Déduction naturelle

$$\frac{}{\Gamma \vdash A}\text{ si } A \in \Gamma \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B}\text{ Intro} \Rightarrow \qquad \frac{\Gamma \vdash A \Rightarrow B \qquad \Gamma \vdash A}{\Gamma \vdash B}\text{ Elim} \Rightarrow$$

Figure 1: Déduction naturelle: logique minimale

$$\frac{}{\Gamma \vdash A}\text{ si } A \in \Gamma \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B}\text{ Intro} \Rightarrow \qquad \frac{\Gamma \vdash A \Rightarrow B \qquad \Gamma \vdash A}{\Gamma \vdash B}\text{ Elim} \Rightarrow$$

$$\frac{\Gamma \vdash \bot}{\Gamma \vdash P}\text{ Elim } \bot$$

$$\frac{\Gamma, A \vdash \bot}{\Gamma \vdash \neg A}\text{ Intro } \neg \qquad \frac{\Gamma \vdash \neg A \qquad \Gamma \vdash A}{\Gamma \vdash \bot}\text{ Elim } \neg$$

$$\frac{\Gamma \vdash P \qquad \Gamma \vdash Q}{\Gamma \vdash P \wedge Q}\text{ Intro } \wedge \qquad \frac{\Gamma \vdash P \wedge Q}{\Gamma \vdash P}\text{ Elim } \wedge\text{g} \qquad \frac{\Gamma \vdash P \wedge Q}{\Gamma \vdash Q}\text{ Elim } \wedge\text{d}$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash P \vee Q}\text{ Intro } \vee\text{g} \qquad \frac{\Gamma \vdash Q}{\Gamma \vdash P \vee Q}\text{ Intro } \vee\text{d}$$

$$\frac{\Gamma \vdash P \vee Q \qquad \Gamma, P \vdash R \qquad \Gamma, Q \vdash R}{\Gamma \vdash R}\text{ Elim } \vee$$

Figure 2: Déduction naturelle: logique intuitionniste

$$\frac{\Gamma, \neg P \vdash \bot}{\Gamma \vdash P}\text{ RAA}$$

Figure 3: Déduction naturelle: logique classique

$$\frac{\Gamma \vdash A}{\Gamma \vdash \forall x\, A}\ \forall\text{ intro } (x \text{ n'est pas libre dans } \Gamma) \qquad \frac{\Gamma \vdash \forall x\, A}{\Gamma \vdash A[x \leftarrow t]}\ \forall\text{ elim}$$

$$\frac{\Gamma \vdash A[x \leftarrow t]}{\Gamma \vdash \exists x\, A}\ \exists\text{ intro} \qquad \frac{\Gamma \vdash \exists x\, A \qquad \Gamma, A \vdash B}{\Gamma \vdash B}\ \exists\text{ elim } (x \text{ n'est pas libre dans } \Gamma \text{ ni } B)$$

Figure 4: Déduction naturelle: logique des prédicats