MASTER THESIS

Profit Optimization of Large Scale Battery Operation in the UK Electricity Markets

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July 7, 2022

Abstract

The transition towards an electricity market with a high penetration of renewable energy is characterized by higher price volatility compared to the more traditional electricity market dominated by conventional generation. The increasing share of intermittent generation requires a method of securing supply and matching demand with supply. A solution to handling this issue is the integration of batteries into the system as a fully controllable energy storage.

The aim of this project is to improve the economic viability of these batteries, in order to increase their incentive to be invested in and participate in the electricity markets. Higher battery capacity allows for increased fluctuations in generation, which in turn allows for the possibility of adding more renewable generation. Thus, it is argued that profitable battery operations will support the green transition.

In this paper, a two-stage stochastic model constitutes the main component in achieving optimized battery operations across the Intra-Day and Day-Ahead electricity markets in the United Kingdom. The model is inspired by an already existing battery operated in the UK by Ørsted, and has the objective of being as commercially viable as possible. This model relies on forecasts of the electricity prices, which are obtained through training Decision Trees and Random Forests on historic prices as well as other features. These forecasts are compared across two test sets each exhibiting different price environments and the best performing forecasts are selected for sampling and clustering. Hereafter, the most promising of these are applied to a validation set.

The models developed in combination with the forecasts generated are able to attain profits in the range of 30-40 % compared to the attained profit with perfect foresight of price developments, i.e. supplying the models with realized prices. Several sensitivity analyses of various aspects are carried out, including price spreads, model parameters, and effect of robustness measures through Conditional Value at Risk (CVaR). In addition, it is investigated how participating in both markets compared to focusing on just one influences the battery scheduling. Based on the findings in this paper, it is concluded that operating a battery in the investigated markets is a profitable endeavour.

Contents

Scope

The aim of this paper is to develop a two-stage stochastic model and optimize the economic performance of the Ørsted-owned Lithium-Ion battery connected to the UK electricity grid across the Intra-Day and Day-Ahead markets.
The hypothesis is that a two-stage stochastic model will perform better than the current sequential optimization model in use by Ørsted.
Consequently, an in-depth research of the electricity market, the sensitivity of the model and price forecasting will be carried out to compare performance and insights, which will be utilized to increase the economic performance of the Ørsted-owned battery.

Motivation and State-of-the-Art

In an electricity market environment that is rapidly increasing its share of intermittent generation capacity mainly in the shape of wind and solar, a need for methods of securing on-demand electricity arises. However, it has proven difficult to preserve the green profile when solving the issues around security of supply, since the majority of Renewable Energy Sources (RES) are inherently dependent on mostly uncontrollable factors.
Here, a symbiotic relationship emerges between storage facilities, e.g. batteries, and RES. While the green electricity supply can not always match the demand, it can always be stored and thus shift the generation profile to match the demand profile. This mechanism can enhance the profitability of a generator connected to a battery and in turn increase the sentiment to invest in more RES, which is an endeavour worth working towards.
However, in order for the battery to increase the profitability of its supplying generator a framework around its operation must exist, which is the objective in this project.

Such a framework will firstly require a method of forecasting prices, as operational decisions mostly have to take place hours if not days ahead of the price being revealed. While the options around forecasting are plentiful, the methods in this paper are narrowed down to Decision Trees and Random Forests through a machine learning approach. This angle of attack was inspired by its ability to predict price spikes, as an analysis of the 2021 price levels, provided later in this paper, shows there is potential in operating during price spikes. The application of this forecast method is demonstrated in the paper [8], where both classification and regression trees were used in combination to forecast price spikes in multiple bidding zones. Furthermore, these methods, especially Decision Trees, have a high degree of versatility, which is reflected in its wide range of usage across different markets and sectors. Its transparent data treatment process makes it a viable option for most forecasting purposes.

Secondly, this paper incorporates a profit maximizing two-stage stochastic model into this framework, which determines the optimal scheduling in the Day-Ahead and Intra-Day market. Stochasticity is introduced to account for the uncertainty in market prices and inter temporal constraints between the markets are used to ensure the scheduling constraints are complied with.
As the electricity markets up until real time operations can be regarded as different derivative markets, various approaches to determine an optimal scheduling can be found in literature. Some of these are sequential optimization [9], robust optimization, dynamic programming [21] and multi stage stochastic optimization [13] [6] [7]. The objective functions observed in literature also vary a lot depending on the goal of the study. Maximizing profit [13] [6] or social welfare [10] as well as determining the optimal bidding curves [9] for bidding in the electricity market. In this paper a battery storage system participates in two markets: Day-Ahead and Intra-Day. In literature, the mentioned approaches are used to optimize the operations in different settings. As an example, the potential of operating a battery storage system jointly with a wind power plant [13] or a micro-grid [7] are common settings.

The vision and aspiration of this framework is to address some of the issues raised around battery operations where the challenges are as numerous as the benefits.
One of these challenges, is how a large amount of storage can change entire market dynamics for better or for worse. One paper argues how a significant share of storage capacity decrease prices in peak hours and smooth out the daily price curves [16], which is an obstacle as these fluctuations often constitute a major advantage for a battery as will be elaborated upon in later sections of this project.

Other concerns raised in publications lie in the profitability of batteries in the current landscape. One such opinion is expressed in a report by EA Energy Analysis from 2020 on storage options in the Danish market. Here, amongst other, it is concluded that "*Participating in the day-ahead market and, more generally, in energy-only markets does not constitute a viable business case for storage today or in the near future*" and that "*Combination of different markets are possible; however, the limited storage capacity creates challenges*" with the latter referring to the obstacle of delivering on multiple contracts while securing a sufficient amount of stored electricity [18].
Both these statements are currently very relevant, but it is hypothesized in this project that a decision model able to account for multiple markets simultaneously could address the concerns raised here, and with these goals in mind, the first step of the way is to understand the markets, presented in the following section.

Market Background

The batteries owned by Ørsted are connected to the electricity grid in Great Britain, and the battery operations are thus affected by its local market structure. In order to ensure the stability of the electrical grid, supply has to equal consumption, otherwise the grid frequency would change and result in instability and as a worst-case scenario lead to major blackouts. Thus, many different markets regarding the sale and purchasing of electricity exist in order to ensure demand meets supply. Some of these are the futures-, reserve-, day-ahead-, intra-day- and balancing market [3].

Figure 1: Illustration of the different markets and their ordering up till real-time operations.

Whether or not to participate in one or multiple markets is up to the individual generator, but participating in multiple markets will impact the flexibility in the concurrent markets participated in. As an example, if one enters both the reserve market and day-ahead market, providing an upward balancing reserve service in the reserve market means that the maximum power output of ones generator needs to be lowered by the same amount in the day-ahead market. If a power deficiency is experienced in the grid, production would potentially need to be increased up to the sold amount of upward reserve in the reserve market. The profit in the day-ahead market would therefore be reduced, as the quantity of electricity one is able produce and sell has lessened. Overall however, participating in both markets could result in total profit being similar or higher, as participating in another market yields more opportunities, which could be more profitable than only participating in one. In addition, participating in multiple markets serve as a hedging strategy, as the risk of the volatile electricity market can be somewhat lessened.

In the day-ahead market, all market participants will compete against each other. All bids are ranked from lowest to highest and the market is cleared by the market operators, where the market clearing price is determined by matching the demand with the supply. In order to participate in the market, one will have to have bid with a price lower than the clearing price. This is known as pool trading. An illustration of how the market clearing price is determined by matching supply and demand offers is seen below.

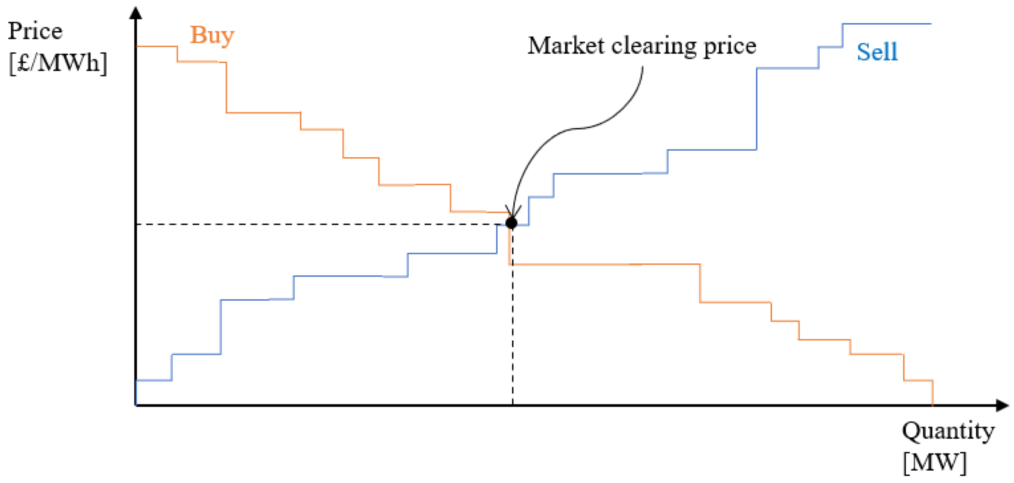


Figure 2: Market clearing in the day-ahead market

In this paper, the Day-Ahead and Intra-Day markets are in focus, and these markets are the ones considered by the developed stochastic model later on.

Settlement Periods and Gate Closures

When participating in either the Day-ahead or the Intra-day market, one is met with the two terms: settlement period and gate closure. The settlement periods are conventions on the time scale of the trades, which vary from market to market. All deliveries and off-takes of energy have to happen within their agreed upon settlement period, in order to assure the balancing between production and consumption. The gate closure is the deadline for all market participants to submit their final bids and offers for either one or multiple settlement periods.

Day-Ahead Market

Each day in the Day-Ahead market is made up of 24 settlement periods with each settlement period being an hour long. In the Day-Ahead market pool trading the gate closure for the following day is set to 11:00 and hereafter the market clearing prices are determined by the electricity system operators for every hour of the following day.

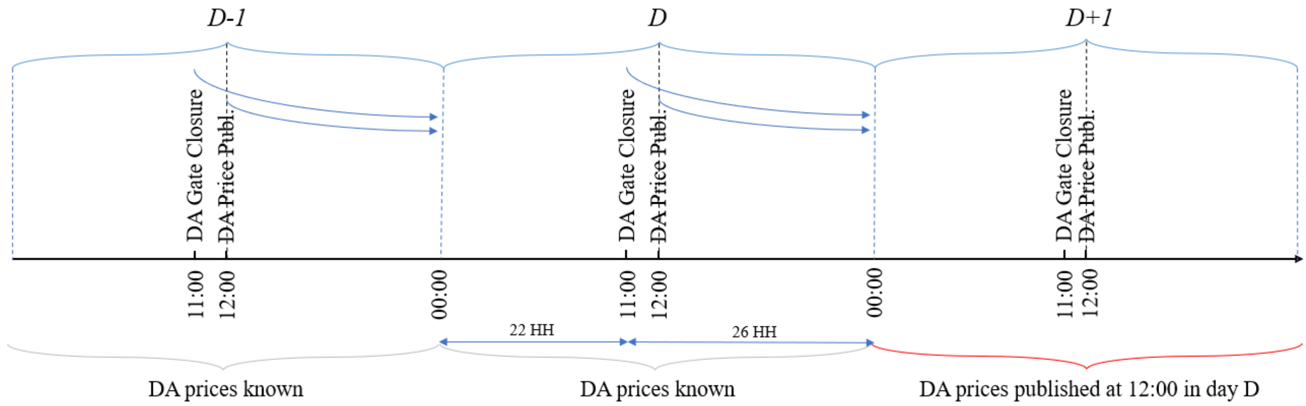


Figure 3: Day-Ahead market illustration for three days (from day D-1 to day D+1). During day D, one has 22 Half-Hours (HH) before the Day-Ahead gate closure for day D+1. In the following hour, at 12:00, the prices determined through market clearing for each settlement in the subsequent day are released.

Observing Figure 3, a period of three days in the Day-Ahead market is illustrated. Each day runs from 00:00 to 00:00 with hourly contracts. At 11:00 the final deadline for bids (the buyer bids for energy) and offers (the seller offers energy) for the next day is reached and they are sent to the market operator. At 12:00 the operators have matched demands with supply and the weighted average market price for each settlement period the following day can be determined. As explained above, the Day-Ahead market functions as a blind auction. All bids and offers are submitted simultaneously and the clearing price is subsequently determined. Due to this, multiple ways of bidding exist. If acting as a price taker, one will in practice bid with the price of 0, and take the determined market clearing price. Alternatively, one can put in a conditional bid of a certain minimum price. The final bidding strategy submitted can therefore be a mix of multiple bids with some having conditions while others do not.

Intra-Day Market

Unlike the Day-Ahead market, the Intra-Day market is continuous. Trading of a settlement period happens up until its beginning, and its gate closure is 30 minutes ahead of delivery. If trading in the settlement period from 9:00 to 9:30, the gate closure is at the start of the previous settlement period 8:30. While the continuous nature of this market allows for trades to be done far into the future, experience shows that the market liquidity is higher six hours before delivery. This builds on the study "Price formation and optimal trading in intraday electricity markets", which shows that "*liquidity starts to appear only 5–6 h before delivery, and grows very quickly at the approach of the delivery date*" [20]. The growing concentration of orders and transactions result in more predictable price levels appearing around six hours ahead of delivery.

In addition to being continuous, the Intra-Day market is split into 48 settlement periods with each settlement period being half an hour long. The granularity is twice as high as in the Day-Ahead market.

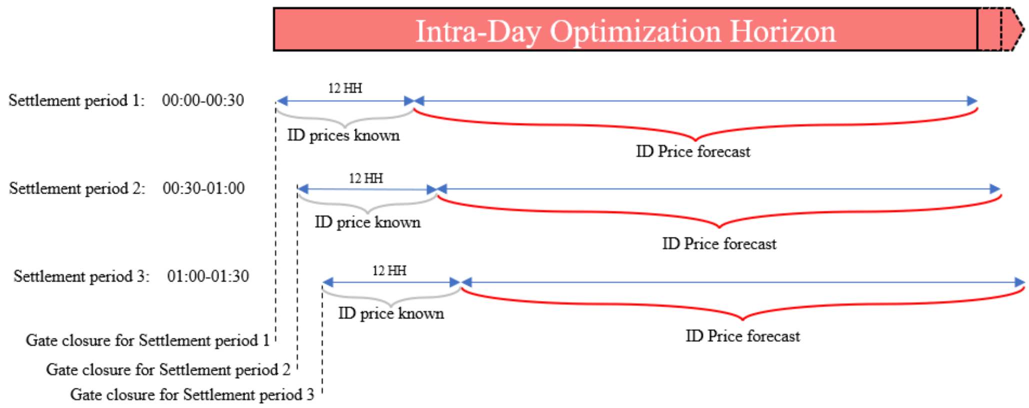


Figure 4: Intra-Day market illustration for three settlement periods. HH is an abbreviation for Half-hour. Trading is continuous with half-hourly settlement periods. The next 12 Half-hours are often fairly well-known while the prices further out in the future are subject of forecast, as the market liquidity is low.

In Figure 4 an overview of the ID market is observed and three half-hourly settlement periods are presented. From practical experience, the market liquidity is high for the next 12 half-hours, which leads to predictable prices. It is therefore assumed that the Intra-Day prices are known for the nearest six hours, while future prices have to be forecasted. The market is continuous, which means revision of contracts could happen every time new information is obtained.

Market Connectivity

In this section the effect of the Day-Ahead- and Intra-Day market on each other will be presented as well as how it impacts the modelling choices made.

The Day-Ahead and Intra-Day market are closely interconnected, as a decision in one would affect possible decisions in the other. As this is also the case for the other markets, Smeers in an article from 2008 argues that the markets are inextricably linked:

"Because of non-storability, the physical trade of electricity only takes place in real-time, which is thus the only true spot market. The other markets are forward markets that trade derivatives products maturing in real-time on the spot market." from (Smeers 2008, page. 57) [3]

As physical trading of electricity in each market takes place in real-time, Smeer argues that the trading of electricity can be viewed as a single trading operation with both the Day-Ahead and Intra-Day market being steps of this operation as both trade in electricity derivatives. Trading electricity for a specific period, the Day-Ahead market precedes the Intra-Day market. Choices made the previous day during the Day-Ahead market would consequently place additional restrictions/constraints on the potential contracts available in the Intra-Day market. Being closer to the settlement period, the forecasted prices for the Intra-Day are more accurate, which enables one to improve profits via e.g. arbitrage and compensating for mistakes in the forecast during the Day-Ahead scheduling. This is also the viewpoint of Dideriksen and Sekkesætter who in 2018 released a report where they wrote the following:

"Zipf and Most (2013) support Weber (2010) and Borggrefe and Neuhoff (2011) in their description of the intraday market. They describe the intraday market as a way to compensate the recognizable forecast errors based on the dayahead forecast errors. Whereas the balancing markets are used to "match real-time electricity demand and supply and guarantee a constant network frequency."" (Dideriksen & Sekkesaeter 2018, page 18) [12]

When trading in the Intra-Day market for day D is being carried out, the Day-Ahead trading has been concluded and contracts have already been signed. The only way to compensate for realized forecast errors in the previous day is through the Intra-Day market. The gate closure for the Day-Ahead market is at 11:00, so from 00:00 to 11:00 the Intra-Day market is only constrained by past decisions. At 11:00 the bids for the Day-Ahead market in the following day are submitted and the Intra-Day market choices are thus additionally constrained by promised upon 'future' trades the following day. However, choices made in the Intra-Day market between 00:00-11:00 can also impact the signed contracts in the Day-Ahead market for the following day. An illustration of this is observed in Figure 5

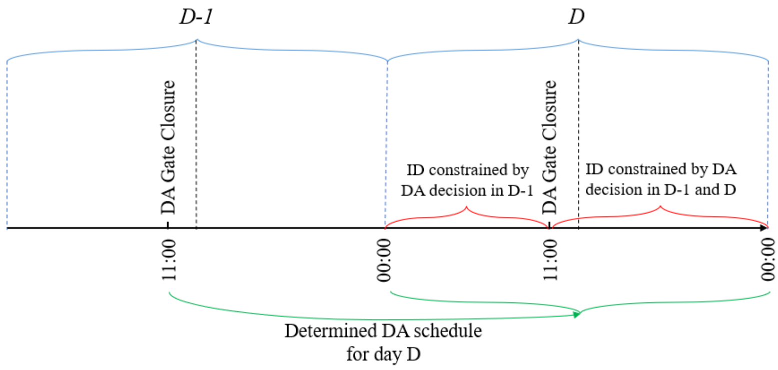


Figure 5: An illustration of how the Day-Ahead- and Intra-Day market influence each other. Decisions made in the Day-Ahead market influence the Intra-Day decisions the following day. These decisions then subsequently affect what one submits at gate closure in the Day-Ahead market for following days settlement periods.

In this paper, the battery will participate in the Day-Ahead and Intra-Day market. The inclusion of the two markets was decided in dialogue with Ørsted. The economic performance when participating in multiple markets as well as their individual performance will be compared to findings in literature, which suggest an increase of profit when multiple markets are considered [11].

Theory on Modelling

Two-Stage Stochastic Model

The general structure of a two-stage stochastic model is that some decisions have to be taken before uncertainty is realized: *Here and Now decisions*, while the remaining decisions have to be taken after the outcome is determined: *Wait and see decisions*. Therefore, two decision stages exist. The following literature was used to create this section: [5], [15] and [2].

Two-Stage Stochastic General Model Formulation:
The general form of the two-stage stochastic model is as follows:

$$\begin{aligned} \underset{x}{\text{Minimize}} \quad & c^\top x + E\{Q(\omega)\} \\ \text{subject to} \quad & Ax = b \\ & x \in X \end{aligned}$$

where c is the first-stage costs. The first term in the objective function represents the first-stage decisions while the second term represents the second-stage decisions. The recourse function, $E\{Q(\omega)\}$, is the expected value for the second-stage decisions and contain $Q(\omega)$, which is given by:

$$Q(\omega) = \left\{ \begin{aligned} \underset{y(\omega)}{\text{Minimize}} \quad & q(\omega)^\top y(\omega) \\ \text{subject to} \quad & T(\omega)x + W(\omega)y(\omega) = h(\omega) \\ & y(\omega) \in Y \end{aligned} \right\}, \forall \omega \in \Omega$$

q is the second-stage costs, $T(\omega)$ is the technology matrix, $W(\omega)$ is the recourse matrix, h are the stochastic requirements. X, Y and Ω are sets containing respectively all the first-stage decisions, second-stage decisions and scenarios.

This can be merged and rewritten into the following:

$$\begin{aligned} \underset{x}{\text{Minimize}} \quad & c^\top x + E\{\underset{y(\omega)}{\text{Minimize}} \quad q(\omega)^\top y(\omega)\} \\ \text{subject to} \quad & Ax = b \\ & T(\omega)x + W(\omega)y(\omega) = h(\omega) \\ & x \in X, y(\omega), \forall \omega \in \Omega \end{aligned}$$

Letting $\omega \in \Omega$ be the finite set of scenarios of uncertainty and $\pi(\omega)$ the probability of scenario ω it is possible to rewire the model above into its deterministic equivalent linear model.

The deterministic equivalent linear model to the two-stage stochastic model is formulated below:

$$\begin{aligned} \underset{x,y(\omega)}{\text{Minimize}} \quad & z = c^\top x + \sum_{\omega \in \Omega} \pi(\omega) q(\omega)^\top y(\omega) \\ \text{subject to} \quad & Ax = b \\ & T(\omega)x + W(\omega)y(\omega) = h(\omega) \qquad \forall \omega \in \Omega \\ & x \in X, y(\omega), \forall \omega \in \Omega \end{aligned}$$

The first-stage decisions, x, are subject to the deterministic technology definitions, A and their respective limits, b, as well as the stochastic technology definitions and limits, T and h, in combination with the second-stage decisions are their definitions, y and W.
Adding the costs of the first-stage decisions together with the weighted costs of the second-stage decisions across each scenario, ω, defines the objective value, z, which in this aimed to be minimized.

CVaR Model

Conditional-Value-at-Risk (CVaR) represents the expected value of the objective value smaller than the $(1-\alpha)$ quantile of the distribution. CVaR is an extension of Value-at-Risk (VaR), and seeks to minimize the losses occured beyond the $(1-\alpha)$. While VaR (Value-at-Risk), η, only considers the value of the quantile for a given α, CVaR also considers the tail. Therefore CVaR is the expected value of the profit, for a given $\alpha \in \{0,1\}$, lower than the $(1-\alpha)$ quantile of the profit distribution, also known as the average value-at-risk [4].

As CVaR is able to quantify the losses in the $(1-\alpha)$ quantile, while being able to be expressed in linear constraints, it is a popular way of managing risk.

The CVaR formulation can be formulated in the following way [1]:

$$CVaR\,(\alpha) = \max\left\{\eta - \frac{1}{1-\alpha}\mathbb{E}\,(\max\{\eta - \text{objective function}\,(\omega)\,,0\})\right\} \qquad \forall \alpha \in \{0,1\}$$

At a given confidence level, α, for the continuous loss distribution, CVaR is the expected loss out of all scenarios, ω. If the objective value obtained from the objective function is larger than the VaR at the given confidence level, the expected loss becomes 0.

Assuming a discrete distribution letting $\omega \in \Omega$ be a finite set of scenarios and $\pi\,(\omega)$ being the corresponding probability for each scenario ω, one gets:

$$CVaR\,(\alpha) = \max\left\{\eta - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\,(\max\{\eta - \text{objective function}\,(\omega)\,,0\})\right\} \qquad \forall \alpha \in \{0,1\} \tag{1}$$

The non-linearity in $\max\{\eta - \text{objective function}, 0\}$ is handled by introducing new variables and constraints:

$$\delta(\omega) \geq 0 \qquad \forall \omega \in \Omega$$

$$\eta \in \mathbb{R}$$

$$\eta - (\text{Objective function}\,(\omega)) \leq \delta\,(\omega) \qquad \forall \omega \in \Omega$$

Where η is VaR and $\delta\,(\omega)$ is a non-negative continuous variable which contains the difference between the VaR for the chosen quantile and the objective value for each respective scenario. If the objective value for a specific scenario is larger than η, $\delta\,(\omega)$ will be 0. If η is higher, $\delta\,(\omega)$ will be the difference.

The objective function part of CVaR is formulated in the following way:

$$\max \quad \eta - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\delta_\omega$$

Observing the CVaR part of the objective function, the model would aim to strike a balance between increasing the value of η and $\delta(\omega)$ and thereby maximize the expected value of the profit lower than the $(1-\alpha)$ quantile of the profit distribution.

This is subsequently worked into the two-stage stochastic model. The resulting changes are observed in the section Model 1 - CVaR.

Model Framework

In this section, the framework behind the developed two-stage stochastic model and the assumptions made will be explained.

Two Model Setup

A two-stage stochastic model which optimizes the economic performance of the Ørsted-owned Lithium-Ion batteries was developed. This model is denoted as Model 1, and is able to determine the optimal scheduling based on the forecasted electricity for both the Day-Ahead and Intra-Day market for all settlements periods during the succeeding day. While the bids and offers in the Day-Ahead market, generated by Model 1, for the succeeding day are binding, the Intra-Day operations are subject to change. The Intra-Day decisions are modelled stochastically to accommodate for the different price developments forecasted. Being in the actual intra-day, more information is known about the price development in the Intra-Day market. When getting closer to real-time operation, revising the Intra-Day decisions made in the previous day is possible, which results in the creation of a secondary model, Model 2. Model 2 is an extension to Model 1 and uses updated electricity price forecast for the Intra-Day market as well as the Day-Ahead contracts and state of charge of the battery at the beginning of each day, determined in Model 1, as inputs and revises the Intra-Day schedule each time the price forecast is updated.
The mathematical formulations for both models are similar to each other with the main difference being the here-and-now and wait-and-see decisions which are respectively modelled as deterministic and stochastic. In Model 1 the here-and-now and wait-and-see decisions are respectively the Day-Ahead and Intra-Day scheduling. In Model 2 the here-and-now decisions are the Intra-Day scheduling, which is modelled deterministically with no wait-and-see decisions. If the Intra-Day schedule is revised on a half-hourly basis, it is possibly to model the first settlement period as a here-and-now decision while the remaining half-hours in the day are wait-and-see decisions but given the assumption of predictable prices in the Intra-Day market, no notable difference was observed between the stochastic and deterministic approach in Model 2. To reduce model complexity the less complex model was therefore preferred. The second model is therefore a one-stage deterministic model with the purpose of revising the Intra-Day market trading strategy.

In practice, the two developed models would be used in the following way: At 11:00 the Day-Ahead scheduling for the following is determined by Model 1 and submitted before gate closure. During the following day, Model 2 is used to revise the Intra-Day schedule of the battery. It is possible to revise the Intra-Day schedule every half-hour (every settlement period). If no revisions are wanted, the initial scheduling determined by Model 1 can be used. This is not an attractive option unless one has perfect foresight, as the Intra-Day market is used to compensate for forecast errors as well as create arbitrage by utilizing price differences between the markets.

In Figure 6, a flow chart has been created to illustrate the process from the gathering of data untill the final operation schedule used in real-time operations have been decided.

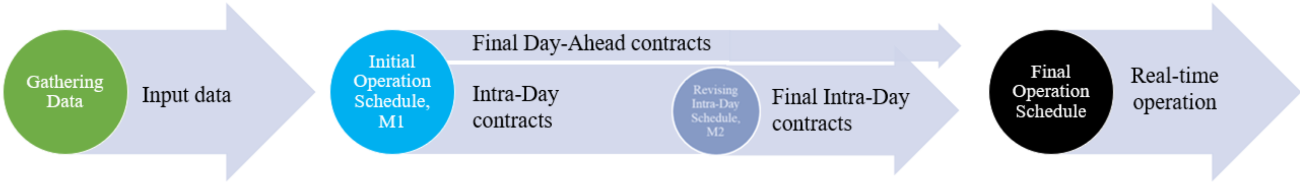


Figure 6: Flow chart showing the interaction between the models. After acquiring the forecasted prices the final Day-Ahead contracts and potential Intra-Day contracts in Model 1 for the following day are decided. After the revision of the Intra-Day contract in Model 2, both the Day-Ahead and Intra-Day schedules have been finalized, and the operational schedule for the day has been determined.

The continuous revisions in Model 2 is illustrated in Figure 7.

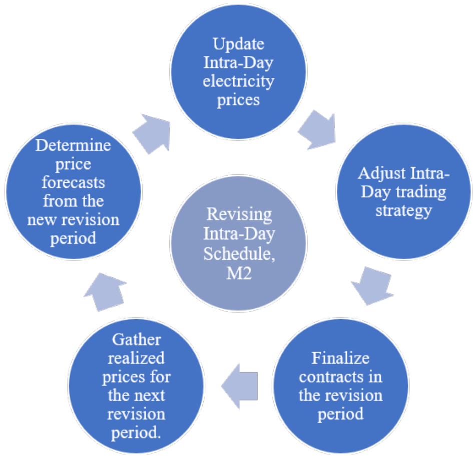


Figure 7: In Model 2, the potential Intra-Day contracts throughout the following day are continuously revised and finalized based on the updated and realised Intra-Day prices. This enables the model to adjust for potential forecast errors. After the revision, the Intra-Day schedule has been finalized, and the operational schedule for the day has been determined.

In Figure 6 and Figure 7 one can observe how the two-model setup works and how M2 is an optional extension to M1 if one wants to revise the initial Intra-Day contracts determined in M1. If one had perfect foresight, the Intra-Day decisions would not have to be revised, as one would not have to adjust to forecasting errors. As this is not the case in the real world, continuous revisions have to be carried out in order to maximize the profit.

Model Outline

Both markets are trading derivative products, with first market traded in being the Day-Ahead market. Model 1 is a two-stage stochastic model which determines when and how much one should trade in the Day-Ahead market. In the second model these contracts are used to fix the value of the Day-Ahead variables. As the real-time delivery of electricity is approaching, the situation in the market becomes clearer and the price forecast become more accurate. One is thus able cope with unforeseen changes in the Intra-Day market and adjust the trading strategy if any forecast errors have occurred. As the final bids and offers submitted in both markets have to be deterministic, both Model 1 and Model 2 output a single trading strategy for the Day-Ahead and Intra-Day market, based on the probabilities of each scenario.

An overview of the inputs and outputs of the two models as well as how they are connected is seen in Figure 8

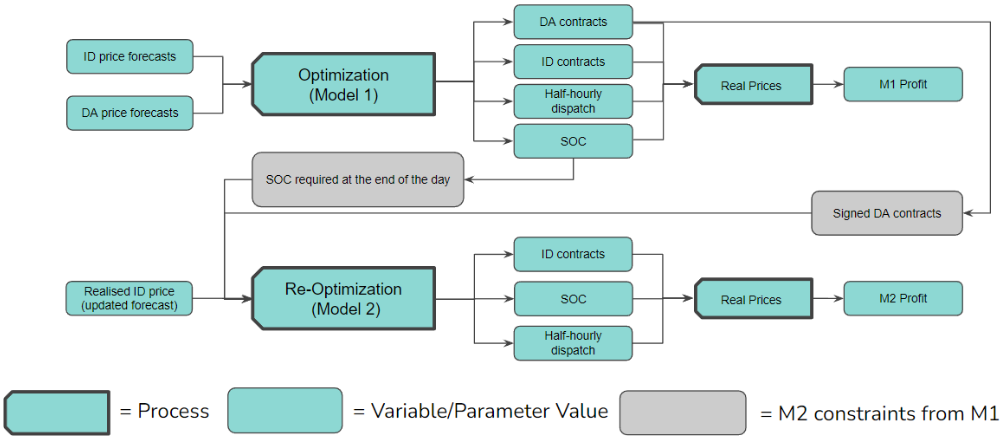


Figure 8: Block diagram of the two models including their input and output. Model 1 takes both Day-Ahead and Intra-Day price forecast as input and from these the Day-Ahead contracts are determined, which are used as input in Model 2, and Intra-Day contracts are forecasted. Based on this, the expected State-of-Charge at the end of the following day is found and set as a requirement for Model 2 to hit. Lastly, the obtained profit from using the determined schedules are calculated in both models, by applying the real prices to the schedules.

In order to ensure that the state of charge in the battery is properly constrained but also able to satisfy the Day-Ahead schedule determined for the following day in Model 1, Model 2 is constrained to always end with the battery's charge set to the initial State-of-Charge for the following day determined in Model 1.

With this implementation, Model 2 compensates for potential forecast errors and maximizes profit by arbitrage in the Intra-Day market without influencing any factors needed in the Day-ahead decisions for the following day, as the initial charge for the following day was decided in Model 1 regardless of what the updated Intra-Day market prices turn out to be. The Intra-Day market is therefore constrained between 00:00 and 11:00, which results in it being more restricted than the actual market presented in Figure 5.

On the subject of battery charge, measures needs to be taken to ensure the model is aware of the value of having charge left at the end of the day to avoid complete discharging each time Model 1 is run. The chosen solution for introducing this continuity between the optimizations, was to optimize over a 48 hour period but only extract the first 24 hours of scheduling. In order to include as recent information as possible in the price forecasts, the forecast period was kept to the first 24 hours with the last 24 hours being a copy of these prices. Predicting a longer period of time limits the data available and also the "lags" you can use, which is covered in the section "Data Set".
This method of duplicating the price forecasts left little to no room for improvement compared to forecasting both days, since it is barely significant how accurate the prices are, as long as they are within reason. Testing the effect on the obtained profits showed both small increases and decreases with no very significant changes.

Assumptions and Simplifications

During the modelling various assumptions were made to reduce model complexity and simplify the modelling process.

Simplifications:

- **Battery degradation** was left out of account, and a daily limit on battery cycles was implemented instead. While the imposed constrained limits the activity of the battery it does not accurately reflect the deterioration in capacity of the battery as well as the associated cost.
- **Capital and operational costs** are disregarded. As the procurement, location, maintenance and operation of the battery all have an associated cost, the resulting profit obtained in the model is higher than in reality. However, the obtained profits are still valuable as they reflect how high the operational costs can be, before turning a loss on the battery. Worth noting here is, that expenses sensitive the operation would benefit from being included in the model rather than calculated afterwards, in order for the optimization to account for these in its planning .

Assumptions:

- **Price-Taker**. The battery is assumed to be a price-taker. This means that the battery is able to purchase/sell any feasible quantity at market price but unable to influence the market clearing price of the market.
- **Average-prices** are assumed for the both the Day-Ahead and Intra-Day market, which are based on recorded trades within each settlement period of each market. This further imply the assumption of bid- and offer price always being the same with no option of arbitrage exploitation within the same settlement period. However, an arbitrage can still exists between markets, as investigated in the section "Profit Distribution", and the impact of including a spread between the bid- and offer price is covered in section "Price Spread".
- **Intra-Day prices** are assumed to be well-known six hours into the future due to practical experience of market liquidity in this period, which reduces the price volatility [20]. Therefore, it is assumed that real Intra-Day prices can substitute the forecasted prices when the real-time delivery is within 6 hours.

Considerations:
Due to the above mentioned circumstances, the following considerations for parameter values as well as trading have been made:

- The model does not accurately reflect the capital and operational expenditures of the battery but only the profit one is able to obtain from participating in both markets and arbitrage.
- Trading done in both markets are limited to a quantity between what the battery is able to charge and discharge in a settlement period. This means that if the battery is at most able to either charge or discharge 1 MW, no contracts signed in the DA market is able to exceed 1 MWh and no contracts in the ID market can exceed 0.5 MWh for each settlement period. This was implemented to avoid the developed model in behaving like a proprietary trader instead of a battery operator.

Mathematical Model Formulation

Model 1

Below, optimization Model 1 is defined:

Sets

T: Set of Half-hours

D: Set of days corresponding to the amount of half-hours

Ω: Finite set of scenarios

Parameters

λ_t^{DA}: Price for electricity during DA at period t

$\lambda_{t,\omega}^{ID,\uparrow}$: Offer price

$\lambda_{t,\omega}^{ID,\downarrow}$: Bid price

$\eta_{battery}$: Battery charge/discharge efficiency

lim_{cycle}: Daily cycle limit

B_{Cap}: Total battery capacity in MWh

$\overline{SOC_t}$: Upper limit on state of charge

$\underline{SOC_t}$: Lower limit on state of charge

$SOC_{initial}$: Initial state of charge

$P_{\text{C-rate}}$: Battery charge/discharge rate in MW

π_ω: Probability corresponding to each scenario

The values for the input parameters which are not determined through price forecasting can be found in the section Model Setup and Scheduling Behaviour under Input Parameters.

Variables

$p_t^{DA,ch}$: Day-Ahead charging at period t

$p_t^{DA,dis}$: Day-Ahead discharging at period t

$p_{t,\omega}^{ID,ch}$: Intra-day charging at period t in scenario ω

$p_{t,\omega}^{ID,dis}$: Intra-day discharging at period t in scenario ω

$soc_{t,\omega}$: State of charge of the battery at period t in scenario ω

$Q_{t,\omega}^{ch}$: Percentage change of the actualized charge of the battery at period t in scenario ω

$Q_{t,\omega}^{dis}$: Percentage change of the actualized discharge of the battery at period t in scenario ω

$Z_t^{DA,ch}$: Binary variable attaining value "1" when charging and "0" when discharging in DA

$Z_t^{DA,dis}$: Binary variable attaining value "0" when charging and "1" when discharging in DA

$Z_{t,\omega}^{ID,ch}$: Binary variable attaining value "1" when charging and "0" when discharging in ID

$Z_{t,\omega}^{ID,dis}$: Binary variable attaining value "0" when charging and "1" when discharging in ID

Model Formulation: Two-Stage Stochastic

$$\underset{p_t^{DA,dis},\, p_t^{DA,ch},\, p_{t,\omega}^{ID,dis},\, p_{t,\omega}^{ID,ch}}{\text{Maximize}} \quad \overbrace{\sum_{t=1}^{T}\left(\lambda_t^{DA}\cdot\frac{1}{2}\cdot\left(p_t^{DA,dis}-p_t^{DA,ch}\right)\right)}^{\text{Day-Ahead trading}} + \overbrace{\sum_{\omega=1}^{\Omega}\sum_{t=1}^{T}\pi_\omega\left(\lambda_{t,\omega}^{ID,\uparrow}\cdot\frac{1}{2}\cdot p_{t,\omega}^{ID,dis}-\lambda_{t,\omega}^{ID,\downarrow}\cdot\frac{1}{2}\cdot p_{t,\omega}^{ID,ch}\right)}^{\text{Intra-day trading}} \tag{2}$$

subject to

$$soc_{t,\omega} = SOC_{initial} + \eta_{battery} \cdot Q^{ch}_{t,\omega} - \frac{1}{\eta_{battery}} \cdot Q^{dis}_{t,\omega} \qquad \forall t = 1, \omega \in \Omega \tag{3}$$

$$soc_{t,\omega} = soc_{t-1,\omega} + \eta_{battery} \cdot Q^{ch}_{t,\omega} - \frac{1}{\eta_{battery}} \cdot Q^{dis}_{t,\omega} \qquad \forall t = 2, \dots, |T|, \omega \in \Omega \tag{4}$$

$$\frac{p^{DA,ch}_{t} - p^{DA,dis}_{t} + p^{ID,ch}_{t,\omega} - p^{ID,dis}_{t,\omega}}{B_{Cap}} \cdot \frac{1}{2} = Q^{ch}_{t,\omega} - Q^{dis}_{t,\omega} \qquad \forall t \in T, \omega \in \Omega \tag{5}$$

$$\sum_{t=1+48\cdot(d-1)}^{d\cdot 48} Q^{ch}_{t,\omega} + Q^{dis}_{t,\omega} \leq lim_{cycle} \qquad \forall t \in T, d \in D, \omega \in \Omega \tag{6}$$

$$p^{DA,ch}_{t} \leq P_{\text{C-rate}} \cdot z^{DA,ch}_{t} \qquad \forall t \in T \tag{7}$$

$$p^{DA,dis}_{t} \leq P_{\text{C-rate}} \cdot z^{DA,dis}_{t} \qquad \forall t \in T \tag{8}$$

$$p^{ID,ch}_{t,\omega} \leq P_{\text{C-rate}} \cdot z^{ID,ch}_{t,\omega} \qquad \forall t \in T, \omega \in \Omega \tag{9}$$

$$p^{ID,dis}_{t,\omega} \leq P_{\text{C-rate}} \cdot z^{ID,dis}_{t,\omega} \qquad \forall t \in T, \omega \in \Omega \tag{10}$$

$$p^{ID,ch}_{t,\omega} \leq P_{\text{C-rate}} - p^{DA,ch}_{t} \qquad \forall t \in T, \omega \in \Omega \tag{11}$$

$$p^{ID,dis}_{t,\omega} \leq P_{\text{C-rate}} - p^{DA,dis}_{t} \qquad \forall t \in T, \omega \in \Omega \tag{12}$$

$$z^{DA,ch}_{t} + z^{DA,dis}_{t} \leq 1 \qquad \forall t \in T \tag{13}$$

$$z^{ID,ch}_{t,\omega} + z^{ID,dis}_{t,\omega} \leq 1 \qquad \forall t \in T, \omega \in \Omega \tag{14}$$

$$soc_{t,\omega} \leq \overline{SOC_{t}} \qquad \forall t \in T, \omega \in \Omega \tag{15}$$

$$soc_{t,\omega} \geq \underline{SOC_{t}} \qquad \forall t \in T, \omega \in \Omega \tag{16}$$

$$p^{DA,ch}_{2\cdot t-1} = p^{DA,ch}_{2\cdot t} \qquad \forall t = 1, \dots, \frac{|T|}{2} \tag{17}$$

$$p^{DA,dis}_{2\cdot t-1} = p^{DA,dis}_{2\cdot t} \qquad \forall t = 1, \dots, \frac{|T|}{2} \tag{18}$$

$$p^{DA,ch}_{t}, p^{DA,dis}_{t}, p^{ID,ch}_{t,\omega}, p^{ID,dis}_{t,\omega} \geq 0 \qquad \forall t \in T, \omega \in \Omega \tag{19}$$

$$0 \leq soc_{t,\omega}, Q^{ch}_{t,\omega}, Q^{dis}_{t,\omega} \leq 1 \qquad \forall t \in T, \omega \in \Omega \tag{20}$$

$$z^{DA,ch}_{t}, z^{DA,dis}_{t}, z^{ID,ch}_{t,\omega}, z^{ID,dis}_{t,\omega} \in \{0,1\} \tag{21}$$

Eq. 2 is the objective function, in which the Day-Ahead and Intra-Day contracts are multiplied by the forecasted prices across all scenarios, with the aim of maximizing the profit obtained.
Eq. 3 and 4 updates the battery state of charge according to the operation of the battery.
Eq. 5 sets the correct charging and discharging by cancelling out Day-Ahead sales with Intra-Day purchases, and Day-Ahead purchases with Intra-Day sales.
Eq. 6 limits the charge that can be moved in and out of the battery daily.
Eq. 7-14 ensures that the battery is not charging and discharging at the same time, aswell as keeping the scheduled contracts within operational limits of charging- and discharging rates.
Eq. 15 and 16 keeps the state of charge below the capacity.
Eq. 17 and 18 ensure that the DA contracting is hourly and nor half-hourly.
Eq. 19-21 defines the variable value ranges.

The model formulation with the variables and parameters replaced by their respective units, is available in the Appendix.

Model 2

The mathematical formulation for M1 and M2 are very similar. The mathematical formulation of model 1, M1, is presented above. M1 determines the Day-Ahead scheduling and is a stochastic optimization model. In model 2, M2, the day-ahead decisions have been determined, thus $p_t^{DA,ch}$ and $p_t^{DA,dis}$ are known and no longer variables. In addition, a constraint is introduced to ensure that the state of charge in the battery at the end of the day is equal to the state of charge in the battery at the beginning of the next day in M1. This is done to guarantee the battery is able fulfill the contracts signed in the Day-Ahead market. If not, one could risk the battery discharging fully at the end of the day, while having signed a contract to discharge electricity during the first hour of the day.

Below, a piece of pseudo code shows the dependencies between the models:

$$\begin{aligned}
&\text{Model 1}\left(\overline{\lambda_1^{DA}},\ \overline{\lambda_1^{ID1}},\ soc_0\right)\\
&\rightarrow \overline{p_1^{DA}},\ soc_1\\
&\textbf{for}\text{ d in Days}\\
&\qquad\textbf{for}\text{ s in Settlement-Periods}\\
&\qquad\qquad\text{Model 2}\left(\overline{\lambda_{d,s}^{ID2}},\ \overline{p_d^{DA}},\ soc_d\right)\\
&\qquad\qquad\quad\rightarrow p_{d,s}^{ID}\\
&\qquad\qquad\textbf{if}\text{ s = 11:00}\\
&\qquad\qquad\quad\text{Model 1}\left(\overline{\lambda_{d+1}^{DA}},\ \overline{\lambda_{d+1}^{ID1}},\ soc_d\right)\\
&\qquad\qquad\quad\rightarrow \overline{p_{d+1}^{DA}},\ soc_{d+1}
\end{aligned} \tag{22}$$

The code shows the timing of inputs and outputs of the models as well as the format of these with $\overline{\text{overlined}}$ terms being vectors and the rest are single number.

The code is initialized by supplying Model 1 with Day-Ahead and Intra-Day price forecasts for tomorrow ($\overline{\lambda_1^{DA}}$, $\overline{\lambda_1^{ID1}}$) as well as the state of charge by the end of the current day (soc_0). This enables the Day-Ahead contracts to be finalized ($\overline{p_1^{DA}}$) and with these in addition to the forecasted Intra-Day operations, the state of charge by the end of tomorrow can be determined (soc_1)

Hereafter, Model 2 is optimized each day for each of the 48 settlement periods in a day. It takes an updated vector of Intra-Day price forecasts ($\overline{\lambda_d^{ID2}}$) as input, which varies in length depending on how much of the day is left. In addition to this, it requires the signed Day-Ahead contracts of the current day and the state of charge which is required to hit by the end of the day. The combination of these allows Model 2 to find the optimal operation of the current settlement period ($p_{d,s}^{ID}$). If operations further into the future than just one settlement period is requested, this is possible by increasing the length of $\overline{\lambda_d^{ID2}}$ and accordingly reducing the number of revisions during a day. However, this entails that decisions are made without the most recent information which is investigated in section "Intra-Day Trading Revision Frequency Sensitivity".

Lastly, at 11:00 Model 1 is run again to meet the gate-closure time of the Day-Ahead market participation.

Worth noting is, that Model 1 solely dictates the state of charge with Model 2 operating within these bounds. This was designed to ensure that Model 1 has certainty about what the filling of the battery will be when submitting the Day-Ahead schedule.

Model 1 - CVaR

Expanding the previously formulated two-stage stochastic model, Model 1, the changes as well as the extensions made, in order to obtain a CVaR model, are covered in the following section.

New parameters:
α: Confidence level: Conditional Value-at-Risk (CVaR) at confidence level, α.
β: The CVaR term of the objective function is weighted with β while the profit term is weighed by $(1-\beta)$.

Additional variables:
η: Represent the VaR (Value-at-Risk)
δ_ω: δ_ω selects the scenarios that have a value lower or equal to VaR.

Extended objective function:
Including the weighting the extended objective function to the two-stage stochastic model now becomes:

$$\underset{\text{Variables}}{\text{Maximize}} \quad (1-\beta)\overbrace{\left(\overbrace{\sum_{t=1}^{T}\left(\lambda_t^{DA}\cdot\frac{1}{2}\cdot\left(p_t^{DA,dis}-p_t^{DA,ch}\right)\right)}^{\text{Day-Ahead trading}}+\overbrace{\sum_{\omega=1}^{\Omega}\sum_{t=1}^{T}\pi_\omega\left(\lambda_{t,\omega}^{ID,\uparrow}\cdot\frac{1}{2}\cdot p_{t,\omega}^{ID,dis}-\lambda_{t,\omega}^{ID,\downarrow}\cdot\frac{1}{2}\cdot p_{t,\omega}^{ID,ch}\right)}^{\text{Intra-day trading}}\right)}^{\text{Profit}}$$
$$+\beta\overbrace{\left(\eta-\frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\delta_\omega\right)}^{\text{CVaR}} \tag{23}$$

Additional constraints:

$$\eta-\left(\sum_{t=1}^{T}\left(\lambda_t^{DA}\cdot\frac{1}{2}\cdot\left(p_t^{DA,dis}-p_t^{DA,ch}\right)\right)+\sum_{t=1}^{T}\left(\lambda_{t,\omega}^{ID,\uparrow}\cdot\frac{1}{2}\cdot p_{t,\omega}^{ID,dis}-\lambda_{t,\omega}^{ID,\downarrow}\cdot\frac{1}{2}\cdot p_{t,\omega}^{ID,ch}\right)\right)\leq\delta_\omega \qquad \forall\omega\in\Omega \tag{24}$$

$$\delta_\omega\geq 0 \qquad \forall\omega\in\Omega \tag{25}$$

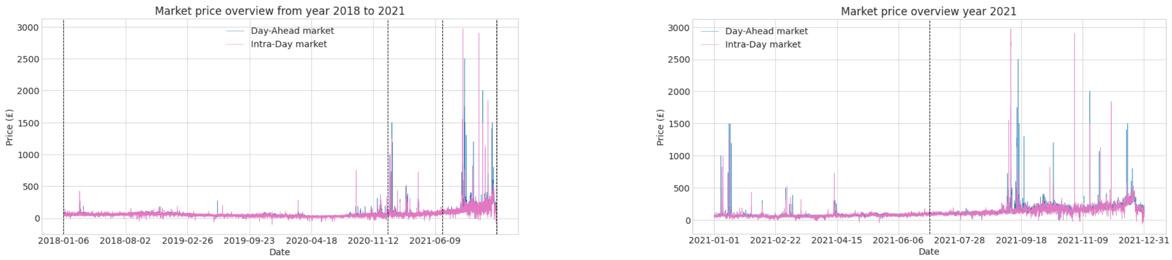
$$\eta\in\mathbb{R} \tag{26}$$

Eq. 23 is the expanded objective function. An additional CVaR part is added to the objective function and weighting is implemented through β. Eq. 24 represents the relation between η and δ. Eq 25 and 26 define the value range of the additional variables.

Descriptive Statistics of Electricity Data

Defining the Test Sets

The Day-Ahead and Intra-Day price development in the UK market are observed in Figure 9, where the prices since 2018 alongside a focused view of the prices in year 2021 are shown.

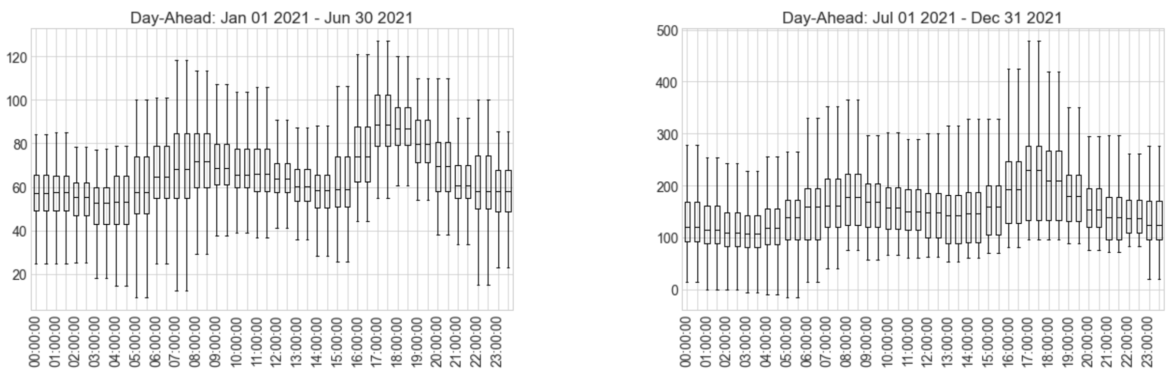


(a) UK market prices (b) UK market prices in 2021

Figure 9: An overview of day-ahead and intra-day prices in the UK market from 2018-2021, with a zoom-in on the more volatile 2021.

During the first three years, 2018-01-06 until 2021-01-01, the market prices behave rather stable. This changes in 2021, where the price fluctuations increase. In the first half of year 2021, the price rises sharply at the beginning, while the sharp price fluctuations are mostly present throughout the second half of 2021.

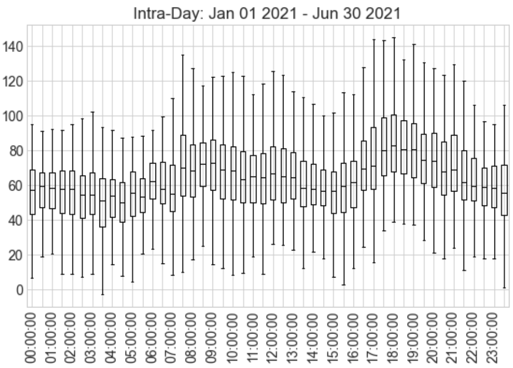
The volatile conditions present in the UK markets, especially towards the end of 2021, stress the importance of good forecast models which are able to catch the new trends in the markets and predict the price developments. If this is not achieved, participating in the markets would result in losing rather than earning money. The UK Day-Ahead market is observed to have clear daily trends with first half year of 2021 being the clearest. The distribution of prices for the Day-Ahead market are observed in Figure 10.



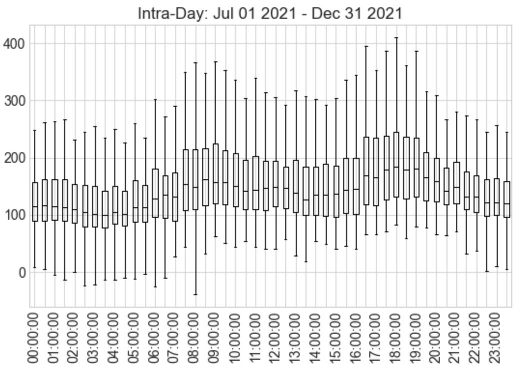
(a) First half-year in 2021 (b) Second half-year in 2021

Figure 10: Hourly price distribution for the first and second half year in 2021 in the Day-Ahead market without outliers. A much more consistent daily curve is observed in the first half-year compared to the second.

The Intra-Day price distribution is observed below in Figure 11.



(a) First half-year in 2021



(b) Second half-year in 2021

Figure 11: Hourly price distribution for the first and second half year in 2021 in the Intra-Day market, without outliers. Generally higher spreads than the Day-Ahead market and less pronounced daily trends.

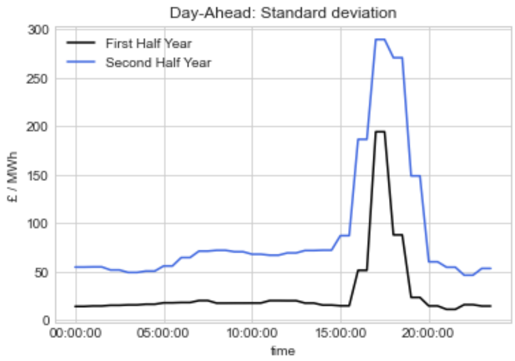
Observing the daily trends for the first and second half year in 2021, electricity prices are highest during the evening around 5 to 6 pm as well as the morning around 8 am. This coincides with when most people respectively get home and get up for work. Observing both Figure 10 and Figure 11 the prices in the second half of year 2021 are considerably higher than in the first half. In addition, the variance shows that the price tends to vary more upwards during the later part of 2021 whereas it seems to favour neither side during the first half. Despite both graphs depicting electricity prices from the same year, the market behaviour is different in both half-years and the forecast models would have to account for this.

For the remainder of this study, the first half-year of 2021 will constitute Test Set 1 and the second half-year will be Test Set 2, which will be the time periods subject for forecasting and model testing. The two sets are chosen due to their difference in volatility and thus exposing the forecast- and optimization models to different price environments.

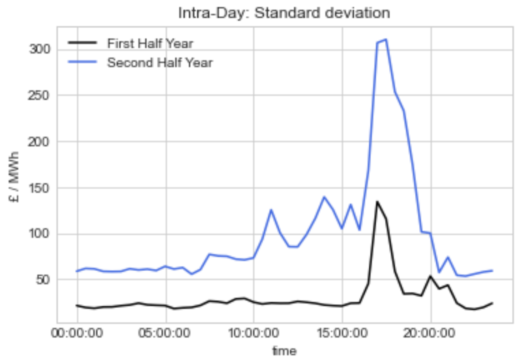
Exploring Price-Spikes

Since an important difference between the sets is the spikes in price, the economic importance of these are investigated in the following section.

Firstly, observing the half-hourly standard deviation for each half-year in Figure 12, large price fluctuation during peak hours are observed. The period between 15:30 and 19:00 is very volatile and therefore has a significant impact on potential profits.



(a) The half-hourly standard deviation in the Day-Ahead market for both the first and second half-year in 2021



(b) The half-hourly standard deviation in the Intra-Day market for both the first and second half-year in 2021

Figure 12: Standard deviation of the electricity prices in each settlement period observed in year 2021 for both the Day-Ahead and Intra-Day market, which the second half-year exhibiting significantly higher deviations and both year-halves peaking in the early evening where prices are highest.

As large price fluctuations are observed in the UK electricity markets, it exhibits a volatile behaviour. On top of the daily patterns, any electricity price forecast would have to take this into consideration. Not only is it important to predict the electricity prices and mimic its general shape throughout the day, but also when it tends to drastically rise in order to maximize ones profit.

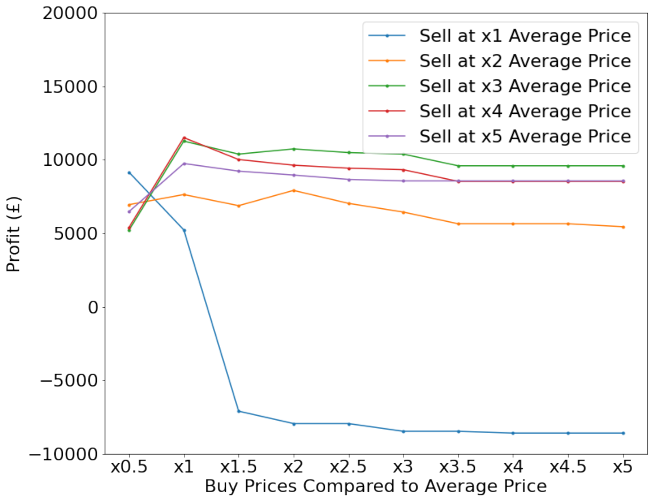
In order to uncover the importance of utilizing price-spikes, an algorithm of intra-day market actions is developed in which the battery actions are solely determined based on the intra-day prices compared against a historical average.

The algorithm goes through each of the two test sets one settlement period at a time, and chooses to fully discharge if the following settlement period price is greater than the historical average by some factor. The discharge will span over 4 half-hourly settlement periods due to the C-rate of the battery being assumed to be 0.6, which means the battery can be fully charged or depleted in $\frac{1\text{ MWh}}{0.6\text{ MW}} = 1.67$ hours, or 1 hour and 40 minutes. Thus, 0.3 MWh is discharged in each of the first 3 settlement periods and 0.1 is discharged in the fourth period.

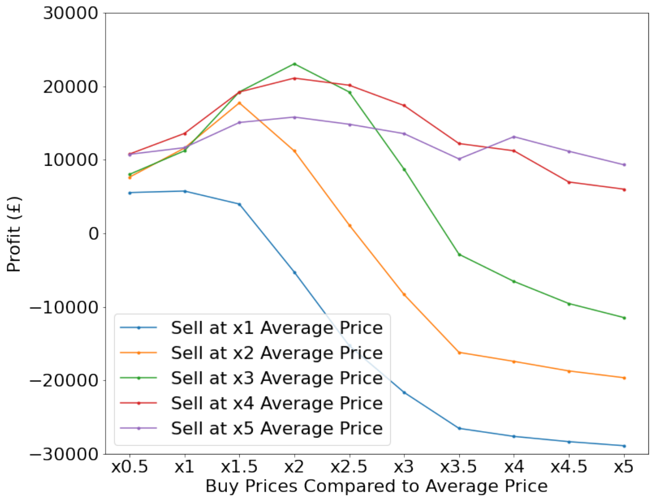
The conditions for charging are the same, with the following settlement period required to be below the historical average by some factor.

In addition to the price conditions, a cycle constraint is implemented to ensure only 2 battery cycles are made daily. Furthermore, when either action is selected, the heuristic skips four settlement periods ahead to avoid charging and discharging at the same time. Lastly, the battery must be fully charged when choosing to discharge and vice versa.

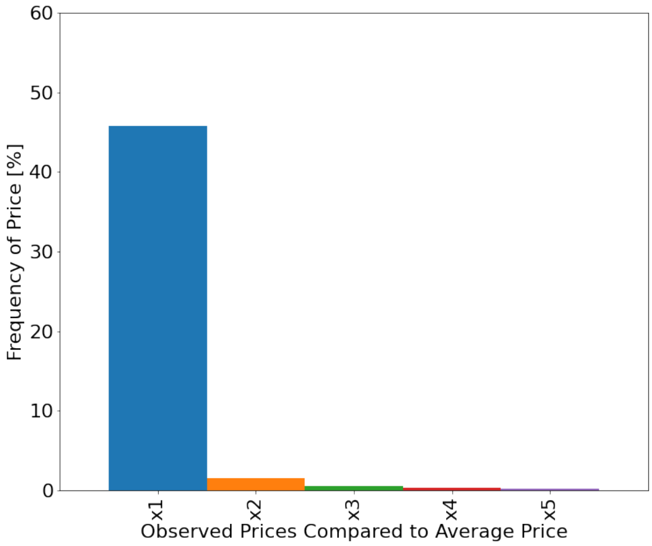
The following profits are observed:



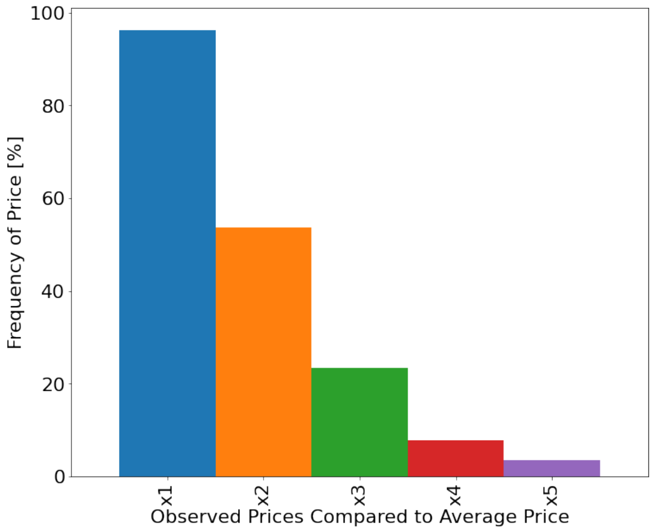
(a) Profits in first half-year 2021



(b) Profits in second half-year 2021



(c) Frequency of prices in first half-year 2021



(d) Frequency of prices in second half-year 2021

Figure 13: Profits obtained in 2021 from the price-spike heuristic, which reflects the highest utilization to lie around 3-4 times the historic average. The plots show the profit of the heuristic with varying sell prices, expressed as factors of the historical average intra-day price, plotted against different buy prices.

Looking at the plot for the first half-year, it is observed that the best performance is obtained when allowing the battery to be filled at prices lower than or equal to the historical average and discharge at price higher than or equal to the 3 or 4 times the historical average. This shows that despite very few occurrences, it is worth preserving the storage until these spikes happen. Furthermore, the curves flatten significantly when selling at x1.5 the historical average or above, except for when selling at or above the average price, which turns a significant loss. This is explained by Figure 13 (c), which displays the scarcity of extreme spikes. Since algorithm discharges over four settlement periods, which is longer than the usual duration of a spike in price, the price will mostly be lower right after discharge. Thus, even though it is allowed to charge at the prices observed in peaks, it is such a rare

occurrence to see those high prices right after a spike, that no loss is incurred. To summarize, with the market landscape observed in the half-year, it is sensible to charge the battery immediately after a spike in price, when the spike was 3-4 times the historic average, with the best performance obtained from charging at or below the historic average.
Looking at the plot for the second half-year, a slightly different narrative is shown. Here, the optimal price to sell at is x3 the historical average, and in order to ensure availability of charge when these spikes are observed, it is beneficial to charge at prices all the way up to x2 the historical average.
Furthermore, due to a generally higher price in this half-year, noticeable losses are incurred when prioritizing availability of storage or in other words, when the allowed purchase price is increased. Interestingly, higher prices required before selling serves as an insurance towards avoiding unwise purchases. The most robust strategies, while not being the most optimal, proves to be the ones waiting for extreme spikes at x4 or x5 the historic average.

With these rough approaches to profit maximization, a baseline of performance has been established. In addition to this, some of the benefits and dangers of working around spikes have been uncovered, for instance how a relatively simple approach to the Intra-Day market can generate seemingly reliable profits, but simultaneously how the same strategy can generate profits under one market behaviour and losses under another, e.g. when selling at x2 the average price and buying at x2.5 or below.

Time-Series Forecasting Theory

Simple Forecasts

In order to establish a baseline, a few simplistic forecasts have been generated, which can be held against the more sophisticated models later in the section.

Last Weeks model

As a simple approach to forecasting, the historic price data has been shifted one week forward, meaning each settlement period in the new data set refers back to the price of the same settlement period one week ago. This is the Last Week model.
Expanding upon this, the Last Weeks model is created by assigning a weight to each of the last four weeks with decreasing probability the farther back they are. This introduces multiple scenarios, which supplies the model with some information of the anticipated price levels and which settlement periods tend to fluctuate in price.
The power of these forecasts are their close resemblance to the patterns of the real data. This feature is obtained at the cost of all predictions being unaware of the direction of the drift in price level.
The forecasts can be defined as:

$$P_i^{LastWeek} = P_{i-7\cdot 48}^{real} \qquad \forall i \in S \tag{27}$$

$$P_{i,w}^{LastWeeks} = P_{i-w\cdot 7\cdot 48}^{real} \qquad \forall w \in W, s \in S \tag{28}$$

Where S is the set of all settlement periods and W is a set of four weeks, i.e. $W = \{1,2,3,4\}$. The probability weight for the Last Weeks forecast is 40% for the most recent week, 30% for the prices two weeks ago, 20% for the second to last week, and 10% for the last week.
This logic is applied to both the intra-day and day-ahead price forecast.

Average Week model

Another simplistic forecast model is created by defining a week consisting of average days. All $48 \cdot 7 = 336$ settlement periods over a week are each averages of their corresponding settlement period across each week of the training period, yielding one average week which is the same for the entire test period.
The forecast can be defined as:

$$P_i^{AverageWeek} = \frac{1}{|W|} \sum_{w=1}^{W} P_{i \bmod 366 + w\cdot 366}^{real} \qquad \forall i \in S \tag{29}$$

Where S is the set of all settlement periods, and W is a set of all weeks in the training data.

Sophisticated Forecast

Previously, the implementation of three simple forecasts were presented. In this section, more sophisticated forecasting methods are introduced.

Decision Tree

Fundamentally, decision trees can be categorized either as a classification tree, in which the target variable is a discrete statement, or as a regression tree, where the target variable is continuous. The regression tree is the model utilized here.

The target variable is the subject of prediction, and the model takes a range of features of various importance as input to achieve the most accurate information possible.

A decision tree consists of a root node, internal nodes, and leaf nodes. With starting point in the root node, a binary statement splits the data in two and directs each subset into a new internal node, where the process is repeated. After an amount of iterations, the data is now categorized into a number of unique groups in the leaf nodes.

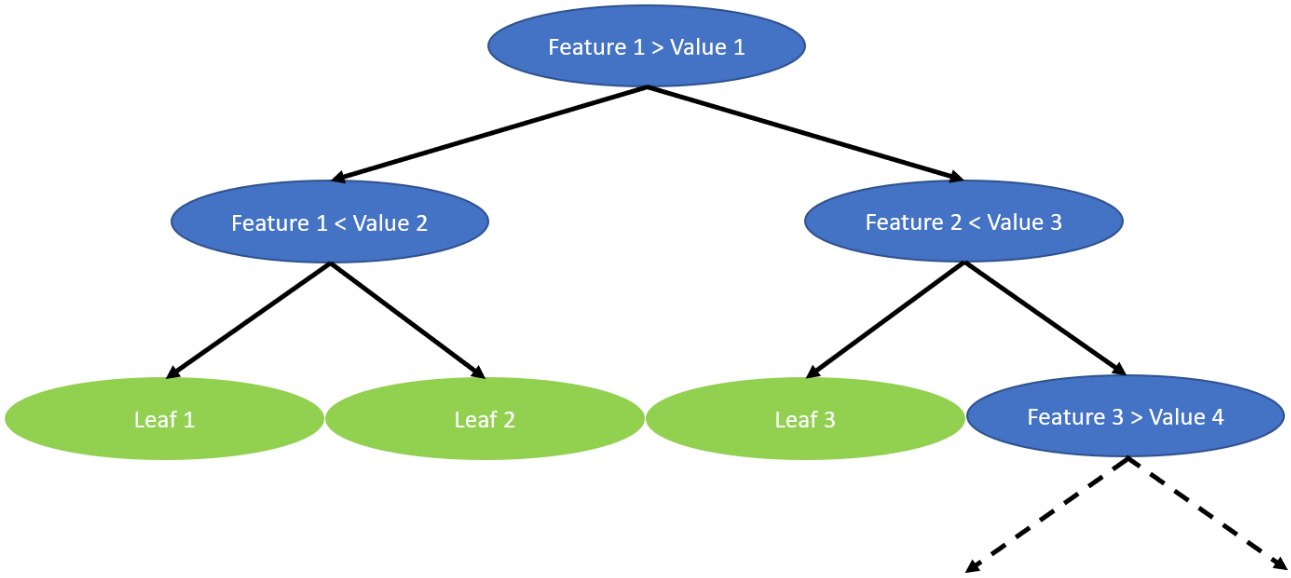


Figure 14: Structure of Decision Tree, with data being split in the root node and each subsequent internal node, based on the value of a specific feature, resulting in a number of leaves which each contain a certain amount of the observations.

The average of the target variable within each leaf node is calculated and now when the model is supplied with values of the features used for training, it can follow the binary paths in the developed tree, and arrive at a prediction equal to the average of the historic values within the given group.

The method behind selecting the thresholds of the root- and internal nodes is based on finding the smallest Mean Squared Error (MSE) for each features. Starting by separating the first observation from the rest, the two average target variable value of the sets are found, and MSE is performed with respect to these. The MSE is saved, next the data is split between the second and third observation, and lastly the average target variable values of the first two observations and the remainder of the set are found. This process is then repeated. This results in one less MSE evaluation compared to the number of observations, and the split of the data is chosen for the lowest

obtained MSE. Thus the root node of the given feature is determined. The subsequent internal node splits are found in a similar fashion, by calculating MSE on the observations that have not been assigned to a group. Here, an important model feature is the smallest number of observations included in each group (leaf). Having few observations can result in over fitting, where the model is not capable of generalizing on the input features, and having many can cause the model to miss patterns that are of a finer granularity. This is tested in later sections. Lastly, the process behind choosing the order the features appear in, is through finding the best candidate from each feature and once again the smallest MSE dictates which will be selected. [17]
Expressed mathematically, given $\theta = (j, t_m)$ for feature j, threshold t_m in root m, feature observations x, and target variables y, the previously described partitioning of the data can be written as a minimization problem, where Q_m^{left} and Q_m^{right} denote the sets on either side of the partition.

$$Q_m^{left}(\theta) = \{(x,y) | x_j \leq t_m\} \tag{30}$$

$$Q_m^{right}(\theta) = Q_m \setminus Q_m^{left}(\theta) \tag{31}$$

With the sets defined, a minimization of the following expression is carried out:

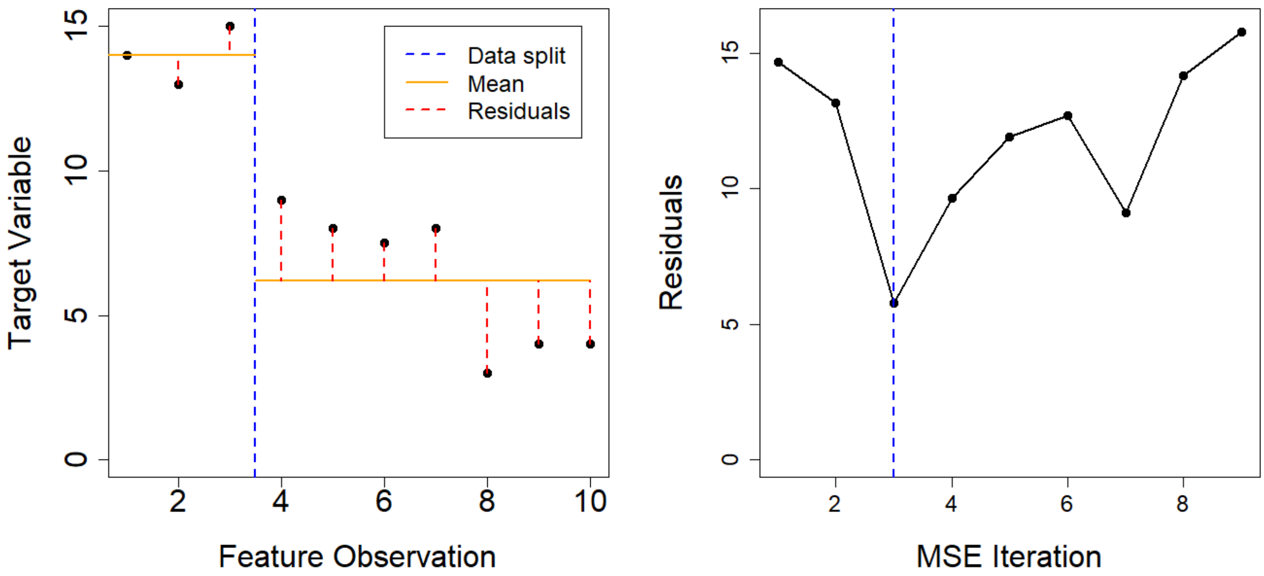
$$\theta^* = \text{Min}_\theta \quad \frac{n_m^{left}}{n_m} H(Q_m^{left}(\theta)) + \frac{n_m^{right}}{n_m} H(Q_m^{right}(\theta)) \tag{32}$$

H is an impurity function i.e. MSE which is the subject of minimization, and expressed as:

$$\bar{y}_m = \frac{1}{n_m} \sum_{y \in Q_m} y \tag{33}$$

$$H(Q_m) = \frac{1}{n_m} \sum_{y \in Q_m} (y - \bar{y}_m)^2 \tag{34}$$

Here, n_m is the number of observations or samples included, y is a list of target variables, and $\bar{y}$ is the average of y. This minimization is repeated for a feature until the data is partitioned so finely that no splits can be made, which satisfy $n_m \geq minimum\ samples\ per\ split$. Alternatively, the maximum depth of the tree could be reached, which refers to the longest path possible from the root node to any leaf node. [22] No max depth is considered in this paper, as the minimum split serves a very similar purpose.
Below an illustrative example is provided of $n_m = 10$, $n_m^{left} = 3$, and $n_m^{right} = 7$.



(a) Third MSE iteration

(b) Complete MSE

Figure 15: Example of determination of root node, where (a) shows the third iteration of the process and (b) shows the results of the complete process.

In the example above, the root node candidate for this feature will be "Feature Observation < 3.5", the first condition the data is split for. Continuing the process and looking at the complete MSE plot it is likely that one more node will be created for "Feature Observation > 7.5". Lastly, in this example a cut of the data is performed with only 10 samples, and thus it can be deducted that the minimum split size must be at least 10.

Random Forest

When utilizing Decision Trees for predictions, the natural extension is to also include a Random Forest model. The core idea of grouping similar data through a binary tree is identical to the previous method, with the main difference being the framework around the selection of data and the fact that multiple trees are generated. Essentially, a number of bootstrapped data sets are generated by randomly selecting samples from the original data, while allowing duplicates, and subsequently choosing only a subset of the available features to create the partitions of the data on. This leads to multiple combinations with repetition of the original data and equally many Decision Tress. When determining the prediction of a target variable, the given features are run through all the generated Decision Trees and the aggregate of all their predictions will be the final outcome. [14]

Model Setup and Scheduling Behaviour

In this section, the profit obtained using the Last Weeks simple forecast compared to perfect information will be presented. The importance of each market and the impact of revising ones schedule during the Intra-Day market when some of the uncertainty is revealed will be explored.

Input Parameters

The electricity price forecasts utilized by the model are generated through forecasting method, which will be elaborated on in later sections.

The values of the remaining input parameters are presented below:

$\eta_{battery} = 0.92$	$B_{Cap} = 1$ MWh	$\overline{SOC_t} = 0.9$ MWh	$C_{rate} = 0.6$ h^{-1}
$lim_{cycle} = 4$ MWh	$SOC_{initial} = 0.4$ MWh	$\underline{SOC_t} = 0.1$ MWh	$P_{\text{C-rate}} = 0.6$ MW

With $\eta_{battery} = 0.92$ for both charging and discharging, the round-trip efficiency becomes approximately 85%. A daily cycle limitation, lim_{cycle}, is imposed upon the battery instead of modelling the degradation. The battery is allowed to fully charge and discharge twice per day; two daily cycles. This corresponds to an accumulated change of 400 % of the nominal capcity of the battery, which is 4 MWh. P_{Crate} is the amount of MW the battery is able to respectively charge or discharge at in a settlement period. This value is derived from a C-rate of 0.6 h^{-1} and the nominal capacity of 1 MWh:

$$C_{rate} = \frac{P_{Crate}}{B_{cap}} \Rightarrow P_{Crate} = C_{rate} \cdot B_{Cap}$$

$$P_{Crate} = 0.6 \text{ h}^{-1} \cdot 1\text{MWh} = 0.6\text{MW}$$

The initial state of charge, $SOC_{initial}$, of the battery is set to 0.4 MWh, with an upper, $\overline{SOC_t}$, and lower, $\underline{SOC_t}$, limit at respectively 0.9 MWh and 0.1 MWh. These constraints were imposed to avoid the degradation of the battery associated with charging or discharging the battery at its limits.

In order to reduce running time, the Intra-Day schedule was only revised four times daily; every six hours. This is consistent throughout the report, unless otherwise stated. Optimally, one would be interested in updating the schedule every time new information is acquired; every half-hour. The resulting difference in profit will be touched upon in the sensitivity analysis.

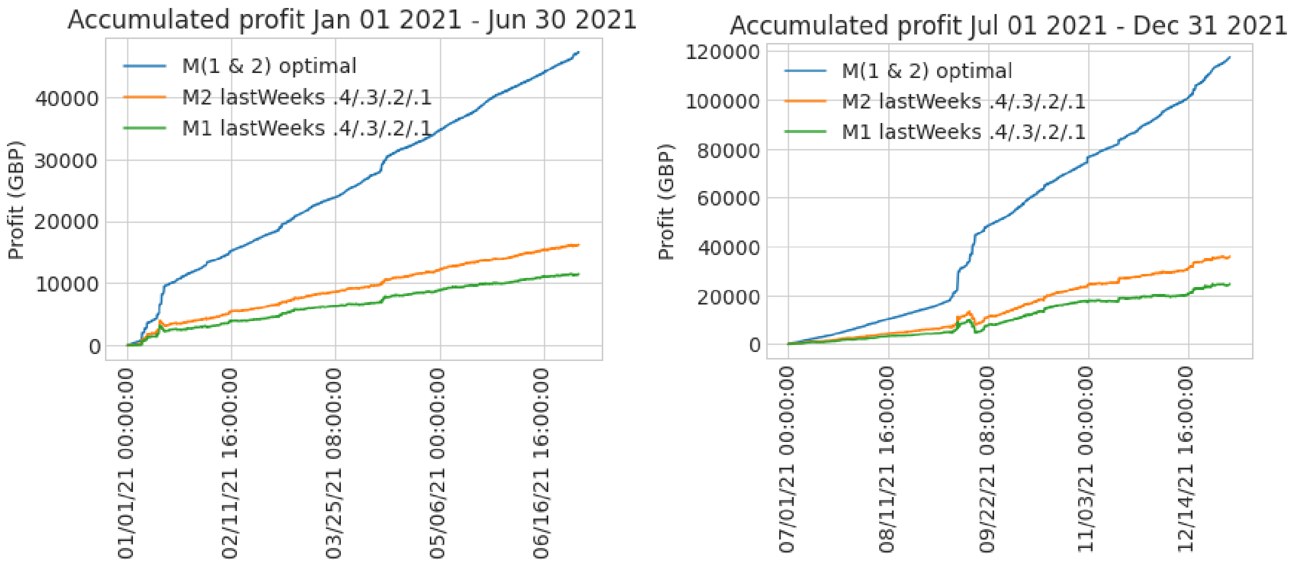
Profit Extraction

The profits obtained from running the models are observed in Table 1.

Table 1: Table containing the profit obtained from the scheduling with either perfect foresight (optimal schedule) or the generated forecasts. The profits obtained in £ as well as the value extracted in percentages for the simple forecast compared to perfect foresight are presented.

	First half-year in 2021 (£)	%	Second half-year in 2021 (£)	%	Year 2021 (£)	%
Perfect foresight	47187	100 %	117352	100 %	164539	100 %
Last Weeks M2 (4,3,2,1)	16219	34.37 %	35917	30.61 %	52136	31.69 %
Last Weeks M1 (4,3,2,1)	11461	24.29 %	24633	20.99 %	36094	21.94 %

A graphical representation of the accumulated profits in year 2021 are shown in Figure 16.



(a) Accumulated profit for the first-half of year 2021

(b) Accumulated profit for the second-half of year 2021

Figure 16: Accumulated profit in year 2021

In Figure 16, the obtained profit is increased in Model 2 compared to Model 1. This shows that revising the Intra-Day schedule decided upon the day prior at 11:00 is beneficial.

Profit Distribution

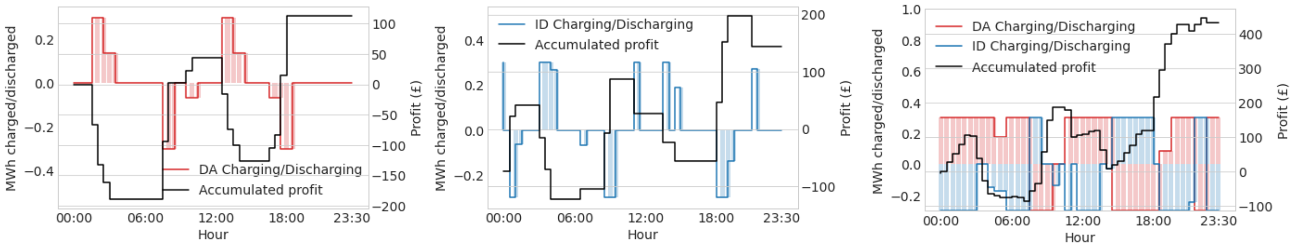
Expanding on the profits determined in the second half-year in Table 1, the effect of participating in the markets together and separately is investigated.

Table 2: Table containing the profit obtained from the scheduling with perfect foresight (optimal schedule) and using the Last Weeks price forecast for the second half-year in 2021. The profit obtained from each market is visible in the columns with market share. The sum of the two market shares yield the second half-year profit.

	Second half-year profit in 2021 (£)	Day-Ahead market share (£)	%	Intra-Day market share (£)	%
Perfect foresight	117352	80798	68.85	36554	31.15
Last Weeks M2	35917	66638	185.54	-30722	-85.54
Last Weeks M1	24633	66638	270.52	-42005	-170.52
Perfect foresight (without ID)	26462	26462	100	-	-
Perfect foresight (without DA)	29506	-	-	29506	100
Last Weeks (without ID)	21785	21785	100	-	-
Last Weeks (without DA)	24180	-	-	24180	100

Observing Table 2, perfect foresight yields the highest profit, while the second highest profit is obtained after revising the Intra-Day trading decisions; Last Weeks M2. Looking at the profit obtained for M1 and M2 using the Last Weeks price forecast, the Day-Ahead market yields the exact same profit, since the final Day-Ahead decision have to be made a day in advance of the actual delivery and are thus unaffected by Intra-Day revisions. The contracts are therefore identical for both models. The Intra-Day market on the other hand shows that revising the decision made the previous day is beneficial, as the profit loss experienced decreased by a significant amount, 12000 £.

While this leads to the suspicion that the Intra-Day market has a negative impact on the final profit and thus only the Day-Ahead market should be participated in, they are both important. Summing the perfect foresight profits obtained for each market separately yields 55968 £, which is slightly less than 50 % of the obtainable profit if both markets were included in the optimization. Both markets are therefore important, as the inter-play between them allow for the extraction of profits unobtainable when the markets are independent of each other. By operating in both markets it becomes possible to buy and sell in either of them for the same settlement periods and thus exploiting potential arbitrages. This enables the battery to earn a profit without changing its state of charge, which was a limiting factor when only participating in one market. As a result, an increase in profit higher than the cumulative profit of both markets separately is observed when one has perfect foresight. The aforementioned behaviour is observed in Figure 17.



(a) Day-Ahead market (b) Intra-Day market (c) Both markets

Figure 17: The operational schedule determined by the Last Weeks forecast and its corresponding profit obtained from participating in the mentioned market(s) during the 11^{th} of September in the second-half year 2021. The left y-axis denotes charging if the bars are above 0, while the battery discharges if a value below 0 is observed. The right y-axis shows the accumulated profit. A higher profit from participating in both markets is observed.

Observing Figure 17, by exploiting arbitrage between both markets, graph (c), it becomes possible to place bids and offers that cancel out within the same settlement period at no expense of the daily cycle limitation, which only tracks changes in the battery state. This is an advantage compared to graph (a) and graph (b), which are limited to trading within the daily cycle constraint and only participating in one market. The final profit obtained during the 11^{th} of September when participating in both markets is therefore larger than the sum of the profits obtained from the individual markets.

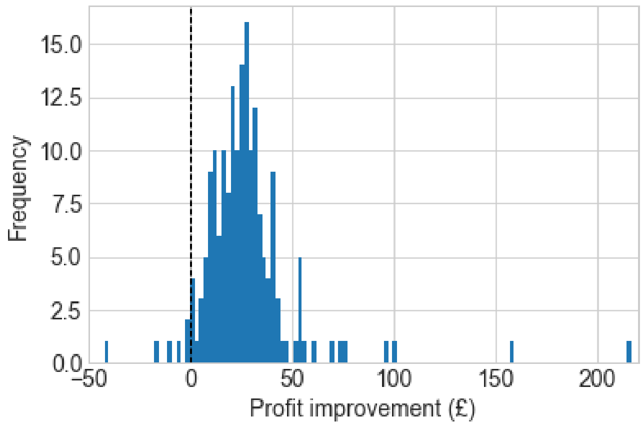
The profits obtained when only one market is considered, are highest in the Intra-Day market, while higher in the Day-Ahead market when both markets are considered. The hypothesized reason for this trend, is based on the fact that the average of the prices in the Day-Ahead market are higher than the Intra-Day market prices while the Intra-Day experiences more price spikes and is more volatile, which can be observed in Figure 9, Figure 10 and Figure 11. Due to the higher average price in the Day-Ahead market, most of the time it is economically preferred to discharge in this market and charge in the Intra-Day market, when both markets are participated in. Futhermore, this leads to price arbitrages generally being carried out as buying Intra-Day and selling Day-Ahead. This moves the obtained profit to the Day-Ahead market, while Intra-Day incurs the costs.
However, due to the higher Intra-Day volatility this market yields a higher stand-alone profits likely due to the exploitation of price spikes, which is elaborated on in the section "Exploring Price-Spikes".

Intra-Day Revision of Schedule

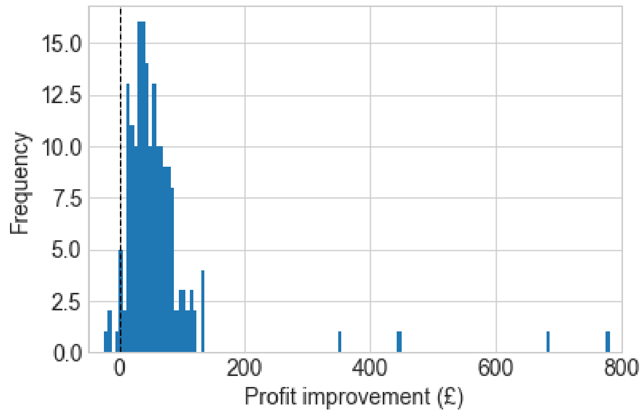
As previously mentioned, revising the Intra-Day schedule when new information is known leads to higher profits. Here, two exemplary days have been chosen to provide an insight into the daily scheduling determined by the models as well as the difference in the Intra-Day market schedule before and after revision.

Exemplary Days

Examining the difference in profit after revising the Intra-Day schedule in Model 2 compared to Model 1 for year 2021, Figure 18 is created.



(a) Profit distribution for the first-half of year 2021



(b) Profit distribution for the second-half of year 2021

Figure 18: Histograms containing the distribution of the change in daily profit when using Model 2 to revise the Intra-Day schedule determined in Model 1 for year 2021. The change in daily profit is determined by subtracting the profit obtained from Model 1 in day D from the profit obtained in day D from Model 2.

Observing the histograms in Figure 18, the majority of days experience an increase in profit. The histograms also show that most days experience a profit increase between 10 and 100 £ depending on the half-year, while a few days experience a drastic increase of profit; the 2nd of November (Day 125 in the second-half of year 2021) experienced the highest profit increase of 782 £. The reason for this increase is the additional information available in the Intra-Day market which enables the model to revise the previously decided upon schedule.

On the 2nd of November, the actual and forecasted prices for both markets are seen in Figure 19.

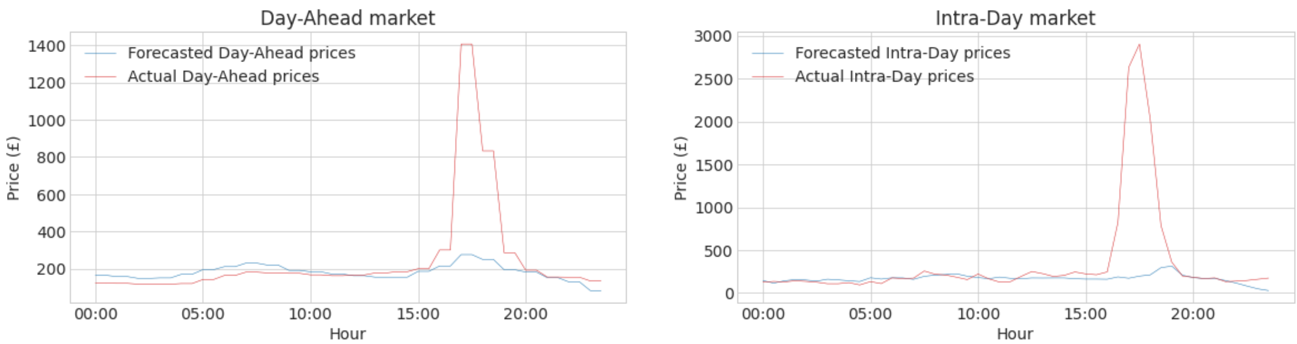


Figure 19: Forecasted (LW (4,3,2,1)) and Actual prices for the 2nd of November 2021

The determined Day-Ahead and Intra-Day schedule for Model 1 as well as the revised Intra-Day schedule for Model 2 can be seen in Figure 20.

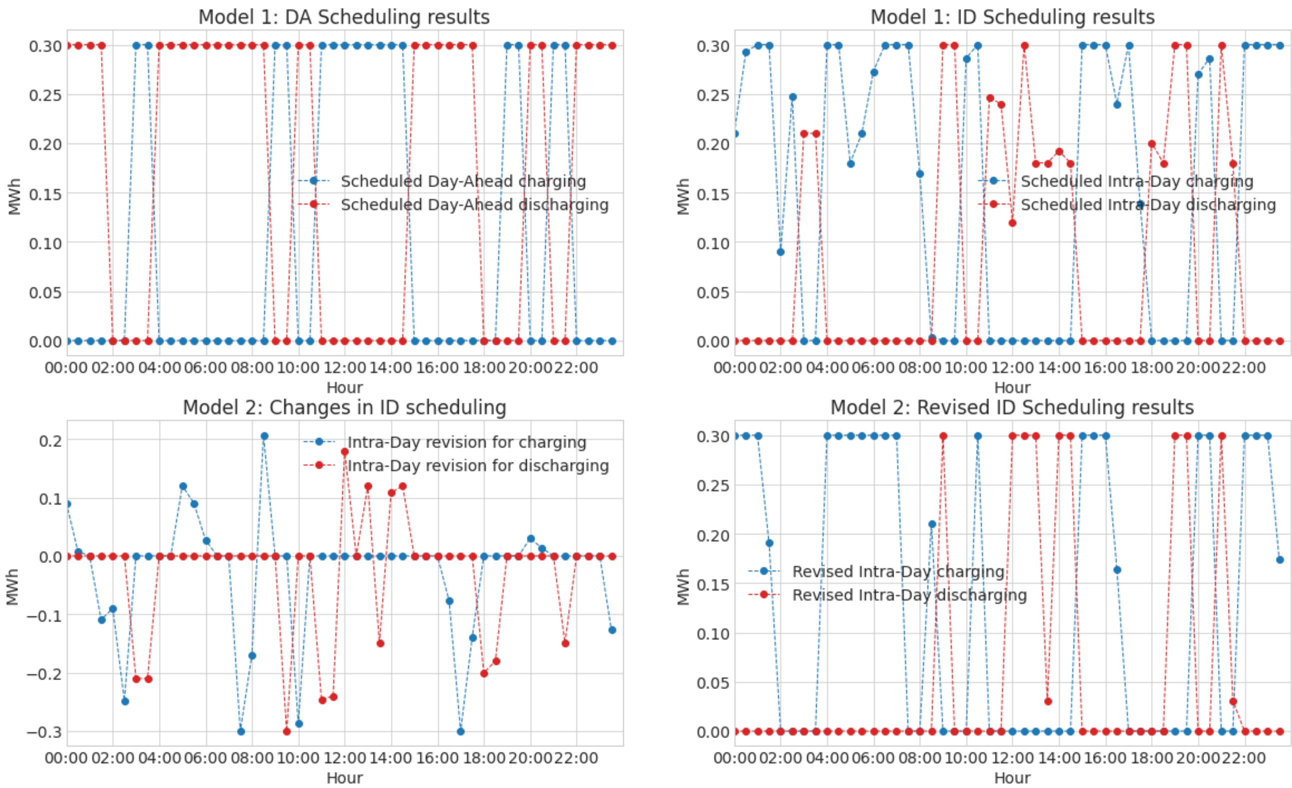


Figure 20: The schedules determined by Model 1 and 2. The top left and right graphs are respectively the scheduled Day-Ahead and Intra-Day trades on the 2nd of November determined by Model 1. The bottom left graph is the changes in Intra-Day trading in Model 2 after revising the previous decision on additional information acquired during the Intra-Day. The changes are determined by subtracting the schedule in Model 1 from the schedule determined by Model 2. The bottom right graph is thus the revised trading schedule in the Intra-Day market determined by Model 2.

The resulting state of charges for Model 1 and 2 are seen below in Figure 21.

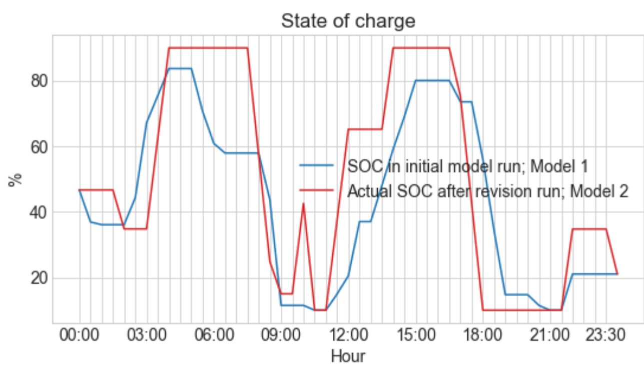


Figure 21: State of charge in the battery using the schedule determined in respectively Model 1 and 2.

In Figure 20 and Figure 21, the change in schedule becomes visible. This is caused by re-optimizing the Intra-Day schedule every six hours with known Intra-Day prices for the following six hours and the model adapting to the large price spike observed in the Intra-Day market in Figure 19. An example of a significant change is that instead of charging the battery in the period between 15:30 to 17:30, where the prices are highest, the Intra-Day contracts are revised so the charging of the battery takes place earlier. The gain in profit is observed in Figure 22, where the accumulated profit during the 2nd of November is graphed.

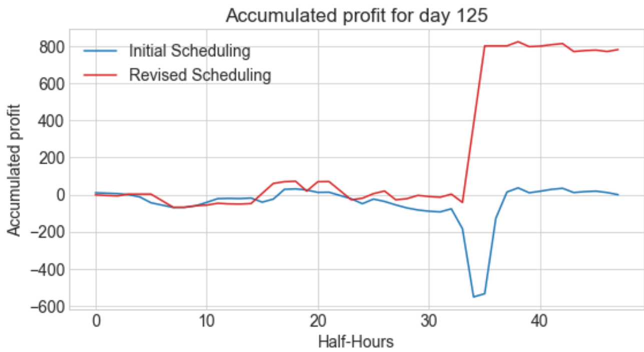


Figure 22: Accumulated half-hourly profit on the 2nd of November 2021.

As observed in Figure 18 and Figure 22, revising the Intra-Day schedule when more information is known increases the overall profit obtained.

As the 2nd of November is the day with the largest profit increase after revising ones Intra-Day decisions, a day in the second half of year 2021 with a typical profit increase is chosen; 11th of September (Day 73 in the second-half of year 2021). The actual and forecasted prices for both markets during the 11th of September 2021 are observed in Figure 23.

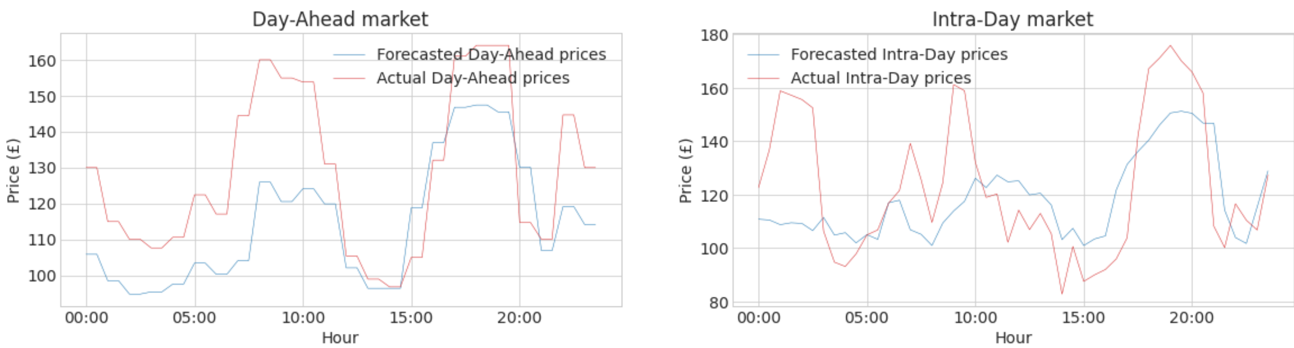


Figure 23: Forecasted, Last Weeks, and Actual prices for the 11th of September 2021.

In Figure 23, the forecasted Day-Ahead price development is observed to be similar to the actual Day-Ahead prices. While this is true, the actual price level predictions are off, as the difference between forecasted and actual price ranges around 0-35 £. The difference between the forecasted and actual Intra-Day prices are more pronounced as most price spikes were not predicted in the forecast.

The battery schedules determined by Model 1 and 2 are observed in Figure 24.

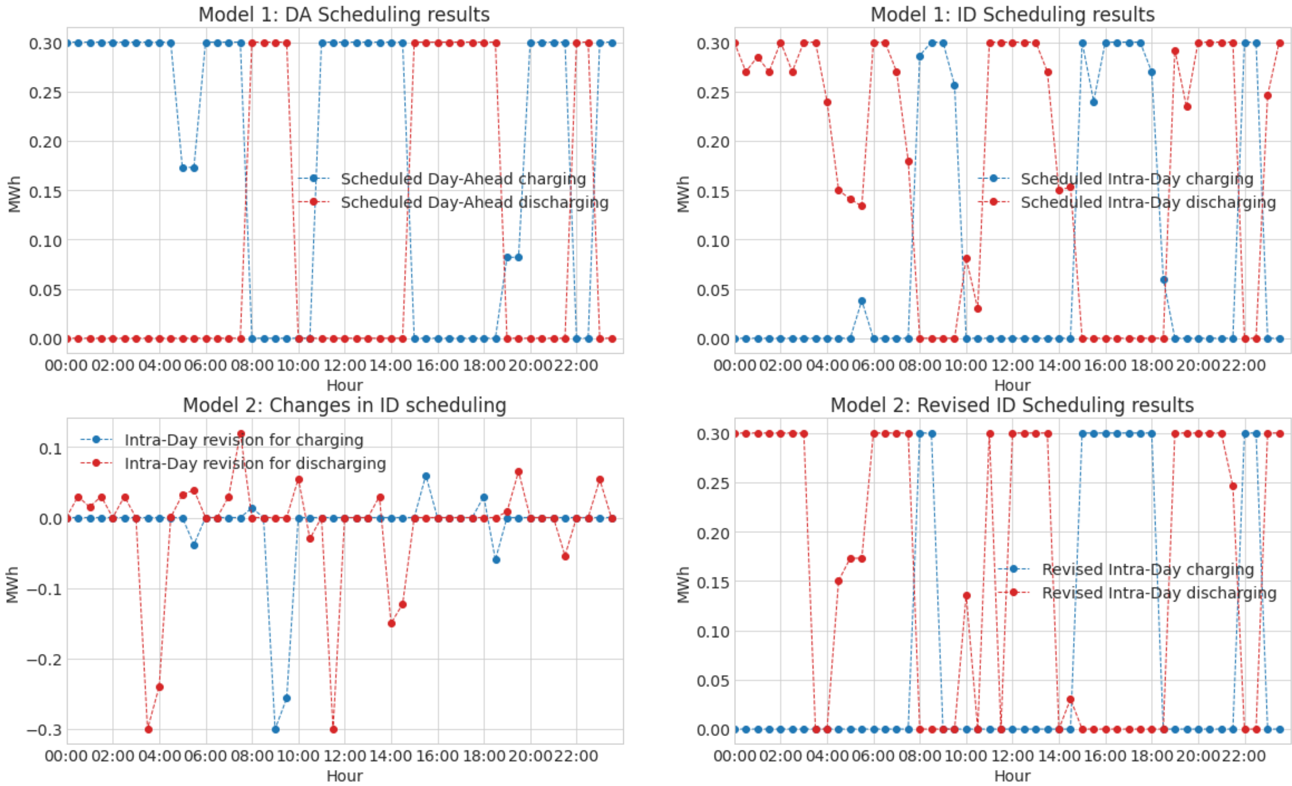
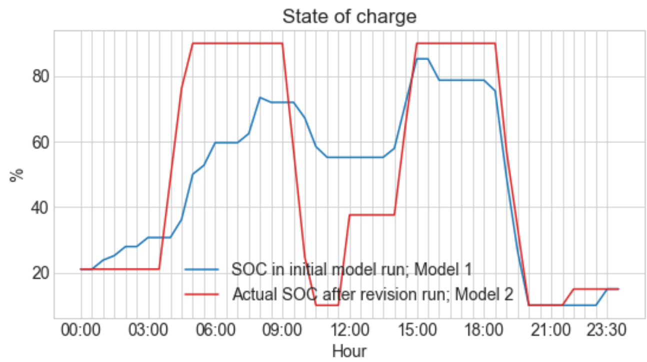


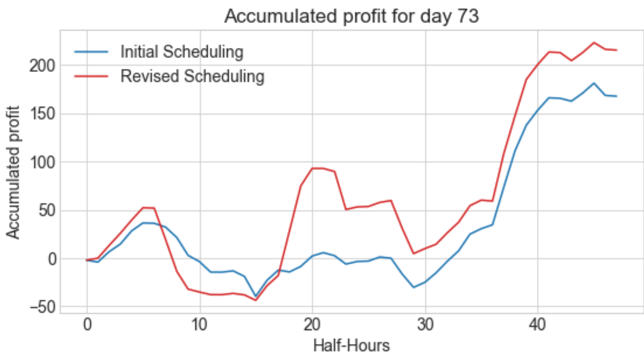
Figure 24: The schedules determined by Model 1 and 2. The top left and right graphs are respectively the scheduled Day-Ahead and Intra-Day trades on the 11th of September determined by Model 1. The bottom left graph is the changes in Intra-Day trading in Model 2 after revising the previous decision on additional information acquired during the Intra-Day. The changes are determined by subtracting the schedule in Model 1 from the schedule determined by Model 2. The bottom right graph is thus the revised trading schedule in the Intra-Day market determined by Model 2.

As previously mentioned, the difference in forecasted and actual Intra-Day prices result in a changed Intra-Day schedule. This can be observed in the bottom left plot of figure Figure 24, where the model chooses to stop discharging at 04:00 and 12:00 due to actual intra-day prices being lower than forecasted and discharging during the price spikes if possible.

The state of charge and accumulated profit before and after revision of the Intra-Day contracts are observed in Figure 25.



(a) State of charge in the battery using the schedule determined in respectively Model 1 and 2.



(b) Accumulated half-hourly profit on the 11th of September 2021.

Figure 25: State of charge in the battery and accumulated profit during the 11th of September 2021 determined by Model 1 and Model 2.

In Figure 25, a profit increase of 48 £ is observed. As the Intra-Day market prices gets revealed, the revision of Intra-Day trades improve the overall profit by charging and discharging at opportune moments.

Forecast Selection

Data Set

A requirement of using machine learning models is a data set with descriptive parameters for the values that are subject of prediction. The values to predict in this instance, are the upcoming day-ahead and intra-day prices. The prediction range varies depending on the model, with the longest period to predict being 48 hours.
The following overview will grant an insight into the descriptive data chosen selected for forecasting, its availability, and the individual components' contribution to describing the variance of the forecast.

Table 3: Origin and formats of descriptive parameter inputs used in the forecast models to predict Day-Ahead price (DA), Intra-Day price in Model 1 (ID1) and Intra-Day price in Model 2 (ID2)

	DA	
Parameter	Source	Availability as of 11:00
Wind Generation Forecast	BMRS API	48+43 settlement periods ahead
Temperature	BMRS API	Up to previous day
Weighted Avg. DA price	Ørsted	Until end of the day
	ID1	
Parameter	Source	Availability as of 11:00
Wind Generation Forecast	BMRS API	48+43 settlement periods ahead
Temperature	BMRS API	Up to previous day
Weighted Avg ID price	Ørsted	12 settlement periods ahead
	ID2	
Parameter	Source	Availability on continuous basis
Wind Generation Forecast	BMRS API	Until the end of the day
Temperature	BMRS API	Up to previous day
Weighted Avg. DA price	Ørsted	Until end of the day
Weighted Avg ID price	Ørsted	12 settlement periods ahead

Furthermore, intrinsic information such as time of the day and whether it is a weekend or holiday, is also added as descriptive parameters to the data set. A complete overview is provided in the following table:

Table 4: Formats of descriptive parameter inputs used in the forecast models derived solely from intrinsic information in the data

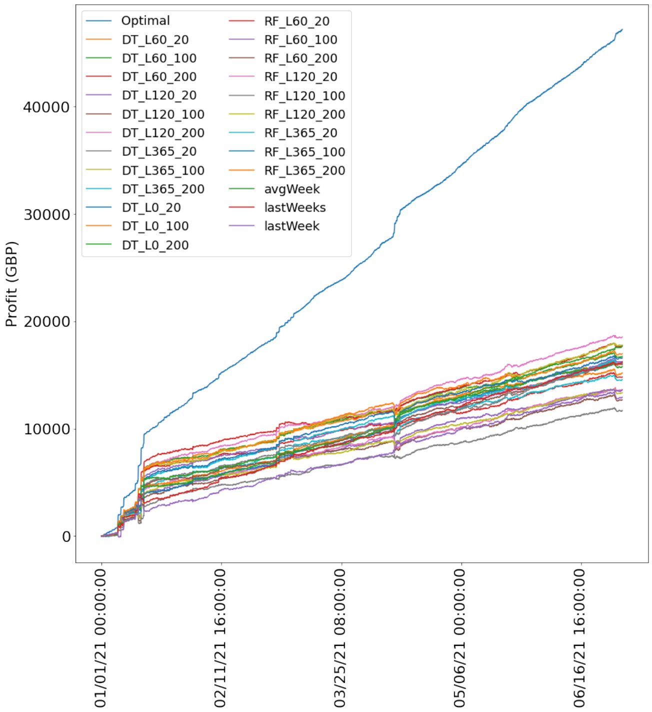
	DA/ID1/ID2	
Parameter	Type	Format
Weekend	Boolean	True if weekend
Holiday	Boolean	True if holiday
Settlement Period	Boolean	Separate columns each true for a given period
Daylight	Boolean	True while daylight savings is in effect
	DA/ID1	
Parameter	Type	Format
Weighted Avg. DA price	Number	Lags from 48 to 96 periods
Weighted Avg. ID price	Number	Lags from 62 to 110 periods
	ID2	
Parameter	Type	Format
Weighted Avg. DA price	Number	Lags from 0 to 48 periods
Weighted Avg. ID price	Number	Lags from 36 to 84 periods

Combining the information in Table 3 and Table 4 yields three different data sets fit for applying forecasting models on.
The DA and ID1 sets are utilized at 11:00 which is the Day-Ahead gate closure. This is also reflected in the lags. The DA and ID1 information are used to respectively predict Day-Ahead and Intra-Day prices from settlement periods 1 through 48 the next day. Since the Day-Ahead prices are known until the end of the current day, at settlement period 48 the next day, the most recent price information will be from 48 settlement periods ago, i.e. 24 hours, thus the smallest feasible lag is 48. To give information of the previous day as well, lag 48 to 96 is used. While ID1 predicts in the same time frame, less recent information is available, since prices are only known 6 hours ahead. Thus, in the last settlement period of the following day, the most recent price is from 17:00 the previous day, i.e. 62 settlement periods. To give a 24 hour window lags from 62 to 110 are provided.
Finally, ID2 can be run multiple times a day starting from the beginning of the day, at 00:00, and predicts the prices for the remainder of the day. In principle it could be updated on a half-hour basis, but due to computing time and the fact that the next 6 hours' real prices are revealed, thus mitigating the forecasting dependency, it is limited to just being trained at the beginning of each day. This explains the Intra-Day lags ranging from 36 to 84 settlement periods, since at the end of the day the earliest known information is from 06:00, i.e. 36 settlement periods ago. The Day-Ahead prices are lagged between 0 and 48, due to the fact that the prices are revealed for the entire day at 12:00 the previous day.
With the data sets defined, the generation of forecasts is now possible.

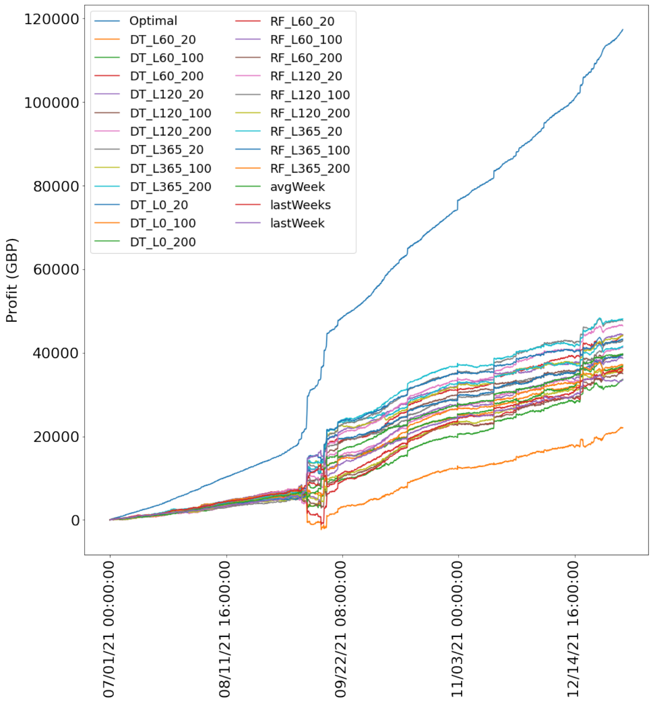
Forecast Screenings

Generation of either Decision Trees or Random Forest models can be done with a wide selection of different model parameters as well as different choices of data information included.

The main factors here have been deemed as the *minimum split samples*, which determines the minimum amount of observations required in order for a model to generate a new node, and the *historic inclusion*, which is a decision on how much of the historic information should be available to the model.
In short, the *minimum split samples* affects the number of leaves generated. A higher requirement of observations in each split generally means less leaves and vice versa. This is the main driver of how well fitted the model is and thus how niche each leaf can be. The risks here are over- and under-fitting, where leaves are either too plentiful and narrow, making the small sample sizes prone to attaining extreme values, or the leaves are too few and broad in scope to add any meaningful information since the range of values is too wide. Alternatively, a *minimum leaf samples* can be defined which serves almost the same purpose. The main difference here is that all leaves will be of a certain size, while *minimum split samples* can have leaves of any size. If the two parameters were set as the same value in two different models, the *minimum split samples* would have as many or more leaves as the *minimum leaf samples*. Both have advantages and disadvantages. Restricting the size of splits allows for small leaves with few samples and thus a greater difference in the values predicted, but also ensures that samples within each leaf are more homogeneous. Directly restricting the size of the leaves can group very different observations together, but this buffer of samples protects against having a few outliers define the value of a leaf.
The *historic inclusion* affect the emphasis on newer data, with a large amount of old data generally causing newer and consistent trends to be diluted and overlooked. Thus, there is a trade-off between ensuring a dynamic and agile model while still including enough data to establish a sound foundation of patterns.
Based on the data set described in the previous section, a set of different forecasts are generated with changes to these two effects. In order to assess how well each forecast performs, a few criteria have been formulated.
The first factor of the screening is simply a profit comparison to see how well the forecasts pick up different market behaviours. For this, every forecast model has been retrained on a daily basis to allow the inclusion of yesterday's data. To decrease running times, no sampling from the forecasts has been performed as of yet, and the single deterministic price forecast from each model will determine which are most worth sampling from, which is elaborated on in later sections. Lastly, the revision of Intra-Day actions determined by Model 2 has the option of being re-optimized every settlement period of the day to include the latest data continuously. This will be utilized once the initial screening has been carried out, but the preliminary results presented below are based on 6-hourly re-optimizations to reduce running time.



(a) Profit of forecasts in the first half-year



(b) Profit of forecasts in the second half-year

Figure 26: Accumulated profit of each forecast over each test set period in order to display the range of performances. The forecasts performing the best during rapid price growths are also the ones ending with the highest profits. This is observed at the start of (a) and middle of (b), where an otherwise very similar profit range is spread out significantly.

[%]	Avg Week	Last Week	Last Weeks
First Half	37.47	28.93	34.37
Second Half	33.82	28.71	30.61

[%]	DT_L60_20	DT_L60_100	DT_L60_200	DT_L120_20	DT_L120_100	DT_L120_200	DT_L365_20	DT_L365_100	DT_L365_200	DT_All_20	DT_All_100	DT_All_200
First Half	32.23	35.48	31.43	27.43	26.92	28.88	24.78	28.32	30.93	33.89	33.97	33.49
Second Half	18.79	28.52	37.62	37.79	33.6	35.18	36.58	37.64	35.44	33.77	31.16	31.44

[%]	RF_L60_20	RF_L60_100	RF_L60_200	RF_L120_20	RF_L120_100	RF_L120_200	RF_L365_20	RF_L365_100	RF_L365_200
First Half	37.58	34.47	34.06	39.33	34.12	37.74	34.29	35.19	36.01
Second Half	30.67	32.94	29.82	39.56	40.61	30.34	41	36.87	31.64

Figure 27: Total profit forecasts in each test set as a percentage of the total best possible performance, with the main take-away of Random Forests being generally better performing, but also with examples of Decision Trees doing better with the same information, e.g. "DT_L60_200" in the second half of 2021.

The top row of Figure 27 is the simple forecasts presented in section "Simple Forecasts", while the second and third row show the Decision Trees and Random Forests respectively. Furthermore, there is a naming convention to distinguish otherwise similar models, e.g. "DT_L120_100" is a Decision Tree including the last 120 days of data with a minimum of 100 samples in each leaf.

Besides economic performance, since the profit analysis only contributes to knowledge about the test set performances, a more generally applicable screening is introduced, where the accuracy of the forecasts are evaluated. The three following indicators are compared:

- The Root-Mean-Square error between real and forecasted price for each settlement period
- The Root-Mean-Square error of the difference in settlement period ordering when sorting the real and forecasted prices from lowest to highest of a day
- The Root-Mean-Square error of the difference in settlement period ordering when sorting the real and forecasted prices from lowest to highest of the day with added weights punishing errors in settlement periods with historically high prices

The price difference, P_m^{RMS}, of a forecast model, m, is computed as follows, with P_i^r and $P_{i,m}^f$ being the real and forecasted price respectively of settlement period i:

$$P_m^{RMS} = \frac{\sum_{i=0}^{I} |(P_i^r - P_{i,m}^f)|}{I} \tag{35}$$

The error in ordering, O^{RMS}, of a forecast model, is computed as follows, with the real, O_d^r, and the forecasted, $O_{d,m}^f$, being lists of indexes of the 48 settlement periods in a day, d, being sorted from lowest to highest price:

$$O_m^{RMS} = \frac{\sum_{d=0}^{D} |(O_d^r - O_{d,m}^f)|}{D} \tag{36}$$

The weighted error in ordering, $O^{RMS-weighted}$, is identical to the previous approach, except that a list of weights, W_d, is multiplied onto the difference in lists of ordering. The weight list is created based on real prices, and thus model insensitive. It is created by summing up the price of each settlement period across 2021 and then normalizing these 48 values. This list of weights is then arranged to match the ordering of settlement periods in list O_d^r for each day, such that a difference in ordering between the real and forecasted prices is multiplied by the associated weight of that settlement period.

$$O_m^{RMS-weighted} = \frac{\sum_{d=0}^{D} |((O_d^r - O_{d,m}^f) \cdot W_d)|}{D} \tag{37}$$

The purpose of the weighted approach is to acknowledge the importance of correctly predicting peaks, as these trends are the ones with the potential of generating the highest profit.

The following figures show the accuracy of the three aforementioned parameters across all three price forecasts, i.e. DA, ID1 and ID2.

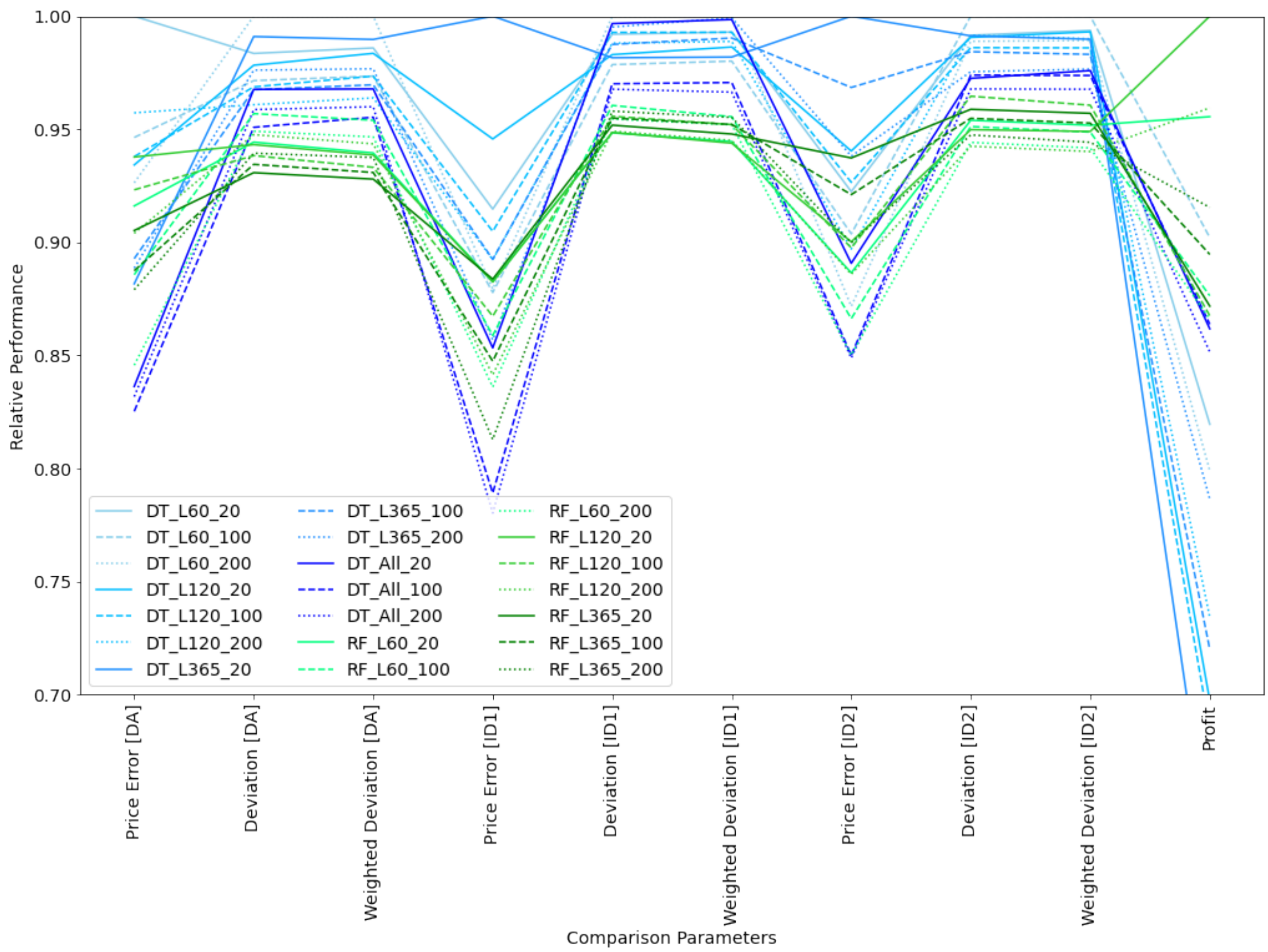


Figure 28: Accuracy measures of forecasts indexed against best performing forecast on each parameter in the first half of 2021. The graph exhibits higher profits for the Random Forests despite generally lower accuracy.

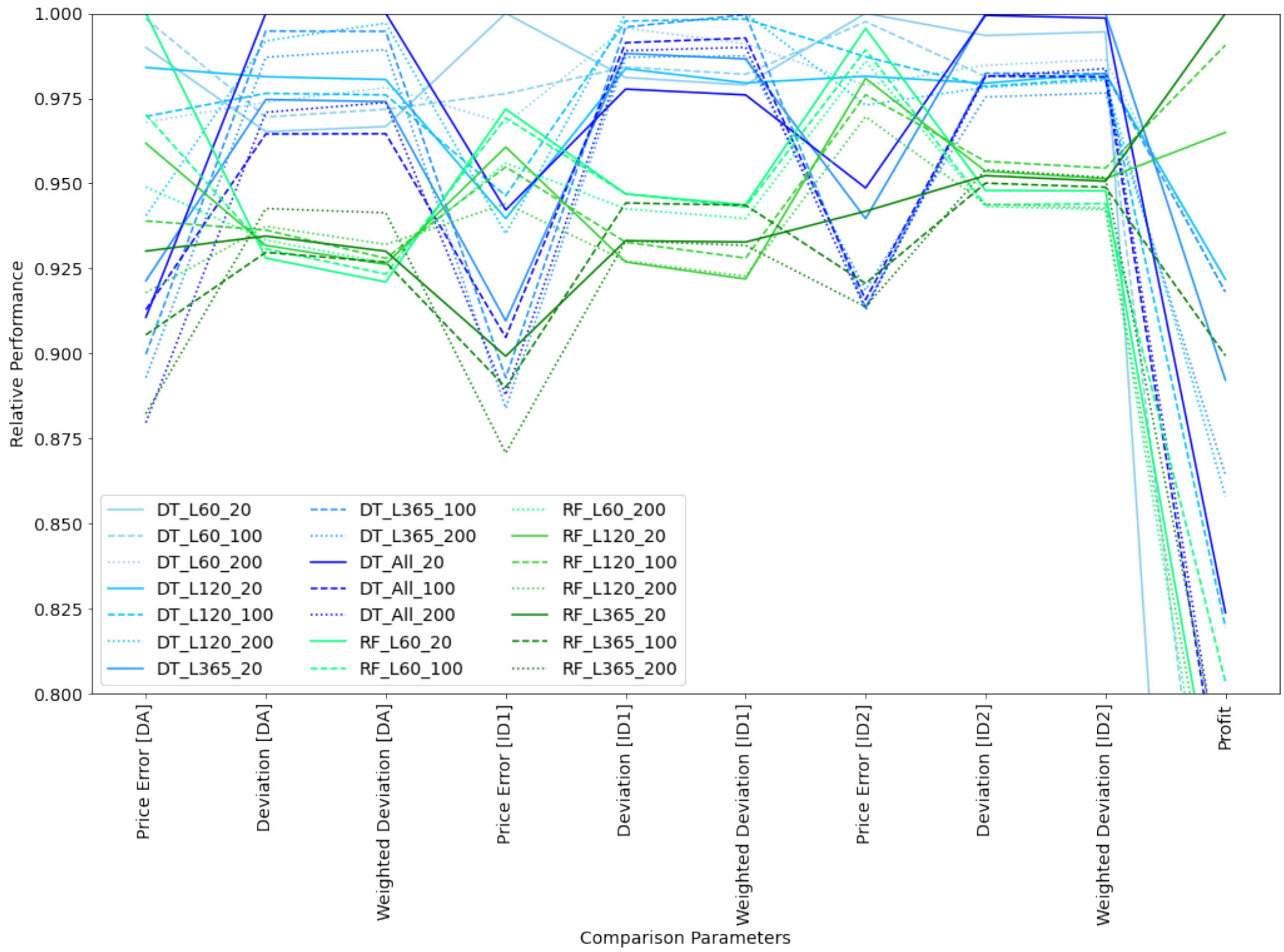


Figure 29: Accuracy measures of forecasts indexed against best performing forecast on each parameter in the second half of 2021, showing a generally better results for the Random Forest models.

The figures show the relative error of each forecast for each parameter, in addition to also showing the realized profit on the far right of the plot.
The relative error is found by indexing all the forecasts against the worst performing one for each parameter, meaning lower values indicate better performance. The only exception here is the "profit", where higher values are preferred. The profit is based on the same information available in figure 27.

Generally, it is observed that the random forests perform slightly better on accuracy in both test sets, and significantly more profitable in the first year-half compared to most decision trees. This changes in the second year-half, where the accuracy is still better, but the majority of random forests see lower profitability.

Keeping all the performance measures in mind, a few forecasts are selected to do sampling from and thus utilize the stochastic model to the fullest in contrast to the current set-up in which each forecast model yields a single deterministic price forecast for each of the three prices (DA, ID1, ID2).
The most promising candidates chosen are the models "DT_L60_200", "DT_L120_20", "DT_L120_100", and

"DT_L365_20".
These were the Decision Trees showing the best economic performances in each test set, as well as having reasonably high accuracies relative to the competing models.
No Random Forest models were selected, despite having the best economic performances especially in the second test set. This is due to the fact that they generally showed lower accuracies, most so in the second test set, which could raise a concern for the ability to perform well in the validation set, which might exhibit much different trends. In addition to this, as explained in section "Random Forest", each Random Forest model bootstraps multiple Decision Trees based on randomly selected portions of the input data. Thus, creating samples based on this method would be done through setting different random states before training the Random Forest, which is very computationally heavy.

Sampling Forecasts

With the most promising forecast models being selected based on accuracy and performance in the first and second half of 2021, the next step is a sampling process, which will generate a wide range of stochastic scenarios from the same model.
The leaves of the Decision Tree models each contain a certain amount observations, which have their value averaged out to one single value of the given leaf. However, by identifying which observations constitute a given leaf, this set of observations can be sampled from rather than using an averaged value. This allows a multitude of different predictions, or scenarios, to be generated each day, in spite of the descriptive parameters being identical for each scenario.
Once being generated, the optimization model requires a probability for each scenario, which is tricky to determine. One possibility would be to find the probability distribution of each leaf being sampled from and computing the likelihood of each possible scenario. An issue with this approach is the sheer amount of possible scenarios. With up to 70,000 rows in the data set of a forecast model and assuming a leaf size of 20 observations, this would result in a model containing $\frac{70{,}000}{20} = 3500$ leaves. The DA and ID prices are forecasted 48 settlement periods at a time, meaning 48 samples would be performed, resulting in $3500^{48} = 1.3^{107}$ scenarios for just one of the prices in a given day, of which some would be the same naturally. As one would only include a fraction of these scenarios in the optimization model, even assuming the 10,000 most likely ones are extracted, it still only covers a small share of the possible outcomes. All this is to say, that though the optimization will find slightly different schedules for each scenario, the final decision is an average of these schedules in which the small variations will be lost, which is the cost of not having 100% accurate forecasts.
So as an alternative to spending a lot of computing power on accurately finding probabilities of an inaccurate forecast model, the scenarios are initially assumed to have uniform probabilities and are sampled just 1000 times. Hereafter, the 1000 scenarios are reduced to 100 through a clustering algorithm, where the number of scenarios attributed to a cluster dictates its assigned probability of being realized and thus reintroducing information of the scenario likelihoods as opposed to the previously assumed uniform distribution. The clustering algorithm for this purpose is the Partitioning Around Medoids" (PAM) [19]. This approach is prefered here due to its ability to only select clusters that are represented in the actual data as opposed to finding averaged price-profiles which have the risk of not resembling the patterns of a day. For example, an averaged approach could create a cluster that reflects the highest price in each settlement period across multiple scenarios, thus generating a very unrealistic

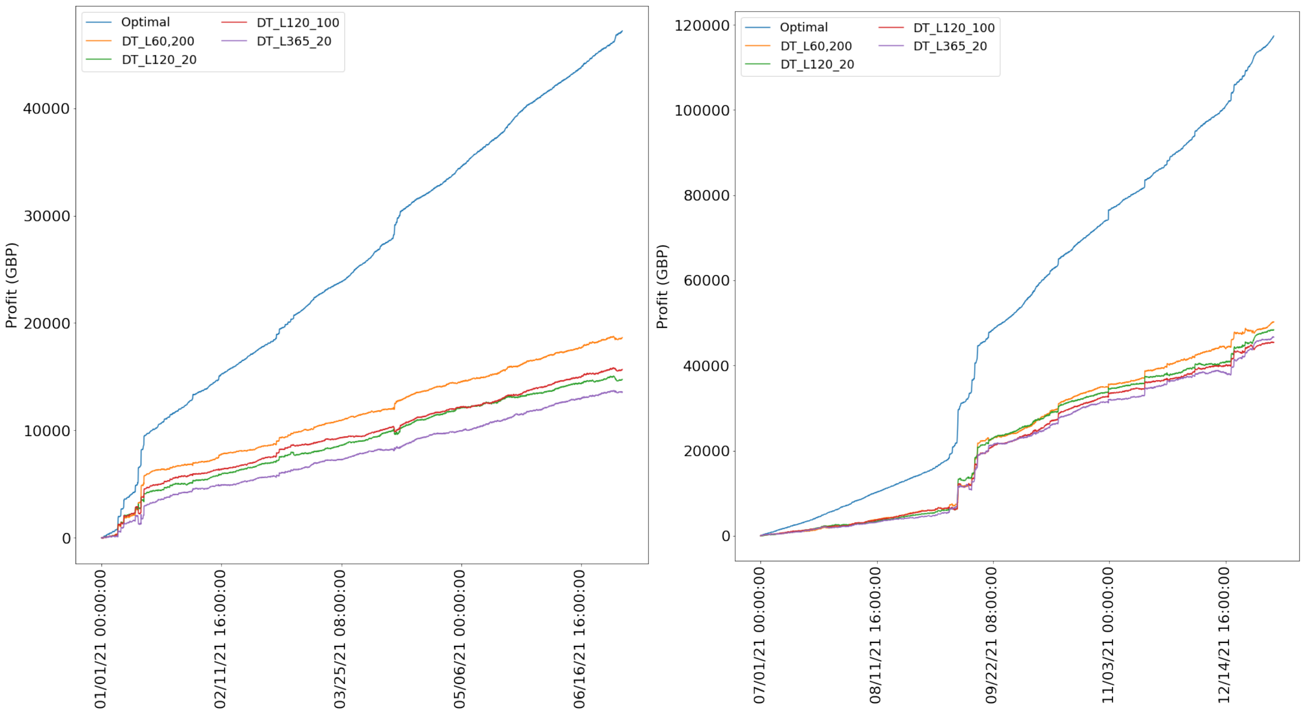
price curve, whereas PAM would be restricted to choose a single of the correlated scenarios as the cluster.

The chosen forecast models were covered in the previous section, where these exhibited high consistent profit yields in both test sets, as well as yielding good accuracy scores relative to their competitors.
These are now being sampled as described and will in the next section be compared among each other to determine the most promising forecast to apply on the validation data set.

Model Results of Selected Forecasts

With the preferred forecasts being narrowed down to just a few, the final test of performance under sampling is carried out, before choosing a single forecast model to apply on the validation data set. These sampled forecast data are revised in Model 2 on a half-hourly basis, which is more computationally heavy, but reflects the possible obtainable profit more realistically.

Test Results



(a) Profit of sampled forecasts in the first half-year

(b) Profit of sampled forecasts in the second half-year

Figure 30: Profit of sampled forecasts in each test set, for which 1,000 samples were reduced to 100 through clustering.

It is observed that "DT_L60_200" yields the best results with a profit utilization of 39.52 % in the first test set and 42.79 % in the second.

Looking at the behaviour of the sampled forecasts, an interesting behaviour is observed during price spikes, compared to the unsampled forecasts. The forecast selection process as well as the sampling of the forecasts, shows to choose forecasts which utilize these spikes in price. This can be observed when comparing the profits of forecasts in Figure 26 and Figure 30, where some forecasts will dip or stall in the profit obtained in the first figure while all forecasts in the last figure fare well during spikes. Since a defining factor for the performance of a forecast is how it fares during the most volatile days, where some will incur drastic losses while others will take advantage and turn a profit, this is a positive effect.

Validation Results

With the forecasts models being narrowed down to a single one, the validation data set is now introduced for the first time. No previous analyses, training or testing has been carried out using this data, with the purpose of avoiding both conscious and subconscious over-fitting to the data.
Consciously, the forecast models could have been trained and tested on this data, which would constitute data leakage, since the objective is to select a forecast that will perform well in future price environments and not just the ones it has been exposed to.
Subconsciously, despite not training the models on the validation data, if one had simply looked at the price developments or had any knowledge of new trends, there could be a bias towards some of the models with the expectation of features currently yielding bad results turning out to be valuable in the validation data.

Therefore, the validation data, which covers the period January 1^{st} 2022 until May 15^{th} 2022, has not been examined in any way.
The profit obtained from the chosen forecast, "DT_L60_200", can be observed in Figure 31. Additional forecasts have been plotted in order to better assess the performance of the chosen forecast and the general range of obtainable profits.

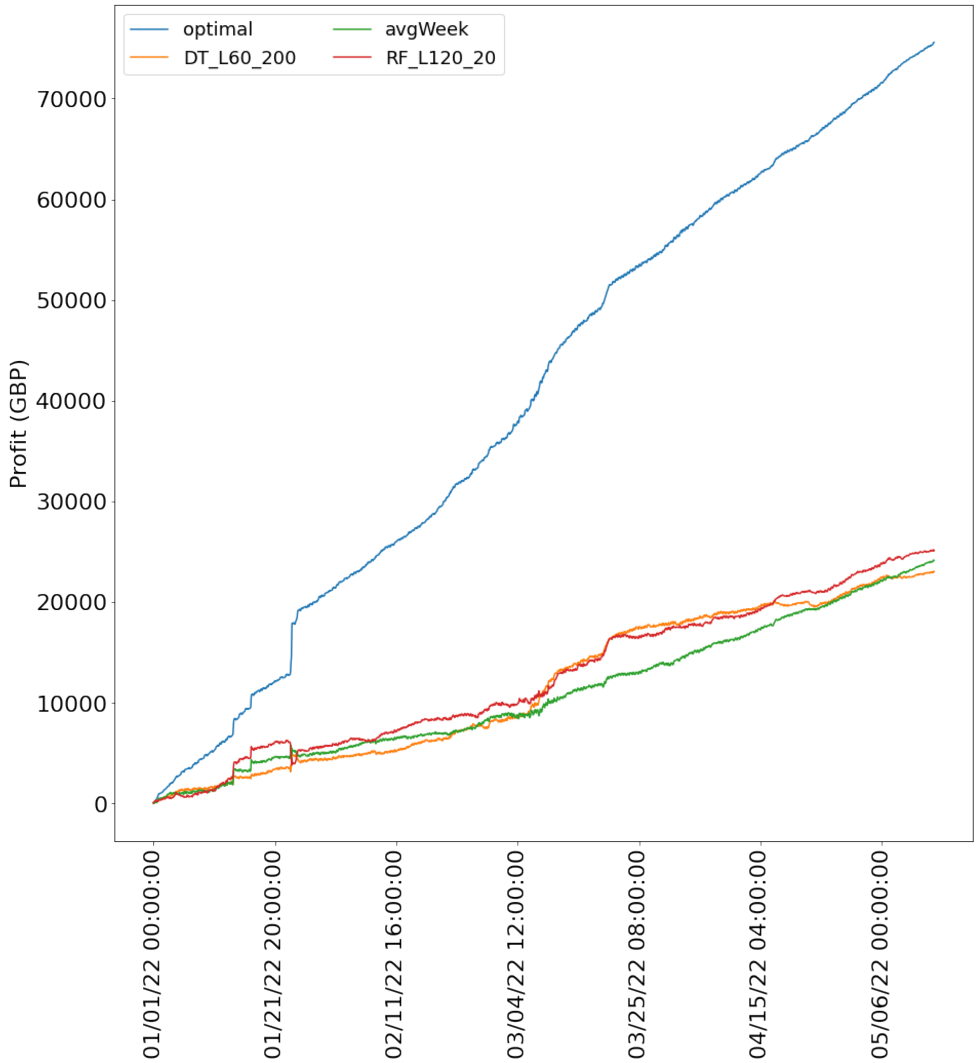


Figure 31: Profit of the chosen sampled forecast alongside others in the validation set, shown with the purpose of affirming that models were not subject to over-fitting of the test data.

The profit utilization obtained by "DT_L60_200" is 30.44 %, which is on the lower side compared to the results obtained from both the test sets, and is comparable to the results obtained from the simple averaged week, 31.94 %. The unsampled "RF_L120_20" outperforms both, with a profit utilization of 33.21 %. This reflects the difficulty and importance of assessing how well a model fits the market patterns. As a way of combating forecast models performing sub-optimal, multiple models should be maintained each day. With frequent intervals, the one showing the most promise in the current market should then be the deciding forecast. This way, rather than trusting the same model to constantly perform for a lengthy period at a time, forecasts can be replaced on a more frequent basis.

Model Sensitivity

The graphs are only shown for the second half year of 2021, as the first half year of 2021 exhibits the same trends.

Parameter Sensitivity

In this section the model sensitivity will be investigated. The investigation is carried out with a battery capacity of 1 MWh, a charging rate of 0.6 MW, the one-way efficiency for both charging and discharging is 0.92 % and the daily cycle limitation set to 2 unless otherwise stated.

The above mentioned parameters are varied in the range between -80 % and 100 %, except for the one-way efficiency, which was varied from -80 % to 8.69 %, as a one-way efficiency above 100 % is unrealistic. This yields the graphs visible in Figure 32 and Figure 33.

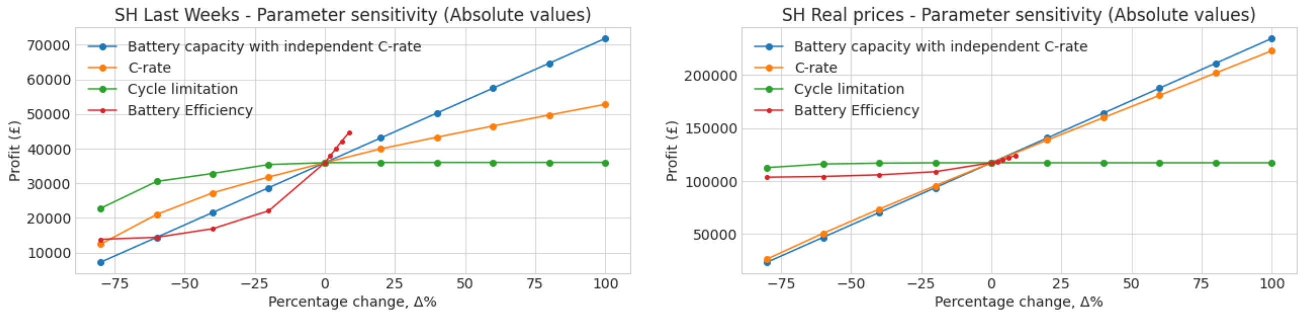


Figure 32: Profit (£) obtained by varying the parameters is shown. On the left graph the profits are based on the forecasted prices from the Last Weeks forecast for the second half of year 2021. In the right graphs, the actual prices for the second half of year 2021 (SH) are used.

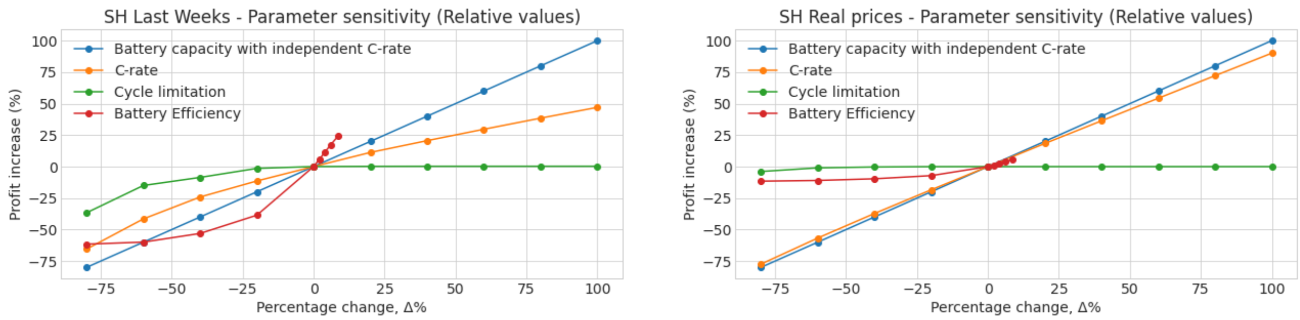


Figure 33: Percentage profit change obtained by varying the parameters is shown. On the left graph the profits are based on the forecasted prices from the Last Weeks forecast for the second half of year 2021. In the right graphs, the actual prices for the second half of year 2021 are used.

The sensitivity analysis is carried out with the profits obtained from using the actual prices, perfect foresight, and the forecasted prices from Last Weeks. Observing the graphs, the model sensitivity to changing the battery capacity is similar for both the actual and forecasted price. Doubling the capacity similarly doubles the obtained

profit. The C-rate exhibits the same behaviour for the actual prices, as a doubling of C-rate increases the obtained profit by 90 %. On the other hand, the obtained profit from using the Last Weeks forecast yields a profit increase of 47 %. This is a significant reduction, which highlights the inaccuracies present in the forecasted prices, which prevents the model from exploiting the increased C-rate optimally.
The battery efficiency is significantly more important for the more inaccurate forecast compared to the real prices as observed in Figure 33. Here, the profit sensitivity to changes in battery efficiency ranges between -65 % to 25 % for the forecasted prices, while it ranges between -11 % to 6 % for the real prices. This is attributed to the fact that more revisions take place in the forecasted Intra-Day market as prices are revealed compared to the real price analysis, where battery operations will be planned perfectly from the beginning. Thus, it is more influential to have a high efficiency for the forecasted prices.

Price Spread

As the battery is assumed price-taker, the trading of electricity is done at the price in the market. In addition, the price used for the electricity markets are average prices. A price spread has therefore been simulated for the Intra-Day market to investigate the sensitivity in case of a more realistic market with a range of prices during Intra-Day trading. A price spread was created by allowing the average market prices to vary by a given percentage. A best case and worst case scenario are then created. The best case scenario is when the discharge and the charge price respectively increase and decrease. For the worst case scenario, the price change is reversed and charging becomes more expensive while the price for discharging is reduced. This creates a span of obtainable profit between the two price spread scenarios, where the two scenarios denote the most and least profitable case. The profit at a given percentage change for the two scenarios can be observed in Figure 34.

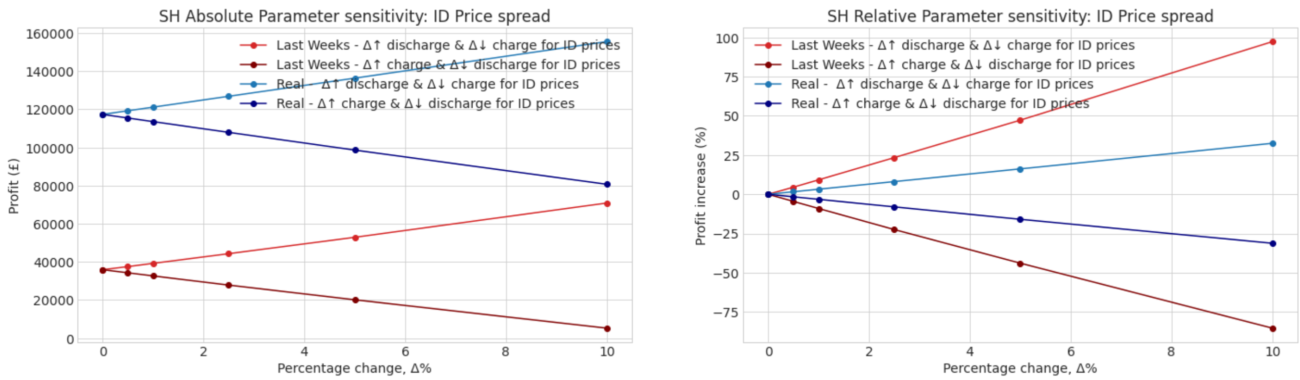


Figure 34: Profit change obtained by implementing a price spread for the forecasted prices from the Last Weeks forecast and the actual prices for the second half of year 2021.

While the model is more sensitive to the price spread when the forecasted prices from Last Weeks is used, the absolute change in £ is similar for both. As the obtained profit for Last Weeks is lower than when using the actual prices, the price spread has a larger impact on the final profit when looking at the relative change. In addition, the profit is affected significantly by implementing the price-spread. At a percentage change of 5% the observed profits may vary from -44% to 47% for the profit obtained when using Last Weeks to forecast the future market prices.

The profits obtained when increasing the cost of purchasing electricity and decreasing the price when selling it again is also able to simulate a market environment where transaction fees are present and their impact on the obtained profit. Thus, the dark blue and dark red line in Figure 34, which represent the worst case scenarios, also show the effect of introducing transaction fees determined as a percentage of the electricity price.

Scenario Sensitivity

The scenario sensitivity is investigated using electricity price forecasts generated via Decision Trees. By applying this method, a thousand different price forecasts were generated.

The scenarios include different forecasts of both the Day-Ahead and Intra-Day prices, however the Day-Ahead is averaged across the included scenarios, due to it being a first-stage decision, while the Intra-Day prices are different each scenario. In Table 5, the profit obtained from respectively using 1 or 100 price forecasts as well as 100 representative price forecast determined via reducing the 1000 scenarios through PAM clustering, is displayed. Due to computational complexity, the model was not solved using 1000 scenarios.

Table 5: Profit obtained in £ for the second half-year in 2021 by respectively using 1- and 100- Intra-Day price forecasts as well as using 100 representative price developments determined based on 1000 generated Intra-Day price forecasts. These three setups are respectively denoted as S1, S2 and S3 in the table.

	Total Profit (£)	%
Optimal scheduling	117352	100 %
S1	46486	39.55 %
S2	46840	39.85 %
S3	48375	41.16 %

Observing Table 5 it is noted that increasing that increasing the number of price forecasts used when determining the Intra-Day decisions in model 1 increases the overall profit.

Intra-Day Trading Revision Frequency Sensitivity

The Intra-Day market is a continuous market with predictable Intra-Day prices six hours into the future. In practice, this means that Intra-Day decision should be revised every half-hour due to the influx of new information. In order to reduce model complexity, all the previous results, except if explicitly noted otherwise, have been determined by revising the Intra-Day decisions every six hours. As this is not the optimal approach when considering profit, a sensitivity analysis on the revision frequency is performed. The sensitivity can be observed in Figure 35.

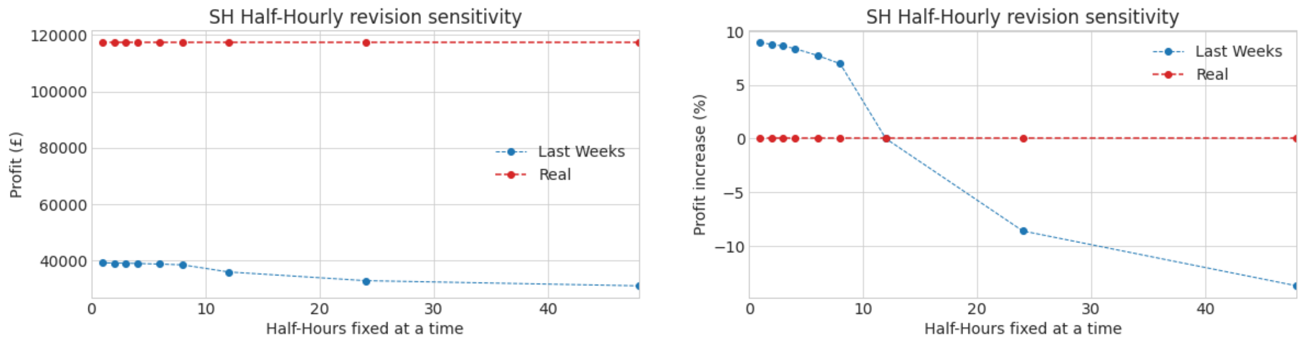


Figure 35: An overview of the profit sensitivity to the frequency of revisions in the Intra-Day market. Fixing fewer half-hours at a time means that the schedule has to be revised more frequently.

Looking at Figure 35, it becomes clear that the more frequent the Intra-Day decisions are revised using the Last Weeks forecast, the higher the obtained profit. This is in accordance to expectations, as the model is able to adjust to new information faster and more frequently. For the Last Weeks price forecast going from a revision frequency of 6 hours (12 half-hours) to half an hour increases the obtained profit by around 9 %. No change is observed when the real prices are used, as having perfect foresight results in the optimal schedule.

CVaR Sensitivity

The CVaR formulation of Model 1 from "Model 1 - CVaR" has a few additions to its set of constraints, but most importantly the objective function now consists of two parts. Firstly the profit across all scenarios and secondly the expected shortfall of the worst performing scenarios, known as the Conditional Value at Risk (CVaR):

$$Maximize \quad z = (1-\beta)\cdot Profit + \beta \cdot CVaR(\alpha) \tag{38}$$

This makes it possible to improve the expected outcome of the worst performing scenarios within the α-quantile. In theory, increasing the weight β, or the significance level α, of CVaR in the objective function causes a decrease in losses occurred in the worst scenarios, but also a decrease in the overall profit. The actual decrease in overall profit can be observed in Figure 36 and the changes in daily profit for the forecasted and realized electricity prices can be observed in Figure 39.

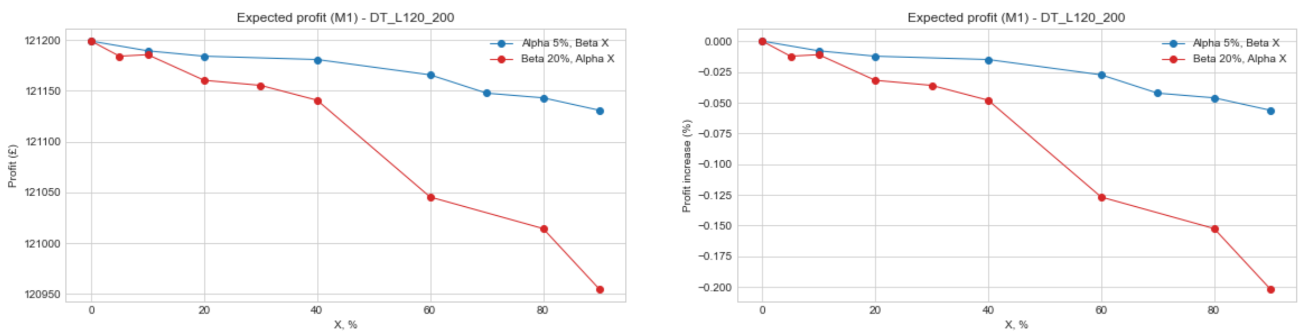


Figure 36: An overview of the relation between weights of β and α and the forecasted profits obtained from Model 1. In the graph to the left, the absolute values are shown, while the right graph shows the change in percentages.

In Figure 36, it is observed that increasing the significance level or the weight of the CVaR part in the objective function will result in minimal decline in forecasted profit. This aligns with the theoretical background of CVaR, in which the ability to avoid the highest loss scenarios is obtained at the cost of having generally lower profits. In spite of this, when evaluating the CVaR-model with real prices as well as Intra-Day revisions of Model 2, the actual profit obtained experiences an increase.

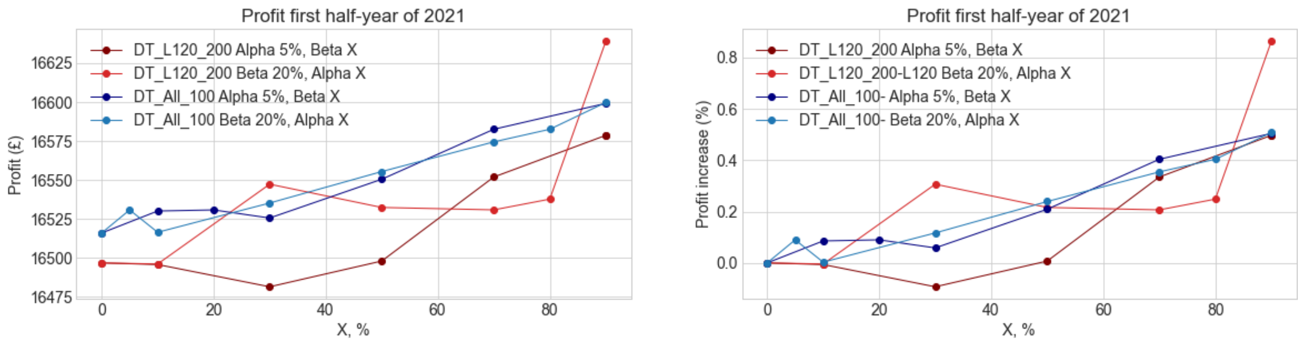


Figure 37: CVaR sensitivity for the first-half of year 2021. The profit obtained from using both models when implementing CVaR in Model 1. The significance level, α, is set to 5% when varying the weights, β, while β is set to 20% when varying α.

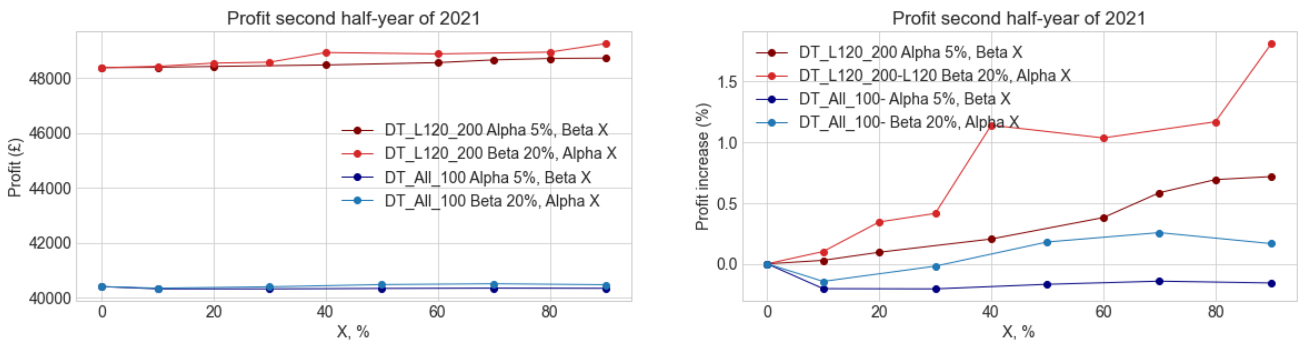


Figure 38: CVaR sensitivity for the second-half of year 2021. The profit obtained from using both model when implementing CVaR in M1. The significance level, α, is set to 5% when varying the weights, β, while β is set to 20% when varying α.

The increase in realized profit observed in Figure 37 and Figure 38 can firstly be explained by the fact that the forecasted and actual electricity prices differ, as shown in Figure 19 and Figure 23, meaning a more robust scheduling strategy based on one set of prices will not have the same effect on another set of prices. Secondly, the model set-up has a second deterministic model, Model 2, which revises the Intra-Day schedule, and since these revisions cannot be accounted for by the CVaR model, the obtained results are unable to solely reflect the effect of CVaR.

To get an understanding of the changes happening when evaluating the bidding strategy, using both forecasted and real prices, Figure 39 was created. Here the daily profits of running Model 1 and Model 2 with and without CVaR are shown for both forecasted and real prices, with the differences introduced by including CVaR highlighted in the bottom two graphs.

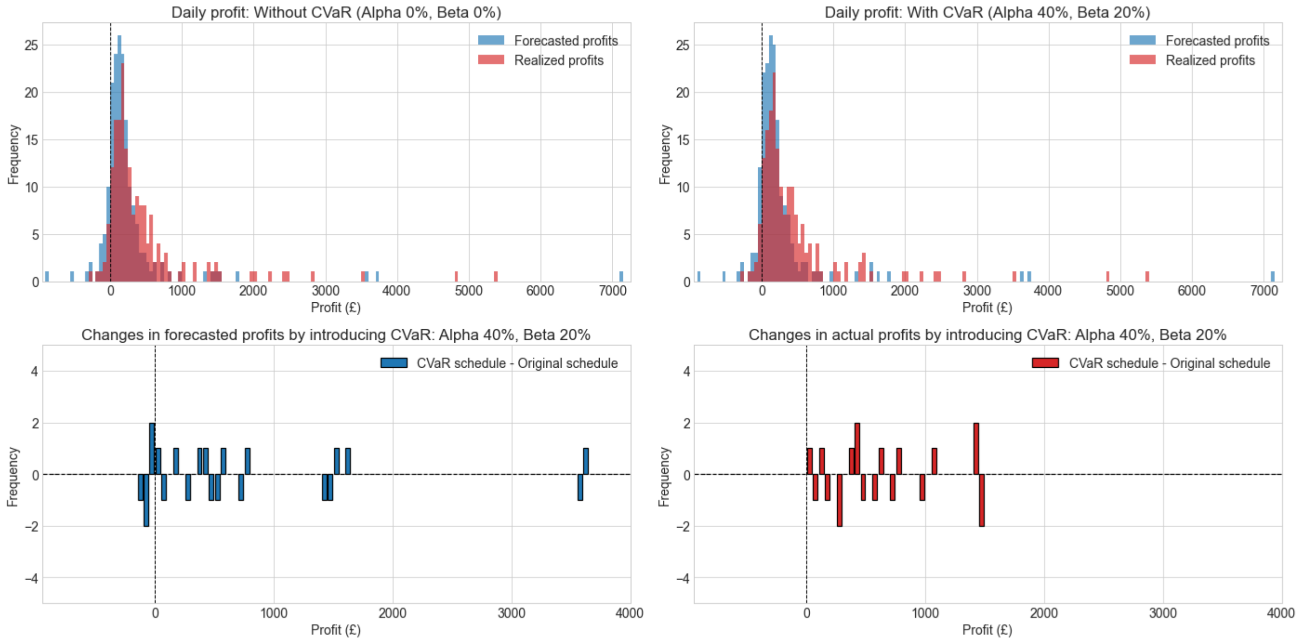


Figure 39: Daily profits based on forecast DT_L120_200. The two figures at the top are the daily profits with and without CVaR for the forecasted and actual prices using Model 1 and 2. The bottom graphs are the frequency difference in daily profits observed between the schedule determined without CVaR subtracted from the schedule determined with CVaR

Focusing on the bottom two graphs in Figure 39, the change in battery scheduling caused by CVaR leads to very few changes in the observed daily profits. However, trends are still visible. The bottom left graph, which is based on changes in forecasted profits shows how 3 negative profit days are removed with CVaR, which is in accordance with theory. However, it also shows a few days with higher profits, likely due to the Model 2 revisions which are outside the control of the CVaR. Looking at the bottom right graph, it is hard to distinguish a clear pattern and the change of scheduling strategy does not exhibit the robustness towards negative profit days.

Thus, it seems difficult to achieve the desired security from applying CVaR, all while the underlying change of strategy indicates possible improvements to the overall profit, which could be subject for further investigation, but from a robustness point of view, other measures will have to be considered.

Discussion

In this section, the obtained results and findings throughout the project will be discussed. Furthermore, the underlying assumptions and simplifications impacting these will be addressed.

Considering the obtained profits in Figure 31, the final profit utilization in the validation year is between 30 % and 34 %. Observing Figure 27 and Figure 30, the unsampled and sampled forecasts exhibit a profit utilization between 30 % and 43 %. Since the obtained profits in both test sets as well as the validation set are similar and within the same range, it alludes towards a stability in obtainable profits with an inclination to be closer to the 30 % rather than the 40 %.

An important consideration to make is that the profit utilization is dependent on the underlying assumptions made. In this paper, two major assumptions are the battery operating as price-taker and that a singular averaged price for each settlement period is sufficient as compared to price spreads. As a small battery has negligible influence on the electricity prices in the market, a price-taker assumption is reasonable. However, different approaches exist such as submitting optimal bidding curves for the battery in the Day-Ahead and Intra-Day market, in which conditions on price can be enforced. While not explored in the Two-Stage Stochastic Model, the price spike algorithm in section "Exploring Price-Spikes" was implemented as a simple bidding curve with a strategy of fully charging or discharging when encountering certain price levels. The profits obtained from this, shown in Figure 13, are generally lower than the ones obtained from forecasting. However, it is able to achieve a profit at much fewer trades and also from purely operating within the Intra-Day market. If one was to introduce transaction fees, as visualized in Figure 34, few impactful trades could prove superior to numerous trades depending on the transaction scheme. In this case, the Two-Stage Stochastic Model set-up could benefit from allowing conditional bidding.

Considering the assumption of average prices without price spreads, an investigation was performed to estimate the sensitivity of profits if price spreads were present. This yielded a span of obtainable profits, which the model would be guaranteed to stay within. Furthermore, the lower bound of the span also reflects the effect of a transaction fee, see Figure 34. From these, it is observed that the simple Last Weeks forecast still generates profits even when trading with 10% spreads working against it, which as mentioned is comparable to a transaction fee of 10% of the traded value. This shows a resilience towards introduction of both price spreads and fees, which can be further strengthened by updating the model logic as it is currently unable to optimize based on price spreads.

Looking into the results obtained from implementing CVaR, a very slight effect is observed when applying forecasted prices. Applying actual prices nearly nullifies the intended effect, as elaborated on in section "CVaR Sensitivity". The time horizon of the optimizations might play a role here, with the CVaR protecting against the worst scenarios on a daily basis, while the more urgent risk to avoid is poor scenarios in the long term. Incurring daily low profits or losses on some occasions is less of a concern than failing to ensure decent performance in the long run. Protecting against this would require a more elaborate method, which could create coherence between the individual daily optimizations.

Conclusion

The main outcome of this study is a framework around the process of operating a large-scale battery in the UK Day-Ahead and Intra-Day electricity markets.

This framework consists of multiple parts including the gathering and processing of data, generation of price forecasts, optimization of charge and discharge scheduling and lastly the submission of contracts. Based on various market data and other descriptive variables, a wide selection of Decision Tree and Random Forest models managed to attain profits mainly in the range of 30-40 % of the total profit obtainable with perfect foresight. This was made possible by firstly supplying the price forecasts to a two-stage stochastic model, which optimizes the planning of the Day-Ahead contracts and predicts the Intra-Day contracts. Secondly, a deterministic model then revises the Intra-Day contracts on a continuous basis as more price information is revealed and the combination of these two models decide the optimal scheduling of the battery.

The study has provided a foundation of tools and processes, which argue that operating a battery in the chosen electricity markets is a commercially viable endeavour. Multiple improvements have been suggested and the impact of simplifications and assumptions made in the study have been discussed, and with those in mind it is believed that battery storage technologies are competitive players in the market.

Future Research

Future work well suited for this subject, would first and foremost include analyses of the model with real spreads on the prices and removing the price-taker assumption, as this opens up a whole new range of possibilities. While increasing the complexity of both the forecasting and optimization, interesting strategies and new aspects of the optimization would most likely emerge, further pushing the utility obtained from having energy readily available for immediate delivery.

Furthermore, the forecasting performed in this paper can be greatly expanded upon, given the wide range of options available. Improvements to the current models can be made through inclusion of more explanatory features or by combining prediction models, as proposed by Marinakis et al. in [8], where a classification Decision Tree is used to label price spikes, which are then used as inputs in the following regression trees.

Another interesting subject to analyse is the impact of increasing the model freedom in the Intra-Day market between 00:00-11:00. During the previous day, the final state of charge for the current day was determined in Model 1, which is used as a fixed endpoint for charge in Model 2 when performing the Intra-Day revisions. This is done in order to ensure the Day-Ahead decisions at 11:00 know the state of charge at the start of the next day. However, it would be possible to delay the introduction of this constraint until after the Day-Ahead decisions have been made. This would require a change in the model setup since information would then be exchanged between the models, rather than only Model 1 imposing constraints on Model 2. In theory, the solution obtained should be either just as good or better, but depending on the quality of the forecasted prices it is not a given.

As stated by Kazemi et al. [11], participating in multiple markets increases the profit obtained. An interesting analysis would then be to increase the amount of markets participated in, as only the Day-Ahead and Intra-Day market were taken into consideration in this paper.

Looking into alternative robustness strategies could prove beneficial, as the CVaR approach does not greatly influence the scheduling. Implementing other measures which operate across a wider time horizon than one day might prove to be more impactful.

Lastly, as the unsampled price forecast "RF_L120_20" in the validation section in Figure 31 has the highest profit utilization and in general does well, it is conjectured that a higher profit utilization could have been obtained if the forecast was sampled.

Appendix

Two-Stage Stochastic Model Formulation - Verifying Units

Veryifying whether the units in the model are correct. In order to provide a better view, ∀ for each constraints has been removed, so the comparison between the constraint and its units is easier observed.

$$\underset{p_t^{DA,dis},\, p_t^{DA,ch},\, p_{t,\omega}^{ID,dis},\, p_{t,\omega}^{ID,ch}}{\text{Maximize}} \quad \overbrace{\sum_{t=1}^{T}\left(\lambda_t^{DA}\cdot\left(p_t^{DA,dis}-p_t^{DA,ch}\right)\cdot\frac{1}{2}\right)}^{\text{Day-Ahead trading}}+\overbrace{\sum_{\omega=1}^{\Omega}\sum_{t=1}^{T}\pi_\omega\left(\lambda_{t,\omega}^{ID,\uparrow}\cdot p_{t,\omega}^{ID,dis}-\lambda_{t,\omega}^{ID,\downarrow}\cdot p_{t,\omega}^{ID,ch}\right)\cdot\frac{1}{2}}^{\text{Intra-day trading}} \tag{39}$$

$$\overbrace{\sum_{t=1}^{T}\left(\frac{£}{MWh}\cdot(MW-MW)\cdot\frac{MWh}{MW}\right)}^{\text{Day-Ahead trading}}+\overbrace{\sum_{\omega=1}^{\Omega}\sum_{t=1}^{T}\%\left(\frac{£}{MWh}\cdot MW-\frac{£}{MWh}\cdot MW\right)\cdot\frac{MWh}{MW}}^{\text{Intra-day trading}} \tag{40}$$

subject to

$$soc_{t,\omega}=SOC_{initial}+\eta_{battery}\cdot Q_{t,\omega}^{ch}-\frac{1}{\eta_{battery}}\cdot Q_{t,\omega}^{dis} \qquad \%=\%+0.92\cdot\%-\frac{1}{0.92}\cdot\%$$

$$soc_{t,\omega}=soc_{t-1,\omega}+\eta_{battery}\cdot Q_{t,s}^{ch}-\frac{1}{\eta_{battery}}\cdot Q_{t,s}^{dis} \qquad \%=\%+0.92\cdot\%-\frac{1}{0.92}\cdot\%$$

$$\frac{p_t^{DA,ch}-p_t^{DA,dis}+p_{t,\omega}^{ID,ch}-p_{t,\omega}^{ID,dis}}{B_{Cap}}\cdot\frac{1}{2}=Q_{t,\omega}^{ch}-Q_{t,\omega}^{dis} \qquad \frac{MW-MW+MW-MW}{MWh}\cdot\frac{MWh}{MW}=\%-\%$$

$$\sum_{t=1+48\cdot(d-1)}^{d\cdot 48}Q_{t,\omega}^{ch}+Q_{t,\omega}^{dis}\le lim_{cycle} \qquad \%+\%\le 4$$

$$p_t^{DA,ch}\le P_{\text{C-rate}}\cdot z_t^{DA,ch} \qquad MW\le MW$$

$$p_t^{DA,dis}\le P_{\text{C-rate}}\cdot z_t^{DA,dis} \qquad MW\le MW$$

$$p_{t,\omega}^{ID,ch}\le P_{\text{C-rate}}\cdot z_{t,\omega}^{ID,ch} \qquad MW\le MW$$

$$p_{t,\omega}^{ID,dis}\le P_{\text{C-rate}}\cdot z_{t,\omega}^{ID,dis} \qquad MW\le MW$$

$$p_{t,\omega}^{ID,ch}\le P_{\text{C-rate}}-p_t^{DA,ch} \qquad MW\le MW$$

$$p_{t,\omega}^{ID,dis}\le P_{\text{C-rate}}-p_t^{DA,dis} \qquad MW\le MW$$

$$z_t^{DA,ch}+z_t^{DA,dis}\le 1$$

$$z_{t,\omega}^{ID,ch}+z_{t,\omega}^{ID,dis}\le 1$$

$$soc_{t,\omega}\le\overline{SOC}_t \qquad \%\le\%$$

$$soc_{t,\omega}\ge\underline{SOC}_t \qquad \%\ge\%$$

$$p_{2\cdot t-1}^{DA,ch}=p_{2\cdot t}^{DA,ch} \qquad MW=MW$$

$$p_{2\cdot t-1}^{DA,dis}=p_{2\cdot t}^{DA,dis} \qquad MW=MW$$

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