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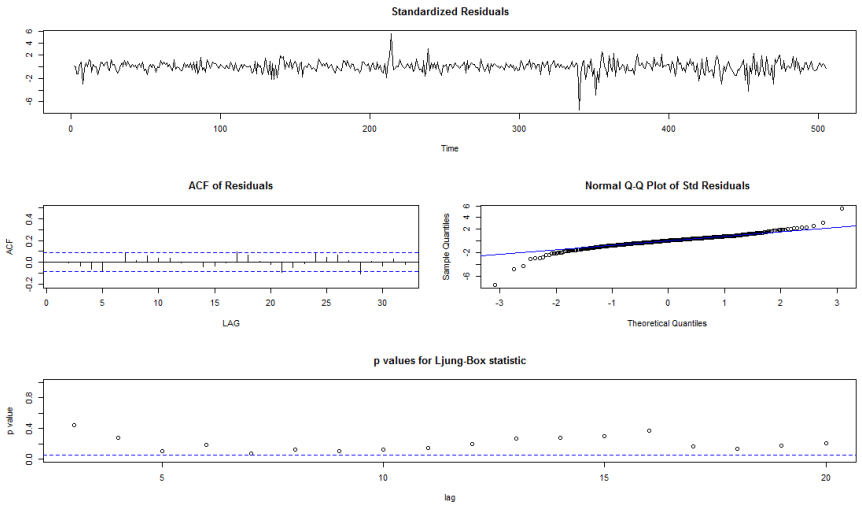
4-20-16
STA4853

FINAL PROJECT

For this project, I chose to analyze stock market closing prices of The Walt Disney Company. I found a data set on the Yahoo Finance website at http://finance.yahoo.com. My data set contained daily values for all days from April 1st, 2014 to April 1st, 2016 where values were given (weekdays that were not holidays). The values in particular that I was analyzing were the close prices listed for each day. Close prices for this set of data ranged from 77.01 to 121.69.

I acquired the daily growth return data by differencing the log of the close prices for each day, and when I plotted those values, there was some noticeable volatility in the graph. When we look at the ACF and the PACF graphs of the growth return data, we see that there is autocorrelation.

I attempted to fit three types of ARMA(p,q) models to this data set: an AR(1), an MA(1), and an ARMA(1,1), and I decided which model to use by looking at the standard errors of the AR(1) and MA(1) portions of each model. The standard error of AR(1) in the AR(1) model was 0.0445, of MA(1) in the MA(1) model was 0.0446, and of AR(1) and MA(1) in the ARMA(1,1) model were 0.7987 and 0.8018, respectively. The AR(1) and the MA(1) models both seemed like good fits, so I decided to use an AR(1) model.



Fitting the data as an AR(1) gave me the output seen to the right. The ACF and the p-values are within an acceptable

range, but there is some noticeable volatility of the residuals and those residuals do not quite fit a normal distribution. When I ran the Ljung-Box Test on the residuals, the output was:

```
Box-Ljung test

data:  r
X-squared = 12.62, df = 9, p-value = 0.1806
```

A p-value of 0.1806 would suggest that I may not need to model a GARCH component, but since that value is not overwhelmingly large, I would still consider modeling a GARCH component and see if I can create a good fit for the data.

I first tried modeling with a normal distribution, and I considered three different GARCH models: GARCH(1,1), GARCH(1,0), and GARCH(2,0). Modeling with GARCH(1,1) gave the following output:

```
Error Analysis:

         Estimate  Std. Error  t value Pr(>|t|)
mu      3.415e-04   5.363e-04    0.637  0.52436
ar1    -1.578e-02   5.659e-02   -0.279  0.78039
omega   2.469e-05   8.331e-06    2.964  0.00304 **
alpha1  1.421e-01   6.535e-02    2.175  0.02965 *
beta1   7.245e-01   8.947e-02    8.098 6.66e-16 ***
```

modeling with GARCH(1,0) gave this output:

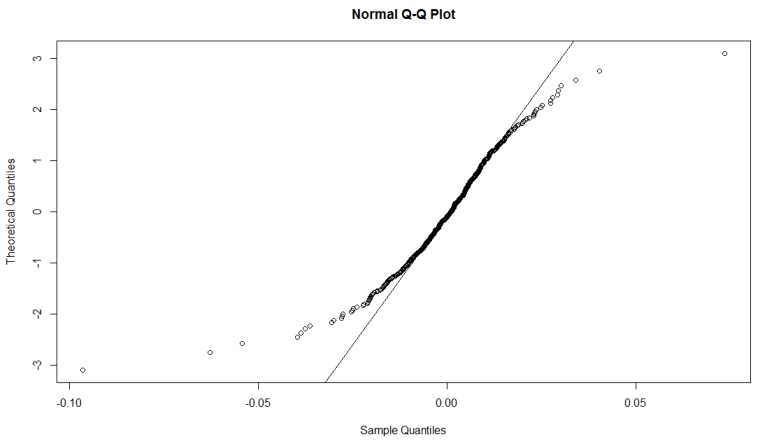
```
Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      2.961e-04   5.856e-04    0.506    0.613
ar1    -2.331e-03   5.992e-02   -0.039    0.969
omega   1.604e-04   1.388e-05   11.561   <2e-16 ***
alpha1  4.831e-02   7.273e-02    0.664    0.507
```

and modeling with GARCH(2,0) printed this:

```
Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      1.542e-05   4.377e-04    0.035    0.972
ar1     6.191e-02   5.148e-02    1.203    0.229
omega   8.461e-05   1.164e-05    7.267 3.67e-13 ***
alpha1 8.633e-02   7.335e-02    1.177    0.239
alpha2 6.110e-01   1.391e-01    4.392 1.13e-05 ***
```

When looking at the p-values of the alpha and beta components of each model, GARCH(1,1) seems to be the best fit: we can see that for alpha1 and beta1, Pr(>|t|) is less than 0.05, suggesting that both components are significant. With GARCH(1,0), alpha1 is not significant since Pr(>|t|) = 0.507 and with GARCH(2,0), alpha1 is not significant since Pr(>|t|) = 0.239, suggesting that those models do not fit the data as well as GARCH(1,1).

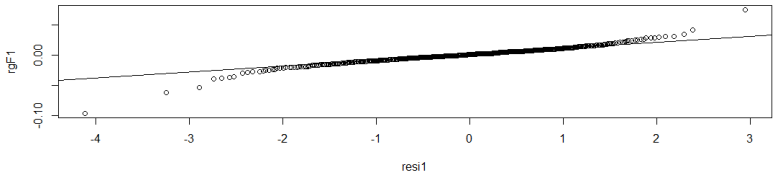
Having chosen to use an AR(1) component and a GARCH(1,1) component, I first attempted a model with a normal distribution of residuals. The normal Q-Q plot shows that the residuals cannot be fit well to a normal distribution. Additionally, the Standardized Residuals Tests indicate that we should not consider the residuals to be normally distributed:

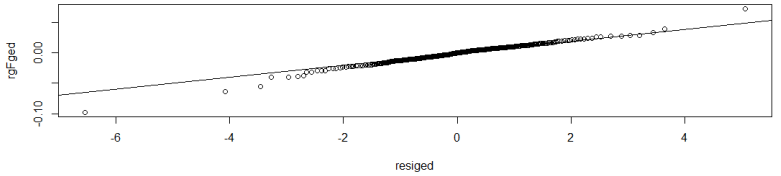



```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  5414.291  0
 Shapiro-Wilk Test  R    W      0.8953559 0
 Ljung-Box Test     R    Q(10)  12.33449  0.263294
 Ljung-Box Test     R    Q(15)  15.77142  0.3974037
 Ljung-Box Test     R    Q(20)  21.65403  0.3595749
 Ljung-Box Test     R^2  Q(10)  0.8227066 0.9999302
 Ljung-Box Test     R^2  Q(15)  11.91063  0.6857805
 Ljung-Box Test     R^2  Q(20)  12.01946  0.9154047
 LM Arch Test       R    TR^2   11.0757   0.5224423
```

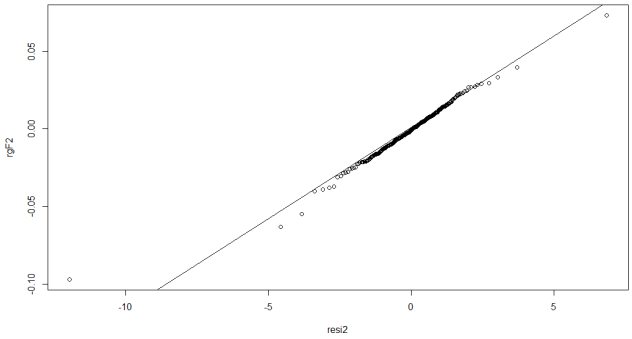
A p-value of near zero for the Jarque-Bera Test highly suggest that this time series does not have the skewness and the kurtosis of a normal distribution, whereas a p-value of near zero for the Shapiro-Wilk Test tells us that this data fails a test of normality.

I then considered a skew normal distribution and a generalized error distribution. The model with a skew normal distribution gave a skewness parameter of approximately 0.7777, and the model with a generalized error distribution gave a shape parameter of approximately 1.0785. However, we can see from the Q-Q plots that neither of these distributions portray the distribution of the residuals well.





The distribution that seemed to be the best fit for the data was the skew student t distribution. Fitted to this data, the model had a shape parameter of approximately 3.7929 and a skewness parameter of approximately 0.8730. The Q-Q plot seen here seems to be the best one of all of the distributions that I have tried, so I will choose to predict future values with this distribution.



The R output for the summary of this model gives these coefficients for the model: `Coefficient(s):`

```
          mu           ar1         omega        alpha1         beta1
 8.5767e-04  -2.4029e-02   8.3123e-06   1.1223e-01   8.5633e-01
```

Which suggests that our model can be written as $x_t \approx 0.0009 - 0.002x_{t-1} + \epsilon_t$, where with $y_t = (x_t - x_{t-1})/(x_{t-1})$, we have that $\sigma^2_t \approx 0.000008 + (0.112 + 0.856)(y_{t-1})^2 + \nu_t + 0.856\ \nu_{t-1}$.

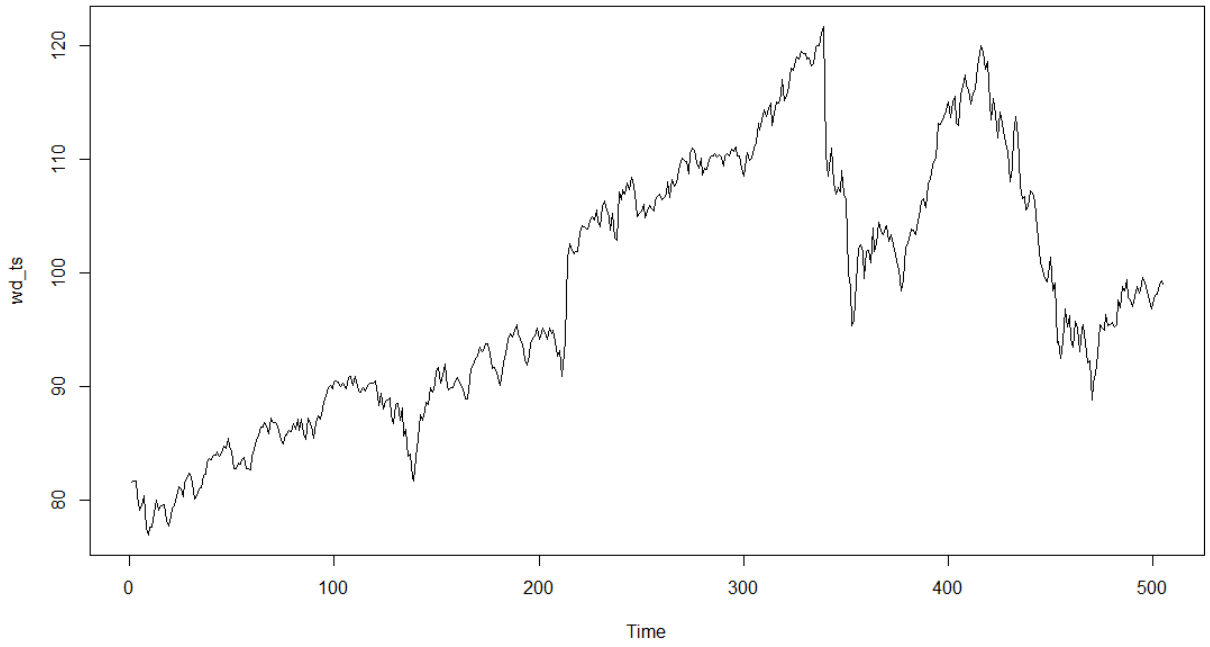
When I run the predict function with this model, I get these values for the difference of the log of the predicted values for the next five days: `[1] 0.0009158121 0.0008356647 0.0008375906 0.0008375443 0.0008375454`. To back transform these values to daily close values, I ran this code which resulted in the following values: `> c=rep(0,5)`

```
> c=predict(gF2,n.ahead=5,trace=FALSE,plot=FALSE)$meanForecast
> wd_vol_pred=c(wd_vol,c)
> wd_ts_log=c(log(wd_ts[1]),log(wd_ts[1])+cumsum(wd_vol_pred))
> wd_ts_pred=exp(wd_ts_log)
> y_hat=wd_ts_pred[(length(wd_ts_pred)-4):(length(wd_ts_pred))]
> y_hat
[1] 99.16077 99.24367 99.32683 99.41006 99.49335
```

The Yahoo finance gave the following values as the actual close values for the following five days (April 4th to April 8th): `[1] 98.68 97.00 97.48 96.16 96.42`, and the correlation between these values and the predicted values is approximately -0.849. It seems that my model did not properly predict the trend: the model predicted the daily values to slowly increase, when in reality, they decreased overall.

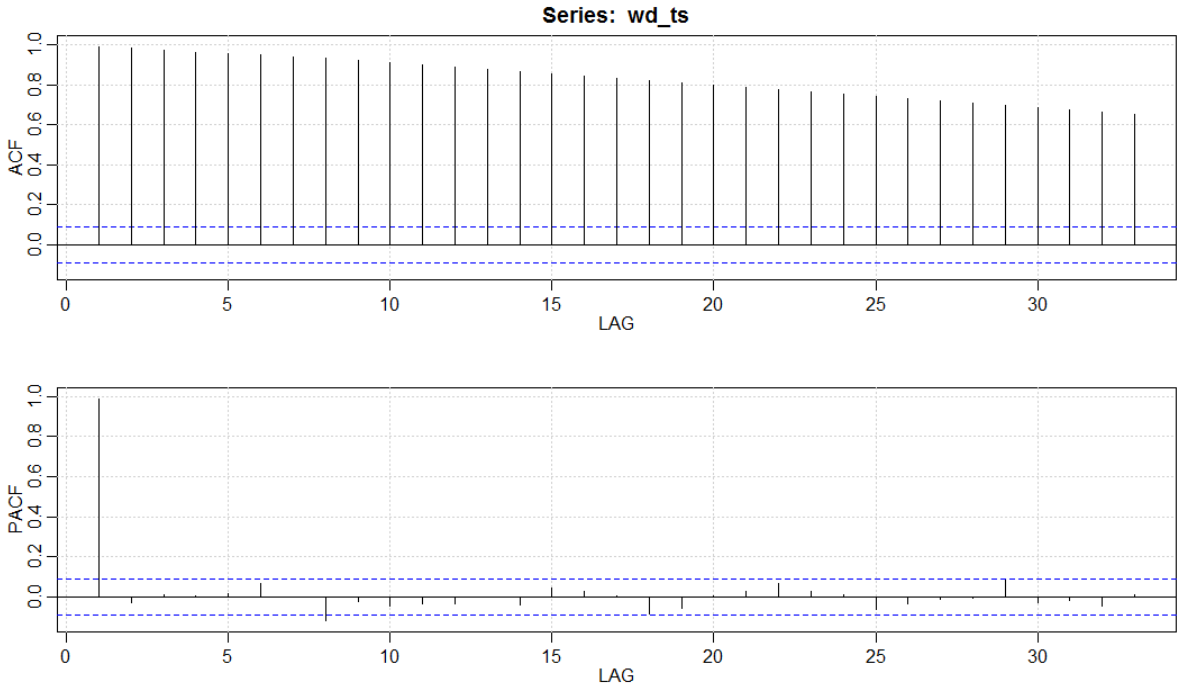
APPENDIX

```
> wd <- read.csv("C:/Users/Jonathan!/Desktop/SCHOOL THINGS!!!/STA4853/wd.csv
")
> wd$Date <- strptime(wd$Date,format="%m/%d/%y")
> format(wd$Date,format="%Y-%m-%d")
> wd=wd[order(wd$Date),]
> wd_ts=ts(wd$Close,frequency=1)
> plot.ts(wd_ts,xy.labels=TRUE)
```

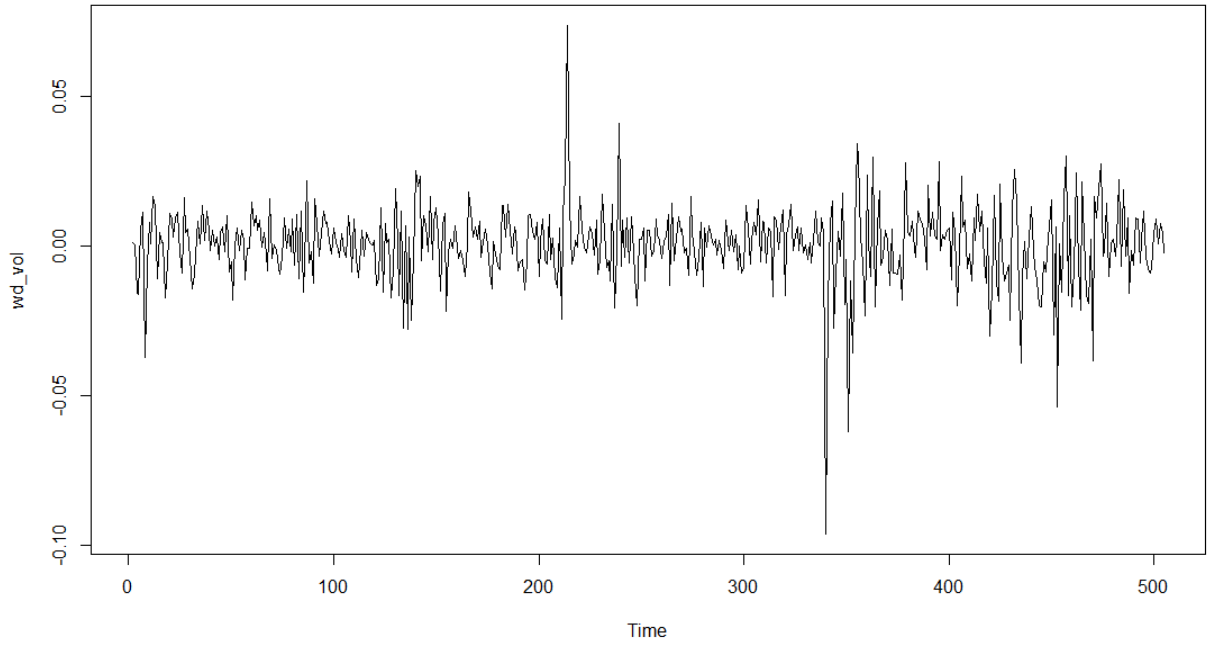



```
> acf2(wd_ts)
       ACF  PACF
 [1,] 0.99  0.99
 [2,] 0.98 -0.03
 [3,] 0.97  0.01
 [4,] 0.96  0.00
 [5,] 0.96  0.02
 [6,] 0.95  0.07
 [7,] 0.94  0.00
 [8,] 0.93 -0.12
 [9,] 0.92 -0.03
[10,] 0.91 -0.05
[11,] 0.90 -0.03
[12,] 0.89 -0.04
[13,] 0.88  0.00
[14,] 0.87 -0.04
[15,] 0.85  0.04
[16,] 0.84  0.03
[17,] 0.83  0.00
[18,] 0.82 -0.09
[19,] 0.81 -0.06
[20,] 0.80  0.00
[21,] 0.78  0.03
[22,] 0.77  0.07
[23,] 0.76  0.03
```

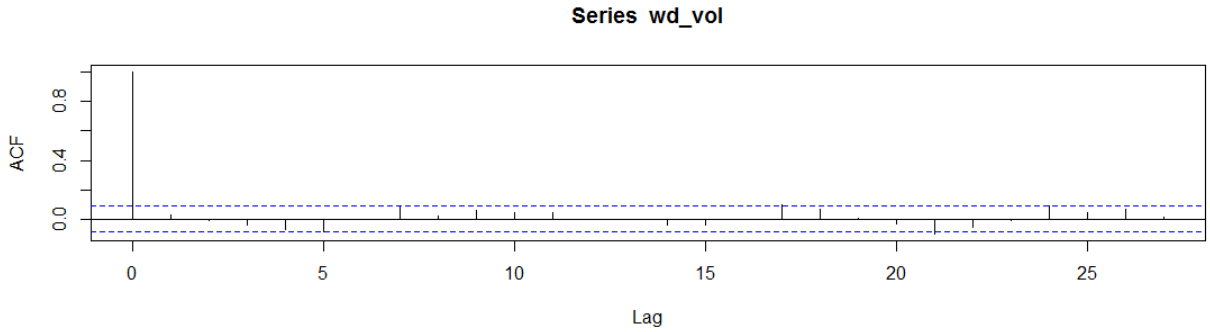
```
[24,] 0.75  0.01
[25,] 0.74 -0.06
[26,] 0.73 -0.03
[27,] 0.72 -0.02
[28,] 0.71 -0.01
[29,] 0.70  0.08
[30,] 0.69 -0.03
[31,] 0.68 -0.02
[32,] 0.66 -0.05
[33,] 0.65  0.01
```

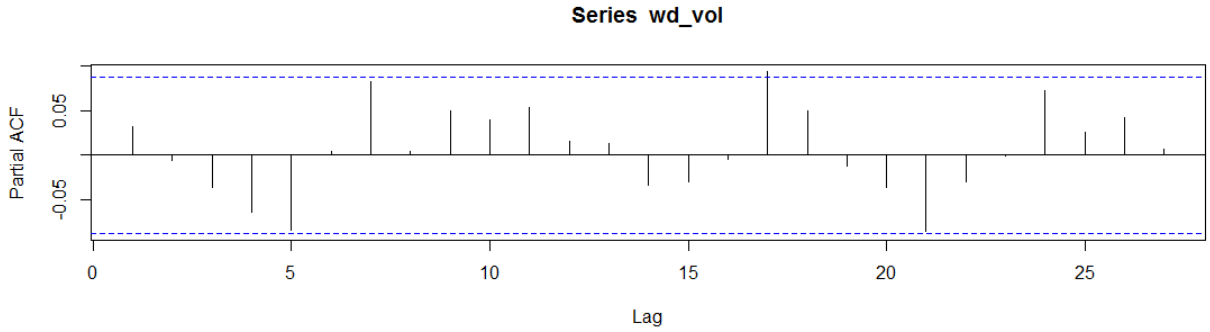



```
> wd_vol=diff(log(wd_ts))
> plot.ts(wd_vol)
```



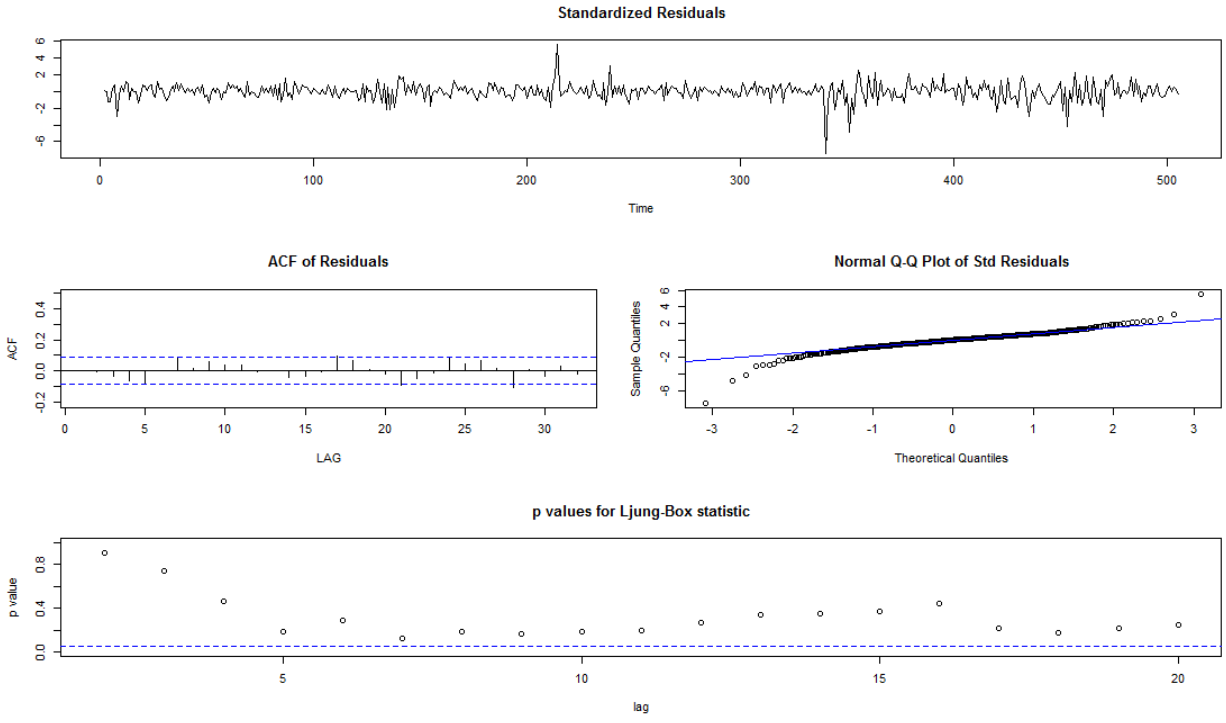

```
> par(mfrow=c(2,1))
> acf(wd_vol)
> pacf(wd_vol)
```





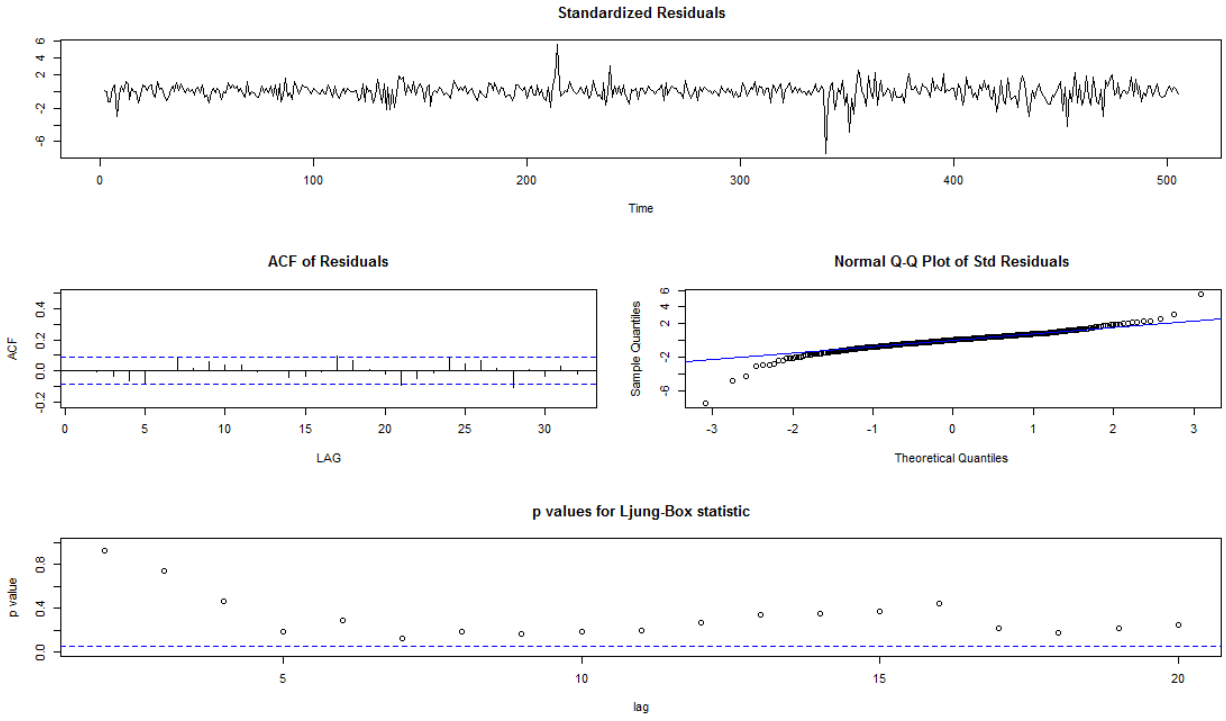

```
> model1=sarima(wd_vol,1,0,0); model1
initial  value -4.345327
iter   2 value -4.345833
```

```
iter   2 value -4.345833
iter   2 value -4.345833
final  value -4.345833
converged
initial  value -4.346821
iter   1 value -4.346821
final  value -4.346821
converged
$fit

Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), xreg = xmean, include.mean = FALSE, optim.control = list
(trace = trc,
    REPORT = 1, reltol = tol))

Coefficients:
         ar1  xmean
      0.0318  4e-04
s.e.  0.0445  6e-04

sigma^2 estimated as 0.0001676:  log likelihood = 1475.65,  aic = -2945.31

$AIC
[1] -7.685708

$AICc
[1] -7.681644

$BIC
[1] -8.668952
```



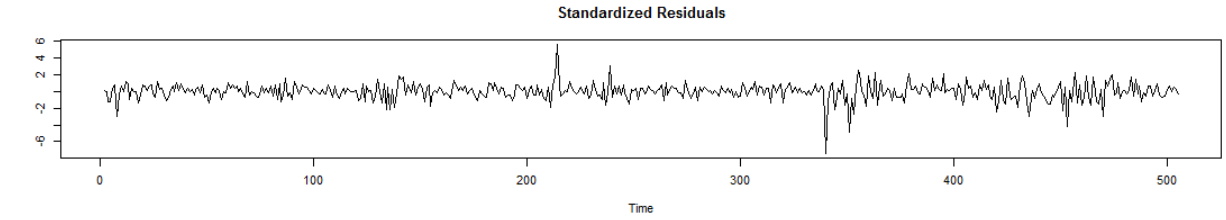

```
> model2=sarima(wd_vol,0,0,1); model2
initial  value -4.346315
iter   2 value -4.346827
iter   2 value -4.346827
```

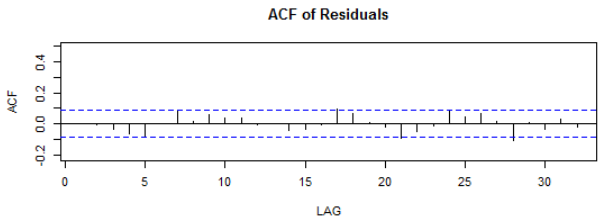
```
iter   2 value -4.346827
final  value -4.346827
converged
initial  value -4.346826
iter   1 value -4.346826
final  value -4.346826
converged
$fit

Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), xreg = xmean, include.mean = FALSE, optim.control = list
(trace = trc,
    REPORT = 1, reltol = tol))

Coefficients:
         ma1  xmean
      0.0320  4e-04
s.e.  0.0446  6e-04

sigma^2 estimated as 0.0001676:  log likelihood = 1475.66,  aic = -2945.31

$AIC
[1] -7.685717

$AICc
[1] -7.681654

$BIC
[1] -8.668961
```

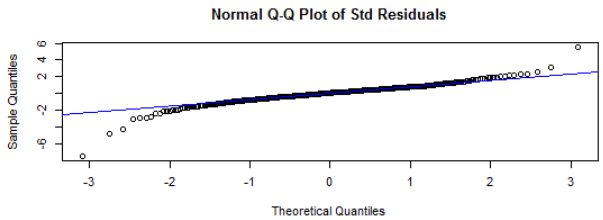


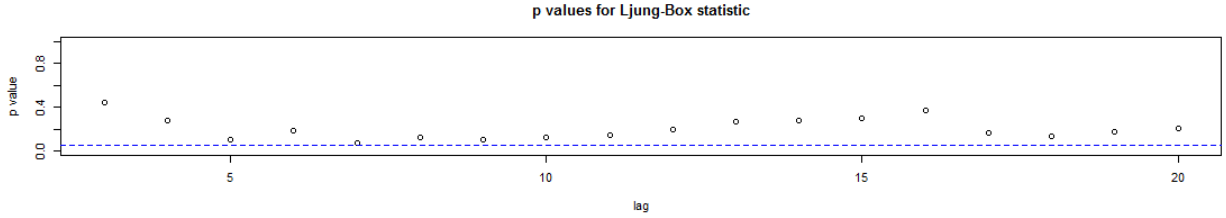

```
> model3=sarima(wd_vol,1,0,1); model3
initial  value -4.345327
iter   2 value -4.345337
iter   3 value -4.345836
iter   3 value -4.345836
```

```
iter   3 value -4.345836
final  value -4.345836
converged
initial  value -4.346824
iter   1 value -4.346824
final  value -4.346824
converged
$fit

Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), xreg = xmean, include.mean = FALSE, optim.control = list
(trace = trc,
    REPORT = 1, reltol = tol))

Coefficients:
         ar1     ma1  xmean
      0.0156  0.0162  4e-04
s.e.  0.7989  0.8018  6e-04

sigma^2 estimated as 0.0001676:  log likelihood = 1475.65,  aic = -2943.31

$AIC
[1] -7.681745

$AICc
[1] -7.677618

$BIC
[1] -8.656611
```

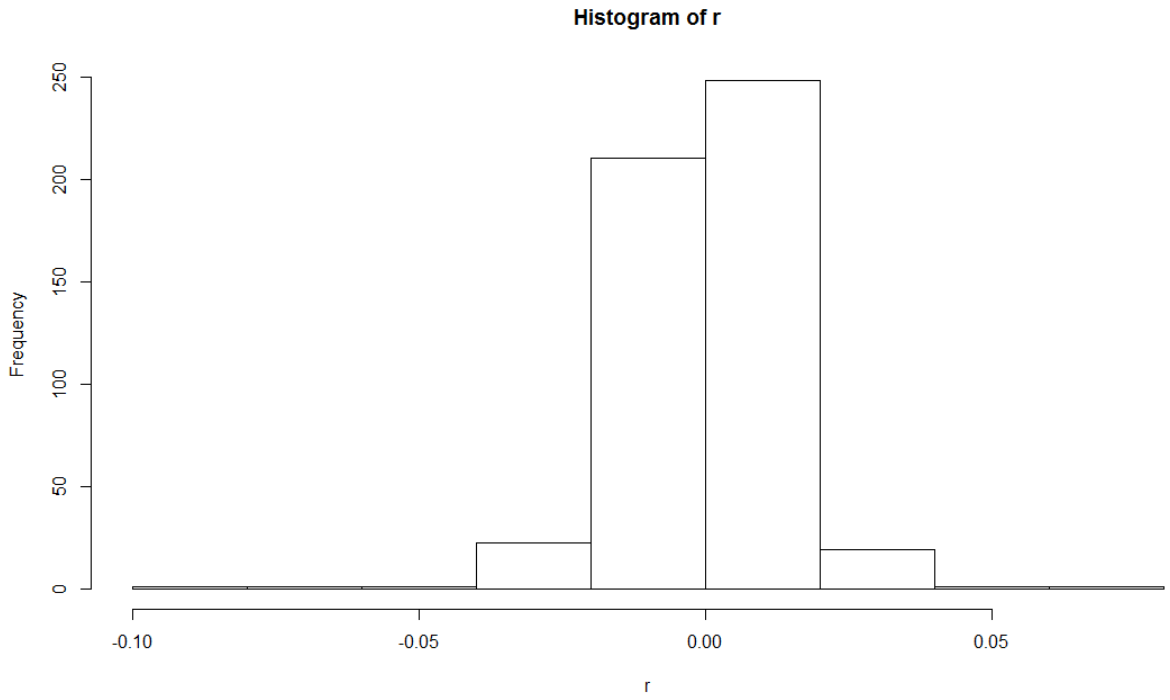




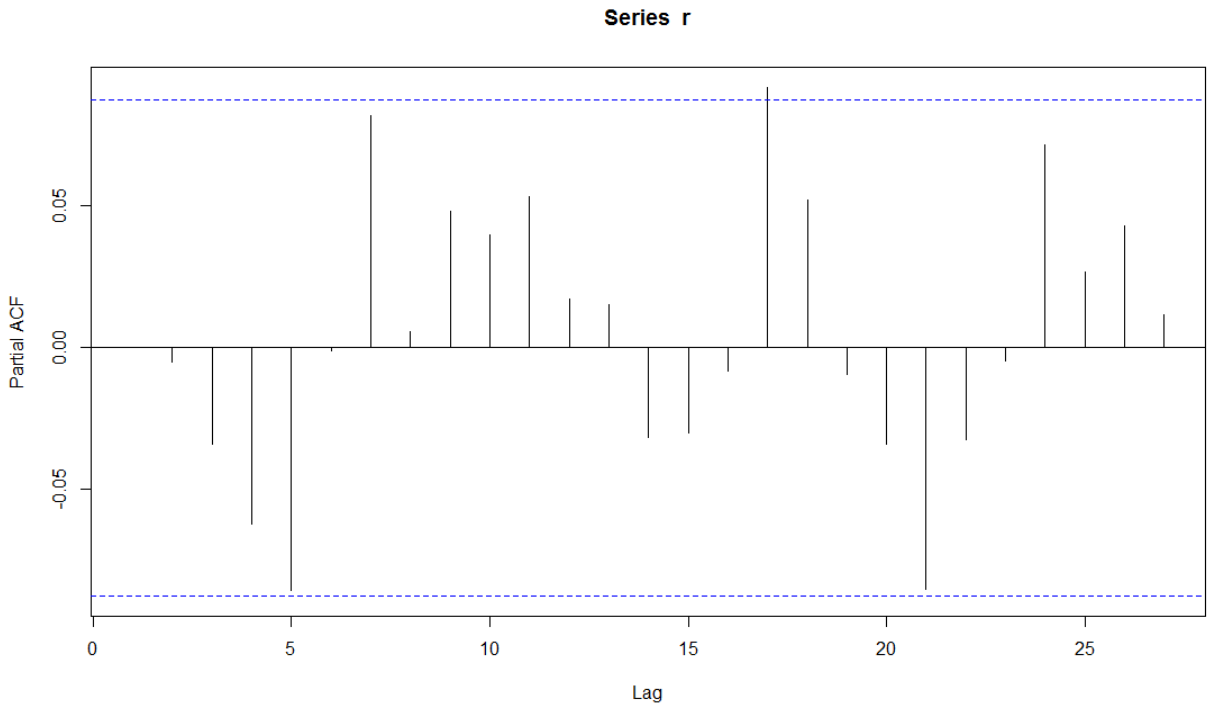





```
> r=residuals(model1$fit)
> par(mfrow=c(1,1))
> hist(r)
```



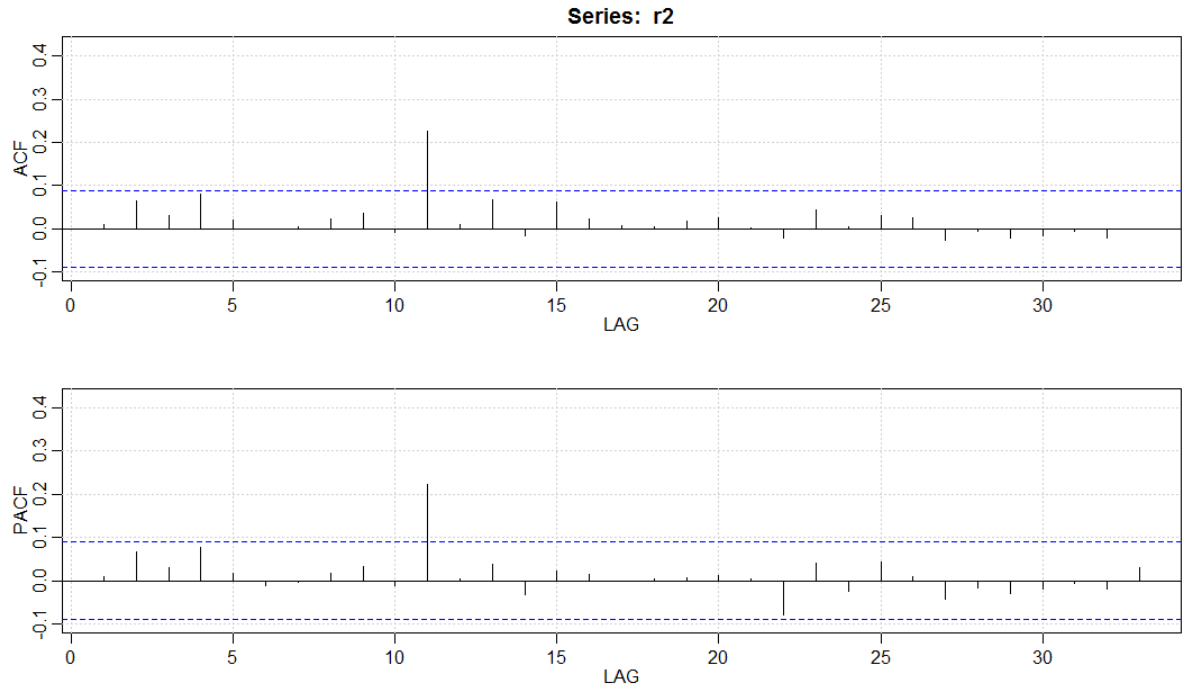

```
> pacf(r)
```




```
> Box.test(r,10,type="Ljung-Box",fitdf=1)

        Box-Ljung test

data:  r
```

```
X-squared = 12.62, df = 9, p-value = 0.1806
```

```
> r2=r^2
> acf2(r2)
        ACF  PACF
 [1,]  0.01  0.01
 [2,]  0.07  0.07
 [3,]  0.03  0.03
 [4,]  0.08  0.08
 [5,]  0.02  0.02
 [6,]  0.00 -0.01
 [7,]  0.00  0.00
 [8,]  0.02  0.02
 [9,]  0.04  0.03
[10,] -0.01 -0.01
[11,]  0.23  0.22
[12,]  0.01  0.01
[13,]  0.07  0.04
[14,] -0.02 -0.03
[15,]  0.06  0.02
[16,]  0.02  0.01
[17,]  0.01  0.00
[18,]  0.00  0.01
[19,]  0.02  0.01
[20,]  0.03  0.01
[21,]  0.00  0.00
[22,] -0.02 -0.08
[23,]  0.04  0.04
[24,]  0.00 -0.02
[25,]  0.03  0.04
[26,]  0.02  0.01
[27,] -0.03 -0.04
[28,] -0.01 -0.02
[29,] -0.02 -0.03
[30,] -0.02 -0.02
[31,] -0.01 -0.01
[32,] -0.02 -0.02
[33,]  0.00  0.03
```




```
> gF.1=garchFit(~arma(1,0)+garch(1,1),wd_vol,cond.dist="norm")

Series Initialization:
 ARMA Model:                 arma
 Formula Mean:               ~ arma(1, 0)
 GARCH Model:                garch
 Formula Variance:           ~ garch(1, 1)
 ARMA Order:                 1 0
 Max ARMA Order:             1
 GARCH Order:                1 1
 Max GARCH Order:            1
 Maximum Order:              1
 Conditional Dist:           norm
 h.start:                    2
 llh.start:                  1
 Length of Series:           504
 Recursion Init:             mci
 Series Scale:               0.01296733

Parameter Initialization:
 Initial Parameters:          $params
 Limits of Transformations:   $U, $V
 Which Parameters are Fixed?  $includes
 Parameter Matrix:
                     U           V     params includes
    mu     -0.29739753   0.2973975 0.02973039     TRUE
    ar1    -0.99999999   1.0000000 0.03176384     TRUE
    omega   0.00000100 100.0000000 0.10000000     TRUE
    alpha1  0.00000001   1.0000000 0.10000000     TRUE
    gamma1 -0.99999999   1.0000000 0.10000000    FALSE
    beta1   0.00000001   1.0000000 0.80000000     TRUE
    delta   0.00000000   2.0000000 2.00000000    FALSE
    skew    0.10000000  10.0000000 1.00000000    FALSE
    shape   1.00000000  10.0000000 4.00000000    FALSE
 Index List of Parameters to be Optimized:
```

```
    mu    ar1  omega alpha1  beta1
     1      2      3      4      6
 Persistence:                  0.9


--- START OF TRACE ---
Selected Algorithm: nlminb

R coded nlminb Solver:

  0:     697.72221: 0.0297304 0.0317638 0.100000 0.100000 0.800000
  1:     697.52219: 0.0297303 0.0308698 0.104143 0.101205 0.802059
  2:     697.43016: 0.0297302 0.0283757 0.105557 0.0984950 0.799215
  3:     697.35387: 0.0297303 0.0220244 0.112841 0.0989164 0.798193
  4:     697.16299: 0.0297310 0.0150627 0.111786 0.100355 0.791638
  5:     697.07697: 0.0297319 0.00850893 0.114747 0.105571 0.787675
  6:     696.93418: 0.0297341 0.00449280 0.121807 0.111880 0.771162
  7:     696.89501: 0.0297245 0.00785481 0.134451 0.118359 0.758309
  8:     696.79934: 0.0297073 -0.00508312 0.131121 0.130306 0.750764
  9:     696.78590: 0.0297069 -0.00602947 0.131773 0.128539 0.749085
 10:     696.77583: 0.0297021 -0.00789439 0.133292 0.128286 0.750263
 11:     696.76436: 0.0296867 -0.00962785 0.133511 0.128186 0.748281
 12:     696.75530: 0.0296720 -0.00835082 0.136204 0.130834 0.744689
 13:     696.73492: 0.0295504 -0.0130852 0.139085 0.134932 0.737481
 14:     696.73439: 0.0295152 -0.0146014 0.139800 0.134450 0.738421
 15:     696.73116: 0.0294802 -0.0136015 0.140178 0.134890 0.736805
 16:     696.72885: 0.0294132 -0.0125635 0.142855 0.135842 0.734143
 17:     696.72654: 0.0291614 -0.0161491 0.143173 0.135146 0.733721
 18:     696.72539: 0.0289060 -0.0158110 0.143331 0.136975 0.730886
 19:     696.72163: 0.0286429 -0.0159024 0.144506 0.138756 0.729319
 20:     696.71951: 0.0275566 -0.0158303 0.146470 0.141709 0.725143
 21:     696.71908: 0.0265508 -0.0156964 0.147079 0.142106 0.724340
 22:     696.71904: 0.0263549 -0.0158464 0.146927 0.142224 0.724368
 23:     696.71904: 0.0263282 -0.0157668 0.146842 0.142118 0.724524
 24:     696.71904: 0.0263321 -0.0157785 0.146838 0.142111 0.724534

Final Estimate of the Negative LLH:
 LLH:  -1493.323    norm LLH:  -2.962943
          mu           ar1         omega        alpha1         beta1
 3.414574e-04 -1.577849e-02  2.469105e-05  1.421110e-01  7.245339e-01

R-optimhess Difference Approximated Hessian Matrix:
                  mu          ar1         omega        alpha1         beta1
mu     -3750065.677  -3994.76770  6.299140e+07 -5.487765e+03  6.117233e+03
ar1       -3994.768   -327.61053  4.391753e+05 -5.520737e+01  2.933576e+01
omega  62991404.955 439175.32430 -1.709123e+11 -1.419975e+07 -2.357218e+07
alpha1    -5487.765    -55.20737 -1.419975e+07 -2.256504e+03 -2.631126e+03
beta1      6117.233     29.33576 -2.357218e+07 -2.631126e+03 -3.802602e+03
attr(,"time")
Time difference of 0.0312531 secs

--- END OF TRACE ---


Time to Estimate Parameters:
 Time difference of 0.1406372 secs
> summary(gF.1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 0) + garch(1, 1), data = wd_vol,
    cond.dist = "norm")
```

```
Mean and Variance Equation:
 data ~ arma(1, 0) + garch(1, 1)
<environment: 0x0000000012b13e30>
 [data = wd_vol]

Conditional Distribution:
 norm

Coefficient(s):
         mu          ar1        omega       alpha1        beta1
 3.4146e-04  -1.5778e-02   2.4691e-05   1.4211e-01   7.2453e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      3.415e-04   5.363e-04    0.637  0.52436
ar1    -1.578e-02   5.659e-02   -0.279  0.78039
omega   2.469e-05   8.331e-06    2.964  0.00304 **
alpha1  1.421e-01   6.535e-02    2.175  0.02965 *
beta1   7.245e-01   8.947e-02    8.098 6.66e-16 ***
---
Signif. codes:  0 ·**·0.001 ·*·0.01 ··0.05 ··0.1 ·· 1

Log Likelihood:
 1493.323    normalized:  2.962943

Description:
 Fri Apr 22 20:54:10 2016 by user: Jonathan!


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  5414.291  0
 Shapiro-Wilk Test  R    W      0.8953559 0
 Ljung-Box Test     R    Q(10)  12.33449  0.263294
 Ljung-Box Test     R    Q(15)  15.77142  0.3974037
 Ljung-Box Test     R    Q(20)  21.65403  0.3595749
 Ljung-Box Test     R^2  Q(10)  0.8227066 0.9999302
 Ljung-Box Test     R^2  Q(15)  11.91063  0.6857805
 Ljung-Box Test     R^2  Q(20)  12.01946  0.9154047
 LM Arch Test       R    TR^2   11.0757   0.5224423

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.906045 -5.864155 -5.906240 -5.889613

> gF.2=garchFit(~arma(1,0)+garch(1,0),wd_vol,cond.dist="norm")

Series Initialization:
 ARMA Model:                 arma
 Formula Mean:               ~ arma(1, 0)
 GARCH Model:                garch
 Formula Variance:           ~ garch(1, 0)
 ARMA Order:                 1 0
 Max ARMA Order:             1
 GARCH Order:                1 0
 Max GARCH Order:            1
 Maximum Order:              1
 Conditional Dist:           norm
 h.start:                    2
 llh.start:                  1
```

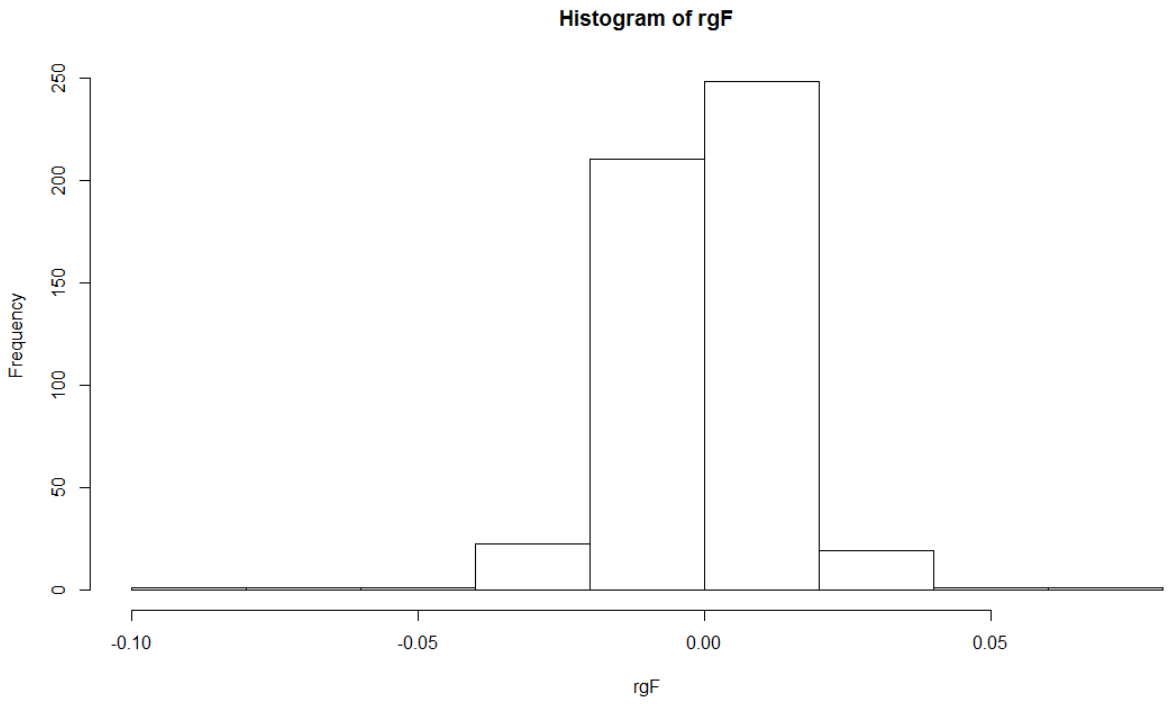
```
 Length of Series:           504
 Recursion Init:             mci
 Series Scale:               0.01296733

Parameter Initialization:
 Initial Parameters:           $params
 Limits of Transformations:    $U, $V
 Which Parameters are Fixed?  $includes
 Parameter Matrix:
                      U           V     params includes
    mu     -0.29739753   0.2973975 0.02973039     TRUE
    ar1    -0.99999999   1.0000000 0.03176384     TRUE
    omega   0.00000100 100.0000000 0.10000000     TRUE
    alpha1  0.00000001   1.0000000 0.10000000     TRUE
    gamma1 -0.99999999   1.0000000 0.10000000    FALSE
    delta   0.00000000   2.0000000 2.00000000    FALSE
    skew    0.10000000  10.0000000 1.00000000    FALSE
    shape   1.00000000  10.0000000 4.00000000    FALSE
 Index List of Parameters to be Optimized:
    mu    ar1  omega alpha1
     1      2      3      4
 Persistence:                  0.1


--- START OF TRACE ---
Selected Algorithm: nlminb

R coded nlminb Solver:

  0:     1728.6092: 0.0297304 0.0317638 0.100000 0.100000
  1:     725.05905: 0.0297123 0.0186317  1.07150 0.336656
  2:     721.57028: 0.0296733 -0.0390232  1.09084 0.232487
  3:     715.64707: 0.0296203 -0.0142096 0.926959 1.00000e-08
  4:     715.55014: 0.0296205 0.0221990 0.957794 0.142068
  5:     714.75999: 0.0295834 -0.00434453 0.888310 0.133080
  6:     714.36548: 0.0295466 -0.0438885 0.910184 0.0733204
  7:     714.10968: 0.0290350 0.00215644 0.964898 0.0589957
  8:     714.09746: 0.0270679 0.00197506 0.974744 0.0251883
  9:     714.03740: 0.0251192 -0.00138942 0.946745 0.0481340
 10:     714.03130: 0.0247102 -0.00341037 0.956400 0.0495691
 11:     714.02897: 0.0238899 -0.00214013 0.954268 0.0481811
 12:     714.02873: 0.0232716 -0.00227627 0.953948 0.0481233
 13:     714.02868: 0.0228636 -0.00228803 0.954013 0.0482177
 14:     714.02868: 0.0228348 -0.00233745 0.953997 0.0483027
 15:     714.02868: 0.0228348 -0.00233134 0.953997 0.0483139

Final Estimate of the Negative LLH:
 LLH:  -1476.014    norm LLH:  -2.928599
          mu           ar1         omega        alpha1
 0.0002961064 -0.0023313423  0.0001604161  0.0483138786

R-optimhess Difference Approximated Hessian Matrix:
                  mu        ar1         omega        alpha1
mu     -3030610.791 -2301.5460     1841011.4    -5179.2765
ar1       -2301.546  -377.2360       50240.4     -154.8573
omega   1841011.409 50240.3993 -9087575859.3 -1110538.9274
alpha1    -5179.276  -154.8573    -1110538.9     -400.0007
attr(,"time")
Time difference of 0.04987192 secs

--- END OF TRACE ---


Time to Estimate Parameters:
```

```
 Time difference of 0.1123769 secs
> summary(gF.2)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 0) + garch(1, 0), data = wd_vol,
    cond.dist = "norm")

Mean and Variance Equation:
 data ~ arma(1, 0) + garch(1, 0)
<environment: 0x00000000104f2418>
 [data = wd_vol]

Conditional Distribution:
 norm

Coefficient(s):
         mu          ar1        omega       alpha1
 0.00029611  -0.00233134   0.00016042   0.04831388

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      2.961e-04   5.856e-04    0.506    0.613
ar1    -2.331e-03   5.992e-02   -0.039    0.969
omega   1.604e-04   1.388e-05   11.561   <2e-16 ***
alpha1  4.831e-02   7.273e-02    0.664    0.507
---
Signif. codes:  0 ·***·0.001 ·**·0.01 ·*·0.05 ··0.1 ·· 1

Log Likelihood:
 1476.014    normalized:  2.928599

Description:
 Fri Apr 22 20:54:11 2016 by user: Jonathan!


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  1944.751  0
 Shapiro-Wilk Test  R    W      0.9116068 0
 Ljung-Box Test     R    Q(10)  13.29421  0.2076844
 Ljung-Box Test     R    Q(15)  15.97062  0.3840069
 Ljung-Box Test     R    Q(20)  24.01359  0.2417991
 Ljung-Box Test     R^2  Q(10)  6.588125  0.7636718
 Ljung-Box Test     R^2  Q(15)  40.42357  0.0003912901
 Ljung-Box Test     R^2  Q(20)  41.03812  0.003683473
 LM Arch Test       R    TR^2   33.33994  0.0008565265

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.841325 -5.807812 -5.841449 -5.828179

> gF.3=garchFit(~arma(1,0)+garch(2,0),wd_vol,cond.dist="norm")

Series Initialization:
 ARMA Model:                 arma
 Formula Mean:               ~ arma(1, 0)
 GARCH Model:                garch
 Formula Variance:           ~ garch(2, 0)
```

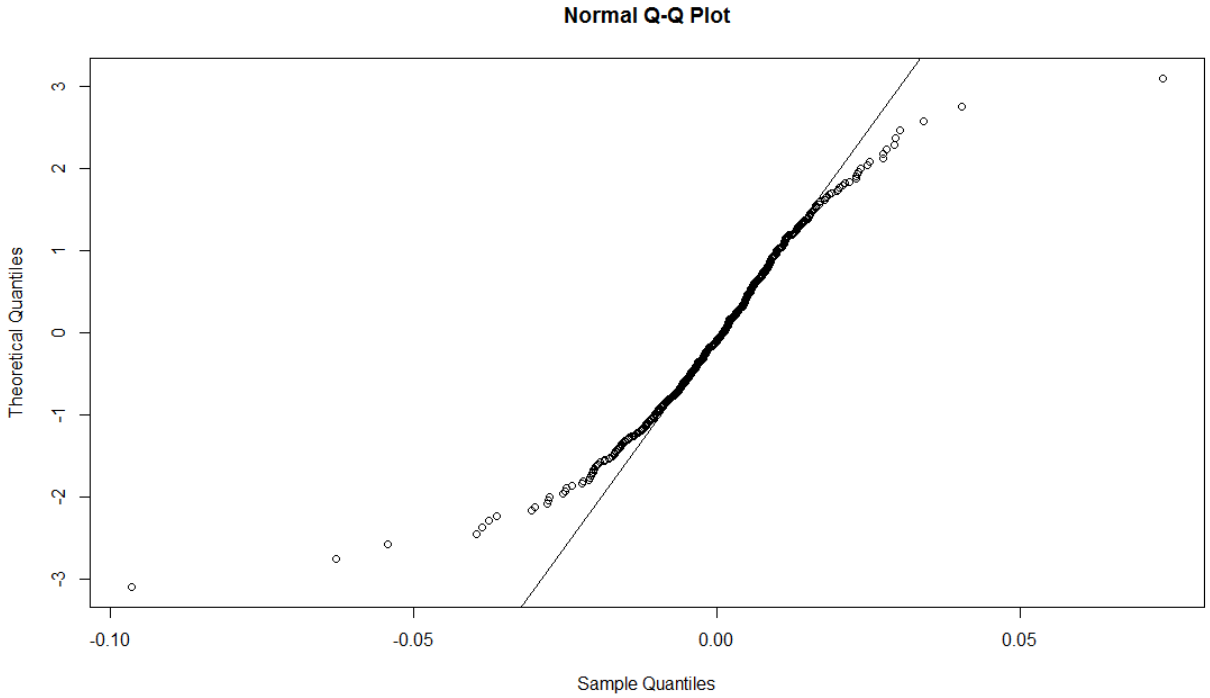
```
 ARMA Order:                 1 0
 Max ARMA Order:             1
 GARCH Order:                2 0
 Max GARCH Order:            2
 Maximum Order:              2
 Conditional Dist:           norm
 h.start:                    3
 llh.start:                  1
 Length of Series:           504
 Recursion Init:             mci
 Series Scale:               0.01296733

Parameter Initialization:
 Initial Parameters:          $params
 Limits of Transformations:   $U, $V
 Which Parameters are Fixed?  $includes
 Parameter Matrix:
                       U           V     params includes
    mu      -0.29739753   0.2973975 0.02973039     TRUE
    ar1     -0.99999999   1.0000000 0.03176384     TRUE
    omega    0.00000100 100.0000000 0.10000000     TRUE
    alpha1   0.00000001   1.0000000 0.05000000     TRUE
    alpha2   0.00000001   1.0000000 0.05000000     TRUE
    gamma1  -0.99999999   1.0000000 0.10000000    FALSE
    gamma2  -0.99999999   1.0000000 0.10000000    FALSE
    delta    0.00000000   2.0000000 2.00000000    FALSE
    skew     0.10000000  10.0000000 1.00000000    FALSE
    shape    1.00000000  10.0000000 4.00000000    FALSE
 Index List of Parameters to be Optimized:
    mu    ar1  omega alpha1 alpha2
     1      2      3      4      5
 Persistence:                  0.1


--- START OF TRACE ---
Selected Algorithm: nlminb

R coded nlminb Solver:

  0:     1422.6094: 0.0297304 0.0317638 0.100000 0.0500000 0.0500000
  1:     718.68092: 0.0297202 0.0266825 0.900509 0.376984 0.552237
  2:     713.50078: 0.0296429 -0.106272 0.815877 0.0421907 0.888492
  3:     700.78431: 0.0268533 0.154499 0.691223 1.00000e-08  1.00000
  4:     697.12828: 0.0158052 0.110068 0.646594 1.00000e-08  1.00000
  5:     692.61885: -0.00942690 0.0758163 0.458651 1.00000e-08  1.00000
  6:     691.55917: -0.0193510 0.0398697 0.409349 0.103389 0.941097
  7:     690.96628: -0.0192950 0.0973143 0.446740 0.101874 0.892330
  8:     690.15059: -0.0211741 0.0727252 0.439778 0.0828161 0.846792
  9:     689.56269: -0.0210413 0.0651371 0.483649 0.0730531 0.776191
 10:     689.42114: -0.0227340 0.0791904 0.458918 0.0642968 0.721813
 11:     689.20176: -0.0248452 0.0737512 0.487689 0.0874925 0.696383
 12:     689.11066: -0.0256045 0.0788602 0.494347 0.0899401 0.616672
 13:     689.04086: -0.0233014 0.0486073 0.500515 0.0786838 0.618831
 14:     688.92599: -0.0208326 0.0613020 0.504304 0.0825857 0.618638
 15:     688.78354: -0.00727853 0.0676487 0.512173 0.0656754 0.614598
 16:     688.71018: 0.000846504 0.0638284 0.500549 0.0859003 0.611671
 17:     688.70798: 0.00117040 0.0622797 0.502948 0.0859453 0.612627
 18:     688.70790: 0.00117727 0.0618928 0.503188 0.0863254 0.610873
 19:     688.70790: 0.00118913 0.0619103 0.503164 0.0863275 0.610967
 20:     688.70790: 0.00118903 0.0619103 0.503163 0.0863293 0.610967

Final Estimate of the Negative LLH:
 LLH:  -1501.335    norm LLH:  -2.978839
          mu          ar1        omega       alpha1       alpha2
```

```
1.541850e-05 6.191034e-02 8.460771e-05 8.632930e-02 6.109672e-01

R-optimhess Difference Approximated Hessian Matrix:
                  mu           ar1         omega        alpha1        alpha2
mu     -5335314.0875    582.40238  2.430526e+07  1.619098e+03    -949.45262
ar1          582.4024   -488.75887 -8.877676e+04 -1.567158e+02      20.20994
omega  24305256.5003 -88776.76298 -1.568743e+10 -1.461549e+06 -718540.68304
alpha1     1619.0983   -156.71577 -1.461549e+06 -3.738135e+02     -78.56835
alpha2     -949.4526     20.20994 -7.185407e+05 -7.856835e+01     -88.38939
attr(,"time")
Time difference of 0.03125 secs

--- END OF TRACE ---


Time to Estimate Parameters:
 Time difference of 0.143261 secs
> summary(gF.3)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 0) + garch(2, 0), data = wd_vol,
    cond.dist = "norm")

Mean and Variance Equation:
 data ~ arma(1, 0) + garch(2, 0)
<environment: 0x0000000014397768>
 [data = wd_vol]

Conditional Distribution:
 norm

Coefficient(s):
        mu         ar1       omega      alpha1      alpha2
1.5418e-05  6.1910e-02  8.4608e-05  8.6329e-02  6.1097e-01

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.542e-05   4.377e-04    0.035    0.972
ar1    6.191e-02   5.148e-02    1.203    0.229
omega  8.461e-05   1.164e-05    7.267 3.67e-13 ***
alpha1 8.633e-02   7.335e-02    1.177    0.239
alpha2 6.110e-01   1.391e-01    4.392 1.13e-05 ***
---
Signif. codes:  0 ·***·0.001 ·**·0.01 ·*·0.05 ··0.1 ·· 1

Log Likelihood:
 1501.335    normalized:  2.978839

Description:
 Fri Apr 22 20:54:12 2016 by user: Jonathan!


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  2119.862  0
 Shapiro-Wilk Test  R    W      0.928311  8.526535e-15
 Ljung-Box Test     R    Q(10)  14.8177   0.1388509
 Ljung-Box Test     R    Q(15)  17.81151  0.2727045
```

```
 Ljung-Box Test      R    Q(20)  22.91192  0.2931401
 Ljung-Box Test      R^2  Q(10)  3.145066  0.977801
 Ljung-Box Test      R^2  Q(15)  9.115363  0.8714086
 Ljung-Box Test      R^2  Q(20)  12.42165  0.9007993
 LM Arch Test        R    TR^2   8.402898  0.7529062

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.937836 -5.895945 -5.938030 -5.921404
> gF=gF.1
> rgF=residuals(gF)
> hist(rgF)
```




```
> qqnorm(rgF,datax=TRUE)
> qqline(rgF,datax=TRUE)
```



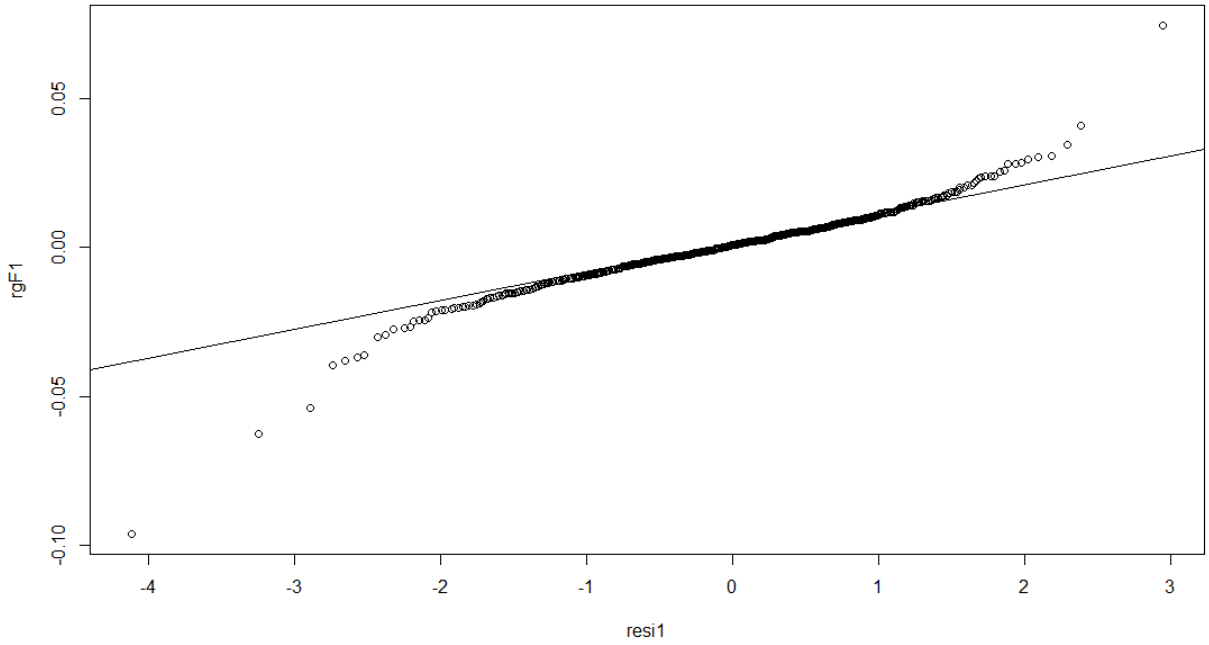

```
> gF1=garchFit(~arma(1,0)+garch(1,1),wd_vol,cond.dist="snorm")

Series Initialization:
 ARMA Model:                 arma
 Formula Mean:               ~ arma(1, 0)
 GARCH Model:                garch
 Formula Variance:           ~ garch(1, 1)
 ARMA Order:                 1 0
 Max ARMA Order:             1
 GARCH Order:                1 1
 Max GARCH Order:            1
 Maximum Order:              1
 Conditional Dist:           snorm
 h.start:                    2
 llh.start:                  1
 Length of Series:           504
 Recursion Init:             mci
 Series Scale:               0.01296733

Parameter Initialization:
 Initial Parameters:          $params
 Limits of Transformations:   $U, $V
 Which Parameters are Fixed?  $includes
 Parameter Matrix:
                      U           V     params includes
    mu     -0.29739753   0.2973975 0.02973039     TRUE
    ar1    -0.99999999   1.0000000 0.03176384     TRUE
    omega   0.00000100 100.0000000 0.10000000     TRUE
    alpha1  0.00000001   1.0000000 0.10000000     TRUE
    gamma1 -0.99999999   1.0000000 0.10000000    FALSE
    beta1   0.00000001   1.0000000 0.80000000     TRUE
    delta   0.00000000   2.0000000 2.00000000    FALSE
    skew    0.10000000  10.0000000 1.00000000     TRUE
    shape   1.00000000  10.0000000 4.00000000    FALSE
 Index List of Parameters to be Optimized:
```

```
    mu    ar1  omega alpha1  beta1   skew
     1      2      3      4      6      8
 Persistence:                  0.9


--- START OF TRACE ---
Selected Algorithm: nlminb

R coded nlminb Solver:

  0:     697.72221: 0.0297304 0.0317638 0.100000 0.100000 0.800000  1.00000
  1:     696.65860: 0.0297302 0.0291481 0.112121 0.103525 0.806024 0.982362
  2:     694.27956: 0.0297300 0.0266526 0.105160 0.0959633 0.794828 0.965737
  3:     691.29615: 0.0297290 0.0202881 0.115998 0.101373 0.795924 0.922541
  4:     688.01368: 0.0297184 0.00766190 0.0928661 0.115965 0.772245 0.840390
  5:     684.96283: 0.0296862 -0.00814506 0.128217 0.173732 0.724153 0.807730
  6:     684.37206: 0.0295488 -0.0349003 0.149415 0.212153 0.650666 0.795315
  7:     684.12130: 0.0294773 -0.0334406 0.144935 0.230860 0.656382 0.790480
  8:     684.11478: 0.0291595 -0.0274723 0.144455 0.222627 0.659333 0.776076
  9:     684.10472: 0.0291553 -0.0278268 0.145216 0.223736 0.660349 0.777076
 10:     684.09811: 0.0291487 -0.0283162 0.143904 0.223953 0.659676 0.778292
 11:     684.09021: 0.0290908 -0.0304416 0.143543 0.226174 0.661213 0.778771
 12:     684.08436: 0.0290048 -0.0326737 0.142009 0.225955 0.660793 0.778945
 13:     684.07920: 0.0288913 -0.0329655 0.142938 0.226245 0.660901 0.779541
 14:     684.07359: 0.0286784 -0.0304947 0.142791 0.227856 0.659179 0.780475
 15:     684.06069: 0.0282030 -0.0344061 0.144054 0.226163 0.658686 0.781758
 16:     684.05995: 0.0281991 -0.0343766 0.144235 0.226557 0.658955 0.781804
 17:     684.05940: 0.0281928 -0.0343298 0.143849 0.226780 0.658772 0.781843
 18:     684.05844: 0.0281660 -0.0344446 0.143920 0.227264 0.659006 0.781825
 19:     684.03976: 0.0269961 -0.0411348 0.140534 0.230644 0.659935 0.780097
 20:     683.80689: 0.00786397 -0.0378379 0.149775 0.239211 0.639813 0.782242
 21:     683.71959: 0.00262133 -0.0324294 0.142582 0.224316 0.661841 0.783279
 22:     683.68509: -0.000210820 -0.0339680 0.146913 0.238137 0.652499 0.7780
81
 23:     683.67433: -0.00309057 -0.0311200 0.144327 0.239072 0.654082 0.77714
7
 24:     683.67389: -0.00395683 -0.0315861 0.143811 0.239276 0.654371 0.77803
9
 25:     683.67382: -0.00389802 -0.0314489 0.143822 0.239059 0.654443 0.77758
8
 26:     683.67382: -0.00383802 -0.0314078 0.143845 0.239085 0.654425 0.77766
1
 27:     683.67381: -0.00385115 -0.0314232 0.143842 0.239085 0.654424 0.77766
0
 28:     683.67381: -0.00385067 -0.0314219 0.143842 0.239085 0.654425 0.77766
0

Final Estimate of the Negative LLH:
 LLH:  -1506.369    norm LLH:  -2.988827
           mu           ar1         omega        alpha1         beta1
  skew
-4.993295e-05 -3.142189e-02  2.418727e-05  2.390846e-01  6.544249e-01  7.7765
96e-01

R-optimhess Difference Approximated Hessian Matrix:
                  mu           ar1         omega        alpha1         beta1
mu     -4.705100e+06  -3951.028825 -1.162206e+08 -1.604947e+04 -2.085207e+04
ar1    -3.951029e+03   -327.685909  6.251140e+05 -5.078687e+01  4.327861e+01
omega  -1.162206e+08 625114.049983 -1.395963e+11 -1.000589e+07 -1.810749e+07
alpha1 -1.604947e+04    -50.786869 -1.000589e+07 -1.284216e+03 -1.724169e+03
beta1  -2.085207e+04     43.278608 -1.810749e+07 -1.724169e+03 -2.861254e+03
skew    2.564292e+03     -4.442523 -2.000127e+05 -1.876150e+02 -1.223295e+02
                skew
mu      2.564292e+03
```

```
ar1    -4.442523e+00
omega  -2.000127e+05
alpha1 -1.876150e+02
beta1  -1.223295e+02
skew   -7.526912e+02
attr(,"time")
Time difference of 0.05507994 secs

--- END OF TRACE ---


Time to Estimate Parameters:
 Time difference of 0.2063909 secs
> summary(gF1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 0) + garch(1, 1), data = wd_vol,
    cond.dist = "snorm")

Mean and Variance Equation:
 data ~ arma(1, 0) + garch(1, 1)
<environment: 0x00000000102db588>
 [data = wd_vol]

Conditional Distribution:
 snorm

Coefficient(s):
         mu          ar1        omega       alpha1        beta1         skew

-4.9933e-05  -3.1422e-02   2.4187e-05   2.3908e-01   6.5442e-01   7.7766e-01


Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu     -4.993e-05   4.747e-04   -0.105 0.916233
ar1    -3.142e-02   5.778e-02   -0.544 0.586587
omega   2.419e-05   6.803e-06    3.555 0.000378 ***
alpha1  2.391e-01   7.327e-02    3.263 0.001102 **
beta1   6.544e-01   7.326e-02    8.933  < 2e-16 ***
skew    7.777e-01   3.820e-02   20.359  < 2e-16 ***
---
Signif. codes:  0 ·**·0.001 ·*·0.01 ··0.05 ··0.1 ··1

Log Likelihood:
 1506.369    normalized:  2.988827

Description:
 Fri Apr 22 20:55:16 2016 by user: Jonathan!


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  6690.299  0
 Shapiro-Wilk Test  R    W      0.8913595 0
 Ljung-Box Test     R    Q(10)  12.69296  0.2413492
 Ljung-Box Test     R    Q(15)  16.58588  0.344217
 Ljung-Box Test     R    Q(20)  21.87417  0.3473773
```

```
 Ljung-Box Test      R^2  Q(10)  0.9313568 0.9998759
 Ljung-Box Test      R^2  Q(15)  12.23232  0.661366
 Ljung-Box Test      R^2  Q(20)  12.32686  0.9043665
 LM Arch Test        R    TR^2   11.39507  0.4953983

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.953844 -5.903575 -5.954123 -5.934125
```

```
> rgF1=residuals(gF1)
> set.seed(6737181)
> resi1=rsnorm(5000,xi=0.7777)
> qqplot(resi1,rgF1)
> qqline(rgF1,distribution=qsnorm)
```

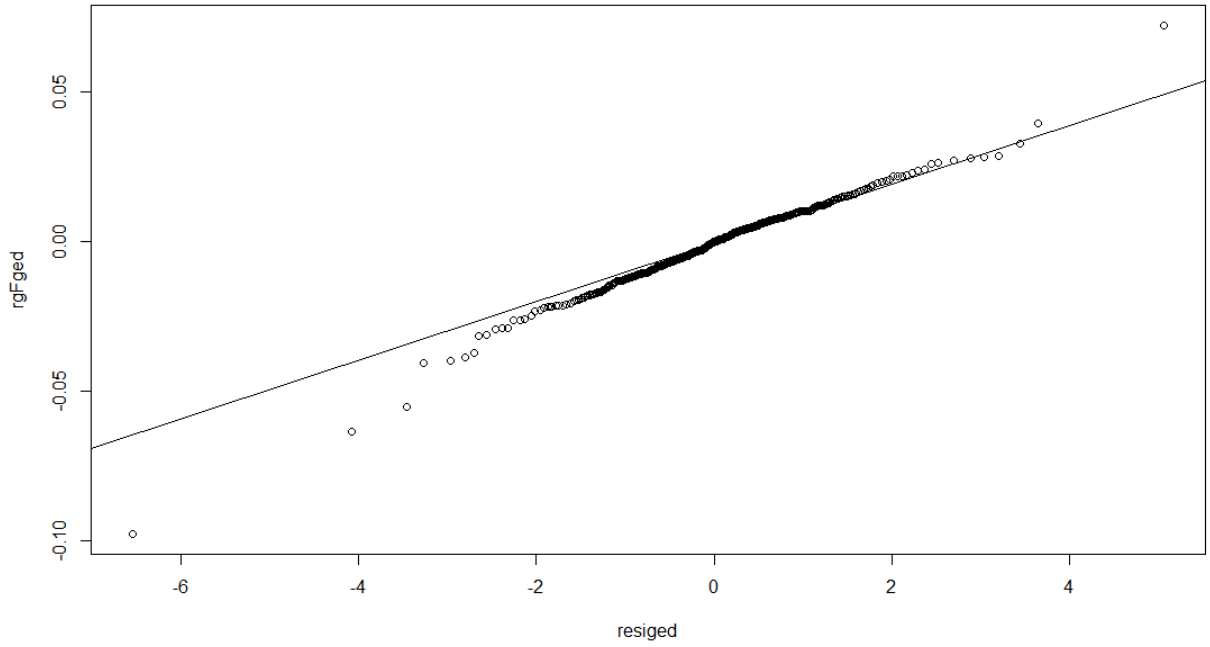



```
> gedFit(rgF1)
$par
       mean          sd          nu
0.001325915 0.012532340 1.078527005

$objective
[1] -1519.822

$convergence
[1] 0

$iterations
[1] 20

$evaluations
function gradient
      43       80

$message
[1] "relative convergence (4)"
```

```
Warning messages:
1: In nlminb(start = start, objective = loglik, lower = c(-Inf, 0,  :
  NA/NaN function evaluation
2: In nlminb(start = start, objective = loglik, lower = c(-Inf, 0,  :
  NA/NaN function evaluation
> n=gedFit(rgF1)$par[3]; n
Warning messages:
1: In nlminb(start = start, objective = loglik, lower = c(-Inf, 0,  :
  NA/NaN function evaluation
2: In nlminb(start = start, objective = loglik, lower = c(-Inf, 0,  :
  NA/NaN function evaluation
      nu
1.078527
> gFged=garchFit(~arma(1,0)+garch(1,1),wd_vol,cond.dist="ged",shape=n,includ
e.shape=FALSE)

Series Initialization:
 ARMA Model:                 arma
 Formula Mean:               ~ arma(1, 0)
 GARCH Model:                garch
 Formula Variance:           ~ garch(1, 1)
 ARMA Order:                 1 0
 Max ARMA Order:             1
 GARCH Order:                1 1
 Max GARCH Order:            1
 Maximum Order:              1
 Conditional Dist:           ged
 h.start:                    2
 llh.start:                  1
 Length of Series:           504
 Recursion Init:             mci
 Series Scale:               0.01296733

Parameter Initialization:
 Initial Parameters:          $params
 Limits of Transformations:   $U, $V
 Which Parameters are Fixed?  $includes
 Parameter Matrix:
                     U           V     params includes
    mu     -0.29739753   0.2973975 0.02973039     TRUE
    ar1    -0.99999999   1.0000000 0.03176384     TRUE
    omega   0.00000100 100.0000000 0.10000000     TRUE
    alpha1  0.00000001   1.0000000 0.10000000     TRUE
    gamma1 -0.99999999   1.0000000 0.10000000    FALSE
    beta1   0.00000001   1.0000000 0.80000000     TRUE
    delta   0.00000000   2.0000000 2.00000000    FALSE
    skew    0.10000000  10.0000000 1.00000000    FALSE
    shape   1.00000000  10.0000000 1.07852700    FALSE
 Index List of Parameters to be Optimized:
    mu    ar1  omega alpha1  beta1
     1      2      3      4      6
 Persistence:                  0.9


--- START OF TRACE ---
Selected Algorithm: nlminb

R coded nlminb Solver:

  0:     656.67937: 0.0297304 0.0317638 0.100000 0.100000 0.800000
  1:     656.47757: 0.0297431 0.0293495 0.0914873 0.103497 0.798155
  2:     656.34286: 0.0297557 0.0270684 0.0921867 0.111243 0.803469
  3:     656.06233: 0.0297939 0.0202550 0.0767562 0.120735 0.803056
  4:     656.03263: 0.0298395 0.0167592 0.0775859 0.125487 0.808801
```

```
  5:     655.95893: 0.0299578 0.0119120 0.0729986 0.122853 0.810682
  6:     655.89944: 0.0306986 0.0141299 0.0659891 0.110359 0.828081
  7:     655.89810: 0.0310976 0.00903262 0.0676866 0.107743 0.830719
  8:     655.86906: 0.0313086 0.00959671 0.0661036 0.107604 0.829138
  9:     655.85090: 0.0315158 0.0110607 0.0661211 0.109802 0.828766
 10:     655.60355: 0.0392328 0.0237111 0.0801810 0.107386 0.811417
 11:     654.57631: 0.0702363 0.0112191 0.0671503 0.132525 0.797605
 12:     654.45967: 0.101200 -0.0449849 0.0787031 0.119747 0.819315
 13:     654.27878: 0.108527 -0.0486754 0.0702237 0.113704 0.809629
 14:     653.78817: 0.101216 -0.0291018 0.0742759 0.121865 0.807225
 15:     653.65514: 0.0974400 -0.0136090 0.0704574 0.120015 0.810775
 16:     653.50341: 0.101150 0.00528547 0.0642024 0.109643 0.828820
 17:     653.47688: 0.104953 0.00176419 0.0633606 0.107005 0.832874
 18:     653.47433: 0.106377 0.00192037 0.0648365 0.105705 0.831390
 19:     653.47326: 0.107015 0.00198381 0.0640812 0.105470 0.832625
 20:     653.47293: 0.107969 0.00193787 0.0638139 0.105407 0.833050
 21:     653.47288: 0.108194 0.00176835 0.0638310 0.105272 0.833150
 22:     653.47286: 0.108311 0.00148875 0.0639114 0.105370 0.832968
 23:     653.47285: 0.108243 0.00126060 0.0639106 0.105330 0.833002
 24:     653.47285: 0.108201 0.00120306 0.0639000 0.105370 0.832981
 25:     653.47285: 0.108195 0.00120580 0.0638933 0.105365 0.832994

Final Estimate of the Negative LLH:
 LLH:  -1536.57    norm LLH:  -3.048749
          mu           ar1         omega        alpha1         beta1
1.402994e-03 1.205796e-03 1.074375e-05 1.053650e-01 8.329942e-01

R-optimhess Difference Approximated Hessian Matrix:
                  mu          ar1          omega         alpha1          beta1
mu      -7830855.373 33742.831457 -1.306858e+07 -5.164757e+03 -2.784870e+03
ar1        33742.831  -332.054148  6.891085e+04 -1.976635e+01  4.046589e+00
omega  -13068577.079 68910.846859 -3.155968e+11 -2.527034e+07 -3.896545e+07
alpha1     -5164.757   -19.766350 -2.527034e+07 -3.583559e+03 -4.152498e+03
beta1      -2784.870     4.046589 -3.896545e+07 -4.152498e+03 -5.694146e+03
attr(,"time")
Time difference of 0.06569886 secs

--- END OF TRACE ---


Time to Estimate Parameters:
 Time difference of 0.2612419 secs
> summary(gFged)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 0) + garch(1, 1), data = wd_vol,
    shape = n, cond.dist = "ged", include.shape = FALSE)

Mean and Variance Equation:
 data ~ arma(1, 0) + garch(1, 1)
<environment: 0x0000000008bf12c0>
 [data = wd_vol]

Conditional Distribution:
 ged

Coefficient(s):
        mu         ar1       omega      alpha1       beta1
1.4030e-03  1.2058e-03  1.0744e-05  1.0537e-01  8.3299e-01

Std. Errors:
```

```
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.403e-03   4.804e-04    2.921  0.00349 **
ar1    1.206e-03   7.379e-02    0.016  0.98696
omega  1.074e-05   5.678e-06    1.892  0.05848 .
alpha1 1.054e-01   5.370e-02    1.962  0.04975 *
beta1  8.330e-01   7.112e-02   11.713  < 2e-16 ***
---
Signif. codes:  0 ·**·0.001 ·*·0.01 ··0.05 ··0.1 ·· 1

Log Likelihood:
 1536.57    normalized:  3.048749

Description:
 Fri Apr 22 20:55:44 2016 by user: Jonathan!


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  10486.49  0
 Shapiro-Wilk Test  R    W      0.8733833 0
 Ljung-Box Test     R    Q(10)  11.30777  0.3340479
 Ljung-Box Test     R    Q(15)  14.39559  0.4957629
 Ljung-Box Test     R    Q(20)  19.06345  0.5177052
 Ljung-Box Test     R^2  Q(10)  0.6028223 0.9999839
 Ljung-Box Test     R^2  Q(15)  3.973419  0.9978231
 Ljung-Box Test     R^2  Q(20)  4.11311   0.9999416
 LM Arch Test       R    TR^2   3.615303  0.9894215

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.077657 -6.035767 -6.077852 -6.061225

> rgFged=residuals(gFged)
> set.seed(6737181)
> resiged=rged(5000,nu=n)
> qqplot(resiged,rgFged)
> qqline(rgFged,distribution=qged)
```

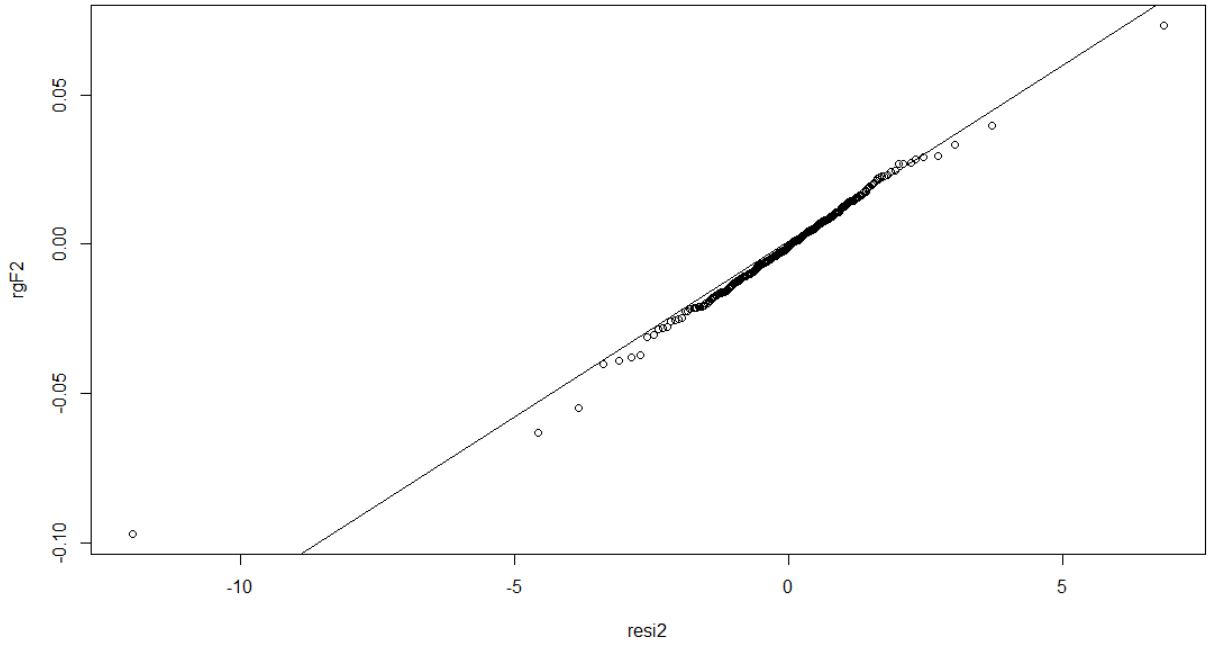



```
> sstdFit(rgF1)
$minimum
[1] -1530.795

$estimate
        mean           sd           nu           xi
0.0004465873 0.0129095970 3.7927380259 0.8730430515

$gradient
         mean            sd            nu            xi
 0.0284479529 -0.3718106313 -0.0001347200 -0.0002689183

$code
[1] 3

$iterations
[1] 37

Warning messages:
1: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
2: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
3: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
4: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
5: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
6: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
7: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
8: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
> sh=sstdFit(rgF1)$estimate[3]; sh
Warning messages:
1: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
2: In nlm(f = loglik, p = p, y = x, ...) :
```

```
  NA/Inf replaced by maximum positive value
3: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
4: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
5: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
6: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
7: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
8: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
      nu
3.792738
> sk=sstdFit(rgF1)$estimate[4]; sk
Warning messages:
1: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
2: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
3: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
4: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
5: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
6: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
7: In log(dsstd(y, x[1], x[2], x[3], x[4])) : NaNs produced
8: In nlm(f = loglik, p = p, y = x, ...) :
  NA/Inf replaced by maximum positive value
      xi
0.8730431
> gF2=garchFit(~arma(1,0)+garch(1,1),wd_vol,cond.dist="sstd",shape=sh,includ
e.shape=FALSE,skew=sk,include.skew=FALSE)

Series Initialization:
 ARMA Model:                 arma
 Formula Mean:               ~ arma(1, 0)
 GARCH Model:                garch
 Formula Variance:           ~ garch(1, 1)
 ARMA Order:                 1 0
 Max ARMA Order:             1
 GARCH Order:                1 1
 Max GARCH Order:            1
 Maximum Order:              1
 Conditional Dist:           sstd
 h.start:                    2
 llh.start:                  1
 Length of Series:           504
 Recursion Init:             mci
 Series Scale:               0.01296733

Parameter Initialization:
 Initial Parameters:          $params
 Limits of Transformations:   $U, $V
 Which Parameters are Fixed?  $includes
 Parameter Matrix:
                     U           V     params includes
    mu     -0.29739753   0.2973975 0.02973039     TRUE
    ar1    -0.99999999   1.0000000 0.03176384     TRUE
    omega   0.00000100 100.0000000 0.10000000     TRUE
    alpha1  0.00000001   1.0000000 0.10000000     TRUE
    gamma1 -0.99999999   1.0000000 0.10000000    FALSE
    beta1   0.00000001   1.0000000 0.80000000     TRUE
    delta   0.00000000   2.0000000 2.00000000    FALSE
    skew    0.10000000  10.0000000 0.87304305    FALSE
    shape   1.00000000  10.0000000 3.79273803    FALSE
 Index List of Parameters to be Optimized:
```

```
    mu    ar1  omega alpha1  beta1
     1      2      3      4      6
 Persistence:                  0.9


--- START OF TRACE ---
Selected Algorithm: nlminb

R coded nlminb Solver:

  0:     642.14893: 0.0297304 0.0317638 0.100000 0.100000 0.800000
  1:     641.50241: 0.0297331 0.0286625 0.104008 0.109139 0.807989
  2:     641.12827: 0.0297367 0.0235983 0.0926001 0.111605 0.804650
  3:     640.31348: 0.0297494 0.0106596 0.0846342 0.129419 0.816638
  4:     640.06250: 0.0297812 -0.00446820 0.0635805 0.131048 0.820664
  5:     639.68197: 0.0298530 -0.0226075 0.0645382 0.125022 0.838553
  6:     639.55257: 0.0300114 -0.0144009 0.0489736 0.117726 0.855900
  7:     639.52730: 0.0300332 -0.0158262 0.0536446 0.113041 0.854065
  8:     639.52616: 0.0300345 -0.0158706 0.0533622 0.112953 0.853920
  9:     639.52535: 0.0300436 -0.0161624 0.0531145 0.113267 0.854378
 10:     639.52332: 0.0300783 -0.0166758 0.0527073 0.112714 0.854677
 11:     639.50671: 0.0308625 -0.0206189 0.0512305 0.107559 0.860303
 12:     639.27432: 0.0440658 -0.00535015 0.0478229 0.102754 0.867875
 13:     639.03356: 0.0572404 -0.0199797 0.0559813 0.121941 0.844779
 14:     639.02347: 0.0572407 -0.0200554 0.0551896 0.121526 0.844312
 15:     639.01578: 0.0572479 -0.0216192 0.0534885 0.120979 0.847578
 16:     639.00971: 0.0574873 -0.0217189 0.0529261 0.120403 0.847487
 17:     638.96636: 0.0650726 -0.0235308 0.0508819 0.110518 0.855703
 18:     638.96167: 0.0663559 -0.0240793 0.0494364 0.112169 0.856278
 19:     638.96163: 0.0661295 -0.0240232 0.0494337 0.112231 0.856342
 20:     638.96162: 0.0661401 -0.0240292 0.0494326 0.112233 0.856330
 21:     638.96162: 0.0661409 -0.0240294 0.0494332 0.112234 0.856329

Final Estimate of the Negative LLH:
 LLH:  -1551.081    norm LLH:  -3.077541
          mu           ar1         omega        alpha1         beta1
 8.576711e-04 -2.402940e-02  8.312274e-06  1.122339e-01  8.563295e-01

R-optimhess Difference Approximated Hessian Matrix:
                  mu           ar1         omega        alpha1         beta1
mu     -5.582765e+06 -7585.073831 -3.326721e+08 -2.638921e+04 -4.318601e+04
ar1    -7.585074e+03  -535.915722  4.816879e+04 -3.132950e+01 -4.703938e+00
omega  -3.326721e+08 48168.793312 -4.397229e+11 -3.473758e+07 -5.751704e+07
alpha1 -2.638921e+04   -31.329498 -3.473758e+07 -4.428263e+03 -5.886331e+03
beta1  -4.318601e+04    -4.703938 -5.751704e+07 -5.886331e+03 -8.989489e+03
attr(,"time")
Time difference of 0.05712914 secs

--- END OF TRACE ---


Time to Estimate Parameters:
 Time difference of 0.197757 secs
> summary(gF2)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 0) + garch(1, 1), data = wd_vol,
    skew = sk, shape = sh, cond.dist = "sstd", include.skew = FALSE,
    include.shape = FALSE)

Mean and Variance Equation:
```

```
 data ~ arma(1, 0) + garch(1, 1)
<environment: 0x0000000011939e28>
 [data = wd_vol]

Conditional Distribution:
 sstd

Coefficient(s):
         mu          ar1        omega       alpha1        beta1
 8.5767e-04  -2.4029e-02   8.3123e-06   1.1223e-01   8.5633e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      8.577e-04   4.376e-04    1.960   0.0500 *
ar1    -2.403e-02   4.370e-02   -0.550   0.5824
omega   8.312e-06   4.211e-06    1.974   0.0484 *
alpha1  1.122e-01   4.696e-02    2.390   0.0169 *
beta1   8.563e-01   5.030e-02   17.024   <2e-16 ***
---
Signif. codes:  0 ·**·0.001 ·*·0.01 ··0.05 ··0.1 ··1

Log Likelihood:
 1551.081    normalized:  3.077541

Description:
 Fri Apr 22 20:56:17 2016 by user: Jonathan!


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  12664.39  0
 Shapiro-Wilk Test  R    W      0.8657818 0
 Ljung-Box Test     R    Q(10)  11.43325  0.3247746
 Ljung-Box Test     R    Q(15)  14.28772  0.5038295
 Ljung-Box Test     R    Q(20)  19.06954  0.5173103
 Ljung-Box Test     R^2  Q(10)  0.5660555 0.999988
 Ljung-Box Test     R^2  Q(15)  2.795261  0.9997414
 Ljung-Box Test     R^2  Q(20)  2.910903  0.9999968
 LM Arch Test       R    TR^2   2.531255  0.9980454

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.135242 -6.093351 -6.135436 -6.118809

> rgF2=residuals(gF2)
> set.seed(6737)
> resi2=rsstd(9000,nu=sh,xi=sk)
> qqplot(resi2,rgF2)
> qqline(rgF2,distribution=qsstd)
```




```
> predict(gF2,n.ahead=5,trace=TRUE,plot=FALSE)

Model Parameters:
           mu            ar1            ma1          omega         alpha1
gamma1
 8.576711e-04 -2.402940e-02  0.000000e+00  8.312274e-06  1.122339e-01  0.0000
00e+00
         beta1          delta           skew          shape
 8.563295e-01  2.000000e+00  8.730431e-01  3.792738e+00

Forecast GARCH Variance:

Forecast ARMA Mean:

Call:
arima(x = object@data, order = c(max(u, 1), 0, max(v, 1)), transform.pars = F
ALSE,
    init = c(ar, ma, mu), optim.control = list(maxit = 0))

Coefficients:
          ar1  ma1  intercept
       -0.024    0      8e-04
s.e.     NaN  NaN      6e-04

sigma^2 estimated as 0.0001684:  log likelihood = 1474.55,  aic = -2941.09

$pred
Time Series:
Start = 505
End = 509
Frequency = 1
[1] 0.0009158121 0.0008356647 0.0008375906 0.0008375443 0.0008375454

$se
Time Series:
```

```
Start = 505
End = 509
Frequency = 1
[1] 0.01297634 0.01298009 0.01298009 0.01298009 0.01298009

  meanForecast   meanError standardDeviation
1 0.0009158121 0.009695078       0.009695078
2 0.0008356647 0.009970266       0.009967544
3 0.0008375906 0.010227332       0.010224525
4 0.0008375443 0.010470300       0.010467415
5 0.0008375454 0.010700371       0.010697413
Warning message:
In sqrt(diag(x$var.coef)) : NaNs produced
> c=rep(0,5)
> c=predict(gF2,n.ahead=5,trace=FALSE,plot=FALSE)$meanForecast
> wd_vol_pred=c(wd_vol,c)
> wd_ts_log=c(log(wd_ts[1]),log(wd_ts[1])+cumsum(wd_vol_pred))
> wd_ts_pred=exp(wd_ts_log)
> y_hat=wd_ts_pred[(length(wd_ts_pred)-4):(length(wd_ts_pred))]
> y_hat
[1] 99.16077 99.24367 99.32683 99.41006 99.49335
> y=c(98.68,97.00,97.48,96.16,96.42); y
[1] 98.68 97.00 97.48 96.16 96.42
> cor(y_hat,y)
[1] -0.8487504
```