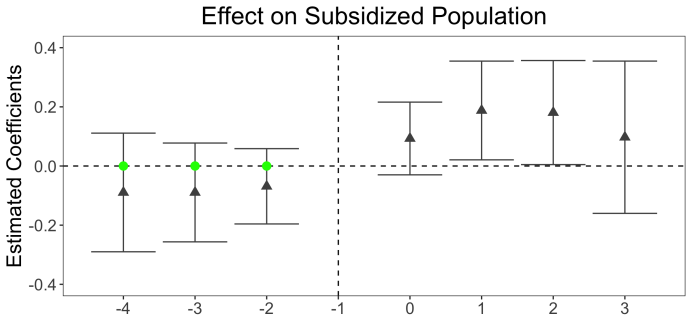

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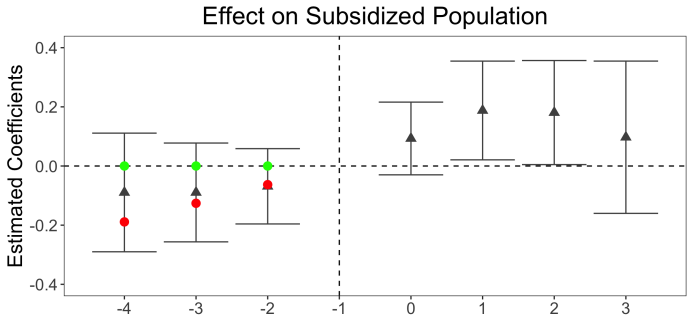

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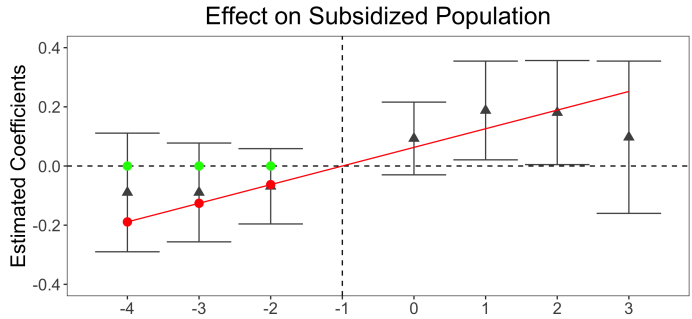

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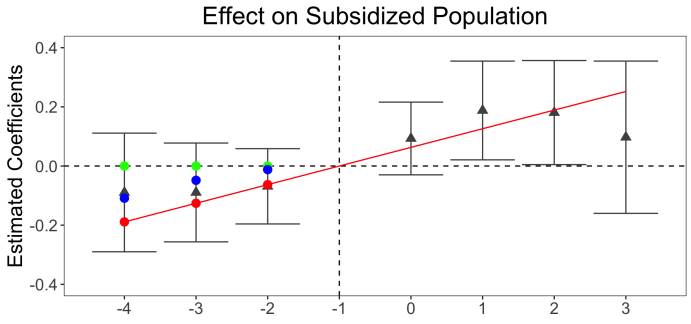

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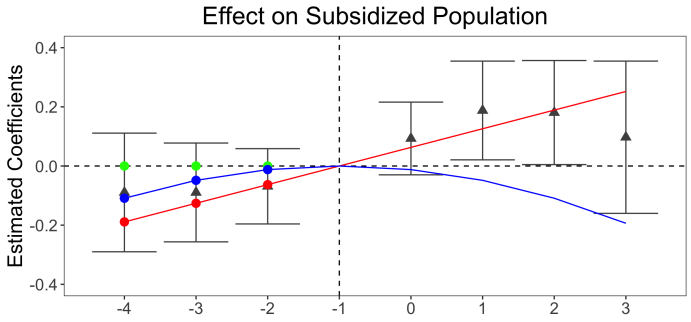

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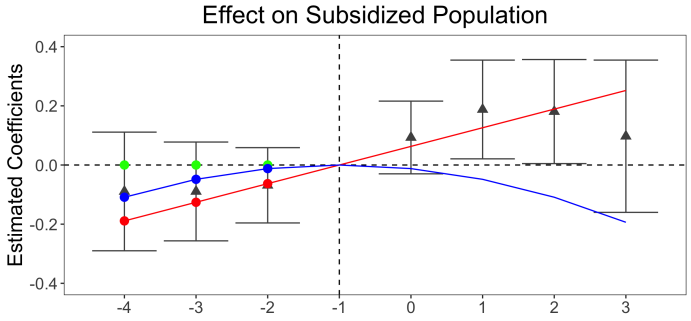

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- We can't reject zero pre-trend, but we also can't reject pre-trends that under smooth extrapolations to the post-treatment period would produce substantial bias

More systematic evidence

- Roth (2022): simulations calibrated to papers published in *AER*, *AEJ: Applied*, and *AEJ: Policy* between 2014 and mid-2018
  - → 70 total papers contain an event-study plot; focus on 12 w/available data

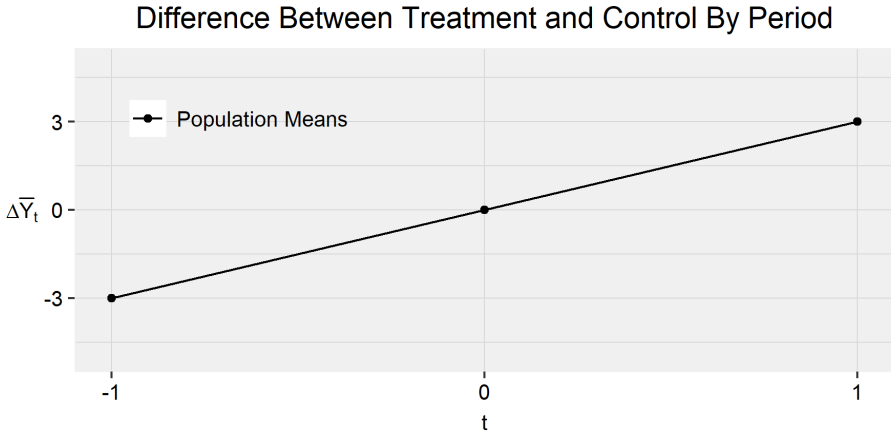
- Evaluate properties of standard estimates/CIs under linear violations of parallel trends against which conventional tests have limited power (50 or 80%):
  1. Bias often of magnitude similar to estimated treatment effect
  2. Confidence intervals substantially undercover in many cases
  3. Distortions from pre-testing can further exacerbate these issues

Issue 2 - Distortions from Pre-testing

- When parallel trends is violated, we will sometimes fail to find a significant pre-trend
- But the draws of data where this happens are a **selected sample**. This is known as *pre-test bias*.
- Analyzing this selected sample introduces additional statistical issues, and can make things worse!

Stylized Three-Period DiD Example

- Consider a 3-period model ($t = -1, 0, 1$) where treatment occurs in last period
- No causal effect of treatment: $Y_{it}(0) = Y_{it}(1)$ in all periods
- In population, treatment group is on a linear trend relative to the control group with slope δ
  - → Control group mean in period t: $E[Y_{it}(0) \mid \text{Control group}] = 0$
  - → Treatment group mean in period t: $E[Y_{it}(0) \mid \text{Treated group}] = \delta \cdot t$
- Simulate from this model with Y_{it} equal to the group mean plus independent normal errors



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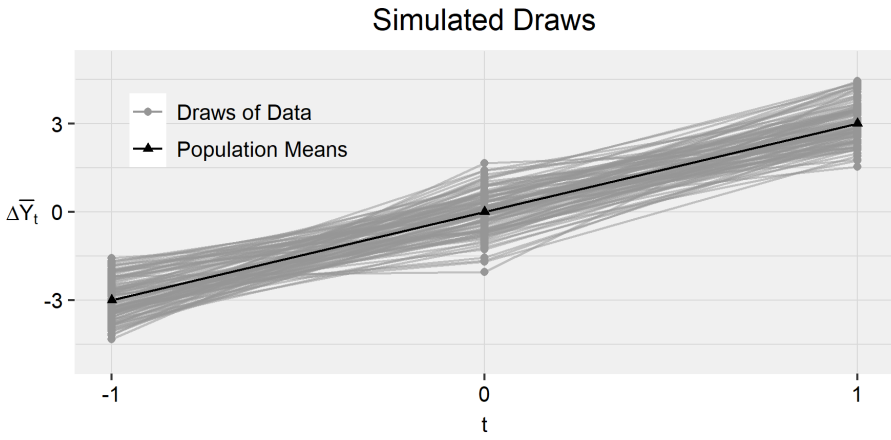

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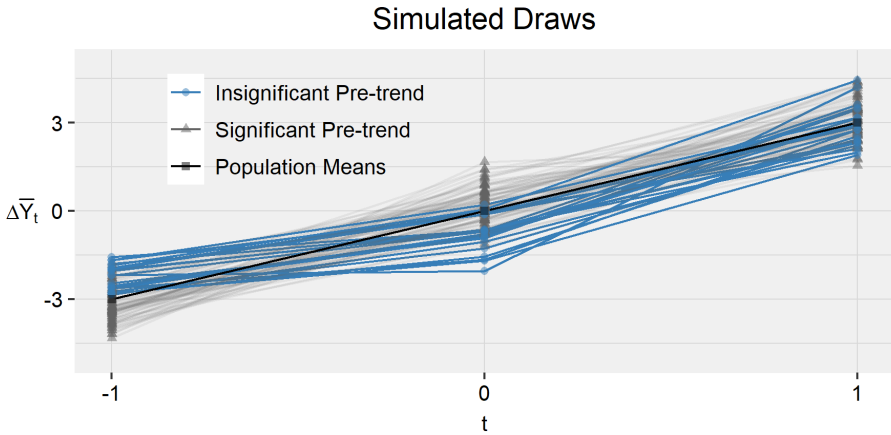
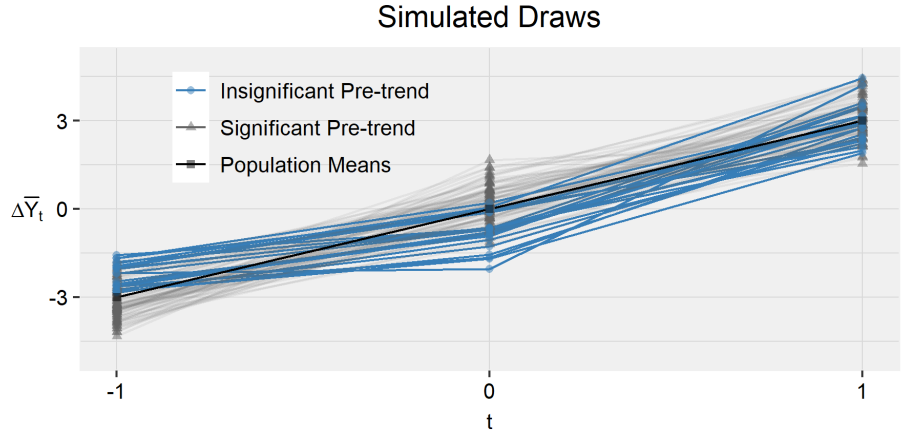
- In actual draws of data, there will be noise around this line



- Example: In population, there is a linear difference in trend with slope 3
- In some of the draws of the data, highlighted in blue, the difference between period -1 and 0 will be insignificant



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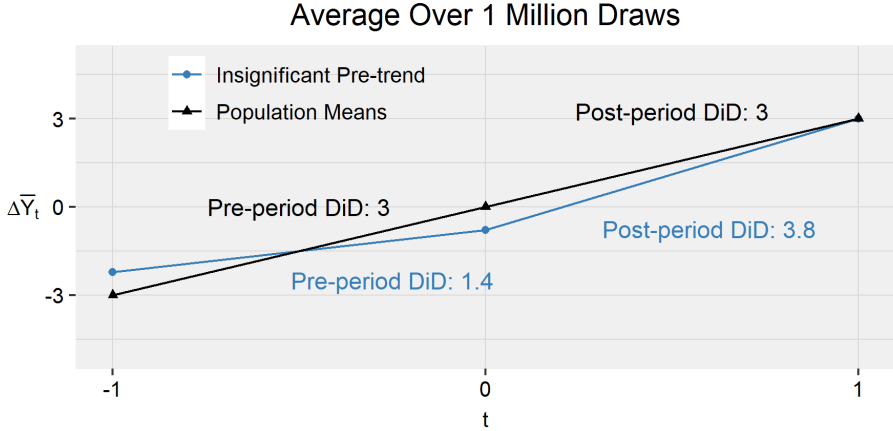
- In the insignificant draws, we tend to underestimate the difference between treatment and control at $t = 0$



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- As a result, the DiD between period 0 and 1 tends to be particularly large when we get an insignificant pre-trend

To Summarize

What are the Statistical Limitations of Pre-trends Testing?

1. Low Power – May not find significant pre-trend even if PT is violated
2. Pre-testing Issues – Selection bias from only analyzing cases with insignificant pre-trend
3. If reject pre-trends test, what comes next?

To Summarize

What are the Statistical Limitations of Pre-trends Testing?

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What Can We Do About It?

1. Diagnostics of power and distortions from pre-testing (Roth, 2022, "Pre-Test with Caution..."). See `pretrends` package. Details
2. Formal sensitivity analysis that avoids pre-testing (Rambachan and Roth, Forthcoming, "A More Credible Approach..."). See `HonestDiD` package.

“A More Credible Approach to Parallel Trends”

- The intuition motivating pre-trends testing is that the pre-trends are informative about counterfactual post-treatment trends
- Formalize this by imposing the restriction that the counterfactual difference in trends can’t be “too different” than the pre-trend
- This allows us to bound the treatment effect and obtain uniformly valid (“honest”) confidence sets under the imposed restrictions
- Enables **sensitivity analysis:** How different would the counterfactual trend have to be from the pre-trends to negate a conclusion (e.g. a positive effect)?

Restrictions on Violations of PT

- Consider the 3-period model ($t = -1, 0, 1$) where treatment occurs in last period
- Let δ_1 be the violation of PT:

$$\delta_1 = \mathbb{E}\left[Y_{i,t=1}(0) - Y_{i,t=0}(0) \mid D_i = 1\right] - \mathbb{E}\left[Y_{i,t=1}(0) - Y_{i,t=0}(0) \mid D_i = 0\right]$$

- We don't directly identify δ_1, but we do identify its pre-treatment analog, δ_{-1}:

$$\delta_{-1} = \mathbb{E}\left[Y_{i,t=-1}(0) - Y_{i,t=0}(0) \mid D_i = 1\right] - \mathbb{E}\left[Y_{i,t=-1}(0) - Y_{i,t=0}(0) \mid D_i = 0\right]$$

- Key idea: restrict possible values of δ_1 given δ_{-1}
  Intuitively, counterfactual trend can't be too different from pre-trend

Examples of Restrictions on δ

- **Bounds on relative magnitudes:** Require that $|\delta_1| \leq \bar{M}|\delta_{-1}|$

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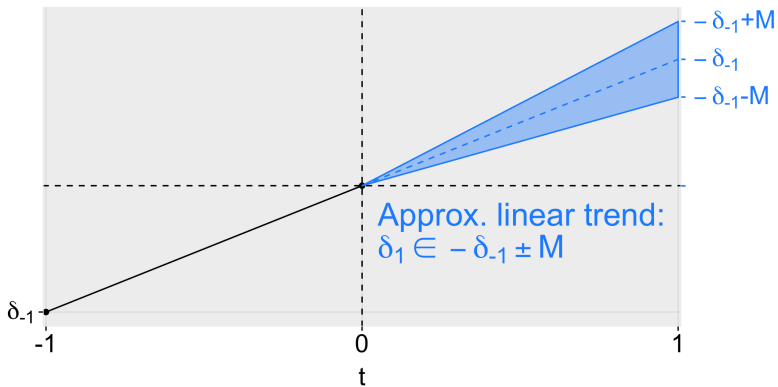
- **Bounds on relative magnitudes:** Require that $|\delta_1| \leq \bar{M}|\delta_{-1}|$
- **Smoothness restriction:** Bound how far δ_1 can deviate from a linear extrapolation of the pre-trend: $\delta_1 \in [-\delta_{-1} - M, -\delta_1 + M]$



Robust confidence intervals

- In the paper, we develop confidence intervals for the treatment effect of interest under the assumptions on δ discussed above
- The CIs account for the fact that we don't observe the true (population) pre-trend δ_{pre}, only our estimate $\widehat{\beta}_{pre}$.
- The robust CIs tend to be wider the larger are the confidence intervals on the pre-trends — intuitive, since if we know less about the pre-trends, we should have more uncertainty
- This contrasts with pre-trends tests, where you're less likely to reject the null that $\beta_{pre} = 0$ when the SEs are larger!

Benzarti & Carloni (2019)

- BC study the incidence of a cut in the value-added tax on sit-down restaurants in France. France reduced the VAT on restaurants from 19.6 to 5.5 percent in July of 2009.

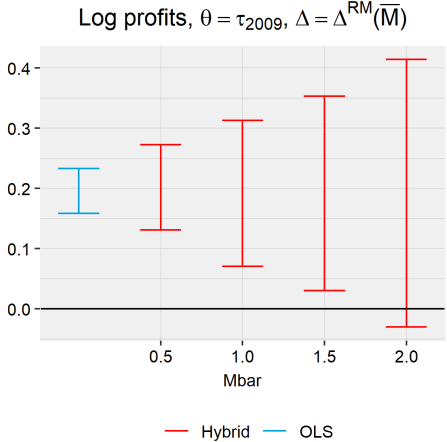
- BC analyze the impact of this change using a difference-in-differences design comparing restaurants to a control group of other market services firms

$$Y_{irt} = \sum_{s=2004}^{2012} \beta_s \times 1[t = s] \times D_{ir} + \phi_i + \lambda_t + \epsilon_{irt}, \quad (1)$$

  - → Y_{irt} = outcome of interest for firm i in region r
  - → D_{ir} = indicator if firm i in region r is a restaurant
  - → Φ_i, λ_t = firm and year FEs

- Outcomes of interest include firm profits, prices, wage bill & employment. We focus on impact on profits in first year after reform.

Event-study coefficients for log profits



- "Breakdown" $\bar{M}$ for null effect using relative magnitudes bounds is ~ 2
- Can rule out a null effect unless allow for violations of PT 2x larger than the max in pre-period

Extension to other estimators

- So far we focused on sensitivity analysis for "simple" event-studies
- However, the key idea was that we were willing to restrict the bias of some post-treatment event-study estimates $\widehat{\beta}_{post}$ using the expectation of some pre-treatment event-study estimates $\widehat{\beta}_{pre}$
- This idea extends to any asymptotically normal event-study estimates $(\widehat{\beta}_{pre}, \widehat{\beta}_{post})$!
- This includes:
  - → Event-studies for settings with staggered treatment timing (e.g. Callaway and Sant'Anna, 2021; Sun and Abraham, 2021)
  - → IV event-studies (Hudson et al., 2017)
  - → Flexible methods for controlling for covariates (Sant'Anna and Zhao, 2020)

So to summarize!

- Tests of pre-trends are intuitive but not a panacea!
- In particular, they may suffer from low power and introduce pre-test bias
- Roth (2022) and Rambachan and Roth (Forthcoming) provide tools for diagnostics and sensitivity analysis
- And these tools play nicely with recent estimators developed for heterogeneous treatment effects; see the bonus question in the coding exercise for an example!

Other Related Papers

- Other bounding exercises (Manski and Pepper, 2018; Ye et al., 2021)
- Non-inferiority approaches to pre-testing (Bilinski and Hatfield, 2018a; Dette and Schumann, 2020)
- Impose structure on the confounds (Freyaldenhoven et al., 2019)

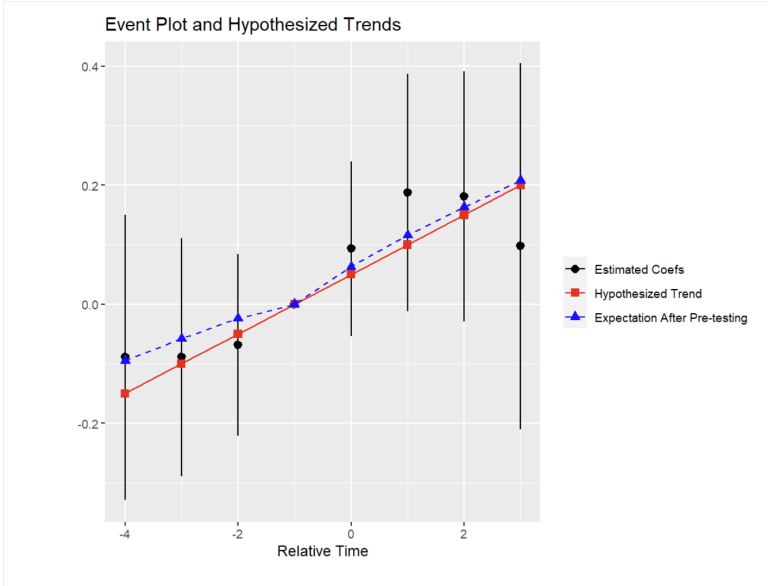
Thank you!

Additional Resources

- Roth (2022 AER:I, “Pre-test with Caution: Event-study Estimates After Testing for Parallel Trends”
  → Paper; `staggered` package ; Shiny app
- Rambachan and Roth (2023 RESTUD), “A More Credible Approach to Parallel Trends”
  → Paper; `HonestDiD` package ; Vignette

Pre-testing Diagnostics

- A “low-touch” intervention is to evaluate the likely power/distortions from pre-testing under *context-relevant* violations of parallel trends
- Enter the `pretrends` package / Shiny app


Event Plot and Hypothesized Trends
0.4
0.2
0.0
-0.2
-4
-2
0
2
Relative Time
Estimated Coefs
Hypothesized Trend
Expectation After Pre-testing

Power	Bayes.Factor	Likelihood.Ratio
0.33	0.76	1.23

- **Power.** Chance find significant pre-trend under hypothesized trend.
- **Bayes Factor.** Relative chance you pass the pre-test under hypothesized trend versus under parallel trends.
- **Likelihood Ratio.** Likelihood of observed pre-trend coefs under hypothesized trend versus under parallel trends.

Pros and Cons

Pros

- Very intuitive, easy to visualize.
- Helps identify when pre-testing may be least effective
- Requires minimal changes from standard practice

Cons

- Power will always be < 1, so no guarantee of unbiasedness/correct inference
- Need to specify the hypothesized trend. Will sometimes be difficult to summarize over many of these.
- Still not clear what to do when reject the pre-test.

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