

# Optimal Policy Without Rational Expectations: A Sufficient Statistic Solution

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- Lack of generality is a problem: no consensus on how expectations are formed (beyond FIRE fails), precisely how they affect the real economy, etc.



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  - **Know how decisions (equilibrium conditions) are directly distorted by non-rational expectations**

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  - But the **belief distortion** is still a sufficient statistic for the optimal policy!
- Work through simple examples for both cases

# General Framework

- General model:

$$B_{X1}\mathbb{E}_t^b[X_{t+1}] = B_{X0}X_t + B_Y Y_t + B_G G_t \quad (1)$$

- $\mathbb{E}_t^b[\cdot]$ : behavioral expectation of type  $b$
- $X_t$ : endogenous variables
- $Y_t$ : exogenous variables
- $G_t$ : policy variables
- **A behavioral expectations equilibrium:**
  1.  $X_t$ ,  $Y_t$ , and  $G_t$  satisfy the equilibrium condition (1)
  2.  $Y_t$ ,  $X_t$  and  $G_t$  are stationary, linear in the history of shocks  $\{\omega_{t-j}\}_{j=0}^{\infty}$
  3.  $G_t$  satisfies a policy rule
- For now: assume FIRE equilibrium  $X_t^*$  is welfare-maximizing, unique

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- See Adams (2023) for technical details



# Belief Distortions

- $\mathbb{E}_t[\cdot]$  (with no  $b$  specified) denotes the *rational expectation*
- Define the **belief distortion** as

$$\mathbb{D}_t^b[X_{t+1}] \equiv \mathbb{E}_t^b[X_{t+1}] - \mathbb{E}_t[X_{t+1}]$$

- In a model, it is specific to the type  $b$  of behavioral expectations
- In the data, requires measuring agents' expectations  $\mathbb{E}_t^b[X_{t+1}]$ , and estimating the rational expectation  $\mathbb{E}_t[X_{t+1}]$

# What Must Optimal Policy Do?

## Lemma

*If there is a time series of policy instruments  $G_t$  such that the non-rational equilibrium is consistent with the policy-less FIRE equilibrium, then  $G_t$  satisfies*

$$B_{X1} \mathbb{D}_t^b[X_{t+1}] = B_G G_t$$

## Proof Outline:

- In the FIRE equilibrium with  $G_t = 0$ , endogenous vector  $X_t^*$  satisfies:

$$B_{X1} \mathbb{E}_t[X_{t+1}^*] = B_{X0} X_t^* + B_Y Y_t$$

- Subtract from the non-rational model to get:

$$B_{X1} \mathbb{E}_t^b[X_{t+1}] - B_{X1} \mathbb{E}_t[X_{t+1}^*] = B_{X0}(X_t - X_t^*) + B_G G_t$$

- Impose  $X_t = X_t^*$ , and rearrange.

# Sentiment Spanning: Definition

- What policy instruments are enough to recover FIRE?
- Some notation:
  - $B_{C1}$  is submatrix of  $B_{X1}$  corresponding to control variables (there is no belief distortion about pre-determined state variables)
  - $P_G \equiv B_G(B'_G B_G)^{-1} B'_G$  is projection onto column space of  $B_G$ .

## Condition (Sentiment Spanning)

*The macroeconomic model defined in (1) is said to satisfy Sentiment Spanning if*

$$(I - P_G) B_{C1} = 0$$

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- Policymaker does not need to know the whole model to evaluate SS! Needs to know:
  - How expectations affect decisions ( $B_{C1}$ )
  - How policy instruments distort economy ( $B_G$ )

## Theorem

*If a model satisfies Sentiment Spanning, then the policy rule*

$$G_t^\dagger = (B_G' B_G)^{-1} B_G' B_{C1} \mathbb{D}_t^b[X_{t+1}^C] \quad (2)$$

*recovers the FIRE equilibrium.*

- The belief distortion  $\mathbb{D}_t^b[X_{t+1}^C]$  is a sufficient statistic!
- Why does Sentiment Spanning matter? Invert the Lemma  $B_{X1} \mathbb{D}_t^b[X_{t+1}] = B_G G_t$

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- Optimal policy: tax capital when agents are overly optimistic about future returns

## Example 1: Decentralized Equilibrium Conditions

- Policymakers have light **information requirements**:

Euler Equation:  $\tau_t = \sigma c_t + \mathbb{E}_t^b[-\sigma c_{t+1} + \bar{R}r_{t+1}]$

Labor Supply:  $w_t = \sigma c_t + \eta n_t$

Production Function:  $y_t = a_t + \alpha k_{t-1} + (1 - \alpha)n_t$

Capital Demand:  $r_t = y_t - k_{t-1}$

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- Optimal policy:  $\tau_t^\dagger = \mathbb{D}_t^b[-\sigma c_{t+1} + \bar{R}r_{t+1}]$

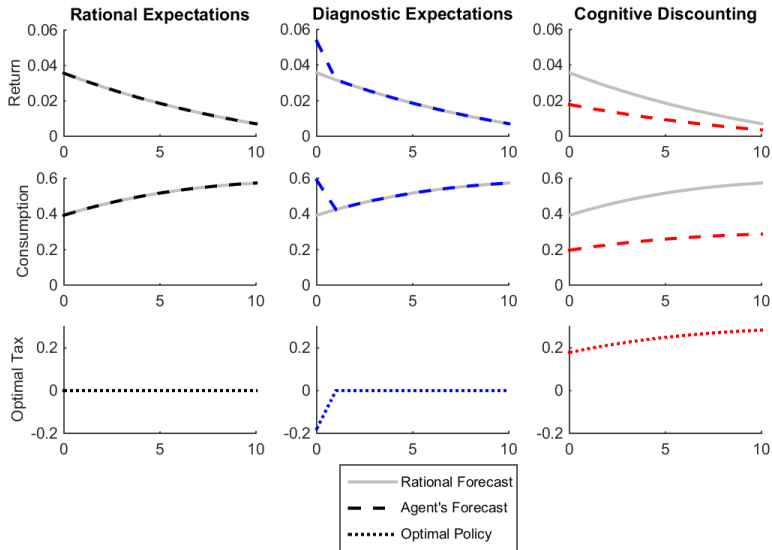
## Example 1: Types of Behavioral Expectations

Rational Expectations:  $\mathbb{E}_t^{RE}[x_{t+1}] = \mathbb{E}_t[x_{t+1}]$

Diagnostic Expectations:  $\mathbb{E}_t^{DE}[x_{t+1}] = (1 + \theta^{DE})\mathbb{E}_t[x_{t+1}] - \theta^{DE}\mathbb{E}_{t-1}[x_{t+1}]$

Cognitive Discounting:  $\mathbb{E}_t^{CD}[x_{t+1}] = \theta^{CD}\mathbb{E}_t[x_{t+1}]$

# Example 1: Response of Expectations to a Productivity Shock



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  - ... but with only monetary, FIRE cannot be recovered
- Both cases: raise interest rates when agents misperceive the economy to be running hot

## Example 2: Optimal Monetary and Fiscal Policy in BNK

New Keynesian Phillips Curve:  $\psi f_t = \kappa y_t - \pi_t - z_t^{PC} + \beta \mathbb{E}_t^b[\pi_{t+1}]$

Euler Equation:  $i_t = -\sigma y_t - z_t^{EE} + \mathbb{E}_t^b[\sigma y_{t+1} + \pi_{t+1}]$

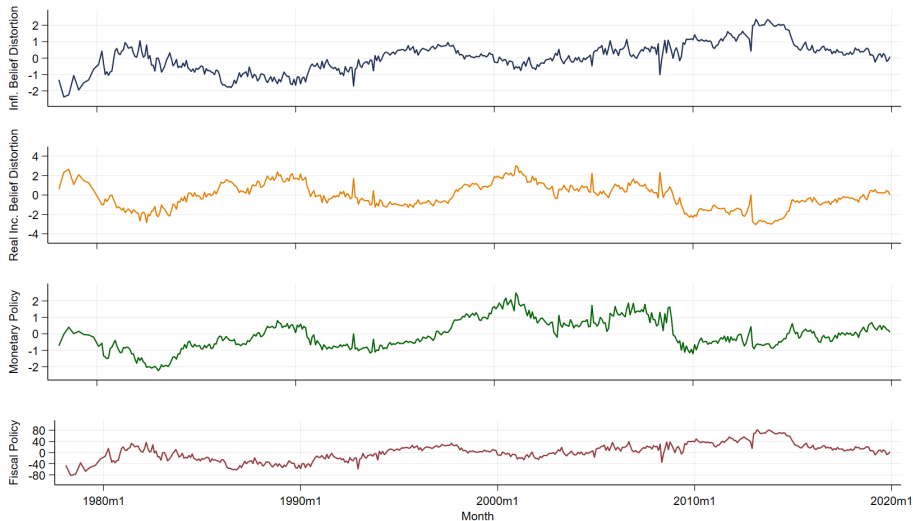
*Expectation components* of optimal policy are:

$$\hat{f}_t^\dagger = \frac{\beta}{\psi} \mathbb{D}_t^b[\pi_{t+1}] \qquad \hat{i}_t^\dagger = \mathbb{D}_t^b[\sigma y_{t+1} + \pi_{t+1}]$$

Implementation:

- Measure agents' expectations  $\mathbb{E}_t^b[\cdot]$ ,
- Estimate the rational expectation, e.g. with a VAR (Adams and Barrett 2024)

## Example 2: Estimated Belief Distortions and Implied Policies



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  3. What if expectation formation is endogenous?
- ... intuition goes through, although implementation may change

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- Policymakers now need to know the whole *economic* model
- ... but they still do not need to know how expectations are formed!

- First-best equilibrium:  $X_t^*$ , with FIRE-optimal policy  $G_t^*$
- Policymakers with no information commit to a policy rule (Rotemberg and Woodford 1997)
- Minimize quadratic loss for some  $W$ :

$$\min \mathbb{E} [(X_t - X_t^*)' W (X_t - X_t^*)]$$



# Optimal Policy Without Sentiment Spanning

## Theorem

*The constrained-optimal policy rule is*

$$G_t^\dagger = \underbrace{B_G^+ P_W B_{C1} \mathbb{D}_t^b[X_{t+1}^C]}_{\text{expectation component}} + G_t^{RE}$$

► *matrix details*

- Economic component  $G_t^{RE}$  follows the *FIRE policy rule*

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- Expectation component  $B_G^+ P_W B_{C1} \mathbb{D}_t^b[X_{t+1}^C]$  has the *same information requirements* as the original case with sentiment spanning

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- $\implies$  if you already have the FIRE optimal policy, adding the response to non-rational expectations requires no additional modeling assumptions, only measuring the belief distortion!

### Example 3: Optimal Monetary Policy Alone in BNK

New Keynesian Phillips Curve:  $0 = \kappa y_t - \pi_t - z_t^{PC} + \beta \mathbb{E}_t^k[\pi_{t+1}]$

Euler Equation:  $i_t = -\sigma y_t - z_t^{EE} + \mathbb{E}_t^b[\sigma y_{t+1} + \pi_{t+1}]$

*Expectation component* of optimal policy is:

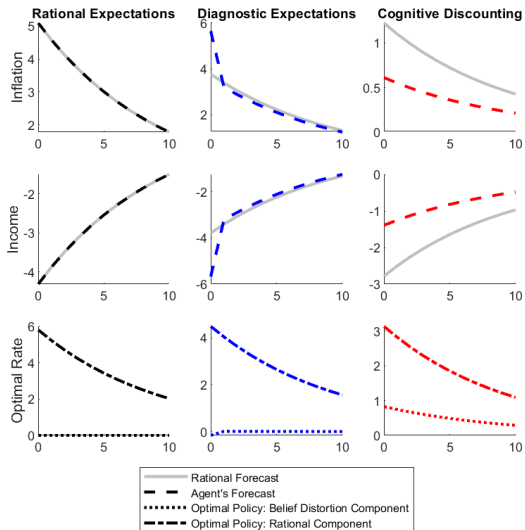
$$\hat{i}_t^\dagger - i_t^{RE} = \sigma \mathbb{D}_t^b[y_{t+1}] + \left(1 - \beta \frac{b_\pi \kappa \sigma}{b_\pi \kappa^2 + b_y}\right) \mathbb{D}_t^b[\pi_{t+1}]$$

which cannot recover FIRE without an additional tool.

If  $\left(1 - \beta \frac{b_\pi \kappa \sigma}{b_\pi \kappa^2 + b_y}\right) > 0$ , raise rates when agents misperceive economy is “running hot”.

► Speedy Conclusion

## Example 3: Response of Expectations to a Cost-Push Shock



## What if Belief Distortions are Measured with Error?

- Policymaker's observation  $D_t$  of the belief distortion is

$$D_t = \xi \mathbb{D}_t^b[X_{t+1}] + v_t$$

with i.i.d. measurement error  $v_t \sim N(0, \sigma_v^2)$

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- Form the *policymaker's* nowcast of the belief distortion  $\mathbb{D}_t^b[X_{t+1}^C]$  conditional on info. set  $\Omega_t$  ( $D_t$  and other observables):

$$\hat{D}_t = \mathbb{E}[\mathbb{D}_t^b[X_{t+1}^C] | \Omega_t]$$

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$$\hat{D}_t = \mathbb{E}[\mathbb{D}_t^b[X_{t+1}^C] | \Omega_t]$$

- **Theorem** *The constrained-optimal policy rule is*

$$G_t^\dagger = B_G^+ P_W B_{C1} \hat{D}_t + G_t^{RE}$$

... same as the solution without Sentiment Spanning, except using  $\hat{D}_t$ !



## What if Expectation Formation is Endogenous? (Framework)

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- Return to simple case: sentiment spanning holds, FIRE is optimal

## What if Expectation Formation is Endogenous? (Results)

- Lemma 1 still true! If  $G_t$  recovers FIRE, it *must* satisfy

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  - $G_t^\dagger$  may not even exist! (example in sec. 6.2.2)



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# Optimal Policy Without Sentiment Spanning

## Theorem

*The constrained-optimal policy rule is*

$$G_t^\dagger = B_G^+ P_W B_{C1} \mathbb{D}_t^b[X_{t+1}^C] + B_G^+ P_W B_{X1} \mathbb{E}_t[X_{t+1} - X_{t+1}^*] + G_t^*$$

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  2. Economic component: optimal policy for FIRE model

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2. What if belief distortions are not perfectly observed?
  - Just use the best *nowcast*
3. What if expectation formation is endogenous?
  - Optimal rule unchanged; lose existence/uniqueness from the main theorem

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