BACHELOR OF COMPUTER SCIENCE
SCHOOL OF COMPUTER SCIENCE
BINA NUSANTARA UNIVERSITY
JAKARTA

ASSESSMENT FORM

Course: MATH6183001 – Scientific Computing

Method of Assessment: Case Study

Semester/Academic Year: 2/2022 – 2023

Name of Lecturer : Nurhanan, S.Si., M.M.

Date : 30 January 2023

Class : Computer Science

Topic : Regression & Interpolation, Taylor Series, Numerical Differentiation, Numerical Integration

Group Members :	1 Jonathan Alvindo Fernandi 2__________ 3__________ 4__________ 5__________ 6__________ 7__________ 8__________

Student Outcomes:

(SO 1) Mampu menganalisis masalah komputasi yang kompleks dan mengaplikasikan prinsip komputasi dan keilmuan lain yang sesuai untuk mengidentifikasi solusi.

Able to analyze a complex computing problem and to apply principles of computing and other relevant disciplines to identify solutions

Learning Objectives:

(LObj 1.1) Mampu menganalisis masalah komputasi yang kompleks
Able to analyze a complex computing problem
(LObj 1.2) Mampu menerapkan prinsip komputasi dan disiplin ilmu terkait lainnya untuk mengidentifikasi solusi
Able to apply principles of computing and other relevant disciplines to identify solutions

Learning Outcomes :
(LO 1) Melakukan komputasi saintifik dasar menggunakan Pyhton
Compute basic scientific computation using Python
(LO 2) Menyelesaikan Sistem Persamaan Linear, Regresi dan Interpolasi menggunakan komputasi saintifik
Solve the System of Linear Algebraic Equations, Regression, and Interpolation through scientific computation
(LO 3) Mengevaluasi penerapan Deret Taylor dan Akar Persamaan dalam komputasi saintifik
Evaluate the application of the Taylor Series and Root of Equations in scientific computation
(LO 4) Menjelaskan konsep dasar dan penerapan Turunan Numerik, Integral Numerik, dan Persamaan Diferensial Biasa dalam komputasi saintifik
Explain the basic concept and application of Numerical Differentiation, Numerical Integration, and Ordinary Differential Equations in scientific computation

No	Related LO – LOBJ-SO	Assessment Criteria	Weight	Excellent (85 – 100)	Good (75 – 84)	Average (65 – 74)	Poor (0 – 64)	Score	(Score x Weight)
1	LO2 – L.Obj.1.2 – SO1	Understanding of Systems of Linear Equations, Regression, and Interpolation	**35%**	Able to clearly explain the concept and solve problems in both numerical and computational approaches without errors.	Able to clearly explain the concept and solve problems in either numerical or computational approaches with some errors	Able to clearly explain the concept but unable to solve problems in both approaches	Only able to poorly explain the concept and unable to solve problems in both approaches	100	35
2	LO3 – L.Obj.1.1 – SO1	Understanding of Taylor Series and Root of Equations	**30%**	Able to clearly explain the concept and solve problems in both numerical and computational	Able to clearly explain the concept and solve	Able to clearly explain the concept but unable to solve problems in both	Only able to poorly explain the concept and unable to	100	30

No	Related LO – LOBJ-SO	Assessment Criteria	Weight	Excellent (85 – 100)	Good (75 – 84)	Average (65 – 74)	Poor (0 – 64)	Score	(Score x Weight)
				approaches without errors.	problems in either numerical or computational approaches with some errors	approaches	solve problems in both approaches		
3	LO4 – L.Obj.1.2 – SO1	Understanding of Numerical Differentiation, Integration, and introductionary ODEs	**35%**	Able to clearly explain the concept and solve problems in both numerical and computational approaches without errors.	Able to clearly explain the concept and solve problems in either numerical or computational approaches with some errors	Able to clearly explain the concept but unable to solve problems in both approaches	Only able to poorly explain the concept and unable to solve problems in both approaches	100	35
		Total Score: ∑(Score x Weight)							100

Remarks:

____

____

____

____

ASSESSMENT METHOD

Instructions

The deadline for this comprehensive assignment is at the end of the semester. Answer the questions below in .pdf format through BINUSMAYA. Attach the manual calculation **AND** script that you use. All the answers **must** be rounded according to the given dataset!

1. The relationship between the average temperature on the earth's surface in odd years between 1981 – 1999, is given by the following below: **(35%)**

Year (y)	**Temperature (x, ºC)**
1981	14.1999
1983	14.2411
1985	14.0342
1987	14.2696
1989	14.197
1991	14.3055
1993	14.1853
1995	14.3577
1997	14.4187
1999	14.3438

   a. Estimate the temperature in even years by linear, quadratic, and cubic interpolation order! Choose the method that you think is appropriate and explain the difference!

Manual calculation:

Linear interpolation:

(i) $y = 1982$

$\hat{x}(1982) = 14.1999 + \frac{(14.2411 - 14.1999)(1982-1981)}{(1983-1981)}$

$\hat{x}(1982) = 14.2205$

(ii) $y = 1984$

$\hat{x}(1984) = 14.2411 + \frac{(14.0342 - 14.2411)(1984-1983)}{(1985-1983)}$

$\hat{x}(1984) = 14.1377$

(iii) $y = 1986$

$\hat{x}(1986) = 14.0342 + \frac{(14.2696 - 14.0342)(1986-1985)}{(1987-1985)}$

$\hat{x}(1986) = 14.1519$

(iv) $y = 1988$

$\hat{x}(1988) = 14.2696 + \frac{(14.197 - 14.2696)(1988-1987)}{(1989-1987)}$

$\hat{x}(1988) = 14.2333$

(v) $y = 1990$

$\hat{x}(1990) = 14.197 + \frac{(14.3055 - 14.197)(1990-1989)}{(1991-1989)}$

$\hat{x}(1990) = 14.2513$

(vi) $y = 1992$

$\hat{x}(1992) = 14.3055 + \frac{(14.1853 - 14.3055)(1992-1991)}{(1993-1991)}$

$\hat{x}(1992) = 14.2454$

(vii) $y = 1994$

$\hat{x}(1994) = 14.1853 + \frac{(14.3577 - 14.1853)(1994-1993)}{(1995-1993)}$

$\hat{x}(1994) = 14.2715$

(viii) $y = 1996$

$\hat{x}(1996) = 14.3577 + \frac{(14.4187 - 14.3577)(1996-1995)}{(1997-1995)}$

$\hat{x}(1996) = 14.3882$

(ix) $y = 1998$

$\hat{x}(1998) = 14.4187 + \frac{(14.3438 - 14.4187)(1998-1997)}{(1999-1997)}$

$\hat{x}(1998) = 14.3813$

Quadratic interpolation

(i) $y = 1982$

$14.1999 = a_0 + a_1(1981) + a_2(1981)^2$

$14.2411 = a_0 + a_1(1983) + a_2(1983)^2$

$14.0342 = a_0 + a_1(1985) + a_2(1985)^2$

$$\begin{bmatrix} 1 & 1981 & 3924361 \\ 1 & 1983 & 3932289 \\ 1 & 1985 & 3940225 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 14.1999 \\ 14.2411 \\ 14.0342 \end{bmatrix}$$

$a_0 = -121853.7257375$

$a_1 = 122.95415$

$a_2 = -0.0310125$

$x(1982) = -121853.7257375 + 122.95415(1982) - 0.0310125(1982)^2$

$x(1982) = 14.2515$

(ii) $y = 1984$

$x(1984) = -121853.7257375 + 122.95415(1984) - 0.0310125(1984)^2$

$x(1984) = 14.1686$

(iii) $y = 1986$

$14.2411 = a_0 + a_1(1983) + a_2(1983)^2$

$14.0342 = a_0 + a_1(1985) + a_2(1985)^2$

$14.2696 = a_0 + a_1(1987) + a_2(1987)^2$

$$\begin{bmatrix} 1 & 1983 & 3932289 \\ 1 & 1985 & 3940225 \\ 1 & 1987 & 3948169 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 14.2411 \\ 14.0342 \\ 14.2696 \end{bmatrix}$$

$a_0 = 217845.0807625$

$a_1 = -219.48425$

$a_2 = 0.0552875$

$x(1986) = 217845.0807625 - 219.48425(1986) + 0.0552875(1986)^2$

$x(1986) = 14.0966$

(iv) $y = 1988$

$14.0342 = a_0 + a_1(1985) + a_2(1985)^2$

$14.2696 = a_0 + a_1(1987) + a_2(1987)^2$

$14.197 = a_0 + a_1(1989) + a_2(1989)^2$

$$\begin{bmatrix} 1 & 1985 & 3940225 \\ 1 & 1987 & 3948169 \\ 1 & 1989 & 3956121 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 14.0342 \\ 14.2696 \\ 14.197 \end{bmatrix}$$

$a_0 = -152071.1078$

$a_1 = 153.0397$

$a_2 = -0.0385$

$x(1988) = -152071.1078 + 153.0397(1988) - 0.0385(1988)^2$

$x(1988) = 14.2718$

(v) $y = 1990$

$14.2696 = a_0 + a_1(1987) + a_2(1987)^2$

$14.197 = a_0 + a_1(1989) + a_2(1989)^2$

$14.3055 = a_0 + a_1(1991) + a_2(1991)^2$

$$\begin{bmatrix} 1 & 1987 & 3948169 \\ 1 & 1989 & 3956121 \\ 1 & 1991 & 3964081 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 14.2696 \\ 14.197 \\ 14.3055 \end{bmatrix}$$

$a_0 = 89553.0348625$

$a_1 = -90.043$

$a_2 = 0.0226375$

$x(1990) = 89553.0348625 - 90.043(1990) + 0.0226375(1990)^2$

$x(1990) = 14.2286$

(vi) $y = 1992$

$14.197 = a_0 + a_1(1989) + a_2(1989)^2$

$14.3055 = a_0 + a_1(1991) + a_2(1991)^2$

$14.1853 = a_0 + a_1(1993) + a_2(1993)^2$

$$\begin{bmatrix} 1 & 1989 & 3956121 \\ 1 & 1991 & 3964081 \\ 1 & 1993 & 3972049 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 14.197 \\ 14.3055 \\ 14.1853 \end{bmatrix}$$

$a_0 = -113303.0364125$

$a_1 = 113.8325$

$a_2 = -0.0285875$

$x(1992) = -113303.0364125 + 113.8325(1992) - 0.0285875(1992)^2$

$x(1992) = 14.274$

(vii) $y = 1994$

$14.3055 = a_0 + a_1(1991) + a_2(1991)^2$

$14.1853 = a_0 + a_1(1993) + a_2(1993)^2$

$14.3577 = a_0 + a_1(1995) + a_2(1995)^2$

$$\begin{bmatrix} 1 & 1991 & 3964081 \\ 1 & 1993 & 3972049 \\ 1 & 1995 & 3980025 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 14.3055 \\ 14.1853 \\ 14.3577 \end{bmatrix}$$

$a_0 = 145265.868825$

$a_1 = -145.7749$

$a_2 = 0.036575$

$x(1994) = 145265.868825 - 145.7749(1994) + 0.036575(1994)^2$

$x(1994) = 14.2349$

(viii) $y = 1996$

$14.1853 = a_0 + a_1(1993) + a_2(1993)^2$

$14.3577 = a_0 + a_1(1995) + a_2(1995)^2$

$14.4187 = a_0 + a_1(1997) + a_2(1997)^2$

$$\begin{bmatrix} 1 & 1993 & 3972049 \\ 1 & 1995 & 3980025 \\ 1 & 1997 & 3988009 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 14.1853 \\ 14.3577 \\ 14.4187 \end{bmatrix}$$

$a_0 = -55523.898675$

$a_1 = 55.6191$

$a_2 = -0.013925$

$x(1996) = -55523.898675 + 55.6191(1996) - 0.013925(1996)^2$

$x(1996) = 14.4021$

(ix) $y = 1998$

$14.3577 = a_0 + a_1(1995) + a_2(1995)^2$

$14.4187 = a_0 + a_1(1997) + a_2(1997)^2$

$14.3438 = a_0 + a_1(1999) + a_2(1999)^2$

$$\begin{bmatrix} 1 & 1995 & 3980025 \\ 1 & 1997 & 3988009 \\ 1 & 1999 & 3996001 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 14.3577 \\ 14.4187 \\ 14.3438 \end{bmatrix}$$

$a_0 = -67724.9446125$

$a_1 = 67.8496$

$a_2 = -0.0169875$

$x(1998) = -67724.9446125 + 67.8496(1998) - 0.0169875(1998)^2$

$x(1998) = 14.3982$

Cubic interpolation

$k_0 = k_9 = 0 \cdots (a), (b)$

$0 + 4k_1 + k_2 = \frac{6}{2^2}(14.1999 - 2(14.2411) + 14.0342)$

$4k_1 + k_2 = -0.37215 \cdots (c)$

$k_1 + 4k_2 + k_3 = \frac{6}{2^2}(14.2411 - 2(14.0342) + 14.2696)$

$k_1 + 4k_2 + k_3 = 0.66345 \cdots (d)$

$k_2 + 4k_3 + k_4 = \frac{6}{2^2}(14.0342 - 2(14.2696) + 14.197)$

$k_2 + 4k_3 + k_4 = -0.462 \cdots (e)$

$k_3 + 4k_4 + k_5 = \frac{6}{2^2}(14.2696 - 2(14.197) + 14.3055)$

$k_3 + 4k_4 + k_5 = 0.27165 \cdots (f)$

$k_4 + 4k_5 + k_6 = \frac{6}{2^2}(14.197 - 2(14.3055) + 14.1853)$

$k_4 + 4k_5 + k_6 = -0.34305 \cdots (g)$

$k_5 + 4k_6 + k_7 = \frac{6}{2^2}(14.3055 - 2(14.1853) + 14.3577)$

$k_5 + 4k_6 + k_7 = 0.4389 \cdots (h)$

$k_6 + 4k_7 + k_8 = \frac{6}{2^2}(14.1853 - 2(14.3577) + 14.4187)$

$k_6 + 4k_7 + k_8 = -0.1671 \cdots (i)$

$k_7 + 4k_8 + 0 = \frac{6}{2^2}(14.3577 - 2(14.4187) + 14.3438)$

$k_7 + 4k_8 = -0.20385 \cdots (j)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 0 \end{bmatrix} \begin{bmatrix} k_0 \\ k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \\ k_7 \\ k_8 \\ k_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -0.37215 \\ 0.66345 \\ -0.462 \\ 0.27165 \\ -0.34305 \\ 0.4389 \\ -0.1671 \\ -0.20385 \end{bmatrix}$$

$k_0 = k_9 = 0$

$k_1 = -0.1582866996300408$

$k_2 = 0.260996799852016281$

$k_3 = -0.222250499445061044$

$k_4 = 0.166005197928227895$

$k_5 = -0.1701202922678506365$

$k_6 = 0.17142259711931425085$

$k_8 = -0.0766835923048164669$

$k_8 = -0.031791601923788383277$

(i) $y = 1982$

$x(1982) = f_{0,1}(1982)$

$$x(1982) = \frac{0}{6}\left[\frac{(1982-1983)^3}{1981-1983} - (1982-1983)(1981-1983)\right] + \frac{0.15828669996300408}{6}\left[\frac{(1982-1981)^3}{1981-1983} - (1982-1981)(1981-1983)\right] + \frac{14.1999(1982-1983) - 14.2411(1982-1981)}{1981-1983}$$

$x(1982) = 14.2601$

(ii) $y = 1984$

$x(1984) = f_{1,2}(1984)$

$$x(1984) = -\frac{0.15828669996300408}{6}\left[\frac{(1984-1985)^3}{1983-1985} - (1984-1985)(1983-1985)\right] - \frac{0.26099679985201628}{6}\left[\frac{(1984-1983)^3}{1983-1985} - (1984-1983)(1983-1985)\right] + \frac{14.2411(1984-1985) - 14.0342(1984-1983)}{1983-1985}$$

$x(1984) = 14.0328$

(iii) $y = 1986$

$x(1986) = f_{2,3}(1986)$

$$x(1986) = \frac{0.26099679985201628}{6}\left[\frac{(1986-1987)^3}{1985-1987} - (1986-1987)(1985-1987)\right] + \frac{0.22225049994450610 44}{6}\left[\frac{(1986-1985)^3}{1985-1987} - (1986-1985)(1985-1987)\right] + \frac{14.0342(1986-1987) - 14.2696(1986-1985)}{1985-1987}$$

$x(1986) = 14.1422$

(iv) $y = 1988$

$x(1988) = f_{3,4}(1988)$

$$x(1988) = \frac{0.2222504999445061044}{6}\left[\frac{(1988-1989)^3}{1987-1989} - (1988-1989)(1987-1989)\right] - \frac{0.166005197928227895 2}{6}\left[\frac{(1988-1987)^3}{1987-1989} - (1988-1987)(1987-1989)\right] + \frac{14.2696(1988-1989) - 14.147(1988-1987)}{1987-1989}$$

$x(1988) = 14.1362$

(v) $y = 1990$

$x(1990) = f_{4,5}(1990)$

$$x(1990) = \frac{0.1660051979282278952}{6}\left[\frac{(1990-1991)^3}{1989-1991} - (1990-1991)(1989-1991)\right] + \frac{0.170120292267850536 5}{6}\left[\frac{(1990-1989)^3}{1989-1991} - (1990-1989)(1989-1991)\right] + \frac{14.147(1990-1991) - 14.3055(1990-1989)}{1989-1991}$$

$x(1990) = 14.2523$

(vi) $y = 1992$

$x(1992) = f_{5,6}(1992)$

$$x(1992) = \frac{0.1701202922678505365}{6}\left[\frac{(1992-1993)^3}{1991-1993} - (1992-1993)(1991-1993)\right] - \frac{0.17142597114317425085}{6}\left[\frac{(1992-1991)^3}{1991-1993} - (1992-1991)(1991-1993)\right] + \frac{14.3055(1992-1993) - 14.1853(1992-1991)}{1991-1993}$$

$x(1992) = 14.16$

(vii) $y = 1994$

$x(1994) = f_{6,7}(1994)$

$$x(1994) = \frac{0.17142597114317425085}{6}\left[\frac{(1994-1995)^3}{1993-1995} - (1994-1995)(1993-1995)\right] + \frac{0.0766835923048464669}{6}\left[\frac{(1994-1993)^3}{1993-1995} - (1994-1993)(1993-1995)\right] + \frac{14.1853(1994-1995) - 14.3577(1994-1993)}{1993-1995}$$

$x(1994) = 14.2978$

(viii) $y = 1996$

$x(1996) = f_{7,8}(1996)$

$$x(1996) = \frac{0.0766835923048464669}{6}\left[\frac{(1996-1997)^3}{1995-1997} - (1996-1997)(1995-1997)\right] + \frac{0.031791601923788383277}{6}\left[\frac{(1996-1995)^3}{1995-1997} - (1996-1995)(1995-1997)\right] + \frac{14.3577(1996-1997) - 14.4187(1996-1995)}{1995-1997}$$

$x(1996) = 14.377$

(ix) $y = 1998$

$x(1998) = f_{8,9}(1998)$

$$x(1998) = \frac{0.031791601923788383277}{6}\left[\frac{(1998-1999)^3}{1997-1999} - (1998-1999)(1997-1999)\right] + \frac{0}{6}\left[\frac{(1998-1997)^3}{1997-1999} - (1998-1997)(1997-1999)\right] + \frac{14.4187(1998-1999) - 14.3438(1998-1997)}{1997-1999}$$

$x(1998) = 14.3733$

Python script:

```
# a.
import numpy as np
import matplotlib.pyplot as plt
from scipy.interpolate import interp1d
from scipy.interpolate import CubicSpline

y = [1981, 1983, 1985, 1987, 1989, 1991, 1993, 1995, 1997, 1999]
x = [14.1999, 14.2411, 14.0342, 14.2696, 14.197, 14.3055, 14.1853, 14.3577, 14.4187, 14.3438]

y_even = [1982, 1984, 1986, 1988, 1990, 1992, 1994, 1996, 1998]
x_est = [0] * len(y_even)

# Linear interpolation
print('Linear interpolation')

f = interp1d(y, x, kind='linear')

for i in range(len(y_even)):
    x_est[i] = np.round(f(y_even[i]), 4)
    print(f'x({y_even[i]}) = {x_est[i]}')

plt.plot(y, x, '-b', label='linear interpolation')
plt.plot(y, x, 'b.', label='average temperature')
plt.plot(y_even, x_est, 'r.', label='temperature estimation')
plt.title('Linear Interpolation')
plt.ylabel(f'Temperature (x, {chr(176)}C)')
plt.xlabel('Year (y)')
plt.legend()
plt.show()
```

```
# Quadratic interpolation
print('Quadratic interpolation')

f = interp1d(y, x, kind='quadratic')

for i in range(len(y_even)):
    x_est[i] = np.round(f(y_even[i]), 4)
    print(f'x({y_even[i]}) = {x_est[i]}')

y_quad = np.linspace(y[0], y[len(y) - 1], 100)
x_quad = f(y_quad)

plt.plot(y_quad, x_quad, '-b', label='quadratic interpolation')
plt.plot(y, x, 'b.', label='average temperature')
plt.plot(y_even, x_est, 'r.', label='temperature estimation')
plt.title('Quadratic Interpolation')
plt.ylabel(f'Temperature (x, {chr(176)}C)')
plt.xlabel('Year (y)')
plt.legend()
plt.show()

# Cubic interpolation
print('Cubic interpolation')

f = CubicSpline(y, x)

for i in range(len(y_even)):
    x_est[i] = np.round(f(y_even[i]), 4)
    print(f'x({y_even[i]}) = {x_est[i]}')
```

```
y_cubic = np.linspace(y[0], y[len(y) - 1], 100)
x_cubic = f(y_cubic)

plt.plot(y_cubic, x_cubic, '-b', label='cubic interpolation')
plt.plot(y, x, 'b.', label='average temperature')
plt.plot(y_even, x_est, 'r.', label='temperature estimation')
plt.title('Cubic Interpolation')
plt.ylabel(f'Temperature (x, {chr(176)}C)')
plt.xlabel('Year (y)')
plt.legend()
plt.show()
```

Analysis:

Based on the calculation above, linear interpolation provides derivatives that are constant on each subinterval. The quadratic interpolation gives derivatives that are not smooth at the data points. However, the cubic interpolation provides smooth derivatives that are very close to the actual data points. Therefore, the most appropriate method to use is the cubic interpolation.

b. Perform a least-square regression of the above data to estimate the temperature in even years!

Manual calculation:

$\bar{y} = \frac{1}{10}(1981+1983+1985+1987+1989+1991+1993+1995+1997+1999)$

$\bar{y} = 1990$

$\bar{x} = \frac{1}{10}(14.1999+14.2411+14.0342+14.2696+14.197+14.3055+14.1853+14.3577+14.4187+14.3438)$

$\bar{x} = 14.2552$

$b = \dfrac{(14.1999(1981-1990))+(14.2411(1983-1990))+(14.0342(1985-1990))+(14.2696(1987-1990))+(14.197(1989-1990))+(14.3055(1991-1990))+(14.1853(1993-1990))+(14.3577(1995-1990))+(14.4187(1997-1990))+(14.3438(1999-1990))}{(1981(1981-1990))+(1983(1983-1990))+(1985(1985-1990))+(1987(1987-1990))+(1989(1989-1990))+(1991(1991-1990))+(1993(1993-1990))+(1995(1995-1990))+(1997(1997-1990))+(1999(1999-1990))}$

$b = 0.012155757 58$

$a = 14.2552-(1990(0.01215575758))$

$a = -9.934757584$

$f(y) = -9.934757584+0.01215575758y$

(i) $y = 1982$

$f(1982) = -9.934757584+0.01215575758(1982)$

$f(1982) = 14.1578$

(ii) $y = 1984$

$f(1984) = -9.934757584+0.01215575758(1984)$

$f(1984) = 14.1823$

(iii) $y = 1986$

$f(1986) = -9.934757584+0.01215575758(1986)$

$f(1986) = 14.2066$

(iv) $y = 1988$

$f(1988) = -9.934757584+0.01215575758(1988)$

$f(1988) = 14.2309$

(v) $y = 1990$

$f(1990) = -9.934757584+0.01215575758(1990)$

$f(1990) = 14.2552$

(vi) $y = 1992$

$f(1992) = -9.934757584+0.01215575758(1992)$

$f(1992) = 14.2795$

(vii) $y = 1994$

$f(1994) = -9.934757584+0.01215575758(1994)$

$f(1994) = 14.3038$

(viii) $y = 1996$

$f(1996) = -9.934757584+0.01215575758(1996)$

$f(1996) = 14.3281$

(ix) $y = 1998$

$f(1998) = -9.934757584+0.01215575758(1998)$

$f(1998) = 14.3524$

Python script:

```
# b.


y = np.array([1981, 1983, 1985, 1987, 1989, 1991, 1993, 1995, 1997, 1999])
```

```
x = np.array([14.1999, 14.2411, 14.0342, 14.2696, 14.197, 14.3055, 14.1853, 14.3577, 14.4187, 14.3438])

A = np.vstack((y, np.ones(len(y)))).T
x = x[:, np.newaxis]

alpha = np.dot(np.dot(np.linalg.inv(np.dot(A.T, A)), A.T), x)

def f(y):
    return ((alpha[0] * y) + alpha[1])

for i in range(len(y_even)):
    x_est[i] = np.round(f(y_even[i]), 4)
    print(f'x({y_even[i]}) = {x_est[i]}')

plt.plot(y, x, 'g.', label='average temperature')
plt.plot(y, f(y), 'r', label='least square regression')
plt.plot(y_even, x_est, 'b.', label='temperature estimation')
plt.title('Least Square Regression')
plt.ylabel(f'Temperature (x, {chr(176)}C)')
plt.xlabel('Year (y)')
plt.legend()
plt.show()
```

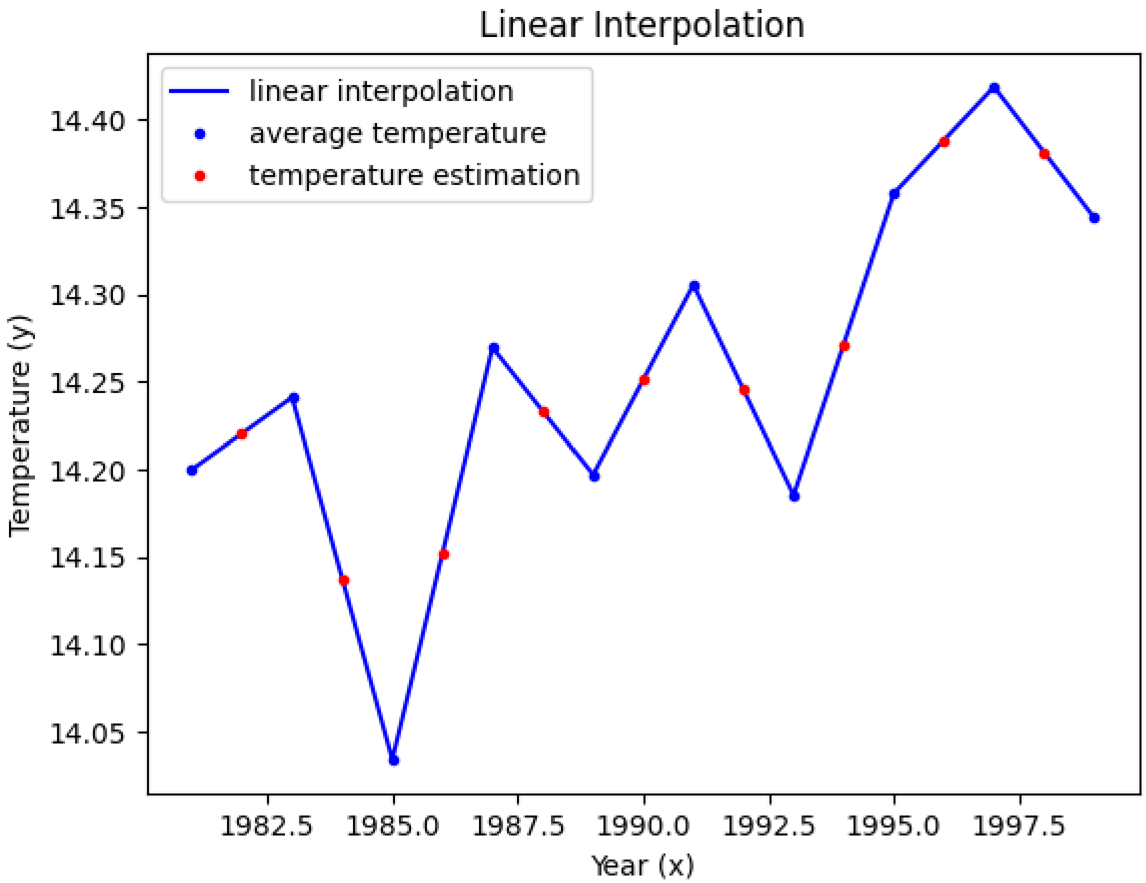
c. Perform an analysis of the difference between the results of the regression and interpolations above, and explain based on the theoretical basis you have learned!

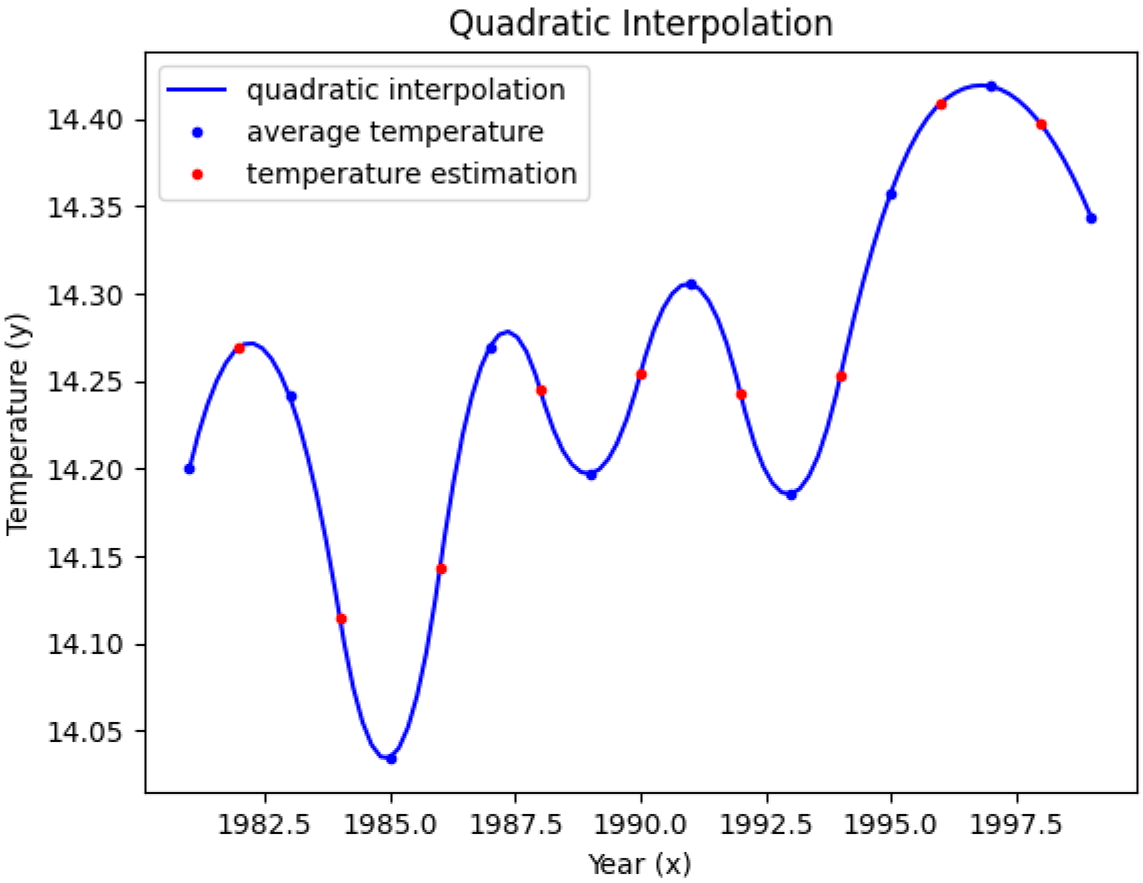
Analysis:

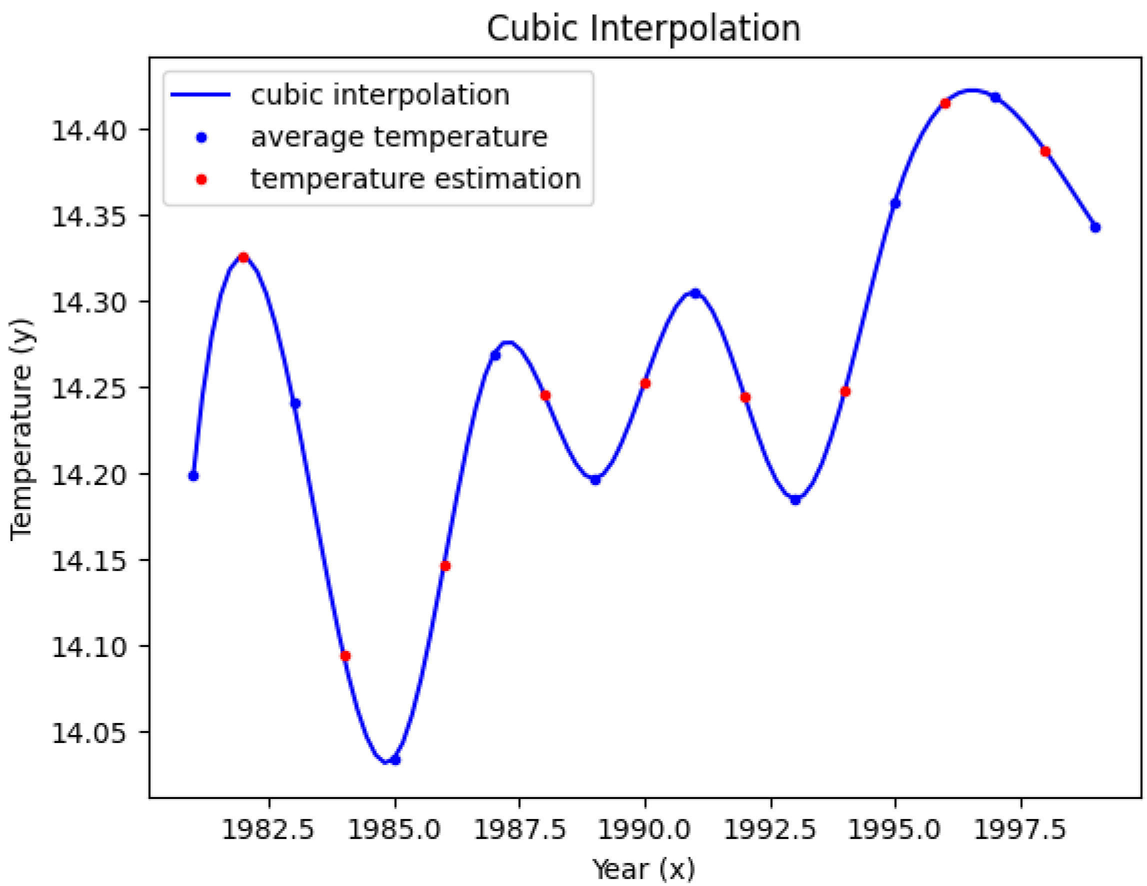
Based on the calculations above, interpolation aims to give a curve that exactly crosses through the data points, while regression aims to provide a curve that approximates the data points, such that the error is minimized. In conclusion, if the data points contain too many errors, regression would be a better choice to estimate the data points.

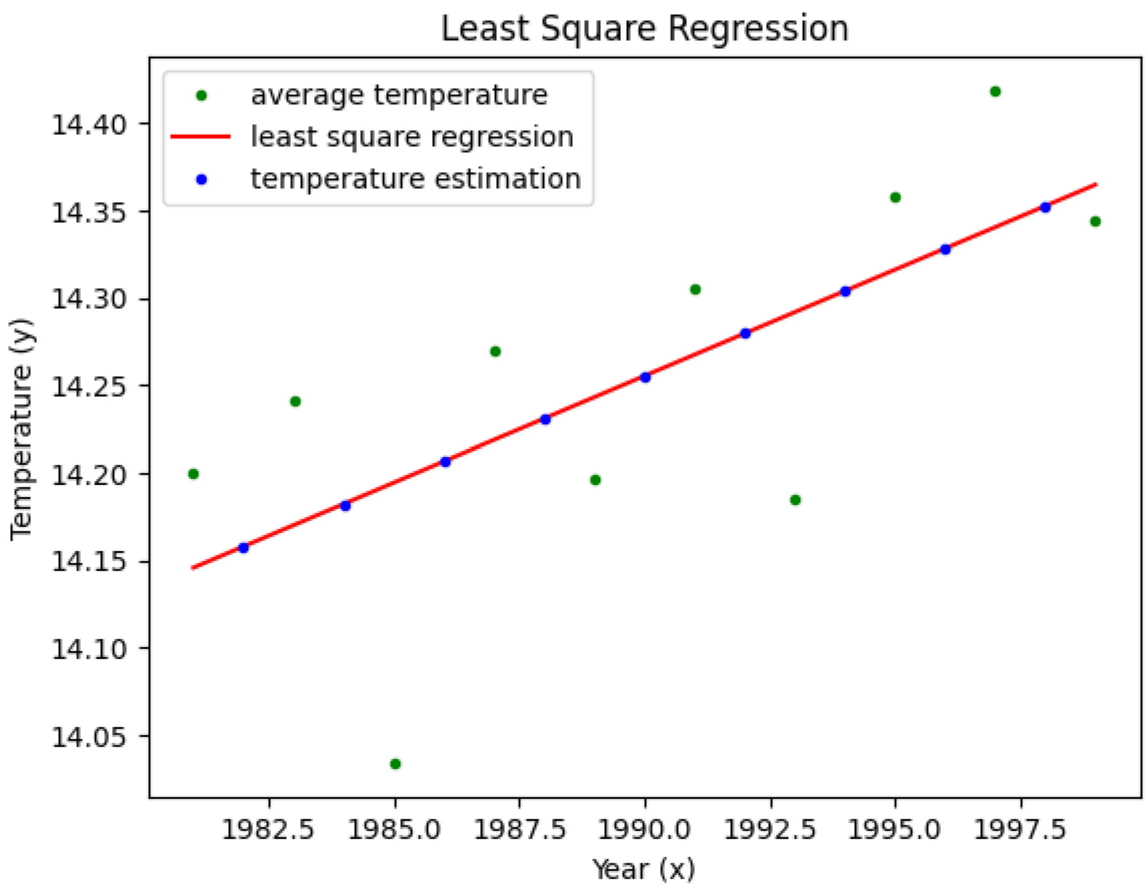
d. Make a plot that describes the relationship between Temperature (y) and Year (x) as informatively as possible for the reader, based on the results of your analysis using the Python library!

Plot:




Quadratic Interpolation
quadratic interpolation
average temperature
temperature estimation
Temperature (y)
14.40
14.35
14.30
14.25
14.20
14.15
14.10
14.05
1982.5
1985.0
1987.5
1990.0
1992.5
1995.0
1997.5
Year (x)


Cubic Interpolation
cubic interpolation
average temperature
temperature estimation
Temperature (y)
14.40
14.35
14.30
14.25
14.20
14.15
14.10
14.05
1982.5
1985.0
1987.5
1990.0
1992.5
1995.0
1997.5
Year (x)



Python script:

```
# d.

x = [1981, 1983, 1985, 1987, 1989, 1991, 1993, 1995, 1997, 1999]
y = [14.1999, 14.2411, 14.0342, 14.2696, 14.197, 14.3055, 14.1853, 14.3577, 14.4187, 14.3438]

x_even = [1982, 1984, 1986, 1988, 1990, 1992, 1994, 1996, 1998]
y_est = [0] * len(x_even)
```

```
# Linear interpolation

f = interp1d(x, y, kind='linear')

for i in range(len(x_even)):
    y_est[i] = np.round(f(x_even[i]), 4)

plt.plot(x, y, '-b', label='linear interpolation')
plt.plot(x, y, 'b.', label='average temperature')
plt.plot(x_even, y_est, 'r.', label='temperature estimation')
plt.title('Linear Interpolation')
plt.ylabel('Temperature (y)')
plt.xlabel('Year (x)')
plt.legend()
plt.show()

# Quadratic interpolation

f = interp1d(x, y, kind='quadratic')

for i in range(len(x_even)):
    y_est[i] = np.round(f(x_even[i]), 4)

x_quad = np.linspace(x[0], x[len(x) - 1], 100)
y_quad = f(x_quad)

plt.plot(x_quad, y_quad, '-b', label='quadratic interpolation')
plt.plot(x, y, 'b.', label='average temperature')
plt.plot(x_even, y_est, 'r.', label='temperature estimation')
plt.title('Quadratic Interpolation')
```

```
plt.ylabel('Temperature (y)')
plt.xlabel('Year (x)')
plt.legend()
plt.show()

# Cubic interpolation

f = CubicSpline(x, y)

for i in range(len(y_even)):
    y_est[i] = np.round(f(x_even[i]), 4)

x_cubic = np.linspace(x[0], x[len(y) - 1], 100)
y_cubic = f(x_cubic)

plt.plot(x_cubic, y_cubic, '-b', label='cubic interpolation')
plt.plot(x, y, 'b.', label='average temperature')
plt.plot(x_even, y_est, 'r.', label='temperature estimation')
plt.title('Cubic Interpolation')
plt.ylabel('Temperature (y)')
plt.xlabel('Year (x)')
plt.legend()
plt.show()

# Least square regression

x = np.array([1981, 1983, 1985, 1987, 1989, 1991, 1993, 1995, 1997, 1999])
y = np.array([14.1999, 14.2411, 14.0342, 14.2696, 14.197, 14.3055, 14.1853, 14.3577, 14.4187, 14.3438])

A = np.vstack((x, np.ones(len(x)))).T
```

```
y = y[:, np.newaxis]

alpha = np.dot(np.dot(np.linalg.inv(np.dot(A.T, A)), A.T), y)

def f(x):
    return ((alpha[0] * x) + alpha[1])

for i in range(len(x_even)):
    y_est[i] = np.round(f(x_even[i]), 4)

plt.plot(x, y, 'g.', label='average temperature')
plt.plot(x, f(x), 'r', label='least square regression')
plt.plot(x_even, y_est, 'b.', label='temperature estimation')
plt.title('Least Square Regression')
plt.ylabel('Temperature (y)')
plt.xlabel('Year (x)')
plt.legend()
plt.show()
```

2. Compute the fourth order Taylor expansion for sin(x) and cos(x) and sin(x)cos(x) around 0! **(30%)**
    a. Write down your manual calculation **AND** Python script to answer the above’s question!

Manual calculation:

$a=0$

(i) $f(x)=\sin(x)$

$$f(x)=\frac{f^{(0)}(0)}{0!}(x-0)^0+\frac{f^{(1)}(0)}{1!}(x-0)^1+\frac{f^{(2)}(0)}{2!}(x-0)^2+\frac{f^{(3)}(0)}{3!}(x-0)^3+\frac{f^{(4)}(0)}{4!}(x-0)^4$$

$$f(x)=\sin 0+x\cos 0-\frac{x^2\sin 0}{2}-\frac{x^3\cos 0}{6}+\frac{x^4\sin 0}{24}$$

$$f(x)=0+x-0-\frac{x^3}{6}+0$$

$$f(x)=x-\frac{1}{6}x^3$$

(ii) $f(x)=\cos(x)$

$$f(x)=\frac{f^{(0)}(0)}{0!}(x-0)^0+\frac{f^{(1)}(0)}{1!}(x-0)^1+\frac{f^{(2)}(0)}{2!}(x-0)^2+\frac{f^{(3)}(0)}{3!}(x-0)^3+\frac{f^{(4)}(0)}{4!}(x-0)^4$$

$$f(x)=\cos 0-x\sin 0-\frac{x^2\cos 0}{2}+\frac{x^3\sin 0}{6}+\frac{x^4\cos 0}{24}$$

$$f(x)=1-0-\frac{x^2}{2}+0+\frac{x^4}{24}$$

$$f(x)=1-\frac{1}{2}x^2+\frac{1}{24}x^4$$

(iii) $f(x)=\sin(x)\cos(x)$

$$f(x)=\frac{f^{(0)}(0)}{0!}(x-0)^0+\frac{f^{(1)}(0)}{1!}(x-0)^1+\frac{f^{(2)}(0)}{2!}(x-0)^2+\frac{f^{(3)}(0)}{3!}(x-0)^3+\frac{f^{(4)}(0)}{4!}(x-0)^4$$

$$f(x)=\sin 0\cos 0+x\cos(2(0))-\frac{\cancel{2}x^2\sin(2(0))}{\cancel{2}}-\frac{\overset{2}{\cancel{4}}x^3\cos(2(0))}{\underset{3}{\cancel{6}}}+\frac{\overset{1}{\cancel{8}}x^4\sin(2(0))}{\underset{3}{\cancel{24}}}$$

$$f(x)=0+x-0-\frac{2x^3}{3}+0$$

$$f(x)=x-\frac{2}{3}x^3$$

Python script:

```
# a.

import numpy as np
import matplotlib.pyplot as plt

plt.style.use('seaborn-poster')

x = np.linspace(-np.pi, np.pi, 200)

# f(x) = sin(x)
y = x - (x**3 / 6)

plt.plot(x, np.sin(x), 'k', label='sin(x)')
plt.plot(x, y, label='fourth order')
plt.grid()
plt.title('Taylor Expansion for sin(x)')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()

# f(x) = cos(x)
y = 1 - (x**2 / 2) + (x**4 / 24)

plt.plot(x, np.cos(x), 'k', label='cos(x)')
plt.plot(x, y, label='fourth order')
plt.grid()
plt.title('Taylor Expansion for cos(x)')
plt.xlabel('x')
plt.ylabel('y')
```

```
plt.legend()
plt.show()

# f(x) = sin(x)cos(x)
y = x - ((2 / 3) * x**3)

plt.plot(x, (np.sin(x) * np.cos(x)), 'k', label='sin(x)cos(x)')
plt.plot(x, y, label='fourth order')
plt.grid()
plt.title('Taylor Expansion for sin(x)cos(x)')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()
```

b. Which produces less error for $x=\pi/2$: computing the Taylor expansion for sin and cos separately then multiplying the result together, or computing the Taylor expansion for the product first and then plugging in x?

Manual calculation:

$x = \frac{\pi}{2}$

Exact value $= \sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)$

Exact value $= (1)(0)$

Exact value $= 0$

$x = 90\left(\frac{3.141592654}{180}\right)$

$x = 1.570796327$

$f(x) = \sin x \cos x$

(i) $\sin\left(\frac{\pi}{2}\right) = 1.570796327 - \frac{1}{6}(1.570796327)^3$

$\sin\left(\frac{\pi}{2}\right) = 0.9248322292$

$\cos\left(\frac{\pi}{2}\right) = 1 - \frac{1}{2}(1.570796327)^2 + \frac{1}{24}(1.570796327)^4$

$\cos\left(\frac{\pi}{2}\right) = 0.01996895758$

Approximation value $= f\left(\frac{\pi}{2}\right)$

Approximation value $= \sin\left(\frac{\pi}{2}\right) \cdot \cos\left(\frac{\pi}{2}\right)$

Approximation value $= (0.9248322292)(0.01996895758)$

Approximation value $= 0.01846793555$

Error $= |0 - 0.01846793555|$

Error $= 0.01846793555$

(ii) $\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) = 1.570796327 - \frac{2}{3}(1.570796327)^3$

$\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) = -1.013060064$

Approximation value $= \sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)$

Approximation value $= -1.013060064$

Error $= |0 - (-1.013060064)|$

Error $= 1.013060064$

$\therefore \text{Error}_{(i)} < \text{Error}_{(ii)}$

Python script:

```
# b.

x = np.pi / 2

exact_val = np.sin(x) * np.cos(x)

x_taylor = np.linspace(-np.pi, np.pi, 200)
y_taylor = np.sin(x_taylor) * np.cos(x_taylor)

# (i) computing the Taylor expansion for sin and cos separately then multiplying the result together

# f(x) = sin(x)
def sin(x):
    return (x - (x**3 / 6))

# f(x) = cos(x)
def cos(x):
    return (1 - (x**2 / 2) + (x**4 / 24))

approx_val_i = sin(x) * cos(x)

error_i = exact_val - approx_val_i
if(error_i < 0):
    error_i = -error_i
print(f'Error(i) = {error_i}')

plt.plot(x_taylor, np.sin(x_taylor), label='sin(x)')
plt.plot(x_taylor, np.cos(x_taylor), label='cos(x)')
plt.plot(x_taylor, y_taylor, 'k', label='sin(x)cos(x)')
plt.plot(x_taylor, (sin(x_taylor) * cos(x_taylor)), label='fourth order')
```

```
plt.plot(x, exact_val, 'b.', label='exact value')
plt.plot(x, approx_val_i, 'r.', label='approximation value')
plt.grid()
plt.title('Taylor Expansion for sin(x)cos(x) (i)')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()

# (ii) computing the Taylor expansion for the product first and then plugging in x

# f(x) = sin(x)cos(x)
def f(x):
    return x - ((2 / 3) * x**3)

approx_val_ii = f(x)

error_ii = exact_val - approx_val_ii
if(error_ii < 0):
    error_ii = -error_ii

print(f'Error(ii) = {error_ii}')

plt.plot(x_taylor, y_taylor, 'k', label='sin(x)cos(x)')
plt.plot(x_taylor, f(x_taylor), label='fourth order')
plt.plot(x, exact_val, 'b.', label='exact value')
plt.plot(x, approx_val_ii, 'r.', label='approximation value')
plt.grid()
plt.title('Taylor Expansion for sin(x)cos(x) (ii)')
plt.xlabel('x')
```

```
plt.ylabel('y')
plt.legend()
plt.show()

# Error analysis
if(error_i < error_ii):
    print('Error(i) < Error(ii)')
elif(error_i > error_ii):
    print('Error(i) > Error(ii)')
else:
    print('Error(i) = Error(ii)')
```

c. Use the same order of the Taylor series to approximate cos (π/4) and determine the truncation error bound. You may include either your manual calculation **OR** Python script for this question!

Python script:

```
# c.

x = np.pi / 4

exact_val = np.cos(x)

x_taylor = np.linspace(-np.pi, np.pi, 200)
y_taylor = np.cos(x_taylor)

# f(x) = cos(x)
def f(x):
    return (1 - (x**2 / 2) + (x**4 / 24))

approx_val = f(x)
print(f'Approximation value = {approx_val}')
```

```
trunc_error = exact_val - approx_val
if(trunc_error < 0):
    trunc_error = -trunc_error

print(f'Truncation error bound = {trunc_error}')

plt.plot(x_taylor, y_taylor, 'k', label='cos(x)')
plt.plot(x_taylor, f(x_taylor), label='fourth order')
plt.plot(x, exact_val, 'bo', label='exact value')
plt.plot(x, approx_val, 'r.', label='approximation value')
plt.grid()
plt.title('Taylor Expansion for cos(x)')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()
```

3. Given that $f(x) = x^3 - 0.3x^2 - 8.56x + 8.448$. **(35%)**

   a. Approximate $\int_0^{2\pi} f(x)dx$ with 20 evenly-spaced grid points over the whole interval using Riemann Integral, Trapezoid Rule, and Simpson’s Rule! Explain the difference behind each of the methods!

Manual calculation:

$f(x) = x^3 - 0.3x^2 - 8.56x + 8.448$

$h = \dfrac{2\pi - 0}{20 - 1}$

$h = \dfrac{2}{19}\pi$

(a) $x_0 = 0\left(\frac{2}{19}\pi\right)$

$x_0 = 0$

$f(x_0) = (0)^3 - 0.3(0)^2 - 8.56(0) + 8.448$

$f(x_0) = 8.448$

(b) $x_1 = 1\left(\frac{2}{19}\pi\right)$

$x_1 = \frac{2}{19}\pi$

$f(x_1) = \left(\frac{2}{19}\pi\right)^3 - 0.3\left(\frac{2}{19}\pi\right)^2 - 8.56\left(\frac{2}{19}\pi\right) + 8.448$

$f(x_1) = 5.620616318$

(c) $x_2 = 2\left(\frac{2}{19}\pi\right)$

$x_2 = \frac{4}{19}\pi$

$f(x_2) = \left(\frac{4}{19}\pi\right)^3 - 0.3\left(\frac{4}{19}\pi\right)^2 - 8.56\left(\frac{4}{19}\pi\right) + 8.448$

$f(x_2) = 2.944602707$

(d) $x_3 = 3\left(\frac{2}{19}\pi\right)$

$x_3 = \frac{6}{19}\pi$

$f(x_3) = \left(\frac{6}{19}\pi\right)^3 - 0.3\left(\frac{6}{19}\pi\right)^2 - 8.56\left(\frac{6}{19}\pi\right) + 8.448$

$f(x_3) = 0.6369443379$

(e) $x_4 = 4\left(\frac{2}{19}\pi\right)$

$x_4 = \frac{8}{19}\pi$

$f(x_4) = \left(\frac{8}{19}\pi\right)^3 - 0.3\left(\frac{8}{19}\pi\right)^2 - 8.56\left(\frac{8}{19}\pi\right) + 8.448$

$f(x_4) = -1.08537362$

(f) $x_5 = 5\left(\frac{2}{19}\pi\right)$

$x_5 = \frac{10}{19}\pi$

$f(x_5) = \left(\frac{10}{19}\pi\right)^3 - 0.3\left(\frac{10}{19}\pi\right)^2 - 8.56\left(\frac{10}{19}\pi\right) + 8.448$

$f(x_5) = -2.005365997$

(g) $x_6 = 6\left(\frac{2}{19}\pi\right)$

$x_6 = \frac{12}{19}\pi$

$f(x_6) = \left(\frac{12}{19}\pi\right)^3 - 0.3\left(\frac{12}{19}\pi\right)^2 - 8.56\left(\frac{12}{19}\pi\right) + 8.448$

$f(x_6) = -1.906047622$

(h) $x_7 = 7\left(\frac{2}{19}\pi\right)$

$x_7 = \frac{14}{19}\pi$

$f(x_7) = \left(\frac{14}{19}\pi\right)^3 - 0.3\left(\frac{14}{19}\pi\right)^2 - 8.56\left(\frac{14}{19}\pi\right) + 8.448$

$f(x_7) = -0.5704333267$

(i) $x_8 = 8\left(\frac{2}{19}\pi\right)$

$x_8 = \frac{16}{19}\pi$

$f(x_8) = \left(\frac{16}{19}\pi\right)^3 - 0.3\left(\frac{16}{19}\pi\right)^2 - 8.56\left(\frac{16}{19}\pi\right) + 8.448$

$f(x_8) = 2.21846206$

(j) $x_9 = 9\left(\frac{2}{19}\pi\right)$

$x_9 = \frac{18}{19}\pi$

$f(x_9) = \left(\frac{18}{19}\pi\right)^3 - 0.3\left(\frac{18}{19}\pi\right)^2 - 8.56\left(\frac{18}{19}\pi\right) + 8.448$

$f(x_9) = 6.677623708$

(k) $x_{10} = 10\left(\frac{2}{19}\pi\right)$

$x_{10} = \frac{20}{19}\pi$

$f(x_{10}) = \left(\frac{20}{19}\pi\right)^3 - 0.3\left(\frac{20}{19}\pi\right)^2 - 8.56\left(\frac{20}{19}\pi\right) + 8.448$

$f(x_{10}) = 13.02403679$

(l) $x_{11} = 11\left(\frac{2}{19}\pi\right)$

$x_{11} = \frac{22}{19}\pi$

$f(x_{11}) = \left(\frac{22}{19}\pi\right)^3 - 0.3\left(\frac{22}{19}\pi\right)^2 - 8.56\left(\frac{22}{19}\pi\right) + 8.448$

$f(x_{11}) = 21.47468697$

(m) $x_{12} = 12\left(\frac{2}{19}\pi\right)$

$x_{12} = \frac{24}{19}\pi$

$f(x_{12}) = \left(\frac{24}{19}\pi\right)^3 - 0.3\left(\frac{24}{19}\pi\right)^2 - 8.56\left(\frac{24}{19}\pi\right) + 8.448$

$f(x_{12}) = 32.24655792$

(n) $x_{13} = 13\left(\frac{2}{19}\pi\right)$

$x_{13} = \frac{26}{19}\pi$

$f(x_{13}) = \left(\frac{26}{19}\pi\right)^3 - 0.3\left(\frac{26}{19}\pi\right)^2 - 8.56\left(\frac{26}{19}\pi\right) + 8.448$

$f(x_{13}) = 45.5563631$

(o) $x_{14} = 14\left(\frac{2}{19}\pi\right)$

$x_{14} = \frac{28}{19}\pi$

$f(x_{14}) = \left(\frac{28}{19}\pi\right)^3 - 0.3\left(\frac{28}{19}\pi\right)^2 - 8.56\left(\frac{28}{19}\pi\right) + 8.448$

$f(x_{14}) = 61.62190681$

(p) $x_{15} = 15\left(\frac{2}{19}\pi\right)$

$x_{15} = \frac{30}{19}\pi$

$f(x_{15}) = \left(\frac{30}{19}\pi\right)^3 - 0.3\left(\frac{30}{19}\pi\right)^2 - 8.56\left(\frac{30}{19}\pi\right) + 8.448$

$f(x_{15}) = 80.65935459$

(q) $x_{16} = 16\left(\frac{2}{19}\pi\right)$

$x_{16} = \frac{32}{19}\pi$

$f(x_{16}) = \left(\frac{32}{19}\pi\right)^3 - 0.3\left(\frac{32}{19}\pi\right)^2 - 8.56\left(\frac{32}{19}\pi\right) + 8.448$

$f(x_{16}) = 102.8859648$

(r) $x_{17} = 17\left(\frac{2}{19}\pi\right)$

$x_{17} = \frac{34}{19}\pi$

$f(x_{17}) = \left(\frac{34}{19}\pi\right)^3 - 0.3\left(\frac{34}{19}\pi\right)^2 - 8.56\left(\frac{34}{19}\pi\right) + 8.448$

$f(x_{17}) = 128.5187227$

(s) $x_{18} = 18\left(\frac{2}{19}\pi\right)$

$x_{18} = \frac{36}{19}\pi$

$f(x_{18}) = \left(\frac{36}{19}\pi\right)^3 - 0.3\left(\frac{36}{19}\pi\right)^2 - 8.56\left(\frac{36}{19}\pi\right) + 8.448$

$f(x_{18}) = 157.7746133$

(t) $x_{19} = 19\left(\frac{2}{19}\pi\right)$

$x_{19} = 2\pi$

$f(x_{19}) = (2\pi)^3 - 0.3(2\pi)^2 - 8.56(2\pi) + 8.448$

$f(x_{19}) = 190.8706219$

Riemann Integral

(i) Left Riemann Integral

$\int_0^{2\pi} f(x)\,dx = \frac{2}{19}\pi(8.448 + 5.620616318 + 2.944602707 + 0.6369443379 + (-1.08537362) + (-2.005365997) + (-1.906047622) + (-0.5704333267)$
$+ 2.21846206 + 6.677623708 + 13.02403679 + 21.47468647 + 32.24655792 + 45.55663631 + 61.62190681 + 80.65935459 +$
$102.8859648 + 128.5187227 + 157.7746133)$

$\int_0^{2\pi} f(x)\,dx = 219.8268$

(ii) Right Riemann Integral

$\int_0^{2\pi} f(x)\,dx = \frac{2}{19}\pi(5.620616318 + 2.944602707 + 0.6369443379 + (-1.08537362) + (-2.005365997) + (-1.906047622) + (-0.5704333267) +$
$2.21846206 + 6.677623708 + 13.02403679 + 21.47468647 + 32.24655792 + 45.55663631 + 61.62190681 + 80.65935459 +$
$102.8859648 + 128.5187227 + 157.7746133 + 190.8706219)$

$\int_0^{2\pi} f(x)\,dx = 280.1529$

(iii) Midpoint Rule

(1) $y_0 = \frac{\frac{2}{19}\pi + 0}{2}$

$y_0 = \frac{1}{19}\pi$

$f(y_0) = \left(\frac{1}{19}\pi\right)^3 - 0.3\left(\frac{1}{19}\pi\right)^2 - 8.56\left(\frac{1}{19}\pi\right) + 8.448$

$f(y_0) = 7.028948473$

(2) $y_1 = \frac{\frac{4}{19}\pi + \frac{2}{19}\pi}{2}$

$y_1 = \frac{3}{19}\pi$

$f(y_1) = \left(\frac{3}{19}\pi\right)^3 - 0.3\left(\frac{3}{19}\pi\right)^2 - 8.56\left(\frac{3}{19}\pi\right) + 8.448$

$f(y_1) = 4.25012668$

(3) $y_2 = \frac{\frac{6}{19}\pi + \frac{4}{19}\pi}{2}$

$y_2 = \frac{5}{19}\pi$

$f(y_2) = \left(\frac{5}{19}\pi\right)^3 - 0.3\left(\frac{5}{19}\pi\right)^2 - 8.56\left(\frac{5}{19}\pi\right) + 8.448$

$f(y_2) = 1.7311675 44$

(4) $y_3 = \frac{\frac{8}{19}\pi + \frac{6}{19}\pi}{2}$

$y_3 = \frac{7}{19}\pi$

$f(y_3) = \left(\frac{7}{19}\pi\right)^3 - 0.3\left(\frac{7}{19}\pi\right)^2 - 8.56\left(\frac{7}{19}\pi\right) + 8.448$

$f(y_3) = -0.3109437657$

(5) $y_4 = \frac{\frac{10}{19}\pi + \frac{8}{19}\pi}{2}$

$y_4 = \frac{9}{19}\pi$

$f(y_4) = \left(\frac{9}{19}\pi\right)^3 - 0.3\left(\frac{9}{19}\pi\right)^2 - 8.56\left(\frac{9}{19}\pi\right) + 8.448$

$f(y_4) = -1.659222079$

(6) $y_5 = \frac{\frac{12}{19}\pi + \frac{10}{19}\pi}{2}$

$y_5 = \frac{11}{19}\pi$

$f(y_5) = \left(\frac{11}{19}\pi\right)^3 - 0.3\left(\frac{11}{19}\pi\right)^2 - 8.56\left(\frac{11}{19}\pi\right) + 8.448$

$f(y_5) = -2.096682227$

(7) $y_6 = \frac{\frac{14}{19}\pi + \frac{12}{19}\pi}{2}$

$y_6 = \frac{13}{19}\pi$

$f(y_6) = \left(\frac{13}{19}\pi\right)^3 - 0.3\left(\frac{13}{19}\pi\right)^2 - 8.56\left(\frac{13}{19}\pi\right) + 8.448$

$f(y_6) = -1.406339038$

(8) $y_7 = \frac{\frac{16}{19}\pi + \frac{14}{19}\pi}{2}$

$y_7 = \frac{15}{19}\pi$

$f(y_7) = \left(\frac{15}{19}\pi\right)^3 - 0.3\left(\frac{15}{19}\pi\right)^2 - 8.56\left(\frac{15}{19}\pi\right) + 8.448$

$f(y_7) = 0.6287926571$

(9) $y_8 = \frac{\frac{18}{19}\pi + \frac{16}{19}\pi}{2}$

$y_8 = \frac{17}{19}\pi$

$f(y_8) = \left(\frac{17}{19}\pi\right)^3 - 0.3\left(\frac{17}{19}\pi\right)^2 - 8.56\left(\frac{17}{19}\pi\right) + 8.448$

$f(y_8) = 4.225698028$

(10) $y_9 = \frac{\frac{20}{19}\pi + \frac{18}{19}\pi}{2}$

$y_9 = \pi$

$f(y_9) = (\pi)^3 - 0.3(\pi)^2 - 8.56(\pi) + 8.448$

$f(y_9) = 9.601362245$

(11) $y_{10} = \frac{\frac{22}{19}\pi + \frac{20}{19}\pi}{2}$

$y_{10} = \frac{21}{19}\pi$

$f(y_{10}) = \left(\frac{21}{19}\pi\right)^3 - 0.3\left(\frac{21}{19}\pi\right)^2 - 8.56\left(\frac{21}{19}\pi\right) + 8.448$

$f(y_{10}) = 16.97277048$

(12) $y_{11} = \frac{\frac{24}{19}\pi + \frac{22}{19}\pi}{2}$

$y_{11} = \frac{23}{19}\pi$

$f(y_{11}) = \left(\frac{23}{19}\pi\right)^3 - 0.3\left(\frac{23}{19}\pi\right)^2 - 8.56\left(\frac{23}{19}\pi\right) + 8.448$

$f(y_{11}) = 26.5569079$

(13) $y_{12} = \frac{\frac{26}{19}\pi + \frac{24}{19}\pi}{2}$

$y_{12} = \frac{25}{19}\pi$

$f(y_{12}) = \left(\frac{25}{19}\pi\right)^3 - 0.3\left(\frac{25}{19}\pi\right)^2 - 8.56\left(\frac{25}{19}\pi\right) + 8.448$

$f(y_{12}) = 38.57075967$

(14) $y_{13} = \frac{\frac{28}{19}\pi + \frac{26}{19}\pi}{2}$

$y_{13} = \frac{27}{19}\pi$

$f(y_{13}) = \left(\frac{27}{19}\pi\right)^3 - 0.3\left(\frac{27}{19}\pi\right)^2 - 8.56\left(\frac{27}{19}\pi\right) + 8.448$

$f(y_{13}) = 53.23131097$

(15) $y_{14} = \frac{\frac{30}{19}\pi + \frac{28}{19}\pi}{2}$

$y_{14} = \frac{29}{19}\pi$

$f(y_{14}) = \left(\frac{29}{19}\pi\right)^3 - 0.3\left(\frac{29}{19}\pi\right)^2 - 8.56\left(\frac{29}{19}\pi\right) + 8.448$

$f(y_{14}) = 70.75554697$

(16) $y_{15} = \frac{\frac{32}{19}\pi + \frac{30}{19}\pi}{2}$

$y_{15} = \frac{31}{19}\pi$

$f(y_{15}) = \left(\frac{31}{19}\pi\right)^3 - 0.3\left(\frac{31}{19}\pi\right)^2 - 8.56\left(\frac{31}{19}\pi\right) + 8.448$

$f(y_{15}) = 91.36045283$

(17) $y_{16} = \frac{\frac{34}{19}\pi + \frac{32}{19}\pi}{2}$

$y_{16} = \frac{33}{19}\pi$

$f(y_{16}) = \left(\frac{33}{19}\pi\right)^3 - 0.3\left(\frac{33}{19}\pi\right)^2 - 8.56\left(\frac{33}{19}\pi\right) + 8.448$

$f(y_{16}) = 115.2630137$

(18) $y_{17} = \frac{\frac{36}{19}\pi + \frac{34}{19}\pi}{2}$

$y_{17} = \frac{35}{19}\pi$

$f(y_{17}) = \left(\frac{35}{19}\pi\right)^3 - 0.3\left(\frac{35}{19}\pi\right)^2 - 8.56\left(\frac{35}{19}\pi\right) + 8.448$

$f(y_{17}) = 142.6802148$

(19) $y_{18} = \frac{\frac{38}{19}\pi + \frac{36}{19}\pi}{2}$

$y_{18} = \frac{37}{19}\pi$

$f(y_{18}) = \left(\frac{37}{19}\pi\right)^3 - 0.3\left(\frac{37}{19}\pi\right)^2 - 8.56\left(\frac{37}{19}\pi\right) + 8.448$

$f(y_{18}) = 173.8290413$

$\int_0^{2\pi} f(x)\,dx = \frac{2}{19}\pi(7.028948473 + 4.25012668 + 1.731167544 + (-0.3109437657)$
$+ (-1.659222079) + (-2.096682227) + (-1.406339038) + 0.6287926571$
$+ 4.225698028 + 9.601362245 + 16.97277048 + 26.5569079 +$
$38.57075967 + 53.23131097 + 70.75554697 + 91.36045283 +$
$115.2630137 + 142.6802148 + 173.8290413)$

$\int_0^{2\pi} f(x)\,dx = 248.4216$

Trapezoid Rule

$\int_0^{2\pi} f(x)\,dx = \frac{\frac{2}{19}\pi}{2}(8.448 + (2(5.620616318 + 2.944602707 + 0.6369443379 + (-1.08537362)$
$+ (-2.005365997) + (-1.906047622) + (-0.5704333267) + 2.21846206 + 6.677623708$
$+ 13.02403679 + 21.47468647 + 32.24655792 + 45.55663631 + 61.62190681 +$
$80.65935459 + 102.8859648 + 128.5187227 + 157.7746133)) + 190.8706219)$

$\int_0^{2\pi} f(x)\,dx = 249.9899$

Simpson's Rule

$h = \frac{2\pi - 0}{20}$

$h = \frac{1}{10}\pi$

(1) $x_0 = 0\left(\frac{1}{10}\pi\right)$

$x_0 = 0$

$f(x_0) = 8.448$

(2) $x_1 = 1\left(\frac{1}{10}\pi\right)$

$x_1 = \frac{1}{10}\pi$

$f(x_1) = \left(\frac{1}{10}\pi\right)^3 - 0.3\left(\frac{1}{10}\pi\right)^2 - 8.56\left(\frac{1}{10}\pi\right) + 8.448$

$f(x_1) = 5.760149152$

(3) $x_2 = 2\left(\frac{1}{10}\pi\right)$

$x_2 = \frac{1}{5}\pi$

$f(x_2) = \left(\frac{1}{5}\pi\right)^3 - 0.3\left(\frac{1}{5}\pi\right)^2 - 8.56\left(\frac{1}{5}\pi\right) + 8.448$

$f(x_2) = 3.189208339$

(4) $x_3 = 3\left(\frac{1}{10}\pi\right)$

$x_3 = \frac{3}{10}\pi$

$f(x_3) = \left(\frac{3}{10}\pi\right)^3 - 0.3\left(\frac{3}{10}\pi\right)^2 - 8.56\left(\frac{3}{10}\pi\right) + 8.448$

$f(x_3) = 0.9510802171$

(5) $x_4 = \overset{2}{\cancel{4}}\left(\frac{1}{\underset{5}{\cancel{10}}}\pi\right)$

$x_4 = \frac{2}{5}\pi$

$f(x_4) = \left(\frac{2}{5}\pi\right)^3 - 0.3\left(\frac{2}{5}\pi\right)^2 - 8.56\left(\frac{2}{5}\pi\right) + 8.448$

$f(x_4) = -0.7981525496$

(6) $x_5 = \cancel{5}\left(\frac{1}{\underset{2}{\cancel{10}}}\pi\right)$

$x_5 = \frac{1}{2}\pi$

$f(x_5) = \left(\frac{1}{2}\pi\right)^3 - 0.3\left(\frac{1}{2}\pi\right)^2 - 8.56\left(\frac{1}{2}\pi\right) + 8.448$

$f(x_5) = -1.862452302$

(7) $x_6 = \overset{3}{\cancel{6}}\left(\frac{1}{\underset{5}{\cancel{10}}}\pi\right)$

$x_6 = \frac{3}{5}\pi$

$f(x_6) = \left(\frac{3}{5}\pi\right)^3 - 0.3\left(\frac{3}{5}\pi\right)^2 - 8.56\left(\frac{3}{5}\pi\right) + 8.448$

$f(x_6) = -2.055781381$

(8) $x_7 = 7\left(\frac{1}{10}\pi\right)$

$x_7 = \frac{7}{10}\pi$

$f(x_7) = \left(\frac{7}{10}\pi\right)^3 - 0.3\left(\frac{7}{10}\pi\right)^2 - 8.56\left(\frac{7}{10}\pi\right) + 8.448$

$f(x_7) = -1.192102126$

(9) $x_8 = \overset{4}{\cancel{8}}\left(\frac{1}{\underset{5}{\cancel{10}}}\pi\right)$

$x_8 = \frac{4}{5}\pi$

$f(x_8) = \left(\frac{4}{5}\pi\right)^3 - 0.3\left(\frac{4}{5}\pi\right)^2 - 8.56\left(\frac{4}{5}\pi\right) + 8.448$

$f(x_8) = 0.9146231235$

(10) $x_9 = 9\left(\frac{1}{10}\pi\right)$

$x_9 = \frac{9}{10}\pi$

$f(x_9) = \left(\frac{9}{10}\pi\right)^3 - 0.3\left(\frac{9}{10}\pi\right)^2 - 8.56\left(\frac{9}{10}\pi\right) + 8.448$

$f(x_9) = 4.450432027$

(11) $x_{10} = \cancel{10}\left(\frac{1}{\cancel{10}}\pi\right)$

$x_{10} = \pi$

$f(x_{10}) = (\pi)^3 - 0.3(\pi)^2 - 8.56(\pi) + 8.448$

$f(x_{10}) = 9.601362245$

(12) $x_{11} = 11\left(\frac{1}{10}\pi\right)$

$x_{11} = \frac{11}{10}\pi$

$f(x_{11}) = \left(\frac{11}{10}\pi\right)^3 - 0.3\left(\frac{11}{10}\pi\right)^2 - 8.56\left(\frac{11}{10}\pi\right) + 8.448$

$f(x_{11}) = 16.55345194$

(13) $x_{12} = \overset{6}{\cancel{12}}\left(\frac{1}{\underset{5}{\cancel{10}}}\pi\right)$

$x_{12} = \frac{6}{5}\pi$

$f(x_{12}) = \left(\frac{6}{5}\pi\right)^3 - 0.3\left(\frac{6}{5}\pi\right)^2 - 8.56\left(\frac{6}{5}\pi\right) + 8.448$

$f(x_{12}) = 25.49273726$

(14) $x_{13} = 13\left(\frac{1}{10}\pi\right)$

$x_{13} = \frac{13}{10}\pi$

$f(x_{13}) = \left(\frac{13}{10}\pi\right)^3 - 0.3\left(\frac{13}{10}\pi\right)^2 - 8.56\left(\frac{13}{10}\pi\right) + 8.448$

$f(x_{13}) = 36.60525739$

(15) $x_{14} = \overset{7}{\cancel{14}}\left(\frac{1}{\underset{5}{\cancel{10}}}\pi\right)$

$x_{14} = \frac{7}{5}\pi$

$f(x_{14}) = \left(\frac{7}{5}\pi\right)^3 - 0.3\left(\frac{7}{5}\pi\right)^2 - 8.56\left(\frac{7}{5}\pi\right) + 8.448$

$f(x_{14}) = 50.07704946$

(16) $x_{15} = \overset{3}{\cancel{15}}\left(\frac{1}{\underset{2}{\cancel{10}}}\pi\right)$

$x_{15} = \frac{3}{2}\pi$

$f(x_{15}) = \left(\frac{3}{2}\pi\right)^3 - 0.3\left(\frac{3}{2}\pi\right)^2 - 8.56\left(\frac{3}{2}\pi\right) + 8.448$

$f(x_{15}) = 66.09415115$

(17) $x_{16} = \overset{8}{\cancel{16}}\left(\frac{1}{\underset{5}{\cancel{10}}}\pi\right)$

$x_{16} = \frac{8}{5}\pi$

$f(x_{16}) = \left(\frac{8}{5}\pi\right)^3 - 0.3\left(\frac{8}{5}\pi\right)^2 - 8.56\left(\frac{8}{5}\pi\right) + 8.448$

$f(x_{16}) = 84.84260012$

(18) $x_{17} = 17\left(\frac{1}{10}\pi\right)$

$x_{17} = \frac{17}{10}\pi$

$f(x_{17}) = \left(\frac{17}{10}\pi\right)^3 - 0.3\left(\frac{17}{10}\pi\right)^2 - 8.56\left(\frac{17}{10}\pi\right) + 8.448$

$f(x_{17}) = 106.508434$

(19) $x_{18} = \overset{9}{\cancel{18}}\left(\frac{1}{\underset{5}{\cancel{10}}}\pi\right)$

$x_{18} = \frac{9}{5}\pi$

$f(x_{18}) = \left(\frac{9}{5}\pi\right)^3 - 0.3\left(\frac{9}{5}\pi\right)^2 - 8.56\left(\frac{9}{5}\pi\right) + 8.448$

$f(x_{18}) = 131.2776905$

(20) $x_{19} = 19\left(\frac{1}{10}\pi\right)$

$x_{19} = \frac{19}{10}\pi$

$f(x_{19}) = \left(\frac{19}{10}\pi\right)^3 - 0.3\left(\frac{19}{10}\pi\right)^2 - 8.56\left(\frac{19}{10}\pi\right) + 8.448$

$f(x_{19}) = 159.3364073$

(21) $x_{20} = \cancel{20}\left(\frac{1}{\cancel{10}}\pi\right)$

$x_{20} = 2\pi$

$f(x_{20}) = (2\pi)^3 - 0.3(2\pi)^2 - 8.56(2\pi) + 8.448$

$f(x_{20}) = 190.8706219$

$$\int_0^{2\pi} f(x)\,dx = \frac{\frac{1}{10}\pi}{3}\Big(8.448 + (2(3.199208338 + (-0.7981525496) + (-2.055781381) + 0.9146231235 + 9.601362245 + 25.49273726 + 50.07704946 + 84.84260012 + 131.2776905) + (4(5.760194152 + 0.9510802171 + (-1.862452302) + (-1.192102126) + 4.450432027 + 16.55345194 + 36.60525739 + 66.09415115 + 106.508434 + 159.3364073) + 190.8706219\Big)$$

$$\int_0^{2\pi} f(x)\,dx = 248.9491$$

Python script:

```
# a.
```

```
import numpy as np

def f(x):
    return (x**3 - (0.3 * x**2) - (8.56 * x) + 8.448)

a = 0
b = 2 * np.pi

n = 20
width = (b - a) / (n - 1)

x = np.linspace(a, b, n)
y = f(x)

# Riemann Integral
# (i) Left Riemann Integral
left_riemann = np.round((width * sum(y[:(n - 1)])), 4)
print(f'Left Riemann Integral = {left_riemann}')

# (ii) Right Riemann Integral
right_riemann = np.round((width * sum(y[1:])), 4)
print(f'Right Riemann Integral = {right_riemann}')

# (iii) Midpoint Rule
x_mid = (x[:(n - 1)] + x[1:]) / 2
y_mid = f(x_mid)

mid_rule = np.round((width * sum(y_mid)), 4)
print(f'Midpoint Rule = {mid_rule}')
```

```
# Trapezoid Rule
trapezoid = np.round(((width / 2) * (y[0] + (2 * sum(y[1:(n - 1)])) + y[n - 1])), 4)
print(f'Trapezoid Rule = {trapezoid}')

# Simpson's Rule
width_simspon = (b - a) / n

x_simpson = np.linspace(a, b, (n + 1))
y_simpson = f(x_simpson)

simpson = np.round(((width_simspon / 3) * (y_simpson[0] + (2 * sum(y_simpson[2:(n - 1):2])) + (4 * sum(y_simpson[1:n:2])) + y_simpson[n])), 4)
print(f'Simpson\'s Rule = {simpson}')
```

Analysis:

The Riemann Integral is a fundamental concept in calculus used as an exact method for evaluating definite integrals. The Riemann Integral involves partitioning the interval of the integration into subintervals and approximating the area under the curve using rectangles. The Riemann Integral becomes more accurate as the number of subintervals increases, approaching the exact value of the integral, and depends on the width of the subintervals and the chosen points within each subinterval (left, right, or midpoint). The Trapezoidal Rule and Simpson’s Rule are numerical approximation methods that provide increasingly accurate results by considering more information about the function being integrated. The Trapezoidal Rule approximates the definite integral by dividing the interval of integration into trapezoids and summing the areas of the trapezoids formed by adjacent points on the curve and the x-axis. The Trapezoidal Rule assumes that the function between two adjacent points can be approximated as a straight line by linear interpolation. It provides a better approximation than the Riemann Integral by accounting for the slope of the curve but still introduces some error. Simpson’s Rule approximates the area under the curve by dividing the interval of the integration into a series of parabolic segments. Simpson’s Rule assumes that the function between three adjacent points can be approximated using a second-degree polynomial. It provides a higher degree of accuracy compared to the Trapezoidal Rule as it captures the curvature of the function more effectively. Simpson’s Rule converges to the exact value of the integral faster as the number of subintervals increases.

b. Compared to the methods above, do you think that analytical integration could be more convenient to be done?

Analysis:

Based on the calculation above, analytical integration could be more convenient to be done. Analytical integration involves finding a closed-form expression for the antiderivative of the function being integrated, which allows us to perform an exact evaluation of the integral. Analytical integration provides exact results for integrals by assuming the antiderivative could be found, which allows us to avoid approximation errors using numerical integration. Once we have found the antiderivative, evaluating the integral using analytical integration is usually more straightforward and computationally efficient as it eliminates the need for iterative calculations. Analytical integration allows symbolic manipulation of the integrand and the resulting integral expression, which would be beneficial for further mathematical analysis, simplification, and algebraic operations. Analytical integration often provides better insight into the behavior and properties of the integrated function, which could be useful to reveal the relationships between different parts of the function and uncover mathematical patterns and symmetries. However, numerical integration methods become useful when dealing with functions that could not be integrated analytically or when the integrand is available only as a set of discrete data points.

c. Use polynomial interpolation to compute $f'(x)$ and $f''(x)$ at $x = 0$, using the discrete data below:

$\boldsymbol{x}$	-1.1	-0.3	0.8	1.9
$\boldsymbol{f(x)}$	15.180	10.962	1.920	-2.040

Manual calculation:

$n=4$

$x=-1.1,\ -0.3,\ 0.8,\ 1.9$

$$\begin{bmatrix} 4 & (-1.1-0.3+0.8+1.9) & ((-1.1)^2+(-0.3)^2+0.8^2+1.9^2) & ((-1.1)^3+(-0.3)^3+0.8^3+1.9^3) \\ (-1.1-0.3+0.8+1.9) & ((-1.1)^2+(-0.3)^2+0.8^2+1.9^2) & ((-1.1)^3+(-0.3)^3+0.8^3+1.9^3) & ((-1.1)^4+(-0.3)^4+0.8^4+1.9^4) \\ ((-1.1)^2+(-0.3)^2+0.8^2+1.9^2) & ((-1.1)^3+(-0.3)^3+0.8^3+1.9^3) & ((-1.1)^4+(-0.3)^4+0.8^4+1.9^4) & ((-1.1)^5+(-0.3)^5+0.8^5+1.9^5) \\ ((-1.1)^3+(-0.3)^3+0.8^3+1.9^3) & ((-1.1)^4+(-0.3)^4+0.8^4+1.9^4) & ((-1.1)^5+(-0.3)^5+0.8^5+1.9^5) & ((-1.1)^6+(-0.3)^6+0.8^6+1.9^6) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} =$$

$$\begin{bmatrix} (15.180+10.962+1.920-2.040) \\ ((15.180(-1.1))+(10.962(-0.3))+(1.920(0.8))-(2.040(1.9))) \\ ((15.180(-1.1)^2)+(10.962(-0.3)^2)+(1.920(0.8)^2)-(2.040(1.9)^2)) \\ ((15.180(-1.1)^3)+(10.962(-0.3)^3)+(1.920(0.8)^3)-(2.040(1.9)^3)) \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1.3 & 5.55 & 6.013 \\ 1.3 & 5.55 & 6.013 & 14.9139 \\ 5.55 & 6.013 & 14.9139 & 23.47573 \\ 6.013 & 14.9139 & 23.47573 & 49.080315 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 26.022 \\ -22.3266 \\ 13.21878 \\ -33.509874 \end{bmatrix}$$

$$\begin{bmatrix} 8.547 & -8.4058552631578947381 & -0.82105263157894736882 & 1.2171052631578947368 \end{bmatrix}^T$$

$f'(0) \approx P_3'(0)$

$f'(0) = -8.4058552631578947381 + 2(-0.82105263157894736882)(0) + 3(1.2171052631578947368)(0)^2$

$f'(0) = -8.4059$

$f''(0) \approx P_3''(0)$

$f''(0) = 2(-0.82105263157894736882) + 6(1.2171052631578947368)(0)$

$f''(0) = -1.6421$

Python script:

```
# c.

import matplotlib.pyplot as plt

plt.style.use('seaborn-poster')

%matplotlib inline

def divided_diff(x, y):
    n = len(y)
    coef = np.zeros([n, n])
    coef[:,0] = y
```

```
    for j in range(1, n):
        for i in range(n - j):
            coef[i][j] = (coef[i + 1][j - 1] - coef[i][j - 1]) / (x[i + j] - x[i])

    return coef

def newton_poly(coef, x_data, x):
    n = len(x_data) - 1
    p = coef[n]
    for k in range(1, (n + 1)):
        p = coef[n - k] + ((x - x_data[n - k]) * p)
    return p

x = np.array([-1.1, -0.3, 0.8, 1.9])
y = np.array([15.180, 10.962, 1.920, -2.040])
a_s = divided_diff(x, y)[0, :]

x_new = np.arange(x[0], (x[len(x) - 1] + .1), .1)
y_new = newton_poly(a_s, x, x_new)

def f_1_poly(x):
    return (-8.40585526231578947381 - (2 * 0.82105263157894736882 * x) + (3 * 1.2171052631578947368 * x**2))

def f_2_poly(x):
    return (-(2 * 0.82105263157894736882) + (3 * 1.2171052631578947368 * x))

f_1_poly_0 = np.round(f_1_poly(0), 4)
print(f"f'(0) = {f_1_poly_0}")
```

```
f_2_poly_0 = np.round(f_2_poly(0), 4)
print(f"f''(0) = {f_2_poly_0}")

plt.plot(x_new, y_new, label='f(x)')
plt.plot(x_new, f_1_poly(x_new), label="f'(x)")
plt.plot(x_new, f_2_poly(x_new), label="f''(x)")
plt.plot(x, y, 'b.', label='data points')
plt.plot(0, f_1_poly_0, 'r.', label="f'(0)")
plt.plot(0, f_2_poly_0, 'k.', label="f''(0)")
plt.title('Polynomial Interpolation')
plt.ylabel('y')
plt.xlabel('x')
plt.legend()
plt.show()
```

d. Calculate the result accuracy compared to the initial $f(x)$!

Manual calculation:

Exact value $f'(0) = 3(0)^2 - 2(0.3(0)) - 8.56$

Exact value $f'(0) = -8.56$

Error $f'(0) = |-8.56 - (-8.4059)|$

Error $f'(0) = 0.1541$

Exact value $f''(0) = 2(3(0)) - 2(0.3)$

Exact value $f''(0) = -0.6$

Error $f''(0) = |-0.6 - (-1.6421)|$

Error $f''(0) = 1.0421$

Python script:

```
# d.
```

```
def f_1(x):
    return ((3 * x**2) - (2 * 0.3 * x) - 8.56)

def f_2(x):
    return ((2 * (3 * x)) - (2 * 0.3))

exact_f_1 = f_1(0)
error_f_1 = np.round(np.abs(exact_f_1 - f_1_poly_0), 4)
print(f"Error f'(0) = {error_f_1}")

exact_f_2 = f_2(0)
error_f_2 = np.round(np.abs(exact_f_2 - f_2_poly_0), 4)
print(f"Error f''(0) = {error_f_2}")

plt.plot(x_new, y_new, label='f(x) by polynomial interpolation')
plt.plot(x_new, f_1_poly(x_new), label="f'(x) by polynomial interpolation")
plt.plot(x_new, f_2_poly(x_new), label="f''(x) by polynomial interpolation")
plt.plot(x, y, 'bo', label='data points')
plt.plot(0, f_1_poly_0, 'r.', label="f'(0) by polynomial interpolation")
plt.plot(0, f_2_poly_0, 'k.', label="f''(0) by polynomial interpolation")
plt.plot(x_new, f(x_new), label='initial f(x)')
plt.plot(x_new, f_1(x_new), label="initial f'(x) by polynomial interpolation")
plt.plot(x_new, f_2(x_new), label="initial f''(x)")
plt.plot(x, f(x), 'g.', label='initial data points')
plt.plot(0, exact_f_1, 'y.', label="initial f'(0)")
plt.plot(0, exact_f_2, 'c.', label="initial f''(0)")
plt.title('Polynomial Interpolation')
plt.ylabel('y')
plt.xlabel('x')
```

```
plt.legend()
plt.show()
```

Analysis:

The error of the f'(0) shows that the f'(0) computed using polynomial interpolation is almost as accurate as the f'(0) calculated using the initial f(x). However, the error of the f''(0) shows that the f''(0) computed using polynomial interpolation is not quite accurate as the f''(0) calculated using the initial f(x).

Note for Lecturers:

1. The lecturers are advised to assess student's understanding towards the topics included in the assignment.
2. The students will submit their answers in .pdf format through BINUSMAYA.
3. The deadline for this comprehensive assignment is at the end of the semester.