Public Financing with Financial Frictions and Underground Economy*

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Abstract

What are the aggregate effects of reducing informality in a financially constrained economy? This paper answers this question by developing and calibrating an entrepreneurship model to data on matched employer-employee from both formal and informal sectors in Brazil. The model distinguishes between informality on the business side (extensive margin) and the informal hiring by formal firms (intensive margin). We find that when informality is eliminated along both margins, aggregate output increases by 7.2%, capital by 13.7%, and TFP by 3.5%. The output and TFP increases would be a factor of 1.4 and 1.9 larger if informality were only eliminated on the extensive margin, a result that supports the view that, in an economy with financial frictions, the informal economy can play a positive role by diminishing the negative effects of costly regulations and institutions on the economy. Finally, we find dramatic differences in the cost of financing social security in our baseline model economy relative to an economy with no frictions.

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1 Introduction

Large informal economies and underdeveloped financial markets are distinguishing features of most developing countries.[1] In this paper, we develop a quantitative theory (and calibrate it to Brazilian micro data) to assess how informality affects capital accumulation, occupational choices, and resource allocation in an economy *with* financial frictions. Moreover, we assess how informality and financial frictions affect the ability of the government to raise taxes and, in particular, the costs of financing a pay-as-you-go social security system.

In our framework, informality acts as a size dependent policy by allowing unproductive entrepreneurs to avoid taxation when using little capital and labor. Financial frictions reduce the scale of operation of high productivity entrepreneurs that lack sufficient resources to operate at their optimal scale. The effects of informality and financial frictions, on the one hand, reinforce each other in creating a competitive advantage for low productivity entrepreneurs, distorting occupational choice and the allocation of capital and labor across entrepreneurs. On the other hand, informality allows financially constrained entrepreneurs to operate at lower costs, speed up the accumulation of capital, and relax borrowing constraints. But the benefits of informality may come at a cost if entrepreneurs in the informal economy are subject to tighter borrowing constraints. In sum, whether the interaction between financial frictions and informality improves or worsens resource allocation in the economy is a quantitative question.

Central to our quantitative findings is the distinction between two margins of informality that we borrowed from Ulyssea (2018): (i) the *extensive margin* represents the entrepreneurial decision of whether to register the business to operate formally or to avoid paying taxes and regulation costs by operating the business in the underground economy; (ii) the *intensive margin* corresponds to the extent to which entrepreneurs, who have registered their business and attain formal status, hire some workers "off the books" to avoid fully complying with their mandatory contributions to the social security system. While the informality literature has focused on the extensive margin alone, the intensive margin of informality is empirically relevant as most informal workers in Brazil are hired by formal businesses. Moreover, we find that the effects of informality on capital accumulation and resource allocation critically depend on financial frictions and that the effects caused by the interaction between informality and financial frictions vary substantially depending on the

[1] For instance, in Brazil, around 70% of businesses and 35% of workers are informal. Similarly, in Mexico, around 60% of workers are in the informal sector. Both countries are characterized by a low financial development when compared to advanced economies. In Brazil, domestic credit to private sector GDP is around 66%, while in Mexico is around 32%. In comparison, the US domestic credit to private sector GDP is 188%. Data from World Bank development Indicators, 2015.

relative importance of the two margins of informality.

Our analysis proceeds in three steps. In the first part of the paper we use matched-employee data to document the key facts on informality in Brazil. Following Ulyssea (2018) we document that informality in Brazil is pervasive (both along the intensive and extensive margin): About 30% of businesses and workers are informal and about 70% of informal paid workers work in formal firms. We provide new evidence on the large differences in capital, investment, debt, and value added between formal and informal entrepreneurs. Conditional on the size of the establishment and industry, the value added in formal businesses is a factor of 2.3 the one of informal businesses. Differences in capital and debt are a factor of 5 and 6. In a second step, we build a theory of occupational choice, financial frictions and informality along the intensive and extensive margin. The government collect social security and sales taxes, and finance a pay-as-you go social security system. The model is calibrated to match Brazilian data on she shares of formal businesses, informal paid workers, and of informal paid workers hired by formal businesses. Moreover, the calibration targets moments on the size distribution of formal and informal businesses, the relative differences in value added, debt and capital intensities across businesses in the formal and informal sector.

In a third step, we use the model to assess the effects of informality in Brazil and to evaluate the interactions between informality and financial frictions. We find that when informality is eliminated along both margins, aggregate output increases by 7.2%, capital by 13.7%, and TFP by 3.5%. The output and TFP increases would be a factor of 1.4 and 1.9 larger if informality were only eliminated on the extensive margin (e.g. an economy with no informal businesses but in which formal entrepreneurs can hire some workers off the books). This result supports the view that, in an economy with financial frictions, the informal economy can play a positive role by diminishing the negative effects of costly regulations and institutions on the economy.

We find that the entry costs in the calibrated model economy are small (about 10% of the wage rate) but nonetheless have important effects on output (5%), capital (5%) and TFP (3.2%). While financial frictions and entry costs play an important role in accounting for the large amount of informal businesses in Brazil, reducing informal paid labor should also involve policies that confront informality on the intensive margin (such as better monitoring or reducing payroll taxes). Finally, we find dramatic differences in the cost of financing social security in our baseline model economy relative to an economy with no frictions (no informality and no financial frictions). While the elimination of the social security system in our baseline economy would lead to an increase in output of 17% *together* with an increase in government tax revenue of 9%, in the absence of frictions the increase in output would be a half (about 9%) and would be associated to a *decrease* in government tax revenue of 19%.

Overall, our results point to the importance of the interaction between financial frictions and informality on both margins for a complete and unbiased assessment of how changes in policies and institutions impact on macroeconomic variables.

Literature. We contribute to different strands of the literature. Broadly, we are connected to the literature that studies aggregate consequences of informality.[2] In recent work, Ulyssea (2018) uses a model of heterogeneous firms to evaluate the result of different formalization policies on output, TFP and welfare. His main contribution is to consider informal hiring by formal firms, the "intensive margin" of informality. Incorporating the intensive margin into the model produces new insights: policies that decrease firms' informality might not decrease labor informality, and lower informality may not be associated to welfare gains. By incorporating financial frictions and an occupational choice, we deliver additional insights based on the incentives to self-finance and the different margins of informality. Other works have used different approaches to study informality. Meghir et al. (2015) analyze the firm productivity distribution through the lens of a wage-posting model. In the equilibrium model studied by de Paula and Scheinkman (2010), the incentives produced by value-added taxes increase informality across the supply chain. Prado (2011) uses cross-country data to calibrate a static industry model with tax, enforcement and entry cost.

Moreover, our paper relates to the large literature that investigates how the misallocation of resources across heteregeneos produces can account for the large cross-country income differences in the data.[3] In particular our paper relates to a large literature assessing the role of financial frictions in models of entrepreneurship (Midrigan and Xu (2014), Buera et al. (2011) and Moll (2014), Erosa (2001) and Allub and Erosa (2019)). We were not the first to study the relationship between financial development and informality. In Ordóñez (2014), the probability of detection is an indicator function that depends on the capital hired by the entrepreneur. This distorts the capital decision of informal firms but not formal firms. D'Erasmo and Moscoso Boedo (2012) explicitly model firms' bankruptcy procedures in equilibrium with the credit market. Antunes and Cavalcanti (2007) uses a static occupational choice model where formal firms have (imperfectly) access to finance.[4] None of these papers account for the large number of informal workers employed at formal firms.

There is a large literature analyzing the effects of tax evasion on public finances. Although the literature spans over theoretical and empirical approaches (Slemrod and Yitzhaki (2002),

[2]For a survey on the current state of the literature, see Ulyssea (2020).

[3]See Restuccia and Rogerson (2008), Guner et al. (2008) and García-Santana and Pijoan-Mas (2014). For a recent survey see Restuccia and Rogerson (2017).

[4]In their paper, entrepreneurial talent has no persistence and is independently drawn every period, hence they abstract for the self-finance motive.

Slemrod (2019)), the work on aggregate effects is somewhat limited. A notable exception is Di Nola et al. (2018), who uses an occupational choice model, in which entrepreneurs can misreport part of their income, to study distributional welfare. They focus on personal income tax evasion, while our work differentiate between payroll and sales taxes. This gives us the opportunity to assess the effect of distinct tax policies.

Finally, there is a large literature studying the effects of soical security on capital accumulation and labor supply (see, for instance, Attanasio et al. (2007), Imrohorolu et al. (1995), Conesa and Krueger (1999), Fuster (1999), and Fuster et al. (2007)). To the best of our knowledge, this literature abstracts from how the financing of social security affects resource allocation across heterogeneous entrepreneurs. Mckiernan (2019) models social security in the presence of an informal sector but her focuses on the worker's labor supply decision, while ours concentrate on occupational choice and resource allocation across entrepreneurs.

2 Empirical Evidence

This section discusses the empirical evidence on the main stylized facts regarding firms, informality and financial frictions. To carry on our empirical analysis, we make use of several Brazilian data sets. The main data comes from the ECINF (*Pesquisa de Economia Informal Urbana*), a cross sectional survey of non-agricultural businesses. The survey is nationally representative for small urban businesses (up to 5 employees) and it was conducted by the Brazilian Bureau of Statistics in 1997 and 2003. The data cover detailed information on the business characteristics (revenue, capital, credit), and workers characteristics - including the owner and non-paid labor. Because of its structure, it provides an unique opportunity to understand the relationship between productivity, credit and hiring decisions in informal production units.

Although ECINF gives a good representation of the characteristics of the informal businesses, where the average size is 1.15 and 97% of the businesses have two workers or less, the size cap of five workers is too small to provide a good representation of the true size distribution of the formal sector. Hence, we use multiple data sets to supplement the ECINF. The formal firm size distribution comes from RAIS, an administrative matched-employer employee that covers the universe of formal firms. Unfortunately, RAIS does not provide any information on informal firms nor informal workers. Therefore, we supplement it with two surveys: PNAD (*Pesquisa Nacional por Amostra de Domiclios*) and PME (*Pesquisa Mensal de Emprego*). PNAD is a national representative household survey and PME is a monthly rotational panel of workers that covers the six largest metropolitan areas in Brazil. Both provides valuable individual level information such as the total share of informal workers, the

share of entrepreneurs in the economy, and the share of informal workers for large business. To keep the data comparable, we look at data from 2003 and maintain the same sample selection whenever possible.[5] Our definition of informality is the usual: a firm is formal when it possesses a tax identification number, and a worker is formal when the labor contract is registered in her worker's booklet - a document that records all formal employment relationships and ensures that workers are entitled to receive all social security benefits.

2.1 Formal Firms and Informal Workers

Many empirical facts about the informal economy have been documented using micro data from a variety of countries. La Porta and Shleifer (2014) suggests that informal firms employ less workers, have lower value added per employee, and pay lower wages than their formal counterparts. Ulyssea (2018) confirms this evidence in Brazil, but adds that formal and informal firms coexists in narrowly defined industries and share a common support in the productivity distribution. Regarding worker characteristics, La Porta and Shleifer (2014) reports that managers of informal firms are, on average, less educated than the ones in formal firms. Yet, there are no clear differences between the human capital of the other employees. This is perhaps surprisingly, since a well known stylized fact is that informal workers are on average less educated than formal workers.[6] Table 1 confirms that, in Brazil, the share of informal firms decreases with firm size. While around 90% of the businesses with one worker are informal, the fraction for businesses of five workers is only 30%. Moreover, the size distribution of informal firms is highly concentrated, with 97% of all informal firms employing two workers or less (including the owner).[7]

Although the most used definition of informality lies on whether the business is formally registered with the tax authorities, recently, the literature has focused on formal firms that can be "partially" informal by hiring informal workers. The hiring of informal workers by formal firms, sometimes referred as the "intensive" margin of informality, potentially accounts for a large share of the informal employees. In Mexico, around 47% of all informal workers are employed in a formal firm (Samaniego de la Parra (2017)), while in Peru, 32% of the informal workers in manufacturing are located in a formal business (Cisneros-Acevedo (2019)). In the context of financial frictions, the intensive margin of informality helps pro-

[5]The sample is selected to be all privately owned firms, including own-account workers, in urban areas.

[6]However, both facts are fully consistent with each other when we consider that a large fraction of informal workers are located in formal firms, especially that informal workers in formal firms are on average low educated workers.

[7]We also show the cumulative distribution of the formal firms in ECINF. Nevertheless, the survey cover small businesses and is representative only for firms up to five employees. For the full distribution of formal firms see Appendix Table A.1.

Table 1: Share of Informal Firms and Informal Workers by Firm Size

Size	Share Inf. Firms	Share Inf. Workers in Formal Firms	Cum. Formal	Cum. Informal
1	0.930	-	0.441	0.898
2	0.657	0.476	0.695	0.972
3	0.449	0.463	0.828	0.988
4	0.344	0.373	0.904	0.994
5	0.296	0.262	0.958	0.998
6	0.311	0.317	0.987	1.000
7	0.069	0.165	0.998	1.000
All (≤ 7)	0.868	0.322		

Notes: Size includes paid employees plus business owners. Share of informal workers in formal firms includes paid employees only. Source: ECINF 2003.

ductive but constrained firms to speed up capital accumulation and grow larger without the size constraints imposed by being fully informal.

Since one needs to know the formality status of both the firm and the worker, knowing the exact extent of the intensive margin is challenging in many countries. Table 1 indicates that, in small Brazilian firms, 32.2% of the informal employment is in formal businesses. Furthermore, the gradient of the intensive margin of informality is decreasing in size. While, formal businesses with at least two workers hire almost 50% of workers informally, formal businesses with five workers hire only half of that. As argued by Ulyssea (2018), given that ECINF only covers small firms, the share of informal employment in formal firms in the economy is likely much larger than 32.2%. Table 2 presents the employment share by each pair of worker and firm formality status using the household survey PNAD. First, out of 22% of informal workers in 2012, almost 14% were employed in formal firms. This means that formal firms account for 62% of the total informal employment.[8] Second, similarly to ECINF, the employment share of informal workers decrease in larger firms. Yet, even in firms with more than 50 employees, 7.5% of the total employment is informal. A possible explanation for this fact is that hiring too many informal workers increases the probability of being detected, hence, the marginal worker in a large firm is likely to be formal.

[8]The formality status of the employer are asked only in the updated PNAD, which started rolling in 2012. Because the share of informal workers decreased 13 p.p. from 2003 to 2012 (see Appendix Table A.2), the number of informal workers in formal firms is presumably higher in 2003. In Appendix A.2, we argue that it can be as high as 75.9%.

Table 2: Employment Share by Worker and Firm Informality Status and Firm Size

Worker-Firm Status	≤ 5	≥ 6 and ≤ 10	≥ 11 and ≤ 50	≥ 51	All Firms
Formal Worker in Formal Firm	42.48	69.99	82.95	91.36	78.02
Informal Worker in Formal Firm	25.76	20.35	13.79	7.54	13.80
Informal Worker in Informal Firm	31.75	9.66	3.27	1.11	8.18
Total Employment Share	17.84	13.85	19.72	48.59	100.00

Notes: Employment share by worker and firm formality status and firm size. Urban paid employees in private firms only. Size is defined by the number of paid employees. Source: PNAD-C 2012.

2.2 Informality, Capital and Debt

In this section, we further explore the relationship between informality, capital and debt. On the one hand, in a world with financial frictions, informality can alleviate the burden of high taxes and allow financially constrained firms to operate. On the other hand, a registered business often has access to better credit conditions as banks may require some form of managerial supervision such as well-developed business plans or accounting books.[9] Using the World Bank Enterprise Surveys, La Porta and Shleifer (2014) shows that access to financing is the most important obstacle to do business for both formal and informal firms. Nevertheless, while 43.8% of informal businesses report financing as the most important issue, just 18.5% of formal businesses argue the same. ECINF directly asked the source of the loan to the entrepreneurs who asked for credit. While 73.6% of the formal firms used public or private banks instead of other loan sources such as friends and family, the same share for informal firms is only 53% (see Appendix Table A.3).

On top of the anecdotal evidence, Appendix Table A.4 displays summary statistics of our ECINF sample conditional on the characteristics of the entrepreneur. On average, formal firms have higher profits, revenues and costs than informal firms. Also, they employ almost five times more capital, hold six times more debt, and invest two times more. Aggregate debt to output (considering only small firms) is 43% in the formal sector, while just 31% in the informal sector. Obviously, a large fraction of these differences is accounted by the fact the formal firms are larger and possibly operate in different sectors than informal firms. Hence, to account for possible differences across sectors, Table 3 exhibits the partial correlations of debt, capital and investment with the formality status conditional on size, sector and value-added per worker.

After differences in the number of workers, sector of activity and value-added are taken

[9] In general, a registered entrepreneur has better loan conditions such as friendlier repayment structure, higher credit limits, and different default options.

Table 3: Partial Correlations of Debt, Capital and Investment with Formality Status

VARIABLES	(1) log(Debt)	(2) log(Capital)	(3) log(Investment)
Informal	-0.538***	-0.658***	-0.505***
	(0.0760)	(0.0500)	(0.0902)
log(VA p/ worker)	0.455***	0.789***	0.673***
	(0.0276)	(0.0164)	(0.0359)
Observations	7,856	32,797	7,696
R-squared	0.414	0.615	0.584
Size FE	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes
State FE	Yes	Yes	Yes

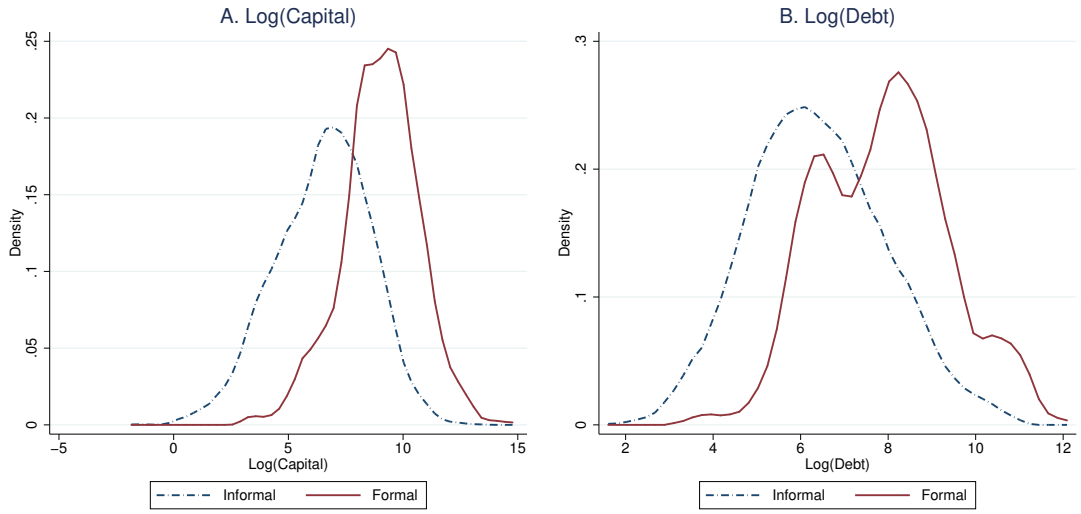
Notes: Size is define as number of paid workers plus business owners. Industry dummies are at 4-digit level. Only firms with positive values of debt, capital and investment are included. Robust standard errors in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. Source: ECINF 2003.

into account, an informal business still holds 53.8% less debt, 65.8% less capital, and invest 50.5% less than formal business. Although this can be seen as evidence that the informal sector faces stronger frictions in the financial market than the formal sector, we cannot argue that there is a direct causal relationship. In fact, one should expect some degree of selection across sector based on initial capital. For instance, an entrepreneur with low asset levels might not have enough economies of scale to operate in the formal sector, and instead, will decide to produce informally. In addition, there could be confounding factors that correlates with both the sector and the financing capacity.

However, even if there is some degree of sector selection based on assets, we argue that it does not account for the full history. Figure 1 shows the distribution of capital and debt in both formal and informal for entrepreneurs with less than one year of operation. The distribution of capital and debt displays a large common support across sectors, illustrating that entrepreneurs with similar asset levels may select into different sectors. One possible explanation is that entrepreneurs self-select based not only on assets but also on their expectation of business success. An entrepreneur who believes she has a successful and large business will select into the formal sector, as opposed to an entrepreneur who wants to operate on a small scale.

In sum, the informality decision interacts in a non-trivial fashion with the degree of financial friction in the economy. The size restriction and possibly different conditions in

Figure 1: Distribution of Capital and Debt



Notes: Smoothed densities of firms with less than one year old, and positive capital and debt by formal and informal. Log capital and debt are conditional on industry. Kernel function is Epanechnikov with bandwidth of 0.22. Source: ECINF 2003.

the financial markets of the informal sector are compensated by the absence of taxes and entry costs. Once we introduce potential hiring off the books by formal firms, and uncertainty regarding business success, the problem becomes even more complicated. Yet, these additional interactions create opportunities for different policy instruments. Informality can be targeted in either sector or in both, as well as financial frictions. Also, social security contributions and sales tax have different implications under different settings. Hence, any quantitative model should acknowledge: (i) plausible levels of informal hiring by formal businesses, (ii) possible heterogeneity in the degree of financial friction across sectors, and (iii) an overlapping distribution of capital and debt in both sectors.

3 Model

We study an economy characterized by a large number of informal firms and informal workers, frictions in the financial markets, and a social security system. The framework builds on Ulyssea (2018) and extends it in two fundamental dimensions. First, we model capital accumulation and financial frictions. Second, there is an occupational choice decision: households decide whether to work for the market wage or to become an entrepreneur in the formal (f) or in the informal sector (i).

These extensions are important for the focus of our paper. By modelling capital, we

can examine how informality - jointly with financial frictions - affects capital accumulation decisions and the allocation of capital across sectors and entrepreneurs. Furthermore, by including an endogenous occupational choice on top of the informality decision, the model allows us to understand how the entrepreneurship rate is affected by changes in the economic environment. Since most of the entrepreneurs at the margin are small and informal, the entrepreneurship decision is potentially responsive to policies targeting informality.

3.1 Environment and Preferences

Time is discrete and the economy is in a steady state competitive equilibrium. The economy is populated by a continuum of households that transit stochastically through two stages in their life: A working stage and a retirement stage. During the working stage, households make occupational choice decisions and are heterogeneous in their assets and in the productivity of their entrepreneurial idea. Every period, with probability π_z individuals keep the same business idea or with probability $1-\pi_z$ they draw a new idea from a fixed distribution Γ_z.

A working age individual faces a retirement shock every period with probability ρ_R. During the retirement stage, which lasts for T periods, individuals collect pensions, make consumption and savings decisions until they die with zero assets. When an individual dies, she is replaced by a newborn individual with zero assets and an initial idea drawn from Γ_z. The size of the population is normalized so that the mass of individuals in the working stage is 1.

3.2 Production Technology

Each period there is a unique output good y that can be consumed or invested. The output can be produced by establishments in the formal entrepreneurial sector (f), in the informal entrepreneurial sector (i), or in the corporate sector (c). An establishment with productivity z in sector $j \in \{c, f, i\}$ produces output according to the following production function:

$$y = zq_j(k, l), \tag{1}$$

where (z, k, l) represent the TFP, capital, and labor of the establishment. The function q_j, which is allowed to vary with the establishment sector, is twice differentiable, strictly increasing, and strictly concave.

Entrepreneurial businesses. Each entrepreneur owns a unique entrepreneurial business, whose productivity is determined by the quality of her entrepreneurial idea z. Entrepreneurs supply inelastically their own labor $\bar{l}$ to their businesses.[10] Following Moll (2014), Buera and Shin (2013), and Midrigan and Xu (2014), the capital used by an entrepreneur with a units of assets, in sector $j \in \{f, i\}$, is limited by the collateral constraint:

$$k \leq \lambda_j a, \qquad \lambda_j \geq 1 \text{ and } a \geq 0. \tag{2}$$

Intuitively, λ controls the degree of credit frictions faced by the entrepreneur, where the limiting case $\lambda = \infty$ corresponds to a perfect capital market, and $\lambda = 1$ corresponds to the situation where all capital has to be self-financed. The degree of credit friction is allowed to differ across entrepreneurs in the formal and informal sectors.

Informal entrepreneurs do not pay payroll taxes nor consumption taxes. Therefore, given factor prices, w and r, the profit function of an informal entrepreneur with assets a and entrepreneurial idea z is:

$$\begin{aligned} \pi_i(a, z; w, r) = \max_{k,l,l_i \geq \bar{l}} & zq_i(k, l) - (r + \delta)k - wl_i + (1 + r)a - c_i, \\ \text{s.t.} \quad & k \leq \lambda_i a, \\ & l = l_i + \bar{l}, \end{aligned} \tag{3}$$

where c_i is the fixed cost of operation in the informal sector and l_i the labor hired by the entrepreneur. Informal entrepreneurs cannot hire formal workers.

Formal entrepreneurs pay payroll and sales taxes and are subject to a fixed cost of operation. We allow formal entrepreneurs to hire informal workers and avoid part of their payroll taxes. As argued before, explicitly modelling the intensive margin of informality is important as it can alleviate credit frictions for formal firms and it is empirically relevant in developing economies. Hiring informal workers, however, is not free of cost. Firms are subject to inspections and may suffer fines for labor laws violation. Intuitively, the higher is the number of informal workers, the higher is the likelihood that the firm is caught and the monetary cost of the fine. Therefore, the cost of hiring informally, $\tau(l_i, l_f)$, is modelled as a convex and increasing function of the number of informal workers, $\partial\tau(\cdot)/\partial l_i > 0$, but possibly decreasing in the number of formal workers (consequently in the size of the firm), $\partial\tau(\cdot)/\partial l_f \leq 0$.[11] Profits of an entrepreneur with assets a and entrepreneurial idea z operating

[10] In the data, 89.8% of the informal entrepreneurs do not employ paid labor. In the next section, we set $\bar{l}$ to one.

[11] The convex cost function acts as a reduced form for the expected cost of being caught and receive a fine. It effectively imposes a limit on informal hiring.

in the formal sector are given by:

$$\begin{aligned}\pi_f(a,z;w,r) = \max_{k,l_i,l_f\geq 0} &(1-\tau_y)zq_f(k,l) - (r+\delta)k - w(l-\bar{l}) - \tau_{ss}wl_f \\ &- w\tau(l_i,l_f) + (1+r)a - c_f, \\ \text{s.t.} \quad & k \leq \lambda_f a, \\ & l = l_i + l_f \geq \bar{l},\end{aligned} \tag{4}$$

where l denotes total labor input (including entrepreneur's own labor), l_i and l_f are the number of informal and formal labor input, k is the capital input, τ_{ss} is the payroll tax used to finance social security, τ_y the sales tax, and c_f a fixed cost of operation incurred by formal entrepreneurs. As in Ulyssea (2018), formal and informal workers are assumed to be perfect substitutes in production. Since formal and informal employees perform the same tasks, there is no wage difference between the two type of workers so that total wage disbursements are given by $w(l-\bar{l})$.[12] Formal entrepreneurs choose the mix between formal and informal labor that minimize total labor costs. As the cost of hiring informal workers is convex, the share of informal labor decreases with firm size.

Corporate firms. The corporate sector is composed by a large number of establishments that are heterogeneous in their productivity and are owned by a representative mutual fund.[13] The distribution of productivities z_c across corporate establishments is described by a fixed distribution Γ_{z_c}. Corporations cannot engage with any informal activity but are not subject to financial frictions. They accumulate capital and are owned by a representative mutual fund that distributes dividends to households. The value of a corporate firm solves:

$$\begin{aligned}V_c(z_c) = \max_{\{k_t,l_t\}_{t=1}^{\infty}} &\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^t d_t, \\ \text{s.t.} \quad & x_t = k_{t+1} - (1-\delta)k_t, \\ & d_t = (1-\tau_y)z_c q_c(k_t,l_t) - wl_t - w\tau_{ss}l_t - c_c - x_t\end{aligned} \tag{5}$$

[12] Also, since we abstract from non-wage benefits perceived by formal workers, there is no compensating wage differential.

[13] Although the literature on entrepreneurship typically abstracts from the corporate sector, a handful number of papers include it in their models (for instance, Quadrini (2000) and De Nardi and Cagetti (2006)). The introduction of the nonentrepreneurial sector comes with two advantages. First, financial frictions depress the demand for capital and the interest rate. By modelling corporations, the equilibrium interest rate will be positive and bounded away from zero. Second, the entrepreneurial decision introduces non-convexities that may generate steps in the aggregate excess demand functions. The corporate sector mitigates this problem by introducing additional demand for capital and labor.

where c_c is the fixed cost of operation of corporate establishments and d_t stands for the dividends distributed. In steady state (constant prices and taxes), the value of a firm with productivity z_c is

$$V_c(z_c) = \frac{d^*(z_c)}{r} \tag{6}$$

where $(k^*(z_c), l^*(z_c))$ solves the problem in (5), given constant factor prices and tax policies, and d^* represents period dividends under the optimal production and investment plan. Note that d^* and $V_c(z_c)$ are increasing in z_c. Given the presence of a fixed cost of operation, there is a threshold value $\overline{z}_c$ such that the value of a firm is positive for all $z > \overline{z}_c$.

Let $\overline{M}$ be the mass of corporations. In every period, the aggregate dividends paid by the representative mutual fund are

$$D = \overline{M} \int_{\overline{z}_c}^{\infty} d^*(z_c) d\Gamma_{z_c}. \tag{7}$$

Finally, in equilibrium, the rate of return of investing in the mutual fund should be equal to the rate of return in deposits r. Denoting the price of one share of the mutual fund by P and normalizing the total number of shares to one, gives the following no arbitrage condition:

$$\frac{P + D}{P} = 1 + r \Rightarrow P = D/r. \tag{8}$$

3.3 Household Problem

We start with the problem of a household recently retired from the labor market. A newly retired household with initial assets a_0 and pension benefit b solves the following deterministic problem:

$$\begin{aligned} V_{ret}(a_0; b) = \max_{\{c_t, a_t\}_{t=1}^{T}} & \sum_{t=1}^{T} \beta^{t-1} u(c_t), \\ \text{s.t.} \quad & c_t + a_t = b + a_{t-1}(1 + r), \ a_0 \text{ given.} \end{aligned} \tag{9}$$

The state of a household in the working stage is given by her assets a, an entrepreneurial idea z, and her initial occupation (the occupational choice is a dynamic decision). The household chooses how much to consume, save, and the occupational choice they will start next period.

The entrepreneurship decision is costly and depends whether entry is into the formal or

informal sector. To enter in the formal (informal) sector a household must pay an entry cost c_e^f (c_e^i). The differential between the entry costs of the formal and informal sector, $c_e^f - c_e^i > 0$, captures the costs of registering and complying with the regulations necessarily to operate a formal business. Let $W_j(a,z)$ be the value of a worker with assets a that chooses to implement the business idea z in the sector $j = \{i, f\}$. This value satisfies the following equation:

$$\begin{aligned} W_j(a,z) = &\max_{c,a'} u(c) + \beta \left[(1-\rho_R)V_j(a',z) + \rho_R V_{ret}(a')\right], \\ &\text{s.t.} \quad c + a' + c_e^j = w + (1+r)a, \end{aligned} \tag{10}$$

where V_j represents the value of an entrepreneur in the sector j and V_{ret} is the value of retirement defined in (9). The value of a worker that chooses to remain a worker next period is given by

$$\begin{aligned} W_w(a,z) = &\max_{c,a'} u(c) + \beta \left[(1-\rho_R)\left(\pi_z W(a',z) + (1-\pi_z)\int W(a',z')d\Gamma_{z'}\right) + \rho_R V_{ret}(a')\right], \\ &\text{s.t.} \quad c + a' = w + (1+r)a. \end{aligned} \tag{11}$$

Note that in the next period the worker might get a new business idea with probability π_z. The value of a worker is the outer envelope over the value of the three occupational choices:

$$W(a,z) = \max\{W_w(a,z), W_f(a,z), W_i(a,z)\} \tag{12}$$

The value of an entrepreneur of type $j = \{i, f\}$ is defined as

$$\begin{aligned} V_j(a,z) = &\max_{c,a'} u(c) + \beta(1-\rho_R)\left[\pi_z \max\left\{V_j(a',z), \int W(a',z')d\Gamma_{z'}\right\} + (1-\pi_z)\int W(a',z')d\Gamma_{z'}\right] \\ &\qquad + \beta\rho_R V_{ret}(a'), \\ &\text{s.t.} \quad c + a' = \pi_j(a,z), \end{aligned} \tag{13}$$

where $\pi_i(a,z)$ and $\pi_f(a,z)$ are the indirect profit functions defined in (3) and (4). The inner maximization in the right hand side states that if an entrepreneur decides to exit, she will become a worker next period with a new business idea drawn from Γ_z. For simplicity we assume that a business cannot directly transit between informal and formal status, an assumption that is consistent with the fact that the vast majority of formal businesses start as formal upon being created and that formal businesses cannot choose to become informal.[14]

[14] La Porta and Shleifer (2014) provides evidence that on average, among 14 Latin American countries,

Finally, with probability $1-\pi_z$, the entrepreneur is forced to shut down the business (e.g. the business idea dies). In this case, she will become a paid worker and draw a new business idea.

3.4 Government Budget and Market Clearing Conditions

The social security system is assumed to pay a fixed pension benefit to all retired households. The excess of government tax revenue (from all sources) over pensions payments is spent on consumption of a public good (G). The public good G does not affect the marginal utility of private consumption and thereby has no consequences on household decisions.

Denote by F the invariant measure of households across states (a,z,j) when production takes place. The output of a type j entrepreneur, net of the fixed cost of operation, can be written as function of the state of the entrepreneur and its optimal production plan according to $y(a,z,j) = zq_j(k(a,z,j),l(a,z,j)) - c_j$. The output (net of the operating fixed cost) of a corporate establishment with productivity z is written as $y_c(z) = zq_c(k(z),l(z)) - c_c$. In a steady state equilibrium the following market clearing conditions hold:

$$\sum_{j=\{i,f\}} \int_{(a,z)} l(a,z,f)dF(a,z,j) + \overline{M}\int_{\bar{z}_c} l_c(z)d\Gamma_{z_c} = 1; \tag{14}$$

$$\sum_{j=\{i,f\}} \int_{(a,z)} k(a,z,j)dF(a,z,j) + P = \sum_{j=\{w,i,f\}} \int_{(a,z)} adF(a,z,j) + A_{ret} \tag{15}$$

$$\begin{aligned}\sum_{j=\{w,i,f\}} \int_{(a,z)} c(a,z,j)dF(a,z,j) + C_{ret} + \delta K + G = \sum_{j=\{i,f\}} \int_{(a,z)} y(a,z,j)dF(a,z,j) + \dots \\ \cdots + \overline{M}\int_{\bar{z}} y_c(z)d\Gamma_{z_c} - \int_{(a,z)} (c_e^f I_w^f(a,z) + c_e I_w^i(a,z))dF(a,z,w),\end{aligned} \tag{16}$$

where C_{ret} and A_{ret} denote aggregate consumption and savings of all retired households, K represents the aggregate stock of capital over all establishments in the economy, G government spending, and $I_w^j(a,z)$ an indicator function that is equal to one if the worker decides be an entrepreneur in sector $j \in \{i,f\}$ in the next period. Equation (14) states that the sum of labor demand over all establishments equals the mass of households in the working stage, which is normalized to 1.[15] Equation (15) states that the sum of the capital across entrepreneurs and the equilibrium value of corporations (P) should be equal to aggregate

more than 90 percent of formal businesses are registered upon creation.

[15] Recall that the entrepreneurs' own labor supply, $\bar{l}$, is set to 1.

savings of retired and non-retired households.[16] The final condition says that the sum of aggregate consumption, investment and government expenditures is equal to the aggregate supply of output net of operating fixed cost and entry cost.

4 Baseline Economy

We now fully specify our baseline economy. First, we specify and motivate the functional forms chosen for the analysis in the paper. Second, we explain our calibration strategy and present the calibration results for the Brazilian economy. We also discuss the performance of our baseline economy along non-targeted dimensions.

4.1 Functional Forms

Before proceeding to the calibration of the model economy, we first specify the functional forms that characterize the model economy.

Preferences. We assume a log utility, $u(c) = \log(c)$. The utility function of public goods is not specified as it is inconsequential for the analysis in the remainder of the paper.

Entrepreneurial ideas. Entrepreneurial ideas are assumed to be drawn from a Pareto distribution, with c.d.f

$$\Gamma_z(x) = \begin{cases} 1 - (\frac{z_0}{x})^{\xi} & x \geq z_0 \\ 0 & x < z_0, \end{cases} \tag{17}$$

where z_0 is the minimum possible entrepreneurial value and ξ governs the tail of the distribution.[17]

Cost of hiring informal workers. The cost function faced by formal entrepreneurs when hiring informal workers is an extension of the one considered by Ulyssea (2018). An entrepreneur that uses l_i informal labor and l_f formal labor incurs the resource cost

$$\tau(l_i, l_f) = \tau_{1,f}(l_i)^2 \left(\frac{l_i}{l_i + l_f} \right)^{\omega} \qquad \omega \geq 0, \tag{18}$$

[16] Since capital in the corporate sector is internally accumulated by firms, it does not appear in the market clearing condition for capital.

[17] Since we discretized the distribution when solving the model numerically, the effective c.d.f is truncated. For more details see Appendix C.

which is assumed to be reduced form for the expected costs of being detected by the government. These costs are assumed to increase in number of informal workers and, if $\omega > 0$ to decrease with the total number of workers hired by the entrepreneur. Formal entrepreneurs choose the optimal mix between formal and informal workers to minimize total labor costs. Equating the marginal cost of formal and informal workers yields the following relationship between the number of informal workers and total employment:[18]

$$\ln(l_i) = \frac{1}{1+\omega} \ln\left(\frac{\tau_{ss}}{\tau_{1,f}(2+\omega)}\right) + \frac{\omega}{1+\omega} \ln(l_i + l_f). \tag{19}$$

The parameter ω controls how the number of informal workers rise with firm size. Conditional on the size of the firm, larger values of ω are associated with more informal workers. Note that under the extreme case where $\omega = 0$, the cost function is exactly the one as in Ulyssea (2018). In this case, the size of the firm has no effect, such that all formal firms hire at most a fixed number l_i^* of informal workers and the first l_i^* workers are always informal. Note that the selected functional form has one convenient property. Even though the number of informal workers is increasing in firm size (if $\omega > 0$), the fraction of informal workers is decreasing.[19] The empirical relationship between the size of an establishment and the number of informal workers implied by equation (19) will be exploited in the calibration of ω.

Production function. The production function is assumed to take the form

$$y = z\left(k^{\alpha_j} l^{1-\alpha_j}\right)^{\theta_j}, \tag{20}$$

where $\theta_c = \theta_f \geq \theta_i$ and $\alpha_c = \alpha_f \geq \alpha_i$. We remark that allowing for establishments in the informal economy to operate with a (relatively) low span of control (θ) and low capital intensity (α) allows the model economy to match important aspects of the data.

Discussion. We find it useful to end this section with a discussion of how our baseline model works. These insights will be useful to develop some intuition on the calibration of the model economy. Note that the capital used by formal and informal establishments

[18] Appendix B.1 provides details of derivations.

[19] Our calibration implies that an establishment with 10, 100, or 1000 workers hires 6.4, 18.4, and 52.1 informal workers respectively.

satisfy:

$$(1-\tau_y)MPK_f = (1-\tau_y)\alpha_f\theta_f Y_f/K_f = r+\delta+\mu_f, \tag{21}$$

$$MPK_i = \alpha_i\theta_i Y_i/K_i = r+\delta+\mu_i, \tag{22}$$

where μ_f and μ_i represent the Lagrange multipliers associated to the borrowing constraint faced by formal and informal entrepreneurs. These two expressions yield

$$\frac{K_f/Y_f}{K_i/Y_i} = (1-\tau_y)\ \frac{r+\delta+\mu_i}{r+\delta+\mu_f}\ \frac{\alpha_f\theta_f}{\alpha_i\theta_i} \tag{23}$$

In Section 2, we documented that the capital to output ratio of small formal businesses in Brazil (with less that 5 workers) is about a factor of 1.32 the one of informal firms. Equation (23) shows that in our model economy this ratio can be expressed as the product of three terms. The first term is less than one since $\tau_y > 0$. The second term will tend (on average across establishments) to be less than 1 as small formal businesses are more likely to be borrowing constrained than informal businesses ($\mu_f > \mu_i$). Hence, the calibration of the baseline economy will set $\alpha_f > \alpha_i$ in order to match the fact that formal firms have a higher capital to output ratio than informal businesses of similar size.

Now consider the labor demand decision of formal and informal entrepreneurs. The marginal worker is chosen so that

$$(1-\tau_y)MPL_f = (1-\tau_y)(1-\alpha_f)\theta_f Y_f/L_f = w(1+\tau_{ss}), \tag{24}$$

$$MPL_i = (1-\alpha_i)\theta_i Y_i/L_i = w, \tag{25}$$

where, for simplicity, we set $\omega = 0$ and assumed that the marginal worker of the formal entrepreneur is formal.[20]

Combining these expressions yield an expression for gross output (including fixed costs of operation) per worker:

$$\frac{Y_f/L_f}{Y_i/L_i} = \frac{1+\tau_{ss}}{1-\tau_y}\ \frac{(1-\alpha_i)\theta_i}{(1-\alpha_f)\theta_f}. \tag{26}$$

Value added per worker in establishments of type j can be expressed as,

$$\frac{VA_j}{L_j} = \frac{Y_j - c_j}{L_j} = \frac{Y_j}{L_j}(1-c_j/Y_j) \tag{27}$$

[20] Assuming that the marginal worker of the formal entrepreneur is informal and $\omega > 0$ does not change the result. In this case, instead of τ_{ss}, equation (24) would have $\partial\tau(\cdot)/\partial l_i$ which is also a positive term.

Combining the last two expressions yields:

$$\frac{VA_f/L_f}{VA_i/L_i} = \frac{1+\tau_{ss}}{1-\tau_y} \frac{(1-\alpha_i)\theta_i}{(1-\alpha_f)\theta_f} \left(\frac{1-c_f/Y_f}{1-c_i/Y_i}\right) \tag{28}$$

Hence, our calibration will set $\theta_f > \theta_i$ to account for the fact that the value added (conditional on the number of workers) is 2.3 times higher for formal when compared to informal entrepreneurs.

4.2 Parameter Values Set Exogenously

The model period is set to a year. The retirement probability is chosen so that the expected working lifetime of a household corresponds to 40 years ($\rho_{ret} = 1/40$). Retired households live for 16 years ($T = 16$).

Entrepreneurs. The parameters of the production function of formal entrepreneurs are set to standard values, $\alpha_f = 0.3$ and $\theta_f = 0.90$.[21] The corresponding parameters for the production function of informal entrepreneurs will be calibrated internally. The depreciation rate of capital is $\delta = 0.06$. The labor services supplied by entrepreneurs in their own businesses is normalized to 1 ($\bar{l} = 1$), so the owner of the businesses is interpreted to supply the same labor as an additional worker. This also implies that aggregate labor supply is equal to the unity and does not change with the share of entrepreneurs in the economy.

The persistence of entrepreneurial ideas is set to a value of $\pi_z = 0.90$, a standard value in the literature. Moreover, this value is roughly consistent with the average business tenure in Brazil which is around 10 years (see Table A.4).

The entry cost of informal entrepreneurs is set to zero, which means that entry into formal entrepreneurship is, effectively, the only dynamic occupational choice in our baseline economy.[22]

Based on equation (19), ω is recovered from the slope of the regression of the number of informal workers on firm size and a constant. Note that, conditional on τ_{ss}, the estimated constant suggests a value for $\tau_{1,f}$. Nevertheless, since the sample covers only small business, it is unlikely that the coefficients jointly match well the aggregate share of informal workers in formal firms. Hence, our strategy involves to fix the estimated value of ω ($\omega = 0.8454$), and calibrate $\tau_{1,f}$ to match the aggregate data.

[21] See, for instance, Midrigan and Xu (2014).

[22] We have also calibrated the model economy allowing for positive entry costs of informal businesses but the estimation implied negligible entry costs without a noticeable improvement in the fit of the data targets. Hence, for simplicity, we eliminated this parameter from the baseline estimation of the model economy.

Taxes. The taxes are assigned their statutory values, specifically $\tau_y = 0.2925$ and $\tau_{ss} = 0.29$.[23] Following the OECD Pension Statistics, the pension replacement rate is set to 70% of the equilibrium wage.

Corporate sector. Productivity in the corporate sector, z_c, is Pareto distributed with a location parameter z_{cmin} and tail parameter ξ_c. These are set to be $z_{cmin} = 2$ and $\xi_c = 3$. We assumed that corporations are subject to a relatively large fixed cost operation ($c_c = 5$), so that establishments in the corporate sector are large. Given these parameters, the mass of corporate firms $\overline{M}$ determines the aggregate market valuation of corporate firms (see (8)).

4.3 Parameter Values Set by Solving the Model Economy

The remaining 12 parameters are chosen to minimize a loss function that consists of the square deviations between some selected model statistics and their data counterparts. In particular, we pin down the parameters of the production function of informal entrepreneurs (θ_i and α_i), the mass of corporate firms $\overline{M}$, the discount factor β, the location and tail parameter of the distribution of entrepreneurial ideas (z_0 and ξ), the fixed cost of operation of formal and informal businesses (c_f and c_i), the entry cost of formal businesses c_e^f, the parameter governing the cost of hiring informal workers by formal businesses ($\tau_{1,f}$), and the parameters on the collateral constraint faced by formal and informal entrepreneurs (λ_f and λ_i).

Although the equilibrium outcomes will be jointly determined by all of the parameters, it is useful to discuss how each of the parameters connect with some moments of interest. The discount factor, β, affects the equilibrium rate of return on capital and hence the K/Y ratio among formal businesses. The parameters θ_i is used to pin down the ratio of value added between formal and informal businesses (with up to 5 employees). As discussed in Section 4.1, the lower θ_i relative to θ_f, the higher will be the ratio of value added between formal to informal businesses (conditional on operating fixed cost). Similarly, α_i is used to match the ratio of K/Y between (small) formal and informal businesses, as it determines the capital intensity of informal firms. The mass of corporate firms, $\overline{M}$, is directly related to the stock market valuation of corporations to GDP and have a first order effect on the equilibrium interest rate. The parameters λ_i and λ_f determine the credit to output ratio of informal and informal entrepreneurs. The entry cost c_e^f affects the share of formal businesses in the economy. The parameter $\tau_{1,f}$ determines the mass of informal workers in formal establishments. The parameters z_0 and ξ, together with the fixed operating costs c_f and c_i,

[23] For a discussion of the tax values see Ulyssea (2018).

determine the size distribution of formal and informal establishments. In addition, the fixed cost of operation of informal entrepreneurs affects the profitability of informal businesses and, hence their mass and the labor force employed by them.

With these connections in mind, our calibration targets the following moments in the Brazilian data:

1. The share of 35% of informal paid workers among total paid workers (Table A.2).
2. The share of informal paid workers hired by formal businesses of 70% (for a discussion see Appendix A.2).
3. The fraction of formal businesses of 0.30 (Ulyssea (2018)).
4. A capital to output ratio of 1.38 among formal entrepreneurs with less than 6 workers (Table A.4).
5. A credit to output ratio of 0.43 among formal entrepreneurs (Table A.4).
6. A capital to output ratio of 1.04 among informal entrepreneurs with less than 6 workers (Table A.4).
7. A credit to output ratio of 0.31 among informal entrepreneurs (Table A.4).
8. A value added per worker ratio between formal and inform firms (with less than 6 workers) of 2.3 (Table A.4).
9. The size distribution of formal establishments (Table 4).
10. The size distribution of informal establishments (Table 4).
11. The value of the stock market to GDP of 40%.[24]

4.4 Calibration Results

The model economy accounts reasonably well for the targeted moments. Table 4 presents the calibration results (parameter values, targets, and model moments). We now describe how the calibrated parameters help to attain the desired targets.

The model captures relative well that most businesses in Brazil are informal. The fraction of formal businesses is 0.27 in the model economy relative to 0.30 in the data. The share of informal paid workers among paid workers is 0.35 in the model and in the data. Moreover, formal businesses hire about 71% of paid informal workers. The model captures that informality is pervasive in the Brazilian economy, both along the intensive and extensive margin of informality.

[24]The stock market to GDP in Brazil was 32% in 2003 and 43% in 2004. We target 40% to avoid business cycle fluctuations.

Table 4: Calibration Results: Baseline Economy

Parameters	Values	Target	Model	Data
θ_i	0.653	Share of Informal Workers	0.349	0.350
$\tau_{1,f}$	0.023	Share of Informal Workers in Formal B.	0.713	0.700
c_e^f	0.089	Share of Formal Firms	0.274	0.300
β	0.931	K/Y Formal (≤ 5)	1.388	1.380
α_i	0.162	K/Y Informal	1.039	1.040
λ_f	1.490	Credit/GDP Formal (≤ 5)	0.440	0.431
λ_i	1.506	Credit/GDP Informal	0.315	0.311
		VA Ratio Formal to Informal (≤ 5)	1.800	2.317
$\overline{M}$	0.625×10^{-13}	Stock Market Value to GDP	0.414	0.400
		Formal Size: ≤ 5	0.775	0.701
z_0	1.351	Formal Size: 6 - 10	0.113	0.141
ξ	7.698	Formal Size: 11 - 20	0.055	0.083
c_f	0.243	Formal Size: 21 - 50	0.035	0.048
c_i	0.635	Informal Size: ≤ 2	0.888	0.957
		Informal Size: ≤ 5	1.000	0.998

The model is consistent with the fact that, conditional on size, there are important differences between formal and informal businesses. First, the ratio of value added between formal to informal businesses (with less than 6 employees) is 1.8 relative to 2.3 in the data. Second, informal businesses are much less capital intensive than formal businesses: The capital to output ratio is 1.04 for the former and 1.40 for the latter. These ratios in the data are 1.04 and 1.38. To account for these observations, the model implies that informal businesses have a low span of control ($\theta_i = 0.65$) and a low capital share ($\alpha_i = 0.16$) relative to formal businesses ($\theta_f = 0.90$ and $\alpha_f = 0.30$). The model accounts for the fact that the credit to output ratio of formal businesses, conditional on size, is higher than that of informal businesses, even though λ_f and λ_i are about the same. The fact that formal businesses are more capital intensive than informal businesses help accounts for the relatively high borrowing of formal businesses.

The model implies that informal businesses tend to be much smaller than formal businesses. While all informal businesses have less than 5 workers, only 76% of formal businesses have less than 5 workers (70% in the data). In the model, the fraction of firms with more than 20 workers is about 14%, relative to 16% in the data.

The stock market value of corporations in the model economy is about 41% of GDP, which is consistent with the data target. This is attained with a relatively low fraction of firms $\overline{M} = 0.6 \times 10^{-13}$ and with an equilibrium return on capital of 3.1%. The model is calibrated so that corporations are large: There are no firm with less than 20 workers, and

most corporations have more than 250 workers.

4.5 Model Performance in Non-target Dimensions

In this section we discuss how the model perform on non-target moments of the economy. Table 5 shows how the model fare along key macroeconomic dimensions. The baseline economy implies a high rate of entrepreneurship, a feature of the Brazilian data. While the model implies that 24% of the working age population is an entrepreneur, in the data this number is about 32%. A notorious characteristic of emerging economies is their low labor share of the national income relative to developed economies. The model is able to replicate this well for the Brazilian data. It predicts the labor share to be roughly 50%, while in the data is around 48%.[25]

Table 5: Model Performance along Selected Macroeconomic Moments

Variable	Model	Data
Fraction of Entrepreneurs	0.240	0.322
Labor Share	0.502	0.480
Social Sec. Contribution/GDP	0.061	0.065
Sales Tax/GDP	0.252	0.168

Notes: Labor share is the wage payments on national income (does not include self-employed income). Social Security Contribution includes payroll taxes plus SS contribution (Table A.5). Sales Tax includes federal, state and local government taxes. Sources: PNAD (2003), Penn World Table 8.0, and IMF Government Finance Statistics (2006).

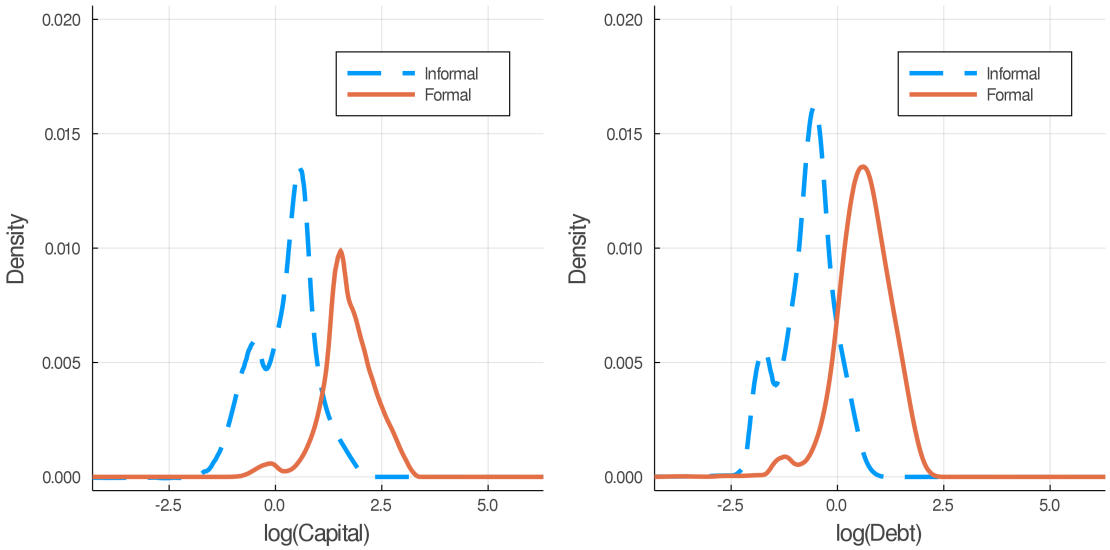
Aggregate tax revenue. Table 5 also shows aggregate tax revenue as a fraction of GDP, both for social security contributions (including other payroll taxes) and for sales tax. The sum of these two revenue sources account for around 64% of the total government revenue and 56% of the federal government revenue (see Table A.5). A fundamental question of this paper is how informality impacts the government capacity to finance a social security system. This requires a good model performance with respect to the aggregate contributions to the

[25]We define labor share as the share of labor compensation of employees (wage payments) over gross domestic product. Since this does not include own-account workers nor entrepreneur's income, it usually serves as a lower bound for the estimate of the labor share in developing economies. We decide to use this measure since it gives a clear mapping of the data into the model. Another way to measure labor share in economies with high rates of entrepreneurship is to include the labor share of income of self-employed individuals. This requires to assume that self-employed individuals use the same proportion of capital and labor as the rest of the economy. In the case of Brazil, once we make this adjustment the labor share increases to 0.530.

social security with respect to GDP. The aggregate revenue from social security contributions and payroll taxes is 6.1% in the model and 6.4% in the data. The fact that the model matches the data quite closely give us confidence to study the financing of social security. Regarding the sales tax, the model predicts that the aggregate revenue to GDP is about 25.2% compared to 16.8% in the data. Since our model economy abstracts from income taxes and informal entrepreneurs are likely to evade income taxes (on top of sales taxes), we believe it is reasonable to view the value added tax in our model economy as representing both income and sales taxes. Under this interpretation, the predictions of our theory are well aligned with the data since the tax revenue from sales and income taxes amount to 24% of GDP in Brazil.

Distributions of capital and debt. In Section 2, we documented that the distribution of capital and debt of small businesses exhibits a common support across the formal and informal sector. Figure 2 replicates the same picture in the model.

Figure 2: Distribution of Capital and Debt: Model



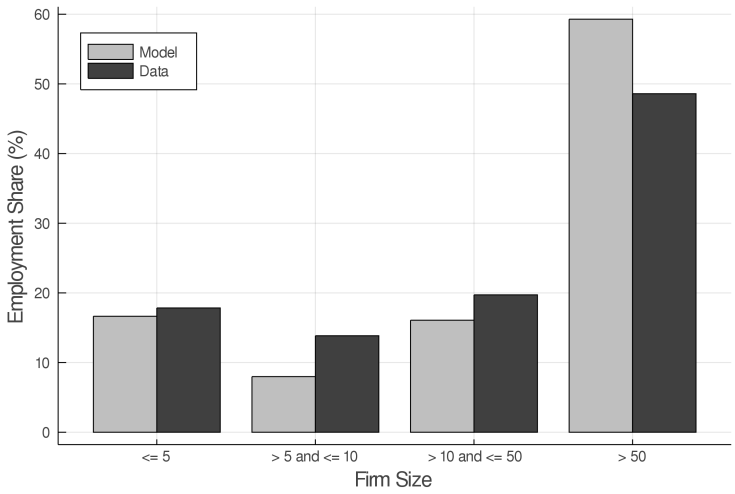
Notes: The figure plots the model invariant distribution of capital (left panel) and debt (right panel) for firms with less than five workers (including the entrepreneur). The distribution is smoothed using a local linear regression with 5% smoothing span.

A question posed in the empirical section is whether the overlapping distribution arises due to differences in collateral constraint, selection, or both. Here we attempt to shed light on this issue. We remark that, in the baseline economy, the estimated parameters of the collateral constraint are roughly the same in both sectors ($\lambda_f = 1.49$ and $\lambda_i = 1.50$). The fact that some informal firms use more capital than some formal firms - despite capital

intensity being higher in the formal sector - points to the coexistence of credit constrained formal entrepreneurs with unconstrained informal entrepreneur.[26] The reason is that high-productivity entrepreneurs self-select into the formal sector in the hope of accumulating capital and, eventually, overcoming their borrowing constraints. In contrast, unconstrained low-productivity entrepreneurs are able to operate at their optimal scale in the informal sector and have higher access to credit than more productive entrepreneurs in the formal sector.

Employment share by firm size. The model is calibrated to match the firm size distribution of the formal and informal sector. One question is whether the two entrepreneurial sectors, together with the corporate sector, imply the correct distribution of workers among different business size. Figure 3 shows the employment share by firm size implied by the model relative to the data from Table 2.

Figure 3: Employment Share by Firm Size



Notes: Aggregate employment share by firm size in the model and in the data (Table 2). Firm size is defined by the number of paid workers.

The model does a good job in predicting that large firms hire most of paid workers in the economy. Although the model slightly overstates the employment fraction in large firms (59% relative to 49% in the data), it correctly predicts that small firms account for about 18% of the paid employees both in the model and in the data.

[26] In the model, 83% of the formal and small entrepreneurs (less or equal five workers) are constrained compared to 15.9% in the informal sector. When considering the entire formal sector, only 6.3% of the entrepreneurs are credit constrained.

5 Quantitative Experiments

We now assess how the high informality of the Brazilian economy affects capital accumulation, occupational choice, resource allocation, and government tax revenue. In particular, we focus on the role played by the interaction of financial frictions with informality along the intensive and extensive margin.

5.1 Assessing the Effects of Informality in Brazil

We now assess how informality in Brazil affects occupational choices, output, capital accumulation, TFP, and government tax revenues. In assessing these effects, close attention is devoted to distinguishing the effects that are coming from firm-level informality versus worker informality (the extensive versus intensive margin of informality). This is done by simulating separately the effects of (i) shutting down informal businesses (no extensive margin), (ii) forbidding formal entrepreneurs to hire informal workers (no intensive margin), and (iii) shutting down both informal businesses and the hiring of informal workers by formal businesses, thereby eliminating informality in the two margins. Our main finding is that shutting down informality has sizable effects on macroeconomic aggregates, occupational choices, and government tax revenues. Moreover, the effects of informality along the intensive and extensive margin tend to work in opposite directions, and the joint effect of two margins can differ substantially from adding the effects of each of them. The results are presented in Table 6.

Macroeconomic aggregates. When informality is eliminated along both margins in the baseline economy, aggregate output increases by 7.2%, capital by 13.7%, and TFP by 3.5%. However, the increases in output and TFP would be a factor of 1.4 and 1.9 larger if informality were only eliminated on the extensive margin (that is, prohibiting informal businesses but allowing formal entrepreneurs to hire informal workers). To put this finding on a different perspective, when formal businesses are not allowed to hire informal workers (e.g. shutting down the intensive margin of informality) aggregate output, capital, and TFP *decrease* by 10%, 11.5%, and 6.5% (Panel 1 in Table 6). In understanding this result, note that informal entrepreneurs in the baseline economy enjoy a competitive advantage over formal entrepreneurs because they do not pay taxes and are less likely to be constrained by financial frictions, as their optimal scale of production is smaller than that of formal entrepreneurs. This logic implies that entrepreneurs with high productivity businesses, ceteris paribus, are more negatively affected by financial frictions (they operate at a suboptimal scale). By hiring some workers off the books, formal entrepreneurs make higher profits, accumulate more

Table 6: Effects of Informality in the Baseline Economy

Panel 1: Change in Macroeconomic Variables (%)

	No Extensive Margin	No Intensive Margin	No Informality
Agg. Output	10%	-10%	7.2%
Agg. Capital	12.3%	-11.5%	13.7%
TFP	6.7%	-6.5%	3.5%
Tax Rev./GDP	0.3%	-10.8%	-8.7%

Panel 2: Occupational Choices

	No Extensive Margin	No Intensive Margin	No Informality
Informal Ent.	0	0.268	0
Formal Ent.	0.182	0.007	0.066
Paid Workers	0.818	0.725	0.934
Frac. Inf. Paid Workers	0.336	0.159	0

Panel 3: Change in Government Tax Revenue (%)

	No Extensive Margin	No Intensive Margin	No Informality
Total Tax Rev.	22%	-17%	35%
S.S. Tax Rev.	-2%	9%	78%
Sales Tax Rev.	28%	-23.2%	24%

Notes: The table displays the changes of removing informality relative to the baseline economy. The baseline economy has a fraction of 0.169 informal entrepreneurs, 0.063 formal entrepreneurs, 0.768 paid workers and 0.271 of informal paid workers. The Fraction of Informal Paid Workers is over all working age household: entrepreneurs plus workers.

savings, and relax the negative effects of financial frictions on their optimal production plan. These effects are reinforced in general equilibrium: The increase in labor demand by productive entrepreneurs, rises the equilibrium wage rage, discouraging entry into the informal economy and improving the allocation of resources across entrepreneurs.

The importance of the interaction between informality and financial frictions can be seen by the fact that the aggregate credit to GDP ratio decreases by more than 10 percent when the intensive margin of informality is eliminated in the baseline economy. Indeed, we find that output and TFP are the highest in the economy with no informal businesses but in which

formal entrepreneurs can hire some workers off the books (see Table 6). This result supports the view that the informal economy can play a positive role by diminishing the negative effects of costly regulations and institutions (e.g. social security and financial frictions) on the economy.

Occupational choices. The fraction of entrepreneurs in the baseline economy is 0.23 and about 70% of them are in the informal sector. When informality is shut down the entrepreneurship rate plummets to a fourth the value in the baseline economy (Panel 2 in Table 6). Part of this effect is mechanical as we have prohibited informal businesses. The key finding, however, is that the elimination of informal entrepreneurs only leads to a negligible increase in the number of formal entrepreneurs (from 0.063 to 0.066). The reason is that in our baseline economy, with financial frictions, the inability to hire informal workers makes it too costly for entrepreneurs to formally operate their businesses. The importance of the intensive margin of informality on occupational choices can be assessed by considering the effects of shutting down informal businesses but keeping the intensive margin of informality. In this case, the fraction of entrepreneurs running formal businesses rises from 0.063 in the baseline economy to 0.182. The increase in labor demand by formal entrepreneurs rises the fraction of paid workers from 0.77 in the baseline economy to 0.82. At the same time, the fraction of *informal* paid workers rises from 0.27 to 0.34 because of the rise in the number of informal workers hired by formal firms, the intensive margin of informality. Hence, the opposite effects along the intensive and extensive margins explains the (surprising) finding that the fraction of informal paid workers can rise when informal businesses are shut down.

Government tax revenue. We find that the intensive and extensive margins of informality have quite different effects on government tax revenue. Moreover, the interaction between the two margins of informality makes their joint effect on government tax revenues different from the sum of their individual effects. Shutting down informal businesses (extensive margin of informality) rises government tax revenue by 22%, while shutting down the intensive margin of informality depresses government tax revenue by 17% (Panel 3 in Table 6). However, the elimination of both margins of informality leads to an increase of government tax revenue of 35%, an increase by a factor of 7 the sum of the individuals effects.

Policies that eliminate the intensive margin of informality depress government revenues for two reasons. First, they lead to a large increase in the number of informal businesses (from 0.17 in the baseline economy to 0.27). Second, in our economy with financial frictions, when formal entrepreneurs are unable to hire informal workers output, capital, and TFP decrease sharply (10%, 11.5%, and 6.5%). These effects lead to a reduction in the sales tax

revenue of more than 20%. The 9% increase in the social security revenue cannot overturn the decrease in government revenue from sales taxes, as the social security tax represents a small share of the government tax revenue in our baseline economy.

Policies that eliminate informal businesses surprisingly reduce the social security revenue by 2%, which is precisely the opposite effect of which these policy recommendations aim to do. The reason is that this policy leads to an increase in the fraction of paid informal workers (from 0.27 in the baseline economy to 0.34). This result underscores the importance of modelling both margins of informality jointly: Reducing informality along one margin may lead to increase of informal paid workers through the other margin. Abstracting from the intensive margin of informality, can lead to misguided policy recommendations.

Policies that shut down informality along *both* margins leads to a large increase in the aggregate government tax revenue. Differently from the previous cases considered, this institutional change increases tax revenue from *both* sales and social security taxation. As before, shutting down informal businesses increase sales tax revenue because these entrepreneurs do not pay sales taxes. Moreover, shutting down the hiring of informal workers by formal employers leads to an increase in social security revenues because entrepreneurs cannot shift their production into the informal economy.

Summary of key findings. In a nutshell, our results point to the importance of modelling both informality margins and analyze their effect separately as a well as jointly to paint an accurate picture of the overall cost of informality. In particular, we highlight two main takeaways. First, policies that eliminate the intensive margin alone have pervasive effects on output and tax revenue. Hiring workers off the books allow small but productive businesses to outgrow borrowing constraints. Without this option, most of entrepreneurs move to the informal sector where they operate in a small scale and use less capital. Second, the joint effect of both informality margins is large and different than the sum of each individual effect. This is true for output and TFP, but it is particularly evident for the change in government tax revenue.

5.2 Institutions, Informality, and the Macroeconomy

In this section, we assess the effects of institutional/policy changes that help reduce the size of the informal economy on occupational choices, macroeconomic variables, and government tax revenue. We consider three scenarios: (i) eliminate entry cost into the formal sector; (ii) eliminate the social security pay-as-you-go system (the social security tax and pension benefits); (iii) eliminate financial frictions (e.g. set $\lambda_f = 100 > \lambda_{baseline} = 1.5$). Results are presented in Table 7.

Table 7: Informality and Institutions

Panel 1: Change in Macroeconomic Variables (%)

	No Entry Costs	No Social Sec.	No Financial Frictions
Agg. Output	5%	17%	35%
Agg. Capital	5%	85%	35%
TFP	3.2%	-1%	23%
Credit/GDP	6.4%	30%	162%

Panel 2: Occupational Choices

	No Entry Costs	No Social Sec.	No Financial Frictions
Informal Ent.	0.104	0.002	0.013
Formal Ent.	0.136	0.194	0.086
Paid Workers	0.759	0.804	0.901
Frac. Inf. Paid Workers	0.298	0	0.298

Panel 3: Change in Government Tax Revenue (%)

	No Entry Costs	No Social Sec.	No Financial Frictions
Total Tax Rev.	8%	9%	55%
S.S. Tax Rev.	-5%	-100%	52%
Sales Tax Rev.	11%	35%	56%

Notes: The table displays the changes of removing entry costs ($c_e^f = 0$), the social security system ($\tau_{ss} = 0$ and $b = 0$) and financial frictions ($\lambda_f = 100$) relative to the baseline economy. The baseline economy has a fraction of 0.169 informal entrepreneurs, 0.063 formal entrepreneurs, 0.768 paid workers and 0.271 of informal paid workers. The Fraction of Informal Paid Workers is over all working age household: entrepreneurs plus workers.

No entry costs. In the baseline economy, the entry cost can be interpreted as the set of regulations faced by the entrepreneur when opening a formal business. Our calibration implies that these costs are about 10% of the equilibrium wage rate. While this estimate does not appear to be particularly large, removing entry costs does have important effects on output (5%), capital (5%), credit to GDP ratio (6.4%), and TFP (3.2%). The change on the entrepreneurship rate is quite small (less than 1 percentage point), although it leads to substantial reallocation of entrepreneurs from the informal to formal sector. When the cost of entry into the formal sector is removed, the mass of informal entrepreneurs is reduced

from 0.17 to 0.10 and the mass of formal employers rises from 0.06 to 0.136. These large change in the formalization of businesses causes a reallocation of productive resources across sectors, and ultimately the sizeable increase of the aggregate variables described above.

While the elimination of entry costs reduce informal business by about 40%, it is important to point that the the reduction in informal businesses is associated with an increase in the number of informal paid workers (from 0.27 in the baseline economy to 0.30). Consistently with our previous findings, the elimination of entry costs has opposite effects on the intensive and extensive margin of informality. The government tax revenue rises because the increase in the tax revenue from the sales tax more than compensates the decrease of revenue from the payroll tax. One takeaway is that entry costs, on their own, do not account for the large shares of informal businesses and of informal paid workers in Brazil. As we shall see, financial frictions and the financing of social security also play an important role in accounting for the large fraction of formal businesses in Brazil. Moreover, reducing informal paid labor requires attacking informality on both the intensive and extensive margin.

No social security system. The elimination of the social security system leads to a large increase in output (17%), a small TFP reduction (-1%), and a very large increase in capital (85%). The rise in the formalization of entrepreneurs is crucial for the large increase in the aggregate capital stock: The mass of formal entrepreneurs increases from 0.063 in the baseline economy to 0.194 (while informal entrepreneurship becomes negligible). The aggregate tax revenue increases by 9%. In sum, a takeaway lesson is that financing the social security system in Brazil is costly and the high labor costs accounts for a large fraction of the informal sector.

Removing financial frictions ($\lambda_f = 100$). The elimination (or, more accurately, the large reduction) of financial frictions increases output by 35%, capital by 39%, credit to GDP by 162%, and TFP by 23%. These large effects are in accordance with the results in the financial frictions literature.[27] In the next section, we contribute to this literature by assessing some interesting interactions between the effects of financial frictions and informality on macroeconomic outcomes in our baseline economy.

The elimination of financial frictions reduces the fraction of informal entrepreneurs by more than a factor of 10 (from 0.17 in the baseline economy to 0.013). The reallocation of resources towards productive entrepreneurs rises the equilibrium wage rate which strongly discourages small-scale informal businesses. Nonetheless, the fraction of paid informal workers remains high at 0.26 (relative to 0.27 in the baseline economy). Once again this result

[27] See Buera et al. (2011), Moll (2014), Midrigan and Xu (2014) and Allub and Erosa (2019).

underscores the importance of modeling the responses of informality both along the intensive and extensive margin to get an unbiased assessment of how changes in the economic environment impact on informality.

5.3 Interactions Between Financial Frictions and Informality

One of the contributions of our paper is to study the effects of financial frictions in a model economy that features informality along the intensive and extensive margins. Our results point that there are important interactions between the effects of financial frictions and informality. First, when formal entrepreneurs can hire informal workers (intensive margin of informality), they accumulate capital more rapidly, which relaxes credit constraints and diminishes the impact of financial frictions. This is precisely the reason why shutting the intensive margin of informality have large negative effects on the macroeconomic aggregates of our baseline economy. Second, the gains from shutting down informal businesses (extensive margin of informality) also depend on the extent of financial frictions. In our baseline economy the presence of informal businesses gives a competitive advantage to entrepreneurs that are less likely to be affected by credit constraints (low productivity entrepreneurs that want to operate at a small scale). However, in the absence of financial frictions, the gains of shutting down informal businesses are smaller because the number of these businesses is negligible to start with. Motivated by these observations, we find it interesting to evaluate the impact of financial frictions under different scenarios on informality (with and without an extensive/intensive margin of informality). Table 8 presents the key findings.

Table 8: Interactions Between Financial Frictions and Informality

	Changers in Macroeconomic Aggregates (%)			
	Baseline	No Extensive Margin	No Intensive Margin	No Informality
Agg. Output	35%	22%	45%	26%
Agg. Capital	39%	23%	53%	22%
TFP	23%	15%	29%	19%

Notes: The table displays the changes of removing financial frictions ($\lambda_f = 100$) relative to the baseline economy and economies without informality.

We find that the gains from removing financial frictions in the baseline economy are larger than in an economy with no informality (both along the intensive and extensive margins): 35% versus 26% for output, 39% versus 22% for capital, and 23% versus 19% for TFP. All

of these changes are smaller when informal businesses are prohibited (no extensive margin), which is consistent with our finding that one of the benefits of eliminating financial frictions is to reduce the number of informal businesses. On the contrary, the changes in macroeconomic aggregates are substantially larger when the removal of financial frictions is done in an economy with no intensive margin of informality: The gains in output, capital, and TFP are 45%, 53%, and 29%, relative to 35%, 39%, and 23% in the baseline economy. Financial frictions distort resource allocation the most in an economy with informal businesses (extensive margin of informality) *and* in which formal businesses cannot hire informal workers (no intensive margin of informality). We conclude that analyzing the impact of financial frictions in an economy with a large informal sector, it is important to model how financial frictions affect informality both along the intensive and extensive margin of informality.

We find interesting to end this subsection noticing the dramatic differences in the cost of financing social security in our baseline model economy relative to an economy with no frictions (no informality and no financial frictions). In our baseline economy, the elimination of the social security system would lead to an increase in output of 17% *together* with an increase in government tax revenue of 9%. The latter result is due to the large expansion of economic activity and the large increase in formalization of entrepreneurs, which boost the sales tax revenue. In the absence of frictions, the increase in output is a half (about 9%) and there is a *decrease* in government tax revenue of 19% when social security taxation is eliminated.

6 Conclusion

This paper builds a theory of occupational choice, financial frictions and informality along the intensive and extensive margin. The theory is disciplined with Brazilian micro data on the informal and formal economy and is used to assess the impact on informality on capital accumulation, entrepreneurship, resource allocation across entrepreneurs, and government tax revenue. In a nutshell, our results point to the importance of modelling both margins of informality and their interaction with financial frictions for assessing the impact of regulations and institutions in Brazil (e.g. entry costs, taxes and social security system, financial frictions). In particular, we highlight two main takeaways. First, policies that eliminate the intensive margin alone have pervasive effects on output and tax revenue. Hiring workers off-the-books allow small but productive businesses to outgrow borrowing constraints. Without this option, most of entrepreneurs move to the informal sector where they operate in a small scale and use less capital. Second, the joint effect of both informality margins is large and different than the sum of each individual effect on macroeconomic aggregates This is true

for output and TFP, but it is particularly evident for the change in government tax revenue.

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Appendix

A Data Appendix

A.1 Additional Tables

Table A.1: Cumulative mass of formal firms and workers by size of formal firms

Size	Mass of Formal Firms	Mass of Formal Workers
≤ 5	0.698	0.142
≤ 10	0.839	0.242
≤ 20	0.922	0.355
≤ 50	0.972	0.498
≤ 250	0.995	0.723
≤ 1000	0.999	0.890

Notes: Size is defined by the number of paid workers (does not include the entrepreneur). Source: RAIS 2003.

Table A.2: Share of Informal Workers and Entrepreneurs

Variable	2003	2012
Share of Informal Workers (out of total paid workers)	0.350	0.220
Share of Formal Workers (out of total paid workers)	0.650	0.780
Share of Entrepreneurs (out of employed population)	0.322	0.319
Share of Workers (out of employed population)	0.678	0.681

Notes: Urban workers and entrepreneurs in private firms. Source: PNAD 2003 and PNAD-C 2012.

Table A.3: Loan Source by Formal and Informal Firms

Loan Source (%)	Formal	Informal
Friends and Family	9.71	18.19
Banks (Public or Private)	73.66	53.05
Own Supplier	10.67	17.7
Other People or Companies	5.97	11.06
N	977	2054
Share who got a loan (%)	15.37	4.86

Notes: Entrepreneurs who got a loan, credit or financing from Aug/03 to Oct/03. Source: ECINF 2003

Table A.4: Conditional Summary Statistics

Variable	Formal (≤ 5)	Informal	Ratio (F/I)
Size	2.00	1.18	1.698
Business Tenure (months)	119.17	112.28	1.061
Business Tenure (months $\geq$12)	127.92	122.98	1.040
Value Added p/ workers	1,589.60	686.08	2.317
Aggregate Debt/Y	0.431	0.311	1.386
Aggregate K/Y	1.377	1.043	1.320

Notes: Summary statistics conditional on sector, state, gender, education and experience of entrepreneur. Size includes paid workers (formal and informal) and business owners. Aggregate K/Y excludes housing and vehicles. Values in 2003 Brazilian Reals. Firms with 5 or less employees. Source: ECINF 2003.

Table A.5: Tax Revenue by Percent of GDP

	Federal Government	All Governments
Tax Revenue		
Income Tax	6.39	6.86
Payroll Tax	0.30	0.30
Property Tax	0.01	1.15
Sales Tax	8.16	16.77
Other Taxes	0.53	0.53
Social Sec. Contribution	5.44	6.24
Other Revenue Sources	4.88	4.76
Total	25.72	36.61

Notes: Government Revenue by Percent of GDP (2006). All governments include federal, state and local administration. Other taxes include revenue from tariffs and other transfers. Other revenue sources include property income, fines, sales of goods and services and other. Source: IMF Government Finance Statistics.

A.2 Share of Informal Workers in Formal Firms in 2003

A key moment used in the calibration of the model is the share of informal workers in formal firms. Unfortunately, such statistic is not available in 2003, the year in which the model is calibrated. To determine an estimate of this moment, we proceed by finding a lower and a upper bound of it. At a first step, we use the updated version of PNAD to calculate the lower bound of the share of informal workers in formal firms. The PNAD-C (*Pesquisa Nacional por Amostra de Domiclios Contnua*) begun to roll in 2012, and asks the worker both the formality status of the firm she is working and her own formality status. Using the first year available of PNAD-C, Table 2 shows that, out of all workers employed informally, 62% were located in formal firms. Given that the overall share of informal workers decreased from 35% in 2003 to 22% in 2012 (see Table A.2), the share of informal workers in formal firms in formal firms is likely higher in 2003 as well. Therefore, we take 62% as the lower bound of the share of informal workers in formal firms in 2003.

To calculate the upper bound, we make use of both ECINF and PME. The initial step involves determining the share of informal workers by business size. Table A.6 indicates that 35.5% of the informal workers are located in micro firms (with less or equal five employees), while 64.5% are located in larger firms. As shown in Table 1, the likelihood of a firm with more than five employees be informal is negligible. Hence, by assuming that all firms with more than five employees are formal, we have that at least 64.5% of all informal workers are in formal firms. The second step requires finding out the share of informal workers employed in formal business conditional that the firm has five employees or less. Using the ECINF, we found that 32.2% of informal workers in micro firms are employed in formal firms. Therefore, the total share of informal workers in formal firms is equal to the share of informal workers in firms larger than five employees (64.5%) plus the share of informal workers in formal micro firms ($32.2\% \times 35.5\% = 11.4\%$).

Table A.6: Informality by Business Size Distribution in 2003

Variable	Micro (≤ 5)	Small (≥ 6 and ≤ 10)	Large (> 10)
Mass of Informal Workers (by size)	0.355	0.115	0.530
Mass of Formal Workers (by size)	0.066	0.074	0.859
Mass of Workers (by size)	0.148	0.086	0.766
Fraction of Informal Workers	0.678	0.379	0.195

Notes: Urban paid employees in private firms only. Source: PME 2003.

Hence, by combining both ECINF and PME, we infer that the share of informal employees in formal business in 2003 is equal to 75.9%. Yet, because PME samples only workers from

the six largest metropolitan regions, it overstates the number of large business with respect all the other data sets. For comparison, in RAIS (in Table A.1), firms with more than 10 employees accounts for 75.8% of all the workers, while in PME this number is equal to 85.9%. We decide to interpret the 75.9% as an upper bound. To find a good compromise between the lower bound (62.9%) and the upper bound (75.9%), we decide to calibrate the share of informal workers in formal business in 2003 to 70%.

B Theory Appendix

B.1 Cost of Hiring Informal Workers

We parametrize the cost of hiring a informal worker by a formal firm to depend of the share of informal workers hired by the firm. The functional form is given by:

$$\tau_f(l_i, l_f) = \tau_{1,f} l_i^{\tau_{2,f}} \left(\frac{l_i}{l_i + l_f} \right)^{\omega} = \tau_{1,f} l_i^{\tau_{2,f}+\omega} (l_i + l_f)^{-\omega} \tag{A.1}$$

The marginal cost of hiring a formal and informal worker:

$$MC_f = \tau_{ss} - \omega \tau_{1,f} l_i^{(\tau_{2,f}+\omega)} (l_i + l_f)^{(-\omega-1)} \tag{A.2}$$

$$MC_i = \tau_{1,f} [(\tau_{2,f} + \omega) l_i^{(\tau_{2,f}-1+\omega)} (l_i + l_f)^{-\omega} - \omega l_i^{(\tau_{2,f}+\omega)} (l_i + l_f)^{(-\omega-1)}] \tag{A.3}$$

The entrepreneur hires informal workers until $MC_i = MC_f$.

$$\tau_{1,f} [(\tau_{2,f} + \omega) l_i^{(\tau_{2,f}-1+\omega)} (l_i + l_f)^{-\omega} - \omega l_i^{(\tau_{2,f}+\omega)} (l_i + l_f)^{(-\omega-1)}] = \tau_{ss} - \omega \tau_{1,f} l_i^{(\tau_{2,f}+\omega)} (l_i + l_f)^{(-\omega-1)}$$

$$\tau_{ss} = \tau_{1,f} l_i^{(\tau_{2,f}-1+\omega)} (l_i + l_f)^{-\omega} (\tau_{2,f} + \omega)$$

$$l_i = \left(\frac{\tau_{ss}}{\tau_{1,f}(\tau_{2,f} + \omega)} \right)^{1/(\tau_{2,f}-1+\omega)} (l_i + l_f)^{\omega/(\tau_{2,f}-1+\omega)}$$

Hence, if $\omega > 0$, the number of informal workers increase with the size. If $\omega = 0$, all firms have the same cutoff. By setting $\tau_{2,f} = 2$ and taking logs, equation (19) follows.

C Computational Appendix

C.1 Discretization and Value Function Approximation

To bring the model to the computer, we discretize state space of the value and policy functions. Specifically, we discretize the asset space in 1200 grid points equidistant over the log space. The entrepreneurial idea, z is discretized in the same 60 points. Again, the grid points are equidistant over the log space, where the initial is point is given by z_0 and final grid point to the value associated to the 0.9999 percentile of the Γ_{z_s}. The distribution of productivity in the corporate sector, z_c, is uniformly discretized over 10000 grid points.

C.2 Computation with Taste Shock

To facilitate the numerical solution of the model and improve convergence to an equilibrium, we smooth out the discrete occupational choice by adding a taste shock. The taste shock simply adds noise to the entrepreneurial decisions of indifferent households, but the calibration of the model and all the results are robust to the inclusion of it and are left mostly unchanged. Here we outline the extended model, and for more details we refer to Iskhakov et al. (2017). The major modification and key assumption is that every period individuals receive a vector of additive-separable taste shocks $\epsilon = (\epsilon_w, \epsilon_f, \epsilon_i)$ to the value of being a worker, a formal entrepreneur and a informal entrepreneur. These shocks are $i.i.d$ according to an Extreme Value type I distribution (Gumbel) with scale parameter σ_ϵ. We calibrate the variance to $\sigma_\epsilon = 0.01$.

The modified value function of a worker in state (a, z_s) is given by

$$W(a, z, \epsilon) = \max\{W_w(a, z) + \sigma_\epsilon \epsilon_w, W_f(a, z) + \sigma_\epsilon \epsilon_f, W_i(a, z) + \sigma_\epsilon \epsilon_i\} \tag{A.4}$$

where:

$$\begin{aligned} W_w(a, z) = &\max_{c, a'} u(c) + \beta \rho_R V_{ret}(a') \\ &+ \beta(1 - \rho_R) \left(\pi_z E_\epsilon W(a', z, \epsilon) + (1 - \pi_z) \int E_\epsilon W(a', z', \epsilon) d\Gamma_z \right), \\ \text{s.t.} \quad & c + a' = w + (1 + r)a, \end{aligned} \tag{A.5}$$

$$W_f(a,z) = \max_{c,a'} u(c) + \beta\rho_R V_{ret}(a') + \beta(1-\rho_R)E_\epsilon V_f(a',z,\epsilon), \tag{A.6}$$
$$\text{s.t.} \quad c + a' + c_e^f = w + (1+r)a,$$

$$W_i(a,z) = \max_{c,a'} u(c) + \beta\rho_R V_{ret}(a') + \beta(1-\rho_R)E_\epsilon V_i(a',z,\epsilon) \tag{A.7}$$
$$\text{s.t.} \quad c + a' + c_e^i = w + (1+r)a,$$

where $W_j(a,z,\epsilon)$ for $j \in \{w,i,f\}$ represents the value function when worker chooses occupational choice j for next period. The E_ϵ denotes the expectation over future taste shocks.

Note that the introduction of the extreme value taste shock smooth out the kink in the entrepreneurial decision. In fact, the binary choice, which in the absence of the shock can described as an indicator function, is now probabilistic function over the relative values of each choice. Denote, $P_w^j(a,z_s)$, the probability that a worker decides occupational choice $j \in \{w,i,f\}$ in state a,z as

$$P_j^w(a,z) = \frac{\exp\{W_j(a,z)/\sigma_\epsilon\}}{\exp\{W_w(a,z)/\sigma_\epsilon\} + \exp\{W_f(a,z)/\sigma_\epsilon\} + \exp\{W_w(a,z)/\sigma_\epsilon\}} \tag{A.8}$$

Intuitively, the taste shock introduces "noise" in the value function such that an individual may decide stays a worker even so the value of being a entrepreneur surpass the value of being a worker as long the preference shock is large enough. Notice the role of σ_ϵ: a large variance generates too much noise, effectively making the values $W_j(a,z)$ unimportant for the entrepreneurial decision. On the other hand, if $\sigma_\epsilon = 0$, the policy function collapses to the binary case without taste shocks.

The taste shock not only smooth out the primary kink given by the discrete choice in the contemporary value function, but also secondary kinks given by the next period value function. Following Iskhakov et al. (2017), we write the expectation with respect to taste shocks using the *log-sum* formula:

$$E_\epsilon W(a',z,\epsilon) = \sigma_\epsilon \log\left(\exp\left\{\frac{W_w(a',z)}{\sigma_\epsilon}\right\} + \exp\left\{\frac{W_i(a',z)}{\sigma_\epsilon}\right\} + \exp\left\{\frac{W_f(a',z)}{\sigma_\epsilon}\right\}\right). \tag{A.9}$$

Similarly, the modified value of an entrepreneur of type $j = \{i,f\}$ is

$$Vj(a,z,\epsilon) = \max\{V_j^j(a,z) + \sigma_\epsilon\epsilon_j, V_j^w(a,z) + \sigma_\epsilon\epsilon_w\}, \tag{A.10}$$

where the value V_j^j is the value function of an entrepreneur that stays operating and V_j^w of an entrepreneur that decides to exit and become a worker in the next period. In recursive form these value functions are given by

$$V_j^j(a,z) = \max_{c,a'} u(c) + \beta(1-\rho_R)\left[(1-\pi_z)E_\epsilon V_j(a',z,\epsilon) + \pi_z \int E_\epsilon W(a',z',\epsilon)d\Gamma_{z'}\right] + \beta\rho_R V_{ret}(a') \tag{A.11}$$

$$V_j^w(a,z) = \max_{c,a'} u(c) + \beta(1-\rho_R)\int E_\epsilon W(a',z',\epsilon)d\Gamma_{z'} + \beta\rho_R V_{ret}(a') \tag{A.12}$$

$$c + a' = \pi_j(a,z). \tag{A.13}$$

The policy function will be, again, given by a logit function:

$$P_j(a,z) = \frac{\exp\{V_j^j(a,z)/\sigma_\epsilon\}}{\exp\{V_j^j(a,z)/\sigma_\epsilon\} + \exp\{V_j^w(a,z)/\sigma_\epsilon\}}. \tag{A.14}$$

where $P^j(a,z)$ is the probability that the entrepreneur in sector j decides to not (endogenously) exit. Finally, the partial expectation (over the taste shock) of a entrepreneur can be written as

$$E_\epsilon V_j(a',z,\epsilon) = \sigma_\epsilon \log\left(\exp\left\{\frac{V_j^j(a',z)}{\sigma_\epsilon}\right\} + \exp\left\{\frac{V_j^w(a',z)}{\sigma_\epsilon}\right\}\right). \tag{A.15}$$

C.3 Numerical Algorithm

1. Guess factor prices (w,r). Compute $V_{ret}(a)$ with an analytical formula.
2. Set the initial guess $E_\epsilon W^n(a,\cdot,\epsilon) = E_\epsilon V_i^n(a,\cdot,\epsilon) = E_\epsilon V_f^n(a,\cdot,\epsilon) = V_{ret}(a)$.
3. Given $E_\epsilon W^n(a,z,\epsilon)$, $E_\epsilon V_i^n(a,z,\epsilon)$ and $E_\epsilon V_f^i(a,z,\epsilon)$, compute $E_\epsilon W^{n+1}(a,z,\epsilon)$:
   (a) Compute $W_j(a,z)$, for $j = \{w,i,f\}$. For that, compute the expectations over Γ_z when applicable, and use the pre-computed $V_{ret}(a)$. The maximization step is carried on using the divide and conquer algorithm of Gordon and Qiu (2018).
   (b) Then, uses the log-sum formula from (A.9) to compute $E_\epsilon W^{n+1}(a,z,\epsilon)$.
   (c) Note that the previous steps define three saving policies, $g_j^w(a,z)$ (one for each of the three occupational choices), and three probability functions, $P_j^w(a,z)$, that describes the probability that a worker choose one of the three occupational choices.

4. Proceed similarly to compute $V_j^{n+1}(a,z)$, for $j = \{i, f\}$. Compute both $V_j^j(a,z)$ and $V_j^w(a,z)$ by taking the expectations over guesses and applying the divide and conquer algorithm in the maximization. Then, apply the associated *log-sum* formula to get $E_\epsilon V_{n+1}^j(a,z,\epsilon)$ and the logit function to calculate $P^j(a,z)$. Again, there will be four associated saving policies (two for formal and two for informal), $(g_f^f(a,z), g_f^w(a,z), g_i^i(a,z), g_i^w(a,z))$.

5. Once $\max\{||E_\epsilon W^{n+1}(a,z,\epsilon) - E_\epsilon W^n(a,z,\epsilon)||, ||E_\epsilon V_f^{n+1}(a,z,\epsilon) - E_\epsilon V_f^n(a,z,\epsilon)||, ||E_\epsilon V_i^{n+1}(a,z,\epsilon) - E_\epsilon V_i^n(a,z,\epsilon)||\} < tol$, stop the value function iteration. Otherwise, update the guess using the values of $n+1$ and go back to step 3. The tolerance specified is equal to 10^{-9}.