Incomplete Information and Irreversible Investment*

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Abstract

How does incomplete information affect irreversible investment at the firm and aggregate levels? We study this question in a stylized continuous-time model of heterogeneous firms facing incomplete information and irreversible investment. We analytically characterize how incomplete information distorts firms' decision rules and stationary distribution when investment is irreversible. The two frictions interact in rich and substantial ways. At the firm level, noisier information shrinks a firm's inaction region and reduces the elasticity of investment to productivity. In the aggregate, it increases steady-state capital, exacerbates capital misallocation, and mitigates the impact of productivity shocks on aggregate investment. Finally, we test and quantify these predictions using Japanese administrative data that match firms' forecasts with their balance sheets, incomes, and expenditures. At the firm level, firms underreact to news as if they face information frictions; those with more extreme underreaction are less inactive, as predicted.

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1 Introduction

Two firm-level frictions are known to substantially distort firm dynamics: investment and information frictions. These frictions are crucial for understanding investment behavior at both the micro and macro levels, which is heterogeneous in the cross-section and much less elastic to aggregate shocks than frictionless models imply. And yet, both frictions have mainly been studied independently, so the natural questions are: Do investment and information frictions interact in economically important ways? If so, does it matter for the aggregate economy?

We approach this question in two steps, beginning with theory. We develop a tractable continuous-time model of heterogeneous firms that face two frictions that are widely recognized to distort firms' decisions. The first friction is that *capital investment is irreversible.* This implies that firms will sometimes choose not to invest, generating "inaction regions" where capital only depreciates. This non-convex adjustment cost has empirically relevant consequences for firm and macroeconomic dynamics (Baley and Blanco, 2022) because many forms of capital are highly illiquid or firm-specific.[1] The second friction is that *firms do not observe their productivity directly.* Instead, they must rely on noisy signals and only learn the truth with a lag, for example, after sales are realized. This delay between investment decisions and feedback on productivity is common in practice and is especially pronounced in capital-intensive industries. These two frictions have been extensively studied separately, but we show they interact in surprising and economically important ways. This is possible because of the tractability of the continuous-time model; we begin by proving analytically the micro implications, and then turn to the macro implications.

First, we find that the information friction reduces a firm's inaction region. All else equal, uncertainty about a firm's current productivity makes the firm more willing to invest. At first glance, this might be surprising given the well-documented relationship between investment and uncertainty over future productivity. When *future* productivity is more uncertain, firms are less willing to invest because uncertainty increases the option value of delaying irreversible investment, also known as the wait-and-see effect (Leahy and Whited, 1996; Hassler, 1996; Bloom, 2009; Bloom et al., 2018). However, when *current* productivity is uncertain, firms prefer to invest more because the marginal value of capital is convex in log productivity, so Jensen's inequality makes firms act as if they are risk-loving over productivity: the Oi-Hartman-Abel effect.[2] This

[1] Ramey and Shapiro (2001) find that upon sale, aerospace plants recover less than 30% of the replacement cost of capital; selling imposes additional wind-down costs and takes years to implement. Kermani and Ma (2023) document that the liquidation value of capital for non-financial firms is only 35% of the net book value.

[2] Strictly speaking, this is only thematically related to the original Oi-Hartman-Abel effects (Oi, 1961; Hartman, 1972; Abel, 1983) because in our simple model, firms' only input is capital which it uses to produce with diminishing returns. However, the term is now commonly used to describe cases in which the marginal value of capital is convex in a random variable. See for example the decomposition in Senga (2025).

effect also holds in traditional models of future uncertainty, but is always dominated by the wait-and-see effect.

Second, the information friction reduces the short-run elasticity of investment to productivity shocks. This standard effect of incomplete information tends to attenuate the effects of shocks by reducing how informative a shock is for forecasting future fundamentals. When firms receive a productivity shock, their noisy signal increases one-for-one, but firms do not know whether this change was due to productivity or noise, so their productivity nowcast increases less than one-for-one. This attenuation effect is potentially valuable for describing aggregate investment behavior. Koby and Wolf (2020) argue that firm-level investment is relatively inelastic to aggregate shocks, which is necessary to match the empirical aggregate investment dynamics over the business cycle or responses to monetary policy shocks (Fang, 2020; Winberry, 2021). These heterogeneous firm models require high fixed capital adjustment costs to deliver this low elasticity but generate unrealistic non-smooth investment distribution at the firm level; this is a challenge for the literature. In sum, a model needs to generate a realistically low investment elasticity to aggregate shocks and realistic investment inaction simultaneously. But non-convex adjustment costs cannot deliver this result; House (2014) argues that firms must face other frictions in order to have a realistic investment elasticity. We show that information frictions can help deliver the required low investment elasticity to aggregate shocks, while simultaneously delivering realistic inaction rates and spike rates.

Turning to the macro implications, we find that the frictions also have surprising effects on the essential aggregate moments: capital accumulation, misallocation, and volatility. First, greater information frictions increase the aggregate capital stock for the same reason that they decrease inaction: uncertainty about current productivity increases firms' willingness to invest. Second, investment irreversibility introduces capital misallocation, and we show analytically that the information friction increases this misallocation. This effect occurs as firms make investment mistakes when their *expected* productivity is higher or lower than the actual level. Third, we find that the information friction actually reduces the riskiness of capital, as measured by sales volatility. Even though firms make more mistakes, their attenuated response to productivity shocks reduces riskiness on net.

Finally, we also find that introducing information frictions affects the conventional relationship between volatility and investment inaction. In full information investment models, the option value effect implies that firms facing higher productivity volatility will be more inactive *ceteris paribus*. However, more severe information frictions dampen this relationship. If information frictions are severe, then raising productivity volatility has only small effects on investment inaction. This is because increasing productivity volatility raises the variance of nowcast errors, leading to more severe information frictions and shrinking the inaction region. In total, this information effect counteracts the classical option value effect, attenuating the relationship between volatility and investment inaction.

In the second step, we test our central theoretical and quantitative predictions using Japanese administrative data. We construct a merged firm-level dataset combining the Business Outlook Survey (BOS) and the Financial Statements Statistics of Corporations (FSS), conducted by the Ministry of Finance and the Cabinet Office of Japan. The BOS provides both realized and forecasted sales for the past and upcoming semi-years, allowing us to estimate industry-level information frictions from the predictability of sales forecast errors. Additionally, firms report investment and investment plans in the BOS, while the FSS provides detailed data on capital stock, employment, and costs, enabling the construction of investment- and productivity-related variables.

We begin our empirical analysis by measuring the severity of firms' information frictions. First, firms are grouped into industries using the industry codes provided by the Ministry of Finance. For each industry, we estimate friction severity by regressing sales forecast errors on past productivity growth. Under full information, this coefficient would be zero, as past productivity would be immediately incorporated into current expectations. In contrast, our incomplete information model predicts a positive coefficient, reflecting delayed and partial incorporation of past information. Consistent with this prediction, the data show that the "underreaction coefficient" is positive across all industries, although it exhibits substantial heterogeneity.

Next, we examine how information frictions affect firms' investment behavior. Consistent with our theory, firms facing more severe information frictions are less likely to remain inactive, conditional on firm-level productivity and size. We test this prediction by regressing a binary investment inaction variable on the industry-level underreaction coefficients. The estimated effect is negative, confirming the model's prediction. A one standard deviation increase in the coefficient reduces the average inaction rate at the industry level by over five percentage points, a quantitatively large effect.[3] Next, we test the predicted attenuated response of investment to productivity shocks by including an interaction between industry-level underreaction and changes in firm-level productivity. This regression allows us to include appropriate fixed effects in order to accounts for all industry-level time-varying and firm-level time-invariant factors affecting investment. Consistent with the model, firms in industries with more severe information frictions are less responsive to productivity shocks.

Across all tests, we directly compare our regression results with analogous outcomes generated from simulated data using our model. Despite its stylized nature, the model provides clear predictions for the signs of key coefficients. Our firm-level empirical results are broadly consis-

[3]The average inaction rate is approximately 36% at the semiannual frequency.

tent with the model's analytical and quantitative predictions.

Literature: First, our theoretical work is closely related to a small but growing literature on incomplete information in continuous time models featuring inaction. In this existing literature, inaction is due to fixed costs; to the best of our knowledge, we are the first to study the interaction between incomplete information and irreversibility as the source of inaction. Verona (2014) studies inattentive firms who pay fixed costs to update information (as in Reis, 2006), which leads to periodic large investment spikes. Alvarez et al. (2011), Alvarez et al. (2016), and Stevens (2020) study price-setting by firms facing high fixed costs to both changing prices and observing fundamentals. Baley and Blanco (2019) consider a model of menu costs where firms observe noisy signals of their productivity; they predict that firms with higher uncertainty change prices more often and learn more quickly. We also join broader literatures studying how investment dynamics are affected by irreversibility and information frictions.[4] Of these, one closely related paper is Senga (2025), which studies how uncertainty over future productivity affects investment behavior (without investment frictions) in a quantitative business cycle model.

Second, our theoretical work joins a broad literature on irreversibility and investment inaction. Early work (Pindyck, 1991; Bertola and Caballero, 1994; Abel and Eberly, 1996; Abel et al., 1996) established the option value of irreversibility, and built the standard theoretical framework for analyzing it. More recent work has developed our understanding of the macroeconomic implications, particularly over the business cycle (Lanteri, 2018; Baley and Blanco, 2024). And new empirical findings have better quantified this friction, suggesting that its contribution to misallocation and productivity are substantial (Caunedo and Keller, 2021; Kermani and Ma, 2023).

Third, our empirical work connects to a burgeoning literature studying predictable errors in firms' forecasts. Most closely related is a set of papers studying whether managers overreact or underreact to news when forecasting their own firms' outcomes. The evidence is mixed. Ma et al. (2020) use Italian sales forecasts from a representative survey run by the Bank of Italy, and find evidence of underreaction, similar to our estimates in the Japanese data. In contrast, Barrero (2022) finds that in the Survey of Business Uncertainty, US CEOs and CFOs overreact to news. Bordalo et al. (2021) estimate overreaction in managers' earnings forecasts in the IBES guidance database. Many factors may contribute to these differences; forecast horizons may matter, the

[4] Some canonical and more recent examples studying irreversibility include Pindyck (1991), Bertola and Caballero (1994), Abel and Eberly (1996), Veracierto (2002), Ottonello (2017), and Baley and Blanco (2022). Stokey (2008) provides a textbook treatment. Papers studying business cycle models of capital investment with information frictions include Townsend (1983), Angeletos and Pavan (2004), Graham and Wright (2010), Angeletos et al. (2018), and Atolia and Chahrour (2020), among many more. In particular, Adams (2023) studies a model with investment frictions that does not induce inaction; instead, firms face investment adjustment costs in the style of Christiano et al. (2005). Instead of firms and investment, some recent papers study how information frictions affect savings heterogeneous agent models, including Adams and Rojas (2024) and Broer et al. (2021).

incentives may be different on earnings calls versus official central bank surveys, and there may be cultural differences between US, Italian, and Japanese managers.[5]

Finally, our paper is linked to the literature that utilizes firm-level survey data to document how micro, industry, and macroeconomic shocks affect firms' expectations formation. Andrade et al. (2022) show that industry-level inflation predicts forecast errors about firms' own prices in a survey of French manufacturers. Massenot and Pettinicchi (2018) and Born et al. (2022) use a survey of German manufacturing firms to document how business conditions and news, respectively, predict forecast errors. Other related papers include Bachmann et al. (2013), Bachmann and Elstner (2015), Bachmann et al. (2021), Born et al. (2023a). Born et al. (2023b) survey additional work in this field, while Candia et al. (2023) survey the larger literature studying biases in firms' expectations of the macroeconomy.

Layout. The remainder of the paper is organized as follows. Sections 2 lay out the stylized firm's investment model, and Section 3 characterizes the aggregate economy. Section 4 validates the model's micro implications using Japanese firm-level data. Section 5 provides a quantitative analysis of the model's macro implications. Finally, Section 6 concludes.

2 A Stylized Investment Model

This section describes the economic environment, investment decisions, and information friction. We derive the value function and optimal decisions and demonstrate analytically how investment decisions depend on different parameters, including those controlling the information friction.

2.1 Firm's Problem

Environment There is a unit measure of atomistic competitive firms. Firms produce using capital K, modified by productivity A. Their production function is $F(A, K) = A^{1-\alpha}K^{\alpha}$ where $\alpha \in (0, 1)$. Investment I is irreversible. If firms invest, they do so at cost ψ. Accordingly, their instantaneous profit is $\pi = A^{1-\alpha}K^{\alpha} - \psi I$. Lowercase letters denote logs of variables, e.g., $a = \ln A$.

[5] This specific literature is also a part of a broader one studying firms' forecast errors of micro and aggregate variables more generally. Seminal work done by Coibion et al. (2018), Tanaka et al. (2020), and Candia et al. (2024) present stylized facts concerning firm-level expectations. Several studies also use data on Japanese firm-level expectations. Using a dataset of multinational firms, Chen et al. (2023b) document heterogeneity in the information frictions firms face that varies by firm size and age. Second, Chen et al. (2023a) document that the degree of information rigidity firms face is higher for aggregate inflation than for firm-specific outcomes. Finally, Charoenwong et al. (2024) show how capital budgeting can alleviate distortions originating from investment frictions and thus improve productivity. Other papers that use Japanese firm-level expectations include Charoenwong et al. (2020), Chen et al. (2020), and Chen et al. (2022).

Log productivity follows a random walk $da = \sigma_a dW^a$ where W^a is a Wiener process. The law of motion for capital is $dK = I - \delta K dt$ where δ is the depreciation rate.

Optimal firm behavior for this type of problem is characterized by an inaction region: above some level of capital (that depends on other state variables), firms choose not to invest. Firms discount the future at a constant rate r, so inside the inaction region, a firm's Hamilton-Jacobi-Bellman (HJB) equation is

$$rV(K, A) = A^{1-\alpha}K^{\alpha} - \delta K V_K(K, A) + \frac{\sigma_a^2 A^2}{2} V_{AA}(K, A) \tag{1}$$

This is the *full information* HJB. Of course, firms will not have full information when forecasting. However, in the inaction region, the firm's true value still follows this PDE. The wrinkle to this model is that when firms *do* make an action, they will not know A exactly.

Information Structure Firms do not know their productivity A exactly. Instead, they receive a noisy signal s of log productivity:

$$s = a + n \tag{2}$$

where the noise n follows a random walk:

$$dn = \sigma_n dW^n \tag{3}$$

where the Wiener process W^n is independent of W^a.

Additionally, we assume that after τ time, the productivity level is revealed to the firm, i.e., at time t, firms learn the productivity that they had at time $t - \tau$. This structure represents the notion that decision-makers do not know exactly how productive their firm is at any moment but learn ex-post after an accounting period is completed.

Expectation Formation To characterize how firms form expectations, it is useful to temporarily introduce time subscripts, which we have suppressed so far. Productivity growth over τ time is distributed $(a_t - a_{t-\tau}) \sim N(0, \tau\sigma_a^2)$, while $(s_t - s_{t-\tau})$ is distributed $N(0, \tau(\sigma_a^2 + \sigma_n^2))$ due to the independent Wiener processes W^a and W^n.

Lemma 1. *For a firm with information set $\Omega_t = \{a_{j-\tau}, s_j\}_{j \le t}$, productivity is conditionally distributed*

$$a_t | \Omega_t \sim N\left(a_{t-\tau} + \gamma\left(s_t - s_{t-\tau}\right), \nu\right)$$

where

$$\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2} \qquad \nu \equiv \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2 + \sigma_n^2}$$

therefore the firm's expected productivity $\hat{a} \equiv \mathbb{E}[a|\Omega]$ *and nowcast error* u *follow the diffusions*

$$d\hat{a} = \sigma_a dW^{\hat{a}} \qquad du = \sigma_u dW^u$$

where

$$dW_t^{\hat{a}} = (1-\gamma)dW_{t-\tau}^a + \gamma dW_t^a + \gamma \frac{\sigma_n}{\sigma_a}(dW_t^n - dW_{t-\tau}^n)$$

$$dW_t^u = (1-\gamma)\frac{\sigma_a}{\sigma_u}(dW_t^a - dW_{t-\tau}^a) + \gamma \frac{\sigma_n}{\sigma_u}(dW_t^n - dW_{t-\tau}^n)$$

$$\sigma_u^2 = 2\frac{\sigma_n^2 \sigma_a^2}{\sigma_a^2 + \sigma_n^2}$$

Proof: Appendix B.1

Lemma 1 describes how firms form expectations of their productivity under incomplete information. Two parameters are worth explaining further. First, the signal coefficient $\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}$ measures how elastic firms' expectations are to the noisy signals. Second, the nowcast error variance $\nu \equiv \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ depends on the information delay τ, the noise volatility σ_n, and productivity volatility σ_a. With longer delays and larger noise, firms make larger nowcast errors.

2.2 Solving the Firm's Problem

The full information HJB equation (1) is homogeneous of degree 1 in (K, A) (Stokey, 2008, Ch. 11) so it is possible to rewrite in terms of a single variable $X \equiv \frac{K}{A}$, which we call "normalized capital":

$$rV(X) = X^\alpha - \delta X V'(X) + \frac{\sigma_a^2 X^2}{2} V''(X) \tag{4}$$

Moreover, it is convenient to express the HJB in terms of log normalized capital $x = k - a$:

$$rv(x) = e^{\alpha x} - \mu v'(x) + \frac{\sigma_a^2}{2} v''(x) \tag{5}$$

where $\mu \equiv \delta + \frac{\sigma_a^2}{2}$. This conversion to writing the HJB as a function of $x = \log(X)$ follows from $v(x) = V(X)$, $v'(x) = V'(X)X$, and $v''(x) = V''(X)X^2 + V'(X)X$.

These are the usual full information HJB equations. How does incomplete information affect the firm's problem if it does not change the HJB equation? It changes the *boundary conditions*, which are the equations characterizing optimal action. The HJB has many solutions; the correct solution is determined by the appropriate boundary conditions.

Information is incomplete, so x is unknown to firms when making investment decisions. The usual optimality conditions of $V(X)$ cannot be applied in this case. Instead, firms have uncertainty; their expected value of $V(X)$ is given by

$$\mathbb{E}[V(X)|\Omega] = \mathbb{E}[V(X)|\hat{X}] \equiv \hat{V}(\hat{X})$$

Because conditional expectations are normally distributed with constant variance (Lemma 1), *expected log normalized capital* $\hat{x} \equiv \mathbb{E}[x|\Omega]$ is a summary statistic for firms' expectations, as is $\hat{X} \equiv e^{\hat{x}}$, which represents the firm's MLE nowcast of X. The firm's goal is thus to maximize $\mathbb{E}[V(X)|\hat{X}]$, which we write as the *expected value function* $\hat{V}(\hat{X})$.[6]

Optimal investment behavior is a threshold strategy, as in the case of full information. Except now, a firm invests only if its expected log normalized capital $\hat{x}$ is less than some boundary $\hat{b}$. Solving the firm's problem involves finding the optimal choice of $\hat{B} \equiv e^{\hat{b}}$. Lemma 2 reports the boundary conditions associated with the optimum. They are analogous to the full information case.

Lemma 2. *Under incomplete information, the boundary conditions consist of two value-matching conditions:*

$$\hat{V}'(\hat{B}) = \psi \qquad\qquad \lim_{\hat{X}\to\infty} \hat{V}'(\hat{X}) = 0$$

and two super contact conditions:

$$\hat{V}''(\hat{B}) = 0 \qquad\qquad \lim_{\hat{X}\to\infty} \hat{V}''(\hat{X}) = 0$$

Proof: Appendix B.2

The proof is standard and follows closely the arguments in Dumas (1991). We include the proof in order to show that we can apply the usual full information optimality conditions to the expected value function $\hat{V}(\hat{X})$.

Lemma 3 below characterizes the solution to the firm's problem. The critical value $\hat{b}$ depends on several parameters: the variance of nowcast errors ν, the returns to scale α, the cost of investment ψ, as well as ϱ and m defined as:

$$\varrho \equiv \frac{\mu - \sqrt{\mu^2 + 2\sigma_a^2 r}}{\sigma_a^2} \qquad\qquad m \equiv \frac{1}{r + \mu\alpha - \frac{\sigma_a^2}{2}\alpha^2}$$

[6] Appendix A describes the general solution to the HJB equation.

Lemma 3. *The critical value of expected normalized capital is*

$$\hat{b} = b^{FI} + \frac{\alpha^2 \nu}{2(1-\alpha)}$$

where b^{FI} is the full information critical value given by:

$$b^{FI} = \frac{1}{(1-\alpha)} \log\left(\frac{m\alpha(\alpha-\varrho)}{\psi(1-\varrho)}\right)$$

Proof: Appendix B.3

Lemma 3 demonstrates how the information friction affects the firm's optimal investment decisions. Conveniently, most of these terms affect the critical values in the same way as in the full information model. The proposition shows that the difference between full and incomplete information critical values depends only on the variance of nowcast errors ν, adjusted by the returns to scale α.[7]

2.3 Micro Implications of Incomplete Information

We can now analytically characterize how the investment and information frictions interact to change firms' investment behavior. First, incomplete information reduces the inaction region. Second, incomplete information attenuates the investment response to productivity shocks.

2.3.1 Investment Inaction

The information friction has a clear effect on the firm's optimal behavior: stronger information frictions *shift* the inaction region to the *right*. This is because the optimal boundaries increase as the variance of the nowcast errors ν gets larger. Proposition 1 formalizes this result.

Proposition 1. *The inaction region bound is increasing in both the noisy signal variance σ_n^2 and the revelation delay τ.*

Proof. The nowcast error variance $\nu = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ is increasing in both τ and σ_n^2. Lemma 3 implies that $\hat{b}$ is increasing in ν. □

Why does the information friction shift the inaction region rightwards? The most intuitive answer is because of an Oi-Hartman-Abel effect: the marginal value of capital is convex in log

[7] This conclusion is not unique to fully irreversible investment. In Appendix D, we extend the model to allow for partial irreversibility, and come to the same conclusion: incomplete information increases inaction region boundaries relative to the full information case.

productivity a. This is true whether information is incomplete or not. The effect of productivity is asymmetric; for the marginal value of capital, the upside of an improvement to a outweighs the downside of a symmetric decrease. Thus a mean-preserving spread in a increases the expected marginal value of capital. And a mean-preserving spread in a is equivalent to a mean-preserving spread in x. This makes firms risk-loving over normalized capital: if they do not know the true value, the expected marginal value exceeds the certainty equivalent, i.e.

$$\mathbb{E}[V(\exp(x))|\Omega] > V(\exp(\mathbb{E}[x|\Omega]))$$

The expected instantaneous return to an additional unit of marginal capital is larger when firms are uncertain about the value of their X. This uncertainty raises a firm's incentive to invest, thus they are willing to do so at higher levels of expected X, raising the lower bound on their inaction region.

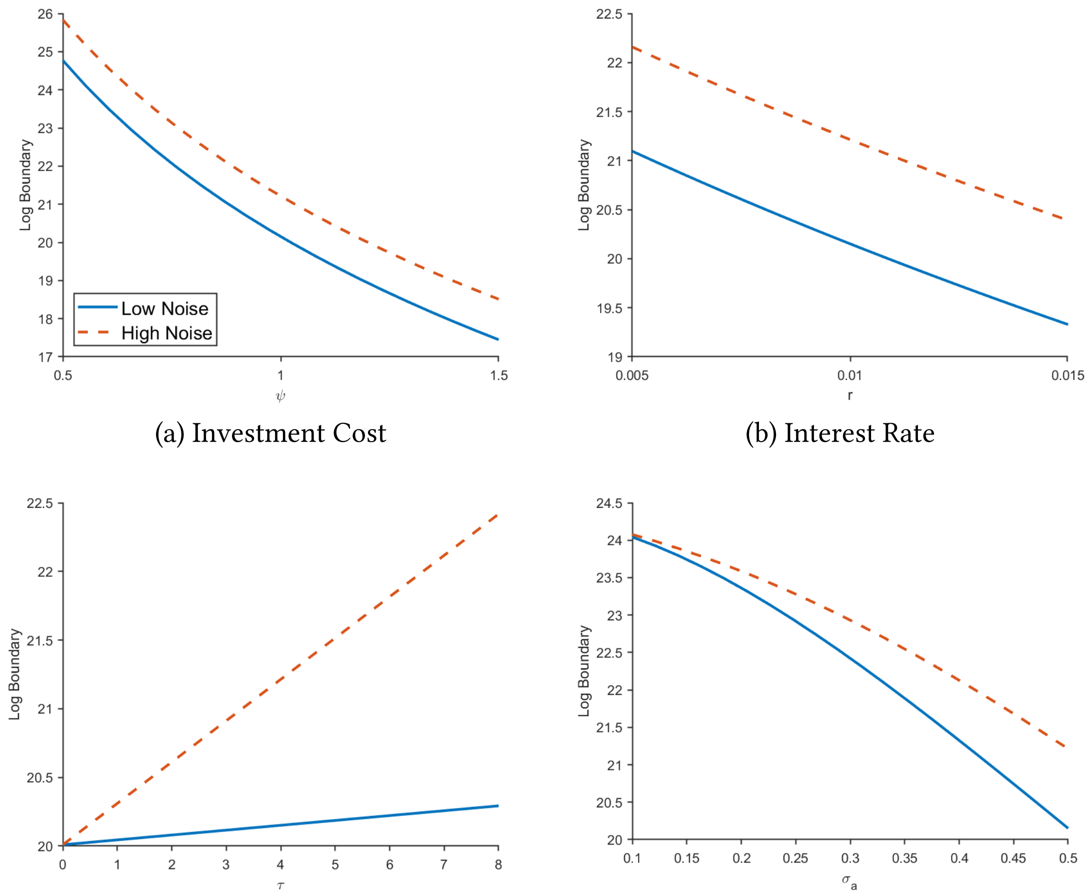
Figure 1 illustrates how different parameters affect firms' decisions. In all cases, the solid blue line plots how $\hat{b}$ depends on the parameter of interest when firms' signals are not particularly noisy ($\sigma_n = 0.25\sigma_a$). The dashed red line plots the $\hat{b}$ sensitivity for a noisier calibration ($\sigma_n = \sigma_a$). In all cases, the dashed red line is above the solid blue line: when firms have *worse* information, they are *more* willing to invest, as implied by Proposition 1.

The first two panels show that investment responds in the usual way to standard parameters. Panel 1a shows that $\hat{b}$ is decreasing in the investment cost ψ: when capital is more expensive, firms are less willing to invest and do so only when their normalized capital is lower. Similarly, 1b shows that $\hat{b}$ is decreasing in the interest rate r: when firms discount the future at a higher rate, they are less willing to invest. The information friction raises the inaction boundary for all values of ψ and r, but does not fundamentally change how inaction depends on these parameters.

The next two panels show how signal noise interacts with parameters. Panel 1c demonstrates the other way that the information friction increases willingness to invest. $\hat{b}$ increases in τ, the amount of time before productivity is revealed to the firm. Larger values of τ make firms less certain about their current productivity level, increasing their nowcast error variance (Lemma 1). When τ is larger, the information friction is worse. Like σ_n, exacerbating the information friction increases the incentive to invest and raises $\hat{b}$. Moreover, when the delay is larger, the signal noise volatility has even stronger effects, because firms accumulate disproportionately more noise in their signals. This is why the curves diverge in Panel 1c.

The information friction has an unusual interaction with productivity volatility. To show this, Panel 1d plots how $\hat{b}$ depends on the productivity standard deviation σ_a. The standard result is that higher productivity volatility should make firms less willing to invest because it increases

Figure 1: How the Boundary $\hat{b}$ Depends on Various Parameters



(a) Investment Cost

(b) Interest Rate

(c) Revelation Delay

(d) Productivity Volatility

Notes: Each plot displays how the optimal inaction boundary $\hat{b}$ depends on a parameter value. The information processes are parameterized in two ways: the blue line is less noisy with $\sigma_n = 0.25\sigma_a$, while the red dashed line is more noisy, with $\sigma_n = \sigma_a$. In both cases $\sigma_a = 0.5$ and $\tau = 4$ so that information is revealed after 4 quarters. The remaining parameters are set as in Table 2a.

the option value of waiting (Leahy and Whited, 1996; Hassler, 1996). This is the case in our model, too: raising volatility reduces the boundary, i.e., making firms less willing to invest. However, volatility also plays a role in information friction. When volatility increases, firms' nowcasts are less accurate (ν decreases in σ_a). This information effect *attenuates* the standard option-value effect of volatility on inaction. And the attenuating information effect is stronger when signals are noisier. Panel 1d makes this clear: if signals are relatively precise (solid blue line), then volatility sharply reduces the boundary, but if signals are relatively noisy (dashed red line), then volatility has less effect on inaction.

This result demonstrates a new channel by which "uncertainty" affects capital investment. The traditional *wait-and-see channel* is that uncertainty over future productivity reduces the incentive to invest by increasing the option value of delaying new capital. In recent work, uncertainty has been documented to be a major driver of business cycles, and this investment wait-and-see channel is considered a central mechanism (Bloom, 2009; Bloom et al., 2018). The new *information channel* is that uncertainty over current productivity increases the incentive to invest. Moreover, these two channels interact as shown by Panel 1d. When information is incomplete, higher productivity volatility still increases the inaction region of investment. However, information frictions change this relationship. If information frictions are severe, then raising productivity volatility has only small effects on investment inaction. This is because increasing productivity volatility raises the variance of nowcast errors, shrinking the inaction region. This information effect counteracts the classical option value effect, attenuating the relationship between volatility and investment inaction.

2.3.2 Investment Sensitivity

The information friction attenuates the short-run impact of shocks by reducing the pass-through from productivity shocks to firms' productivity expectations. With full information, productivity shocks affect forecasts one-for-one because productivity follows a random walk. This is not the case when information is incomplete.

Lemma 4 shows that the short-run attenuation depends on $\gamma = \frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}$, as defined in Lemma 1. When the noise variance σ_n^2 is large, then signals are noisy, and productivity shocks $da = \sigma_a dW^a$ have little effect on firms' immediate nowcasts. But as $\sigma_n^2 \to 0$, firms more accurately nowcast their productivity and approach the full information case.

Lemma 4.

$$\frac{d}{da_{t-h}}\mathbb{E}[a_t|\Omega_t] = \begin{cases} \gamma & 0 \leq h < \tau \\ 1 & h \geq \tau \end{cases}$$

Proof: Appendix B.4

Lemma 4 above implies that the immediate pass-through of a productivity shock to the firm's expected log normalized capital ($\hat{x} = k - \hat{a}$) is $-\gamma$. Thus, any quantity that depends on $\hat{x}$ will be similarly attenuated in the short run, e.g., the average time until leaving the inaction region, the expected investment over a time period, and so forth. However, because the actual productivity level is eventually revealed to firms, Lemma 4 also says that the long-run pass-through from productivity to nowcasts and forecasts is one-for-one.

What does this result imply for tests of incomplete information? In Section 4.2, we estimate a standard *underreaction coefficient* (Kohlhas and Walther, 2021), which measures how future forecast errors responds to current information. Proposition 2 derives how this coefficient – the effect of productivity shock $da_t = \sigma_a dW_t^a$ on forecast error $(a_{t+h} - \mathbb{E}[a_{t+h}|\Omega_t])$ – depends on the information friction parameters in the model.

Proposition 2. *A firm's underreaction coefficient is*

$$\frac{d(a_{t+h} - \mathbb{E}[a_{t+h}|\Omega_t])}{da_t} = 1 - \gamma$$

Proof. Log productivity a_t follows a random walk, which implies

$$\frac{d(a_{t+h} - \mathbb{E}[a_{t+h}|\Omega_t])}{da_t} = \frac{da_{t+h}}{da_t} - \frac{d\mathbb{E}[a_t|\Omega_t]}{da_t} = 1 - \frac{d\mathbb{E}[a_t|\Omega_t]}{da_t}$$

with $\frac{d\mathbb{E}[a_t|\Omega_t]}{da_t} = \gamma$ by Lemma 4. □

The coefficient γ controls how sensitive firms' decisions are to productivity shocks. But it is even more informative than this: the next section shows that γ also controls the sensitivity of the macroeconomic response to aggregate shocks.

3 The Aggregate Economy

To characterize the aggregate economy, we first must make several assumptions about the distribution of firms. We derive the partial differential equations governing the dynamics of firm distributions and solve explicitly for the stationary distribution. Then, we characterize how the information friction affects macroeconomic aggregates.

3.1 Firm Entry and Exit

We assume that a measure η of firms enter the economy at every moment. Entering firms do not know their productivity. They are as uncertain as existing firms, i.e., their Bayesian prior is that their log productivity is normally distributed with variance ν.

Across entering firms, expected log productivity $\hat{a}$ is distributed by $\hat{a}_{enter} \sim N(0, \varsigma)$. Thus, the entering distribution of actual log productivity is $a_{enter} \sim N(0, \varsigma + \nu)$. Firms' log expected normalized capital $\hat{x}$ enters at the critical value $\hat{b}$: $\hat{x}_{enter} = \hat{b}$. This is a natural assumption for the entering distribution of capital: firms are born with some unknown productivity level and acquire capital until they are no longer willing to invest. Firm exit is random, independent of other variables. We assume that when firms exit, their value is returned to shareholders, so the exit risk does not change the firm's HJB equation. For the probability distribution to integrate into one, firms must exit at the same rate they enter: η.

3.2 The Stationary Distribution of Normalized Capital

Lemma 1 implies that expected normalized capital $\hat{x} = k - \hat{a}$ follows the diffusion

$$\hat{x} = -\delta dt - \sigma_a dW^{\hat{a}}$$

Firms exit at rate η, so the Kolmogorov Forward Equation (KFE) for the distribution $h(\hat{x}, t)$ of expected normalized capital in the inaction region is

$$\partial_t h(\hat{x}, t) = \delta \partial_{\hat{x}} h(\hat{x}, t) + D \partial_{\hat{x}}^2 h(\hat{x}, t) - \eta h(\hat{x}, t) \tag{6}$$

where $D \equiv \frac{\sigma_a^2}{2}$. PDFs written without time arguments denote stationary distributions. The KFE for the stationary distribution of expected normalized capital is thus

$$0 = \delta h'(\hat{x}) + D h''(\hat{x}) - \eta h(\hat{x}) \tag{7}$$

The boundary condition is that $h(\hat{x})$ must integrate to one on the interval $[\hat{b}, \infty)$. Lemma 5 below gives the solution.

Lemma 5. *The stationary distribution of expected normalized capital $h(\hat{x})$ for $\hat{x} \geq \hat{b}$ is*

$$h(\hat{x}) = \rho e^{-\rho(\hat{x} - \hat{b})}, \quad \textit{where} \quad \rho \equiv \frac{\delta}{\sigma_a^2} + \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$$

and the stationary distribution of log-normalized capital is

$$f_x(x) = h(x) e^{\frac{\nu \rho^2}{2}} \Phi\left(\frac{x - (\hat{b} + \nu\rho)}{\sqrt{\nu}}\right)$$

where $\Phi(\cdot)$ is the standard normal CDF.

Proof: Appendix B.5

Lemma 5 shows that the information friction does not affect the shape of the stationary distribution $h(\hat{x})$. The root ρ is determined from purely economic fundamentals: depreciation δ, productivity volatility σ_a, and exit risk η; ν never appears. The information friction only shifts the stationary distribution left or right by determining the lower bound $\hat{b}$.

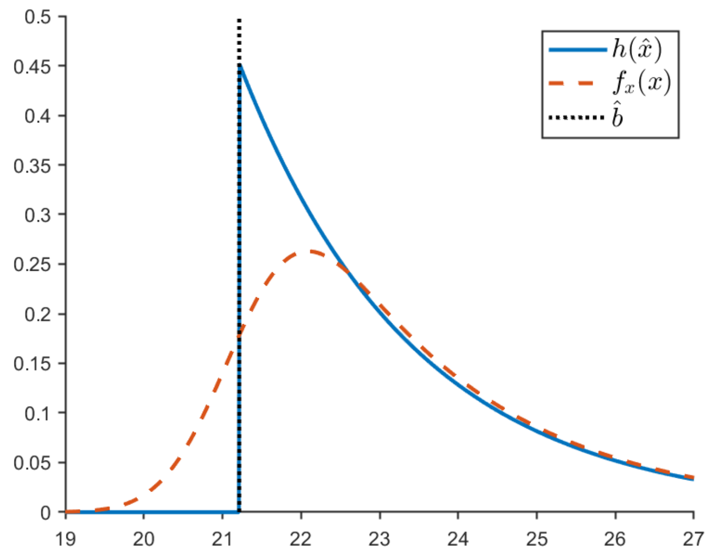
The joint distribution $f_{\hat{x},u}(\hat{x}, u)$ of expected normalized capital $\hat{x}$ and productivity nowcast errors $u = a - \hat{a}$ follow straightaway from Lemma 5, because nowcast errors must be independent of nowcasts. Thus, their joint distribution is the product of their marginal distributions:

$$f_{\hat{x},u}(\hat{x}, u) = h(\hat{x})\phi(\frac{u}{\sqrt{\nu}}) \tag{8}$$

where $\phi(\cdot)$ is the standard normal pdf. From the joint distribution, it is straightforward to calculate the marginal distribution distribution $f_x(x)$ of actual normalized capital $x = \hat{x} - u$.

We illustrate the differences between the two distributions. Figure 2 plots how the stationary distribution of *realized* normalized capital $f_x(x)$ compares to the distribution $h(\hat{x})$ of *expected* normalized capital. $h(\hat{x})$ is monotonic and has a discrete lower bound at the barrier $\hat{x}$. In contrast, normalized capital $x = \hat{x} - u$ is smoothed out because it subtracts an independent Gaussian. Instead of featuring a lower bound, it has an infinite domain. And Lemma 5 implies that the larger the nowcast error variance ν is, the smoother the distribution.

Figure 2: Stationary Distributions for Expected and Realized Normalized Capital



Notes: The plot compares the marginal distributions of expected log normalized capital $\hat{x}$ and actual log normalized capital x. The vertical line denotes the boundary of the inaction region $\hat{b}$. The parameters match the "High Noise" case from Figure 1: $\sigma_a = 0.5$, $\sigma_n = \sigma_a$, and $\tau = 4$. Figure 12 shows an analog of this plot, simulated from a calibrated model.

What is the firm size distribution? We have characterized the distribution of normalized capital; to answer this question, we need to decompose normalized capital into its capital and productivity components. To do this, we need to solve the KFE in multiple dimensions. In the inaction region, the KFE for the distribution of capital and expected productivity $g(k, \hat{a}, t)$ is

$$\partial_t g(k, \hat{a}, t) = \delta \partial_k g(k, \hat{a}, t) + D \partial^2_{\hat{a}} g(k, \hat{a}, t) - \eta g(k, \hat{a}, t) \tag{9}$$

With the stationary distribution satisfying the partial differential equation

$$0 = \delta \partial_k g(k, \hat{a}) + D \partial^2_{\hat{a}} g(k, \hat{a}) - \eta g(k, \hat{a}) \tag{10}$$

This distribution is more challenging to characterize analytically than the univariate normalized capital distribution, because it requires solving a PDE with an unusual boundary condition. Appendix C does so. After calibrating the model in Section 5, Figure 6 plots the joint distribution.

3.3 Macro Implications of Incomplete Information

Having a closed-form solution for the steady-state distribution of expected normalized capital is valuable because it allows us to characterize in closed form how various macroeconomic aggregates depend on the parameters of the information friction. In this section, we show that information frictions increase both capital misallocation and average normalized capital, and attenuate the short-run effects of aggregate shocks.

3.3.1 Capital Accumulation

Aggregate normalized capital $\bar{X}$ in the economy is given by

$$\bar{X} \equiv \int_{-\infty}^{\infty} e^x f_x(x) dx$$

where $f_x(x)$ is the stationary distribution of log normalized capital as defined in Proposition 5.

We documented in Section 2.2 that the information friction increases firms' willingness to build capital. Noisier information raised the lower bound on firms' inaction region. This effect increases aggregate normalized capital, as Proposition 3 demonstrates.

Proposition 3. *If $\rho > 1$, then steady state normalized capital is finite and increasing in both the noisy signal variance σ_n^2 and the revelation delay τ.*

Proof: Appendix B.6

3.3.2 Capital Misallocation

We measure misallocation as the variance of the log marginal product of capital:

$$\mathrm{Var}\left[\log \frac{\partial F(A,K)}{\partial K}\right] = (1-\alpha)^2 \mathrm{Var}[x]$$

The information friction increases the above capital misallocation in a straightforward manner. Misallocation can be decomposed into two components: dispersion in expected capital $\mathrm{Var}[\hat{x}]$ and dispersion in nowcast errors ν. The former is due to endogenous decisions, while the latter is due to mistakes made by firms that do not know their productivity precisely. Proposition 4 shows that the information friction increases misallocation entirely due to mistakes.

Proposition 4. *Steady-state capital misallocation is increasing in both the variance of the noisy signal σ_n^2 and the revelation delay τ.*

Proof. Normalized capital is decomposed into nowcast errors by $x = \hat{x} - u$. $\hat{x}$ and u are independent, so

$$\mathrm{Var}[x] = \mathrm{Var}[\hat{x}] + \nu$$

where $\nu = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ is the nowcast error variance. Lemma 5 implies that $\mathrm{Var}[\hat{x}]$ is independent of the information friction parameters. Thus misallocation is increasing in ν, which is increasing in σ_n^2 and τ. □

3.3.3 Aggregate Shock Attenuation

A typical effect of information frictions is to attenuate aggregate shocks. This model is no different. Firms are slow to invest after productivity improvements because they do not know the improvement with certainty. This dampens the endogenous capital response to the shock, at least until information is revealed.

In this section, we describe the attenuation result theoretically. The endogenous response of capital is *exactly* attenuated by the nowcasting coefficient $\gamma < 1$ described in Lemma 4. This result holds to first order after a permanent productivity shock. We numerically explore the attenuation further in the quantitative Section 5.

Consider an aggregate productivity shock to an economy in the steady state. This shock permanently raises the log productivity a of all firms by ε at time $t = 0$. Accordingly, this shock lowers log normalized capital x, shifting the distribution $h(x, 0)$ to the left by ε. Any firms that are pushed outside of their inaction region immediately invest up to the boundary. However, Lemma

4 implies that the information friction causes the distribution of expected normalized capital $\hat{x}$ to only shift left by $\gamma\varepsilon$, with $\gamma < 1$. This is how the information friction attenuates the effects of aggregate shocks.

The model's tractability allows us to characterize precisely how the economy-wide impulse response function (IRF) is attenuated. To do so, let $f_x(x, t, \varepsilon)$ denote the dynamic distribution of log normalized capital x given the ε shock at time $t = 0$. Define the IRF of average log normalized capital relative to the steady state by

$$IRF_x(t, \varepsilon) \equiv \int_x x f_x(x, t, \varepsilon) dx - \bar{x}$$

where $\bar{x}$ denotes the steady state averages. Following from $k - a = x$, the IRF for average log capital to the permanent ε shock is

$$IRF_k(t, \varepsilon) = IRF_x(t, \varepsilon) + \varepsilon$$

The IRF is nonlinear in ε, so to make progress analytically we consider the marginal effect of a small productivity shock, following Borovička et al. (2014) and Alvarez and Lippi (2022). Let $\widehat{IRF}_k(t)$ denote the *marginal IRF* of average log capital k, i.e.

$$\widehat{IRF}_k(t) \equiv \frac{\partial}{\partial \varepsilon} IRF_k(t, \varepsilon)\bigg|_{\varepsilon=0}$$

and let $\widehat{IRF}_k^{FI}(t)$ denote the counterfactual marginal IRF under full information.

Proposition 5. *The marginal IRF of average log capital k is attenuated relative to the full information marginal IRF by*

$$\widehat{IRF}_k(t) = \gamma \widehat{IRF}_k^{FI}(t)$$

for time $t < \tau$.

Proof: Appendix B.8

Proposition 5 proves that the information friction attenuates the endogenous response of capital in the short run. Combined with Proposition 2, this result also reveals that firms' underreaction coefficient $1 - \gamma$ is a *sufficient statistic* for the aggregate attenuation. If underreaction is large, as Ma et al. (2020) find in Italian data, then the information friction can severely distort the dynamic effects of aggregate shocks.

4 Validation with Microdata

In this section, we use Japanese firm-level data to test our key theoretical predictions at the micro level. We proceed in three steps. First, we estimate the extent of information incompleteness by measuring the industry-specific underreaction coefficient in the data. Second, we numerically simulate a model economy of firms following the optimal decision-making derived in Section 2. Third, we run the same regressions on both the microdata and the simulated data and compare the results to illustrate the model's micro-level implications, showing that several key coefficients estimated from the firm-level data align with those from the simulated data.

4.1 Japanese Firm-level Data

We utilize the Business Outlook Survey (BOS) and the Financial Statements Statistics of Corporations (FSS), conducted by the Ministry of Finance and the Cabinet Office of Japan, which span approximately 15 years from 2004 to 2018. These two quarterly datasets include all large firms and a representative sample of small and medium-sized firms across both manufacturing and non-manufacturing sectors. Large firms are surveyed every quarter, while medium-sized and small firms are sampled on a rotating basis.[8]

Sample of Firms The sample size of the BOS is about 11,000 with a response rate of more than 75%, and the sample size of the FSS is about 21,000 with a response rate of about 70%. The FSS covers basic financial statement information in the balance sheet and profit and loss account. At the same time, the BOS contains firm-level forecasts of sales and profit at the semiannual frequency, as well as firms' investment and investment plans at the quarterly frequency. Fortunately, both datasets share standard time-invariant firm identifiers for firms with registered capital exceeding 0.1 billion JPY, which is approximately 0.7 million US dollars. As a result, we merge the two datasets and construct a panel dataset that only contains firms with registered capital exceeding 1 billion JPY for the period 2004-2018.

A unique feature of the BOS is that it provides quantitative forecasts of sales and profits, which allows us to compute forecast errors. Reporting of both realized and expected sales and operating profits occurs on a semiannual basis, whereas investment plans are reported quar-

[8]In the BOS, all firms with registered capital above 2 billion JPY are surveyed every quarter. For firms with registered capital between 0.5 billion and 2 billion JPY, 50% are randomly sampled each quarter. For firms with registered capital between 0.1 billion and 0.5 billion JPY, 10% are randomly sampled each quarter. For firms with registered capital below 0.1 billion JPY, 1% are randomly sampled each quarter. The random sample is redrawn at the beginning of each fiscal year, so once a firm is selected, it appears in all four quarters of that fiscal year (if it answers the survey every time). In the FSS, all firms with registered capital above 5 billion JPY are surveyed quarterly, while for firms with registered capital between 1 billion and 5 billion JPY, 50% are randomly sampled each quarter.

terly.[9] Because the forecasting targets (i.e., sales and profits) are semiannual, we define related variables—such as sales, investment, usage of intermediate goods, and productivity—at the semiannual frequency in our analysis. Furthermore, the BOS directly collects firms' investment data, enabling us to measure investment directly rather than inferring it indirectly from capital stock. Because the variables in our regressions are defined at the semiannual frequency, we use firm-level semiannual observations. Table 1a reports the summary statistics for observations in our merged dataset and the original FSS dataset. First, the merged dataset contains about 6,500 firms per quarter, compared with roughly 21,000 in the original FSS. Second, firms in both datasets are, on average, relatively large: average employment exceeds 490 workers, and average sales are above 8.5 billion JPY (about 7 million USD) per quarter.

Table 1: Summary Statistics of Japanese Firm-level Data

(a) Sample Comparison at Quarterly Frequency

Moments	Merged Dataset	Entire Sample (FSS)
Number of obs. (Non-missing sales)	392,158	1,260,836
Average employment	1040.582	491.6123
Average sales (million JPY)	19991.75	8541.767
Average fixed capital stock	59919.34	24842.79

(b) Investment Moments Using Fixed Capital at Both Frequencies

Frequency	Exit Rate	Agg. Inv. Rate	Inv. Rate Mean	Inv. Rate S.D.	Inaction Rate	Spike Rate
Quarterly	2.00%	1.23%	2.27%	6.10%	60.00%	0.90%
Semiannual	3.96%	2.64%	4.00%	8.3%	36.6%	2.45%

(c) Untrimmed Forecast Errors at Semiannual Frequency

Variable	Obs.	Mean	Median	S.D.	Min.	Max.
Log forecast error of sales	119,335	-.0106	-.0005	0.199	-8.472	5.759
Percentage forecast error of sales	119,359	.0198	-.0005	1.556	-1	316

Notes: The sample period covers 2004–2018 (15 years, or 29 semiannual observations). In Sub-table (b), all statistics are based on variables defined at the quarterly frequency. Investment refers to the total of expenditures on equipment, machinery, and land, while capital denotes the stock of fixed capital. An investment spike is defined as an investment rate greater than 20%, and investment inaction as an investment rate of 1% or less. In Sub-table (c), the log forecast error is defined as the log deviation of realized sales in period t from the sales forecast made in period $t-1$. The percentage forecast error is defined as the percentage deviation of realized sales from the forecasted sales reported at the beginning of each semiannual period.

Calculating Variables of Interest We construct firm-level variables of interest in a manner closely aligned with our model. First, we compute the forecast errors using the forecasts reported

[9] For example, in the October survey, a firm reports its realized sales from April to September and its projected sales for the upcoming October–March period. In the same survey, the firm also provides investment plans for October–December and January–March of the following year.

at the beginning of each semiannual period. Second, we calculate the firm-level investment rate at the quarterly level, defined as the ratio of fixed investment to the fixed capital stock, and use it to identify investment inaction and spikes.[10] Similarly, we compute the semiannual investment rate by summing investment expenditures over two consecutive quarters. Third, we aggregate firms into 30 industries based on their reported industry affiliations (originally 47 industries), since some industries contain too few observations in a given quarter. Fourth, we measure labor productivity using revenue per worker as our baseline productivity metric.[11] The dataset provides information on total sales, cost of goods sold (including capital appreciation and labor costs), capital depreciation, and labor costs (wages, salaries, and benefits). Using these components, we infer the purchase of intermediate goods, which serves as an input in our TFP estimation.

Next, we describe how we construct industry-level variables, which are used as independent variables in our regressions. First, we define firm-level capital intensity as the ratio of capital stock to the sum of capital stock, labor costs, and purchases of intermediate goods. We then average these firm-level measures within each industry to create the industry-level capital intensity.[12] Next, we compute the semiannual sales growth rate of each firm.[13] We then compute industry-level sales growth volatility and average sales growth using all firm-semiannual observations within the same industry.

Tables 1b and 1c report summary statistics for our constructed investment-related variables (based on fixed capital) and for our untrimmed forecast errors, respectively. Table 1b shows that firm investment in the merged dataset is highly lumpy: both the average (and aggregate) investment rate and the average investment spike rate are low, while the average inaction rate is substantial. Table 1c indicates that although the median of the two forecast errors is close to zero, their standard deviations are large, reflecting substantial heterogeneity across firms and over time, as well as the potential presence of outliers. To address this, we trim the top and bottom 1% of sales forecast errors in our empirical regressions.

[10]An investment spike is defined as an investment rate greater than 20%, while investment inaction is defined as an investment rate of 1% or less.

[11]We use labor productivity instead of TFP in the baseline analysis for two reasons. First, estimating TFP requires assumptions about the production function. Second, because estimated production-function coefficients carry confidence intervals, the resulting TFP measure would be imprecisely estimated rather than directly observed.

[12]We also constructed an alternative industry-level measure using a weighted average of firm capital intensity. The results are largely unchanged when using this measure.

[13]To account for firm entry and exit, we follow Davis et al. (1998) and use the mid-point growth rate: $sg_{it} = \frac{sales_{i,t} - sales_{i,t-1}}{(sales_{i,t} + sales_{i,t-1})/2}$, where i indexes firms and t refers to time. The denominator is the average sales in semi-years t and $t-1$. Growth rates are bounded between -200% and 200%, with firm entry (exit) corresponding to 200% (-200%) growth. Using a growth rate that ignores entry/exit yields similar empirical results. We do not have sufficient observations to reliably calculate growth volatility at finer industry-region or industry-semiannual levels.

4.2 Information Incompleteness Estimation in the Data

The first step is to quantify how the severity of information frictions influences firms' investment decisions To do so, we estimate the industry-specific underreaction coefficient ξ_s using forecast errors and lagged productivity shocks calculated from our data:

$$e_{it+1} = \xi_s w_{it} + \Gamma z_{it} + \gamma_{st} + \gamma_{rt} + \gamma_{gt} + \epsilon_{it+1} \tag{11}$$

Here, $e_{it+1} = y_{it+1} - \widehat{y}_{it+1}$ denotes the firm's forecast error, calculated as the difference between the logarithm of realized sales (y_{it+1}) at time $t+1$ and the logarithm of forecasted sales ($\widehat{y}_{it+1}$) made at time t. The term $w_{it} = a_{it} - a_{it-1}$ represents the productivity shock, where a_{it} and a_{it-1} are the logarithms of the firm's measured productivity at times t and $t-1$, respectively. Our coefficient of interest is the industry-specific underreaction coefficient, ξ_s, estimated for 30 industries. We include lagged productivity, a_{it-1}, in z_{it} as a firm-level control to remain consistent with the model regressions below. Additionally, we control for industry-time (γ_{st}), region-time (γ_{rt}), and size-time (γ_{gt}) fixed effects.[14]

Figure 3 presents the estimated coefficients and their 95% confidence intervals.[15]. The figure shows that the estimated industry-specific coefficient, ξ_s, is positive and statistically significant for most industries, indicating the presence of information frictions even among large and mature firms.[16] The coefficient varies across industries, from near zero to about 0.05, with manufacturing industries exhibiting higher frictions, likely due to exposure to domestic and international shocks. At the industry level, information frictions are negatively correlated with capital intensity and growth volatility, consistent with the idea that firms in more volatile or capital-intensive environments are more sensitive to frictions and may act to mitigate them.

4.3 Model-Based Simulations

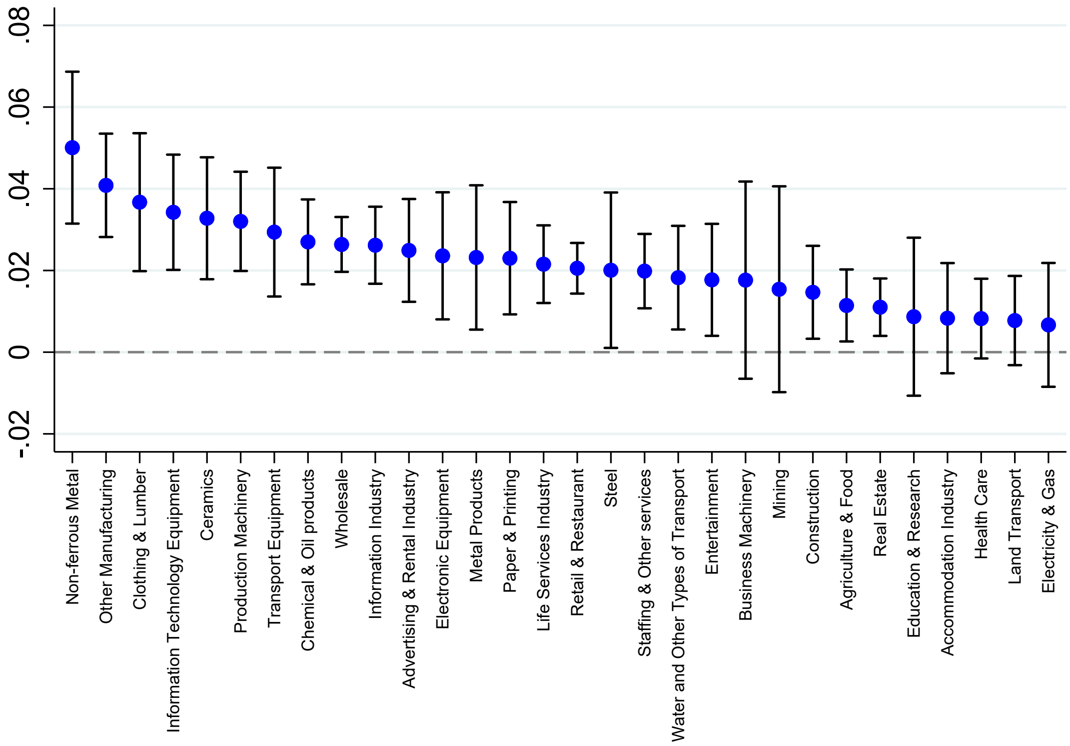
In the second step, to compare the model qualification to the data, we calibrate and simulate the model for 30 different industries, which yields a similar industry-specific underreaction coefficient ξ_s as estimated from equation (11) and presented in Figure 3. Since the model is stylized, we fix most parameters to match corresponding moments directly, but we leave the standard

[14] Regions are defined by the survey agency. In total, there are 53 survey regions, roughly corresponding to Japan's 47 prefectures. For large prefectures, such as Tokyo or Hokkaido, multiple survey regions exist.

[15] We also estimated the industry-level attenuation coefficients while including firm fixed effects; the results are shown in Figure 10 Most estimated coefficients remain positive and are highly correlated with those obtained without firm fixed effects (correlation coefficient: 0.71). However, including firm fixed effects can introduce downward bias in the presence of measurement errors. For this reason, we prefer the estimates obtained without firm fixed effects. For further details, see Section F.1.

[16] See Chen et al. (2023a) for evidence that small and young firms face even higher degrees of information frictions.

Figure 3: Estimated Attenuation Coefficients across Industries: Labor Productivity



Notes: This figure illustrates how the coefficient governs the impulse response of the (log) sales forecast error in period $t + 1$ to realized (log) productivity shocks in period t. Each dot represents an industry estimate, with 95% confidence intervals, across all 30 industries. The top and bottom 1% of observations are trimmed to remove outliers. The data are semiannual, and productivity is measured using labor productivity.

deviation of noise, σ_n^s, to vary across industries to generate the corresponding $\xi_s \in [0, 0.055]$.

Parametrization We calibrate our model quantitatively in Table 2a to roughly match our Japanese firm-level data, which is presented quarterly in Table 1b. We first fix the quarterly interest rate r to be 1% to match an annual rate of 4%, returns to scale $\alpha = 0.85$ as in Winberry (2021), and a one-period revelation delay $\tau = 1$ before past productivity is revealed, consistent with quarterly accounting reports. We normalize the investment cost $\psi = 1$ and then choose the exit rate η of 2% to match the quarterly exit rate and a depreciation rate $\delta = 1.23\%$ to match the aggregate quarterly investment rate of 1.23%. We choose the entry productivity volatility $\varsigma = 0$ for simplicity. We then choose the standard deviation of the productivity process, $\sigma_a = 0.15$, to roughly match the average standard deviation of the investment rate, which is 6.1%.

Simulation We simulate 30 industries with different degrees of information friction, $\{\sigma_n^s\}_{s=0}^{30}$, to illustrate the effects of incomplete information on various moments of investment dynamics. We

Table 2: Parametrization and Investment Moments at Quarterly Frequency

(a) Parametrization of the Stylized Model

Parameter	r	α	τ	ψ	η	ς	δ	σ_a	σ_n^0	σ_n^{30}	$\Delta\sigma_n$
Value	1%	0.85	1	1	2%	0	1.23%	0.15	0.00	$0.75\sigma_a$	$0.025\sigma_a$

(b) Information Incompleteness and Investment Moments

Industry	σ_n	ξ_s	Inv. Rate Mean	Inv. Rate S.D.	Inaction Rate	Spike Rate
Full Information	0.000	0.000	2.37%	6.7%	81.0%	3.9%
Median Noise	$0.375\sigma_a$	0.018	2.29%	6.1%	79.8%	3.3%
Highest Noise	$0.75\sigma_a$	0.055	2.20%	5.53%	77.7%	2.4%

Notes: The investment rate is defined as investment over capital stock. The inaction rate is defined as the frequency at which the investment rate is below 1%. The spike rate is defined as the frequency at which the investment rate exceeds 20%.

include a 31st artificial industry ($s = 0$) that has full information $\{\sigma_n^0 = 0, \xi_0 = 0\}$ for comparison. Each industry has 10,000 firms for 50 quarters. For each industry, we calculate the corresponding ξ_s from the regression equation (11) and make sure our range of $\{\sigma_n^s\}_{s=1}^{30}$ delivers the range of $\{\xi_s\}_{s=1}^{30}$ from $\xi_0 = 0$ to $\xi_{30} = 0.055$.

Our simulation is quarterly. As usual in discrete time, we assume capital is built one period in advance. This allows us to calculate the ξ_s coefficients directly from the model parametrization:

Corollary 1. *In the discrete time simulation, the ξ_s underreaction coefficient for output forecast errors is given by*

$$\xi_s = (1-\alpha)(1-\gamma_s)$$

Proof. The underreaction coefficient is

$$\xi_s = \frac{d(y_{t+1} - \mathbb{E}[y_{t+1}|\Omega_t])}{da_t} = (1-\alpha)\frac{d(a_{t+1} - \mathbb{E}[a_{t+1}|\Omega_t])}{da_t} + \alpha\frac{d(k_{t+1} - \mathbb{E}[k_{t+1}|\Omega_t])}{da_t}$$

because $y_t = (1-\alpha)a_t + \alpha k_t$. In discrete time k_{t+1} is known at time t, so $\frac{d(k_{t+1} - \mathbb{E}[k_{t+1}|\Omega_t])}{da_t} = 0$, and Proposition 2 implies $\frac{d(a_{t+1} - \mathbb{E}[a_{t+1}|\Omega_t])}{da_t} = 1 - \gamma_s$. $\square$

From Corollary 1, we directly calculate $\xi_s = (1-\alpha)(1-\gamma_s)$ for each industry, using $\gamma_s = \sigma_a^2/(\sigma_a^2 + (\sigma_n^s)^2)$. Thus the noisiest industry in our sample has $\sigma_n^{30} \equiv \sigma_a\sqrt{\xi_{30}/(1-\alpha-\xi_{30})} \approx 0.75\sigma_a$. As a result, we choose $\{\sigma_n^s\}_{s=0}^{30} = \{0, 0.025\sigma_a, 0.05\sigma_a, ..., \sigma_n^{30}\}$ with a step size $\Delta\sigma_n = 0.025\sigma_a$ to match a corresponding empirical underreaction coefficient $\{\xi_s\}_{s=0}^{30} \in [0, 0.055]$ estimated from the regression equation (11) and as presented in Figure 3. Table 2b illustrates how the investment moments evolve with the noise level, ranging from full information to the noisiest industry.

Although our model predicts higher aggregate investment and capital stock (with a constant aggregate investment rate) under noisier information, Table 2b shows that the average firm-level investment rate is lower in this case.[17] The difference between the aggregate and the average firm-level investment rates reflects a compositional effect, and the lower average firm-level investment rate under noisier information is consistent with recent empirical evidence.[18]

4.4 Validation of the Micro Implications

In the third step, we run the same regressions using the microdata and the simulated data and compare the results directly to illustrate the micro implications.

Investment Inaction First, we run regressions to examine whether the industry-level coefficient ξ_s negatively affects the probability of firm-level investment inaction using semiannual observations:

$$\text{inaction}_{it} = \alpha \xi_s + \Gamma z_{it} + \Lambda \gamma_s + \gamma_t + \epsilon_{it} \tag{12}$$

where inaction_{it} is a binary variable equal to one if the investment rate is $\leq 1\%$ and zero otherwise. z_{it} includes firm-level controls such as lagged log capital stock (k_{it-1}), log labor productivity (a_{it}), and intermediate goods usage per worker (m_{it}). γ_s captures industry-level controls, including capital intensity and growth volatility, while γ_t represents time (semiannual) fixed effects. We standardize ξ_s (mean zero, standard deviation one) to facilitate interpretation. Since the attenuation coefficient varies only at the industry level, firm- or industry-fixed effects cannot be included. Standard errors are clustered at the industry level. We also replicate the regression using the simulated firm sample of 31 industries.

Table 3 presents the estimation results. Specifically, the first six columns show the regression results using our firm-level data, while the last two present the results using the simulated data. Consistent with the theory's predictions, the industry-specific attenuation coefficient has a negative impact on investment inaction at the firm level. Moreover, one standard deviation increase in this measure reduces the inaction probability by 5.1%. This is a substantial change: the average inaction rate is 36.6% at the semiannual frequency and 20.7% annually. The inclusion of industry-level confounding factors barely changes the estimated coefficient of α, as revealed in columns

[17] Since aggregate investment compensates for aggregate capital depreciation in the steady state, the aggregate investment rate is unaffected by the degree of information frictions in the steady state.

[18] When signals become noisier, many small firms—which typically have high investment rates—reduce their investments (i.e., a drop in the spike rate), while many large firms—which tend to be inactive or invest at lower rates—either become active or increase their investment (i.e., a drop in the inaction rate). Because large firms carry more weight in aggregate calculations, the average firm-level investment can decline even as aggregate investment rises. Kumar et al. (2023) find that average *firm-level* investment increases when firms have better information about macroeconomic variables, based on a randomized controlled trial.

Table 3: Incomplete Information and Investment Inaction

	inaction = 1							
	Data						Model	
ξ_s	-0.076**	-0.079***	-0.054**	-0.069**	-0.039*	-0.051**	-0.013	-0.011
	(0.028)	(0.026)	(0.025)	(0.026)	(0.020)	(0.021)	(——)	(——)
$a_{i,t}$	0.039	0.059*	0.104***	0.113***	0.091**	0.099***	-0.206	-0.298
	(0.034)	(0.031)	(0.038)	(0.033)	(0.033)	(0.032)	(——)	(——)
$k_{i,t-1}$		-0.050***	-0.049***	-0.044***	-0.041***	-0.039***		-0.458
		(0.009)	(0.009)	(0.007)	(0.008)	(0.007)		(——)
$m_{i,t}$			-0.026	-0.045***	-0.015	-0.030**		
			(0.021)	(0.016)	(0.019)	(0.014)		
cap share$_s$				-0.549*		-0.366		
				(0.314)		(0.304)		
growth vol$_s$					1.016***	0.870***		
					(0.279)	(0.278)		
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	99027	99027	86294	86294	86294	86294	14291997	14291997
adj. R^2	0.038	0.069	0.063	0.089	0.078	0.095	0.116	0.180

Notes: Standard errors are clustered at the industry level. Significance levels: * 0.10 ** 0.05 *** 0.01. The degree of information friction, ξ_s, is estimated at the industry level and normalized to mean zero and standard deviation one, both in the model and the data. The top and bottom 1% of productivity observations are winsorized.

four to six. Finally, the industry-level volatility is estimated to increase the firm-level inaction rate, consistent with the findings from the previously discussed literature on uncertainty shocks. Although we have controlled for two important industry-level confounding factors in the above regression, other factors at the industry level can potentially affect the average inaction rate. Since ξ_s is measured at the industry level, we cannot include industry fixed effects in regression equation (12). To address this concern, in our next test we estimate the firm level effects of the industry-wide information friction.

Investment Sensitivity In order to account for industry and firm-level fixed effects, we estimate interaction regressions to examine how a realized and unexpected productivity shock, w_{it}, affects a firm's investment inaction differently across industries with varying degrees of information frictions. The key variable of interest is the interaction term, $w_{it} \times \xi_s$. Specifically, we run the following regression:

$$\text{inaction}_{it} = \beta(w_{it} \times \xi_s) + \gamma w_{it} + \Gamma z_{it} + \gamma_i + \gamma_t + \gamma_{st} + \epsilon_{it} \tag{13}$$

where inaction$_{it}$ is a binary variable equal to one if the investment rate is $\leq$ 1% and zero otherwise. z_{it} includes firm-level controls such as lagged log capital stock (k_{it-1}), lagged log productivity (a_{it-1}), and intermediate goods usage per worker (m_{it}). γ_s represents firm fixed effects,

γ_t represents time (semiannual) fixed effects, and γ_{st} represents industry-time fixed effects. The industry-level coefficient, ξ_s, is standardized for interpretability. The advantage of this interaction regression is that including firm-level fixed effects allows us to control for firm-level time-invariant factors affecting investment. We also replicate the regression using the same simulated firm sample.

Table 4: Incomplete Information and Investment Sensitivity

	inaction = 1					
	Data				Model	
$\xi_s \times w_{i,t}$	0.010**	0.011**	0.011**	0.010**	0.012	0.013
	(0.005)	(0.005)	(0.005)	(0.005)	(——)	(——)
w_{it}	-0.036	-0.030	-0.036	-0.029	-0.188	-0.188
	(0.031)	(0.031)	(0.032)	(0.032)	(——)	(——)
a_{it-1}	-0.028**	-0.015	-0.029**	-0.016	-0.670	-0.670
	(0.012)	(0.012)	(0.011)	(0.011)	(——)	(——)
Productivity	Rand. Walk	Rand. Walk	Auto. Reg(1)	Auto. Reg(1)	Rand. Walk	Rand. Walk
Firm FE	Y	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y	Y
Industry-Time FE	N	Y	N	Y	N	Y
N	84656	84656	84313	84313	14274640	14274640
adj. R^2	0.446	0.451	0.446	0.451	0.450	0.450

Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. The degree of information friction, ξ_s, is estimated at the industry level and standardized to have a mean of zero and a standard deviation of one in both the model and the data. The top and bottom 1% of productivity observations are winsorized.

The interaction regression confirms that firms in industries with more severe information frictions are more likely to remain inactive following a positive productivity shock, compared with firms in industries with lower frictions. Table 4 reports the regression results.[19] The first four columns use firm-level data, while the last two columns present results from the simulated data. Our estimation allows for flexible inclusion of various fixed effects, and productivity innovations are computed both from a random walk process (consistent with the model) and from an AR(1) process. Across specifications, the coefficient β is positive and statistically significant (or very close) for investment inaction in the first four columns, matching the model qualitatively and quantitatively. The magnitude of the productivity coefficients is smaller in magnitude than in the model. Still, they have the correct sign when estimated using the firm-level interaction regression, which was not the case with the industry-level regression.

Robustness We implement a variety of robustness checks for our empirical analysis. Appendix F reports details and results. Specifically, we first estimate the underreaction coefficients in Section F.1 using alternative specifications that include firm-level fixed effects, and find that they are

[19]Tables 5 and 6 in Appendix F.2 present detailed estimation results (columns 2 and 4 of the two tables).

similar to our baseline estimates reported in Figure 3. The coefficients are generally attenuated, but still show variation across industries and consistent evidence of underreaction, as in Ma et al. (2020). Appendix F.2 presents the full details of our main regressions using labor productivity. Finally, we revisit all of our main regressions in Section F.3 using estimated total factor productivity instead of the simple productivity measures. With this approach, our key estimates are quantitatively similar to the main results we presented in Figure 3, and Tables 3 and 4. Across all these alternatives, we conclude that the central conclusions from our empirical analysis are robust.

5 Quantification of the Macro Implications

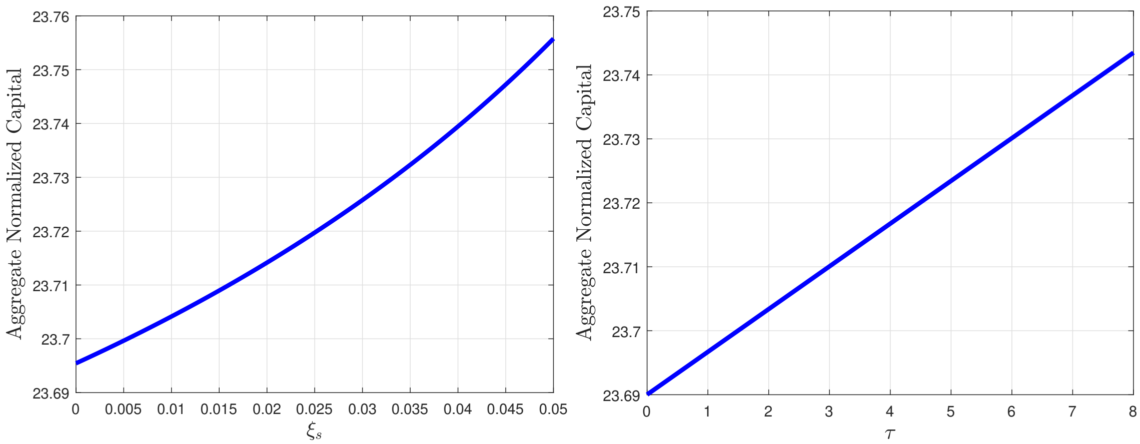
Having estimated the information frictions in the data, we now quantify the macroeconomic implications of incomplete information and irreversibility. This section evaluates the theoretical predictions presented in Section 3.3, regarding capital accumulation, capital misallocation, and aggregate shock attenuation. In addition to the effects of variations in the underreaction coefficient ξ_s, which can be estimated from our data, we can also demonstrate the effects of revelation delay, a variation that is not observed in the data, on the macro implications.

5.1 Capital Accumulation

The first macro implication of the information friction is that steady-state aggregate normalized capital should increase with both the noisy-signal variance, σ_n^2, and the revelation delay, τ (Proposition 3). Intuitively, noisier information raises the lower bound of firms' inaction region, leading to higher steady-state capital holdings. We document this prediction in our calibrated model by varying the underreaction coefficient, ξ_s, from 0 (full information, $\sigma_n^0 = 0$) to 0.055 (highest noise, $\sigma_n^{30} = 0.75\sigma_a$), as well as by varying the revelation delay (holding ξ_s fixed at 0.018, the median value in our data, for this exercise). For each case, we compute the steady-state level of normalized capital. The quantitative results are reported in Figure 4.

Panel (a) in Figure 4 shows that for larger underreaction coefficients (i.e., larger variance of the noisy signal), steady state aggregate normalized capital increases. This result is mainly driven by the fact that the boundary is increasing in ξ_s, which ultimately induces firms to accumulate more capital. Panel (b) demonstrates that more severe information frictions, such as longer delays in revealing actual productivity (i.e., larger τ), lead to a larger steady state level of normalized capital. Overall, the calibrated model aligns with the theoretical prediction that steady-state normalized capital increases with the severity of the information friction.

Figure 4: Information Incompleteness and Aggregate Normalized Capital



(a) Underreaction Coefficient ξ_s

(b) Revelation Delay τ

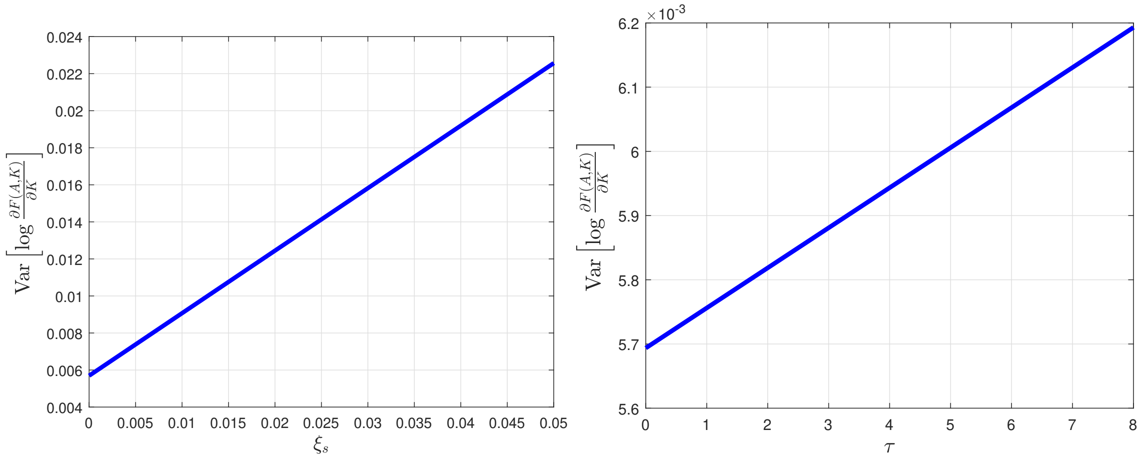
Notes: These figures illustrate how information incompleteness affects steady state aggregate normalized capital. Panel (a) varies the underreaction coefficient $\xi_s = (1-\alpha)\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}$, which goes from 0 (full information) to 0.055 (noisiest empirical measure). Panel (b) increases the revelation delay τ, which goes from 0 (full information) to 8 (8 quarters of revelation delay), with $\xi_s = 0.018$, the median value in our data.

5.2 Capital Misallocation

The second macro implication of the information friction is that steady-state capital misallocation should increase with both the noisy-signal variance, σ_n^2, and the revelation delay, τ (Proposition 4). Intuitively, noisier information increases the nowcast errors of firms, leading to greater mistakes and higher steady-state capital misallocation. Again, we document this prediction in our calibrated model by varying the underreaction coefficient, ξ_s, from 0 (full information, $\sigma_n^0 = 0$) to 0.055 (highest noise, $\sigma_n^{30} = 0.75\sigma_a$), as well as by varying the revelation delay (holding ξ_s fixed at 0.018, the median value in our data, for this exercise). For each case, we compute the steady-state level of capital misallocation as the variance of log MPK.

Figure 5 shows the quantitative results of increments in capital misallocation due to information incompleteness. Specifically, we plot the changes in the variance of the log marginal product of capital $\text{Var}\left[\log\frac{\partial F(A,K)}{\partial K}\right] = (1-\alpha)^2\text{Var}[x]$ relative to the full information case. According to Proposition 4, such changes equal to $(1-\alpha)^2(\text{Var}[x] - \text{Var}[\hat{x}]) \equiv (1-\alpha)^2\nu$, which is solely governed by the nowcast error $\nu = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$. In Panel (a), we show that a higher noise level increases capital misallocation. From the full information sector to the noisiest sector, the variance of the log marginal product of capital increases from 0.006 to over 0.022. In Panel (b), we demonstrate that a longer delay in revelation also leads to increased capital misallocation. From no revelation delay to a revelation delay of eight quarters, the variance of the log marginal product of capital

Figure 5: Information Incompleteness and Increments in Capital Misallocation



(a) Underreaction Coefficient ξ_s (b) Revelation Delay τ

Notes: These figures illustrate how information incompleteness affects capital misallocation, measured as the variance of the log marginal product of capital $\mathrm{Var}\left[\log \frac{\partial F(A,K)}{\partial K}\right]$. Panel (a) varies the underreaction coefficient $\xi_s = (1-\alpha)\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}$, which goes from 0 (hypothetical full information industry) to 0.055 (noisiest industry in our empirical measure). Panel (b) increases the revelation delay τ, which goes from 0 (full information) to 8 (8 quarters of revelation delay), with $\xi_s = 0.018$, the median value in our data.

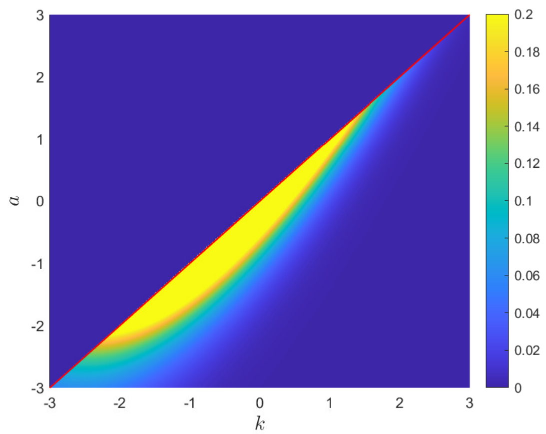
increases from 0.0057 to about 0.0062. This quantification shows that both the level of signal noise and revelation delay play essential roles in increasing capital misallocation.

To further visualize the capital misallocation caused by information incompleteness, we also present the joint stationary densities for log capital and log productivity in Figure 6 for various cases.[20] In each panel, the red line represents the expected log productivity $\hat{a}$ such that, for a given log capital k, the firm's expected normalized capital is exactly at the boundary $\hat{b}$. Panel (a) presents the full information case: investment irreversibility induces misallocation by leaving firms in the interior of the inaction region (i.e. below the red line) where they hold too much capital but cannot divest it.

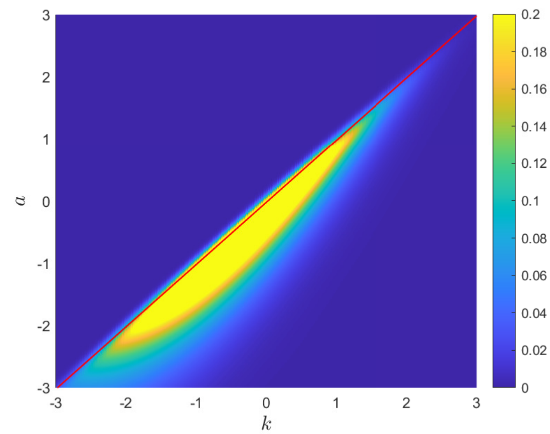
Panels (b) and (c) illustrate how the information friction increases capital misallocation, by smearing out the distribution of firms relative to panel (a). Both panels show the incomplete information case with the highest noise ($\xi_{30} = 0.055$) and panel (c) increases the revelation delay to 4 quarters ($\xi_{30} = 0.055$ and $\tau = 4$). These plots illustrate the role that information frictions play in determining how firms adjust their capital stock in noisy scenarios. Specifically, we observe that firms tend to remain closer to the boundary when there is incomplete information, compared to when there is complete information. Moreover, we observe that when information

[20] Appendix C solves the Kolmogorov PDE to derive these densities analytically.

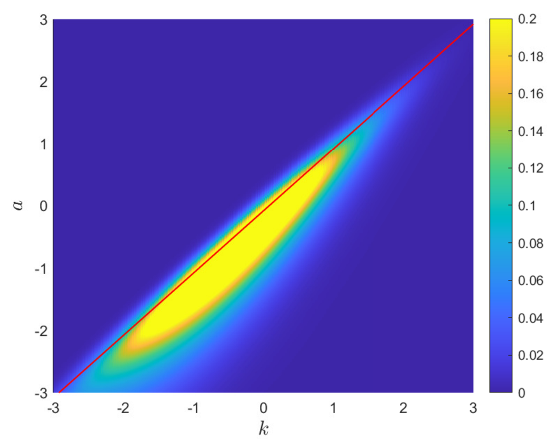
Figure 6: Information Incompleteness and Joint Stationary Densities



(a) Full Information ($\xi_0 = 0$)



(b) Highest Noise ($\xi_{30} = 0.055$)



(c) Highest Noise ($\xi_{30} = 0.055$) + Long Delay ($\tau = 4$)

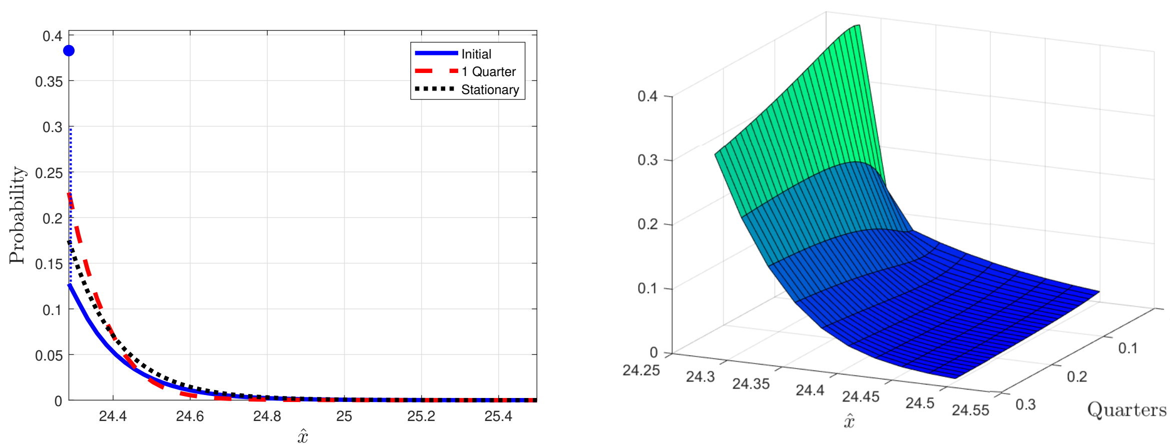
Notes: The red solid line denotes the inaction boundary for expected log productivity $\hat{a} = k - \hat{b}$. Grids for log capital (k) and log productivity (a) are normalized around 0. See Appendix C for the analytical derivation of the joint stationary densities.

becomes incomplete, firms begin to make mistakes: there is a relatively greater mass to the left of the boundary. Lastly, we observe that these patterns are exacerbated as the revelation delay increases. The two-dimensional distributions validate Proposition 4, which states that more severe information frictions lead to greater capital misallocation.

5.3 Aggregate Shock Attenuation

Finally, we demonstrate how aggregate productivity shocks impact the expected log-normalized capital $\hat{x}$. Under incomplete information, an aggregate shock to expected productivity differs from one to *actual* productivity.[21] We define an aggregate shock as an exogenous increase of one quarterly standard deviation σ_a in the log productivity of all firms. The aggregate shock induces a shift in the distribution of normalized capital, which is illustrated in Figure 7.

Figure 7: Response of the Normalized Capital Distribution to an Aggregate Productivity Shock



(a) Distribution on Impact and After One Quarter

(b) Distribution Over Time

Notes: These figures show how the normalized capital distribution evolves after the economy experiences an aggregate productivity shock. Panel (a) presents the distribution on impact ("Initial"), one quarter after the shock ("1 Quarter"), and the stationary distribution ("Stationary"). Panel (b) shows how the distribution is within the first quarter after the shock.

Figure 7 shows that the aggregate shock immediately pushes many firms to the boundary. If the aggregate shock pushes a firm's normalized capital past the boundary $\hat{b}$, the firm immediately invests in capital to remain at the barrier. Then after the shock, firms start adjusting their expected log normalized capital levels in several directions: we observe firms moving *away* from the boundary where the is excess mass, but we also observe firms moving *toward* the boundary, as the mass on higher values of normalized capital is lost relative to the initial equilibrium.

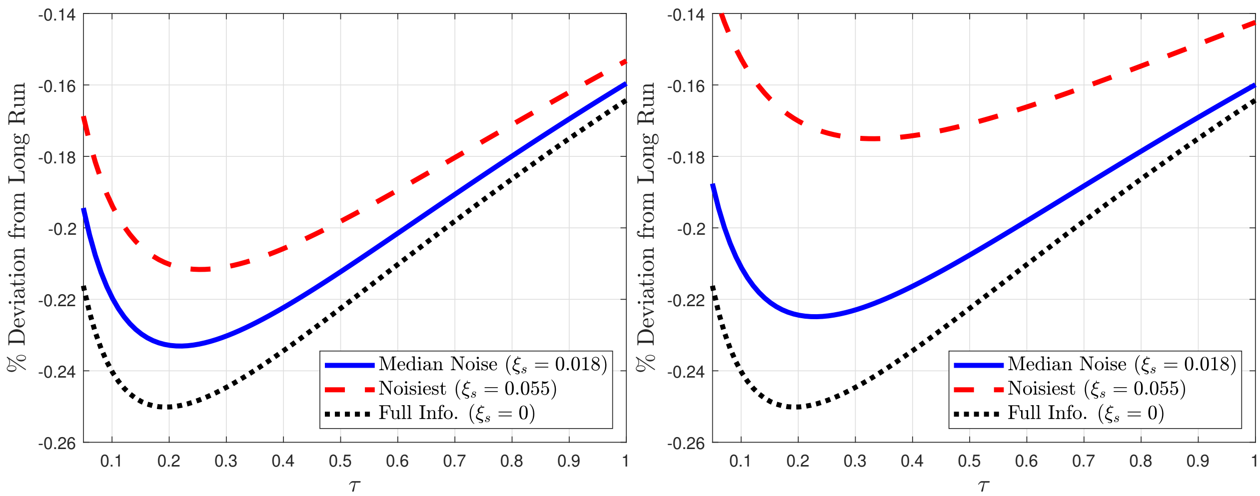
To analyze the response of average expected normalized capital, we define the impulse response function (IRF) as

$$IRF_{\hat{x}}(t) = \int_{\hat{x}} \hat{x} h(\hat{x}, t) d\hat{x}$$

where $h(\hat{x}, 0)$ is the perturbation to the distribution caused by the aggregate shock. Panel (a)

[21] Notice that the two responses are equivalent under full information.

Figure 8: Aggregate Impulse Response Function to a Productivity Shock



(a) $\tau = 1$ (b) $\tau = 4$

Notes: This figure presents the impulse response function of the average expected normalized capital to a positive productivity shock. Panel (a) presents the response for different degrees of information friction with a revelation delay of one quarter ($\tau = 1$). Panel (b) presents similar responses but for a revelation delay of 4 quarters ($\tau = 4$).

in Figure 8 presents the average response of expected normalized capital (solid blue line). We observe that the aggregate shock reduces the average expected capital, but after nearly two years, it returns to its long-run average.

To illustrate the role of the information friction, we study two exercises: (1) we consider a noisier signal, characterized by a larger ξ_s (or increased variance of the noise σ_n), and (2) we allow for a longer horizon in terms of the revelation delay τ. In particular, we examine the cases where $\xi_s = 0.018$ (the median level of noise in our data) and $\xi_s = 0.055$ (the highest level of noise in our data). Additionally, we also explore cases where the revelation delay τ is equal to one quarter ($\tau = 1$) or 4 quarters ($\tau = 4$). For completeness, we also add the full information case. The red dashed line in Figure 8 presents the IRF of average log expected normalized capital $\hat{x}$ for the noisiest signal, and the blue solid line presents a similar result for the case of median noise. The black dotted line corresponds to the average log expected normalized capital response under the full information case, for $\tau = 1$ and $\tau = 4$.

Figure 8 shows the role of the information friction: higher noise variance attenuates the response of expected normalized capital. Panel (b) shows that the discrepancy mentioned above is exacerbated when the revelation delay is one year rather than a quarter. Intuitively, the response of average expected capital is more severely attenuated the longer it takes the average firm to realize whether the shock it is experiencing is noise or a truly productive one. Lastly, the responses of expected normalized capital are identical across revelation delays when firms have full information; in this case, the information delay τ does not matter for firms' decisions.

6 Conclusion

This paper presents a framework for a better understanding of the roles of two key frictions that impact investment dynamics: irreversibility and information frictions. Our framework is tractable and generates testable theoretical predictions, many of which can be derived analytically. This stylized model can serve as a building block for further studies, incorporating additional features, enhanced realism, and more complex information problems.

We first learned that investment irreversibility and information frictions interact in meaningful ways in a stylized continuous time model. Two results stand out. First, information frictions introduce a new type of uncertainty that raises firms' willingness to invest, in contrast to the current effects of uncertainty in the literature; this effect reduces inaction and increases capital. Second, information frictions lessen the elasticity of investment to aggregate shocks, a valuable property for investment frictions that have macroeconomic effects in larger models.

Finally, we disciplined our stylized model with rich Japanese firm-level data. Sectoral heterogeneity in information frictions allowed us to test the model's predictions. We found that firms in sectors that face worse information frictions are less inactive and less elastic in their responses to productivity shocks. This confirms the theory's characteristic prediction.

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Appendix A The General Solution to the HJB

This intermediate result is used in multiple proofs that follow.

Lemma 6. *The normalized HJB (5) is solved by*

$$v(x) = me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}$$

for some c_1 and c_2.

Proof. The normalized HJB (5) has a particular solution

$$v_p(x) = me^{\alpha x}$$

with m solved by

$$rme^{\alpha x} = e^{\alpha x} - \mu\alpha me^{\alpha x} + \frac{\sigma^2}{2}\alpha^2 me^{\alpha x}$$

$$rm = 1 - \mu\alpha m + \frac{\sigma^2}{2}\alpha^2 m$$

$$m = \frac{1}{r + \mu\alpha - \frac{\sigma^2}{2}\alpha^2}$$

The homogeneous solution is

$$v_h(x) = c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}$$

where ϱ_1 and ϱ_2 are roots of the polynomial $-\frac{\sigma^2}{2}\varrho_j^2 + \mu\varrho_j + r = 0$. Thus the value function is

$$v(x) = me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}$$

□

One result of Lemma 6 is that the expected value function $\hat{v}(\hat{x})$ has a similar form.

Corollary 2. *The expected value function $\hat{v}(\hat{x})$ satisfies*

$$\hat{v}(\hat{x}) = me^{\alpha\hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c_1 e^{\varrho_1 \hat{x}} e^{\frac{\varrho_1^2 \nu}{2}} + c_2 e^{\varrho_2 \hat{x}} e^{\frac{\varrho_2^2 \nu}{2}} \tag{14}$$

for some c.

Proof. The firm's expectation of the value function derived in Lemma 6 is

$$\hat{v}(\hat{x}) = E[v(x)|\hat{x}] = E[me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}|\hat{x}]$$

The firm's conditional expectation of x is $x \sim N(\hat{x}, \nu)$:

$$= \int_{-\infty}^{\infty} (me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x})\, \phi(\frac{x - \hat{x}}{\sqrt{\nu}}) dx$$

$$= \int_{-\infty}^{\infty} \left(me^{\alpha(x-\hat{x})} e^{\alpha \hat{x}} + c_1 e^{\varrho_1 (x-\hat{x})} e^{\varrho_1 \hat{x}} + c_2 e^{\varrho_2 (x-\hat{x})} e^{\varrho_2 \hat{x}}\right) \phi(\frac{x - \hat{x}}{\sqrt{\nu}}) dx$$

Then use that $e^{\alpha(x-\hat{x})}$, $e^{\varrho_1(x-\hat{x})}$ and $e^{\varrho_2(x-\hat{x})}$ are log-normal, where the associated normal distributions have zero mean and variance $\alpha^2 \nu$, $\varrho_1^2 \nu$ and $\varrho_2^2 \nu$ respectively:

$$= me^{\alpha \hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c_1 e^{\varrho_1 \hat{x}} e^{\frac{\varrho_1^2 \nu}{2}} + c_2 e^{\varrho_2 \hat{x}} e^{\frac{\varrho_2^2 \nu}{2}}$$

□

Appendix B Proofs

B.1 Proof of Lemma 1

Proof. The firm's conditional expectation of a_t is

$$E[a_t | \Omega_t] = a_{t-\tau} + E[a_t - a_{t-\tau} | \Omega_t]$$

From the firm's perspective, $s_t - s_{t-\tau}$ is a noisy signal of $a_t - a_{t-\tau}$:

$$s_t - s_{t-\tau} = a_t - a_{t-\tau} + n_t - n_{t-\tau}$$

the noise $n_t - n_{t-\tau}$ is independent of productivity and distributed $N(0, \tau\sigma_n^2)$, while $a_t - a_{t-\tau}$ is distributed $N(0, \tau\sigma_a^2)$. Therefore:

$$E[a_t - a_{t-\tau} | \Omega_t] = \frac{\text{Cov}(a_t - a_{t-\tau}, s_t - s_{t-\tau})}{\text{Var}(s_t - s_{t-\tau})}(s_t - s_{t-\tau}) = \frac{\tau\sigma_a^2}{\tau\sigma_a^2 + \tau\sigma_n^2}(s_t - s_{t-\tau})$$

and the definition $\gamma = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2}$ implies

$$E[a_t | \Omega_t] = a_{t-\tau} + \gamma\,(s_t - s_{t-\tau})$$

The nowcast errors $u_t = a_t - E[a_t|\Omega_t]$ are normally distributed and have variance

$$\text{Var}\left(a_t - E[a_t|\Omega_t]\right) = \text{Var}\left(a_t - a_{t-\tau} - \gamma\left(s_t - s_{t-\tau}\right)\right) = \text{Var}\left((1-\gamma)(a_t - a_{t-\tau}) - \gamma\left(n_t - n_{t-\tau}\right)\right)$$

$$= \text{Var}\left((1-\gamma)(a_t - a_{t-\tau})\right) + \text{Var}\left(\gamma\left(n_t - n_{t-\tau}\right)\right) = (1-\gamma)^2\tau\sigma_a^2 + \gamma^2\tau\sigma_n^2$$
$$= \left(\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}\right)^2\tau\sigma_a^2 + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2\tau\sigma_n^2 = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2} = \nu$$

The conditional distribution implies that the diffusion for $\hat{a}_t$ is given by

$$d\hat{a}_t = da_{t-\tau} + \gamma(ds_t - ds_{t-\tau}) = (1-\gamma)da_{t-\tau} + \gamma da_t + \gamma dn_t - \gamma dn_{t-\tau}$$

$$= (1-\gamma)\sigma_a dW_{t-\tau}^a + \gamma\sigma_a dW_t^a + \gamma\sigma_n dW_t^n - \gamma\sigma_n dW_{t-\tau}^n$$

The right-hand side is the sum of independent innovations, so they can be recomposed as innovations to a single Wiener process:

$$d\hat{a}_t = \sigma_{\hat{a}} dW^{\hat{a}}$$

It remains to show that $\sigma_{\hat{a}} = \sigma_A$. The independence of the innovations imply

$$\sigma_{\hat{a}}^2 = (1-\gamma)^2\sigma_a^2 + \gamma^2\sigma_a^2 + 2\gamma^2\sigma_n^2$$

$$= \left(\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}\right)^2\sigma_a^2 + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2\sigma_a^2 + 2\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2\sigma_n^2$$
$$= \frac{\sigma_n^2\sigma_a^2}{\sigma_a^2+\sigma_n^2} + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2\sigma_a^2 + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2\sigma_n^2$$
$$= \frac{\sigma_n^2\sigma_a^2}{\sigma_a^2+\sigma_n^2} + \frac{\sigma_a^4}{\sigma_a^2+\sigma_n^2} = \sigma_a^2$$

As a result, innovations to nowcast errors follow

$$du_t = da_t - d\hat{a}_t = \sigma_a dW_t^a - ((1-\gamma)\sigma_a dW_{t-\tau}^a + \gamma\sigma_a dW_t^a + \gamma\sigma_n dW_t^n - \gamma\sigma_n dW_{t-\tau}^n)$$

$$= (1-\gamma)(\sigma_a dW_t^a - \sigma_a dW_{t-\tau}^a) - \gamma(\sigma_n dW_t^n - \sigma_n dW_{t-\tau}^n) = \sigma_u dW_t^u$$

Again, independence of the innovations implies

$$\sigma_u^2 = 2(1-\gamma)^2\sigma_a^2 + 2\gamma^2\sigma_n^2$$

$$= 2\left(\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}\right)^2\sigma_a^2 + 2\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2\sigma_n^2 = 2\frac{\sigma_n^2\sigma_a^2}{\sigma_a^2+\sigma_n^2}$$

□

B.2 Proof of Lemma 2

Proof. The value-matching and super contact conditions at infinity are standard.

Corollary 2 gives the firm's expectation of the value function in terms of two roots ϱ_1 and ϱ_2. The conditions at infinity imply that the coefficient on the positive root is zero. We write the remaining negative root ϱ and coefficient c without subscripts; the expected value function becomes

$$\hat{v}(\hat{x}) = me^{\alpha\hat{x}}e^{\frac{\alpha^2 v}{2}} + ce^{\varrho\hat{x}}e^{\frac{\varrho^2 v}{2}}$$

which in levels is

$$\hat{V}(\hat{X};\hat{B}) = m\hat{X}^{\alpha}e^{\frac{\alpha^2 v}{2}} + c(\hat{B})\hat{X}^{\varrho}e^{\frac{\varrho^2 v}{2}} \tag{15}$$

This solution is written as a function of the boundary $\hat{B}$, to be clear about how the choice of $\hat{B}$ determines which solution to the HJB is the true value function.

To derive the value-matching condition at the boundary, use that firms are indifferent between applying the infinitesimal regulator dI at the boundary $\hat{B}$:

$$\hat{V}(\hat{B}) = \hat{V}(\hat{B}+dI) - \psi dI$$

$$\hat{V}(\hat{B}) = \hat{V}(\hat{B}) + \hat{V}'(\hat{B})dI - \psi dI$$

$$\implies \psi = \hat{V}'(\hat{B})$$

To derive the super-contact condition at the boundary, first consider the problem of a firm: their only decision is to select the critical value $\hat{B}$ that maximizes their value (15). The first order condition for this problem is

$$c'(\hat{B})\hat{X}^{\varrho}e^{\frac{v}{2}} = 0 \tag{16}$$

Next, take the derivative of the value matching condition $\psi = \hat{V}'(\hat{B})$ with respect to $\hat{B}$:

$$0 = m\alpha\hat{B}^{\alpha-1}e^{\frac{\alpha^2 v}{2}} + c(\hat{B})\varrho\hat{B}^{\varrho-1}e^{\frac{\varrho^2 v}{2}} + c'(\hat{B})\hat{B}^{\varrho}e^{\frac{\varrho^2 v}{2}}$$

then substitute using (16) to find the super contact condition:

$$0 = m\alpha\hat{B}^{\alpha-1}e^{\frac{\alpha^2 v}{2}} + c(\hat{B})\varrho\hat{B}^{\varrho-1}e^{\frac{\varrho^2 v}{2}} = \hat{V}''(\hat{B})$$

□

B.3 Proof of Lemma 3

Proof. Per Corollary 2 , the first derivative of the value function in expected log normalized capital $\hat{v}(\hat{x})$ is

$$\hat{v}'(\hat{x}) = m\alpha e^{\alpha\hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho e^{\varrho\hat{x}} e^{\frac{\varrho^2 \nu}{2}}$$

Apply this to the value-matching condition from Lemma 2 (using $\hat{V}'(\hat{X}) = \frac{d\hat{V}(\hat{X})}{d\hat{X}} = \frac{d\hat{V}(\hat{X})}{d\hat{x}}\frac{d\hat{x}}{d\hat{X}} = \hat{v}'(\hat{x})e^{-\hat{x}}$):

$$\psi = \hat{v}'(\hat{b})e^{-\hat{b}} = m\alpha e^{(\alpha-1)\hat{b}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho e^{(\varrho-1)\hat{b}} e^{\frac{\varrho^2 \nu}{2}} \tag{17}$$

Before evaluating the super contact condition, it is helpful to rewrite $\hat{V}''(\hat{X})$ in terms of $\hat{x}$:

$$\hat{V}''(\hat{X}) = \frac{d\hat{V}'(\hat{X})}{d\hat{X}} = \frac{d\hat{V}'(\hat{X})}{d\hat{x}}\frac{d\hat{x}}{d\hat{X}} = \frac{d\hat{v}'(\hat{x})e^{-\hat{x}}}{d\hat{x}}\frac{1}{\hat{X}} = (\hat{v}''(\hat{x})e^{-\hat{x}} - \hat{v}'(\hat{x})e^{-\hat{x}})\frac{1}{\hat{X}}$$

$$= (\hat{v}''(\hat{x}) - \hat{v}'(\hat{x}))e^{-2\hat{x}} = m\alpha(\alpha-1)e^{(\alpha-2)\hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho(\varrho-1)e^{(\varrho-2)\hat{x}} e^{\frac{\varrho^2 \nu}{2}}$$

The super contact condition from Lemma 2 becomes

$$0 = \hat{V}''(\hat{b}) = m\alpha(\alpha-1)e^{(\alpha-2)\hat{b}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho(\varrho-1)e^{(\varrho-2)\hat{b}} e^{\frac{\varrho^2 \nu}{2}} \tag{18}$$

Equations (17) and (18) imply

$$\psi(1-\varrho) = m\alpha(\alpha-\varrho)e^{(\alpha-1)\hat{b}} e^{\frac{\alpha^2 \nu}{2}}$$

$$\implies \hat{b} = \frac{1}{1-\alpha}\log\left(\frac{m\alpha(\alpha-\varrho)}{\psi(1-\varrho)}\right) + \frac{\alpha^2 \nu}{2(1-\alpha)}$$

□

B.4 Proof of Lemma 4

Proof. Per Lemma 1:

$$\frac{d}{da_{t-h}}\mathbb{E}[a_t|\Omega_t] = \frac{d}{da_{t-h}}\left(a_{t-\tau} + \gamma\left(s_t - s_{t-\tau}\right)\right)$$

There are two cases. In both, $\frac{ds_t}{da_{t-h}} = 1$. But if $0 \le h < \tau$, then $\frac{da_{t-\tau}}{da_{t-h}} = \frac{ds_{t-\tau}}{da_{t-h}} = 0$:

$$[0 \le h < \tau]: \qquad \frac{d}{da_{t-h}} \mathbb{E}[a_t|\Omega_t] = \gamma \frac{d}{da_{t-h}} s_t = \gamma$$

If $h \ge \tau$, then $\frac{da_{t-\tau}}{da_{t-h}} = \frac{ds_{t-\tau}}{da_{t-h}} = 1$:

$$[h \ge \tau]: \qquad \frac{d}{da_{t-h}} \mathbb{E}[a_t|\Omega_t] = 1 + \gamma - \gamma = 1$$

□

B.5 Proof of Lemma 5

Proof. The general solution to the ODE (7) is

$$h(\hat{x}) = c_{h1} e^{-\rho_1 \hat{x}} + c_{h2} e^{-\rho_2 \hat{x}}$$

where ρ_1 and ρ_2 are roots of the equation

$$0 = D\rho_j^2 - \delta\rho_j - \eta$$

Using $D = \frac{\sigma_a^2}{2}$, the roots are given by

$$\rho_j = \frac{\delta}{\sigma_a^2} \pm \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$$

To satisfy the boundary condition at infinity, only the positive root can have a non-zero coefficient. Therefore, the solution simplifies to

$$h(\hat{x}) = c_h e^{-\rho \hat{x}}$$

where ρ (without subscript) denotes the positive root $\frac{\delta}{\sigma_a^2} + \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$. The coefficient c_h is yet to be found.

The remaining boundary condition is that $h(\hat{x})$ integrates to one:

$$1 = \int_{\hat{b}}^{\infty} c_h e^{-\rho \hat{x}} d\hat{x} = \frac{c_h}{\rho} e^{-\rho \hat{b}}$$

which implies

$$c_h = \rho e^{\rho \hat{b}}$$

The joint distribution $f_{\hat{x},u}(\hat{x}, u)$ given by equation (8) implies

$$f_x(x) = h(x) * \phi(-\frac{x}{\sqrt{\nu}})$$

$$= \int_{-\infty}^{\infty} h(\hat{x})\phi(\frac{\hat{x}-x}{\sqrt{\nu}})d\hat{x}$$

$h(\hat{x}) = 0$ for $\hat{x} < \hat{b}$, so the convolution becomes

$$= \int_{\hat{b}}^{\infty} h(\hat{x})\phi(\frac{\hat{x}-x}{\sqrt{\nu}})d\hat{x} = \int_{\hat{b}}^{\infty} \rho e^{-\rho(\hat{x}-\hat{b})} \frac{1}{\sqrt{2\pi\nu}} e^{-\frac{(\hat{x}-x)^2}{2\nu}} d\hat{x}$$

$$= e^{-\frac{x^2}{2\nu}} \int_{\hat{b}}^{\infty} \rho e^{\rho\hat{b}} \frac{1}{\sqrt{2\pi\nu}} e^{-\frac{\hat{x}^2-2(x-\nu\rho)\hat{x}}{2\nu}} d\hat{x} = e^{-\frac{x^2-(x-\nu\rho)^2}{2\nu}} \int_{\hat{b}}^{\infty} \rho e^{\rho\hat{b}} \frac{1}{\sqrt{2\pi\nu}} e^{-\frac{\hat{x}^2-2(x-\nu\rho)\hat{x}+(x-\nu\rho)^2}{2\nu}} d\hat{x}$$

$$= \rho e^{-\rho(x-\hat{b})} e^{\frac{\nu\rho^2}{2}} \int_{\hat{b}}^{\infty} \frac{1}{\sqrt{2\pi\nu}} e^{-\frac{(\hat{x}-(x-\nu\rho))^2}{2\nu}} d\hat{x} = \rho e^{-\rho(x-\hat{b})} e^{\frac{\nu\rho^2}{2}} \int_{\hat{b}}^{\infty} \phi(\frac{\hat{x}-(x-\nu\rho)}{\sqrt{\nu}})d\hat{x}$$

$$= \rho e^{-\rho(x-\hat{b})} e^{\frac{\nu\rho^2}{2}} \left(1 - \Phi\left(\frac{\hat{b}+\nu\rho-x}{\sqrt{\nu}}\right)\right) = h(x) e^{\frac{\nu\rho^2}{2}} \Phi\left(\frac{x-(\hat{b}+\nu\rho)}{\sqrt{\nu}}\right)$$

□

B.6 Proof of Proposition 3

Proof. Decompose normalized capital into the independent nowcasts and errors by $x = \hat{x} - u$:

$$\int_{-\infty}^{\infty} e^x f_x(x)dx = \int_{\hat{b}}^{\infty} \int_{-\infty}^{\infty} e^{\hat{x}-u} f_{\hat{x},u}(\hat{x}, u)dud\hat{x}$$

$$= \int_{\hat{b}}^{\infty} \int_{-\infty}^{\infty} e^{\hat{x}-u} h(\hat{x})\phi(\frac{u}{\sqrt{\nu}})dud\hat{x} = \int_{\hat{b}}^{\infty} e^{\hat{x}} h(\hat{x}) \int_{-\infty}^{\infty} e^{-u}\phi(\frac{u}{\sqrt{\nu}})dud\hat{x}$$

Use that $\int_{-\infty}^{\infty} e^{-u}\phi(\frac{u}{\sqrt{\nu}})du = e^{\frac{\nu}{2}}$ is the mean of a log-normal distribution:

$$= e^{\frac{\nu}{2}} \int_{\hat{b}}^{\infty} e^{\hat{x}} h(\hat{x})d\hat{x} = e^{\frac{\nu}{2}} \int_{\hat{b}}^{\infty} e^{\hat{x}} \rho e^{-\rho(\hat{x}-\hat{b})} d\hat{x}$$

$$= \frac{e^{\frac{\nu}{2}+\hat{b}}\rho}{\rho - 1}$$

which is increasing in ν, per Proposition 1, and $\nu = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ is increasing in σ_n^2 and τ. $\square$

B.7 Proof of Proposition 11

Proof. Per Corollary 2, the firm's expectation of the value function derived in Lemma 6 is

$$\hat{v}(\hat{x}) = me^{\alpha\hat{x}}e^{\frac{\alpha^2\nu}{2}} + c_1e^{\varrho_1\hat{x}}e^{\frac{\varrho_1^2\nu}{2}} + c_2e^{\varrho_2\hat{x}}e^{\frac{\varrho_2^2\nu}{2}}$$

which in levels is

$$\hat{V}(\hat{X};\hat{B}) = m\hat{X}^{\alpha}e^{\frac{\alpha^2\nu}{2}} + c_1(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + c_2(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_2}e^{\frac{\varrho_2^2\nu}{2}} \tag{19}$$

This solution is written as a function of the boundaries $(\hat{B}_L,\hat{B}_U)$, to be clear about how the boundary choice determines which solution to the HJB is the true value function.

To derive the value-matching condition at the lower boundary, use that firms are indifferent between applying the infinitesimal regulator dI at the boundary $\hat{B}_L$:

$$\hat{V}(\hat{B}_L) = \hat{V}(\hat{B}_L + dI) - \psi_+ dI$$

$$\hat{V}(\hat{B}_L) = \hat{V}(\hat{B}_L) + \hat{V}'(\hat{B}_L)dI - \psi_+ dI$$

$$\implies \psi_+ = \hat{V}'(\hat{B}_L)$$

and a similar argument gives the value-matching condition at the upper boundary:

$$\psi_- = \hat{V}'(\hat{B}_U)$$

To derive the super-contact condition at the boundary, first consider the problem of a firm: their only decision is to select the critical values $\hat{B}_L$ and $\hat{B}_U$ that maximize their value (15). The first order conditions for this problem are

$$\partial_{\hat{B}_L}c_1(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + \partial_{\hat{B}_L}c_2(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_2}e^{\frac{\varrho_2^2\nu}{2}} = 0 \tag{20}$$

$$\partial_{\hat{B}_U}c_1(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + \partial_{\hat{B}_U}c_2(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_2}e^{\frac{\varrho_2^2\nu}{2}} = 0 \tag{21}$$

Next, take the derivative of the value matching condition $\psi_+ = \hat{V}'(\hat{B}_L)$ with respect to $\hat{B}_L$:

$$0 = m\alpha\hat{B}_L^{\alpha-1}e^{\frac{\alpha^2\nu}{2}} + c_1(\hat{B}_L,\hat{B}_U)\varrho_1\hat{B}_L^{\varrho_1-1}e^{\frac{\varrho_1^2\nu}{2}} + c_2(\hat{B}_L,\hat{B}_U)\varrho_2\hat{B}_L^{\varrho_2-1}e^{\frac{\varrho_2^2\nu}{2}} + \partial_{\hat{B}_L}c_1(\hat{B}_L,\hat{B}_U)\hat{B}_L^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + \partial_{\hat{B}_L}c_2(\hat{B}_L,\hat{B}_U)\hat{B}_L^{\varrho_1}e^{\frac{\varrho_2^2\nu}{2}}$$

then substitute using (20) to find the super contact condition:

$$0 = m\alpha \hat{B}_L^{\alpha-1} e^{\frac{\alpha^2 \nu}{2}} + c_1(\hat{B}_L, \hat{B}_U)\varrho_1 \hat{B}_L^{\varrho_1 - 1} e^{\frac{\varrho_1^2 \nu}{2}} + c_2(\hat{B}_L, \hat{B}_U)\varrho_2 \hat{B}_L^{\varrho_2 - 1} e^{\frac{\varrho_2^2 \nu}{2}} = \hat{V}''(\hat{B}_L)$$

Again, a similar argument taking the derivative of the value matching condition $\psi_- = \hat{V}'(\hat{B}_U)$ with respect to $\hat{B}_U$ gives the super contact condition at the upper boundary:

$$0 = \hat{V}''(\hat{B}_U)$$

□

B.8 Proof of Proposition 5

Before proving the proposition, some notation and a lemma are needed.

Just as $f_x(x, t, \varepsilon)$ denotes the dynamic distribution of log normalized capital x after a time 0 permanent productivity shock of size the ε, let $f_x^{FI}(x, t, \varepsilon)$ denote the distribution of x under the counterfactual economy if firms were to have full information. Define the IRF of average log normalized capital relative to the steady state in each scenario by

$$IRF_x(t, \varepsilon) \equiv \int_x f_x(x, t, \varepsilon)dx - \bar{x} \qquad IRF_x^{FI}(t, \varepsilon) \equiv \int_x f_x^{FI}(x, t, \varepsilon)dx - \bar{x}^{FI}$$

where $\bar{x}$ and $\bar{x}^{FI}$ denote the steady state averages under incomplete and full information respectively.

Lemma 7.

$$IRF_x(t, \varepsilon) = IRF_x^{FI}(t, \gamma\varepsilon) - (1 - \gamma)\varepsilon$$

Proof. A $\gamma\varepsilon$ shock to the counterfactual full information economy shifts $f^{FI}(x, t, \varepsilon)$ left by $\gamma\varepsilon$, the same amount as the ε shock shifts $h(\hat{x}, t, \varepsilon)$. The dynamic KFE (6) is the same regardless of the severity of the information friction; the information friction only affects the boundary condition. Therefore (for $t < \tau$) the distribution of $\hat{x} - \hat{b}$ responds to a ε shock the same as the distribution of $x - b^{FI}$ responds to a $\gamma\varepsilon$ shock in the full information counterfactual:

$$h(\hat{x}, t, \varepsilon) = f^{FI}(\hat{x} - \hat{b} + b^{FI}, t, \gamma\varepsilon) \qquad t < \tau \tag{22}$$

where b^{FI} denotes the full information boundary.

Log normalized capital $x = \hat{x} - u$ has PDF

$$f_x(\hat{x} - u, t, \varepsilon) = h(\hat{x}, t, \varepsilon) f_u(u, t, \varepsilon)$$

where $f_u(u, t, \varepsilon)$ is the marginal distribution of the productivity nowcast error u, because u and $\hat{x}$ are independent. Ordinarily this distribution is mean zero, but after the aggregate productivity shock, all firms' nowcast errors increase by $(1 - \gamma)\varepsilon$:

$$f_u(u, t, \varepsilon) = \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) \qquad t < \tau$$

Therefore, for $t < \tau$, the impulse response function of average log normalized capital $x = \hat{x} - u$ is

$$IRF_x(t, \varepsilon) = \int_{\hat{x}} \int_u (\hat{x} - u) h(\hat{x}, t, \varepsilon) \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) du d\hat{x}$$

$$= \int_{\hat{x}} \int_u \hat{x} h(\hat{x}, t, \varepsilon) \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) du d\hat{x} - \int_{\hat{x}} \int_u u h(\hat{x}, t, \varepsilon) \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) du d\hat{x}$$

$$= \int_{\hat{x}} \hat{x} h(\hat{x}, t, \varepsilon) d\hat{x} - \int_u u \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) du = IRF_{\hat{x}}(t, \varepsilon) - (1 - \gamma)\varepsilon$$

where $IRF_{\hat{x}}(t, \varepsilon) \equiv \int_{\hat{x}} \hat{x} h(\hat{x}, t, \varepsilon) d\hat{x}$ denotes the IRF of average expected log normalized capital. We can relate this term back to the full information counterfactual by equation (22):

$$IRF_{\hat{x}}(t, \varepsilon) = \int_{\hat{b}}^{\infty} \hat{x} f^{FI}(\hat{x} - \hat{b} + b^{FI}, t, \gamma\varepsilon) d\hat{x}$$

apply the change of variable $x^{FI} = \hat{x} - \hat{b} + b^{FI}$:

$$= \int_{b^{FI}}^{\infty} (x^{FI} + \hat{b} - b^{FI}) f^{FI}(x^{FI}, t, \gamma\varepsilon) dx^{FI} = IRF_x^{FI}(t, \gamma\varepsilon) + \hat{b} - b^{FI}$$

Reintroduce the $(1 - \gamma)\varepsilon$ term to express the IRF as

$$IRF_x(t, \varepsilon) = IRF_x^{FI}(t, \gamma\varepsilon) + \hat{b} - b^{FI} - (1 - \gamma)\varepsilon$$

and $\bar{x} - \bar{x}^{FI} = \hat{b} - b^{FI}$ implies the desired expression. □

Proof. Because $x = k - a$, and the aggregate change to a is simply ε, the IRF for capital is

$$IRF_k(t, \varepsilon) = IRF_x(t, \varepsilon) + \varepsilon = IRF_x^{FI}(t, \gamma\varepsilon) + \gamma\varepsilon$$

for $t < \tau$ by Lemma 7. To find the marginal IRF, take the derivative with respect to ε under both incomplete and full information:

$$\frac{\partial}{\partial \varepsilon} IRF_k(t,\varepsilon) = \frac{\partial}{\partial \varepsilon} IRF_x^{FI}(t,\gamma\varepsilon) + \gamma \qquad \frac{\partial}{\partial \varepsilon} IRF_k^{FI}(t,\varepsilon) = \frac{\partial}{\partial \varepsilon} IRF_x^{FI}(t,\varepsilon)\gamma + 1$$

then evaluate at $\varepsilon = 0$

$$\widehat{IRF}_k(t) = \gamma \widehat{IRF}_x^{FI}(t) + \gamma \qquad \widehat{IRF}_k^{FI}(t) = \widehat{IRF}_x^{FI}(t) + 1$$

Combine these equations to yield

$$\widehat{IRF}_k(t) = \gamma \widehat{IRF}_k^{FI}(t)$$

□

B.9 Proof of Lemma 7

Before the proof, we prove a lemma that is independently useful for computing the model:

Lemma 8. *The difference between the upper and lower log bounds of the inaction region $\Delta \equiv \hat{b}_H - \hat{b}_L$ solves the implicit equation*

$$\frac{\psi_- e^{(1-\varrho_2)\Delta} - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+}{\psi_- e^{(1-\varrho_2)\Delta} - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+} = \frac{\left(e^{(\alpha-\varrho_2)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)}{\left(\alpha e^{(\alpha-\varrho_2)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)} \tag{23}$$

and the lower bound $\hat{b}_L$ is given in terms of Δ by

$$\hat{b}_L = \frac{\alpha^2 \nu}{2(1-\alpha)} - \frac{1}{1-\alpha}\log\left(\frac{1}{m\alpha}\frac{\psi_- - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\varrho_2-1)\Delta}}\right) \tag{24}$$

Proof. Per Corollary 2, the first derivative of the value function in expected log normalized capital $\hat{v}(\hat{x})$ is

$$\hat{v}'(\hat{x}) = m\alpha e^{\alpha\hat{x}} e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1 e^{\varrho_1\hat{x}} e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{x}} e^{\frac{\varrho_2^2\nu}{2}}$$

Apply this to the value-matching conditions (using $\hat{V}'(\hat{X}) = \frac{d\hat{V}(\hat{X})}{d\hat{X}} = \frac{d\hat{V}(\hat{X})}{d\hat{x}}\frac{d\hat{x}}{d\hat{X}} = \hat{v}'(\hat{x})e^{-\hat{x}}$):

$$\psi_+ e^{\hat{b}_L} = \hat{v}'(\hat{b}_L) = m\alpha e^{\alpha\hat{b}_L} e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1 e^{\varrho_1\hat{b}_L} e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_L} e^{\frac{\varrho_2^2\nu}{2}} \tag{25}$$

$$\psi_- e^{\hat{b}_H} = \hat{v}'(\hat{b}_H) = m\alpha e^{\alpha\hat{b}_H} e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1 e^{\varrho_1\hat{b}_H} e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_H} e^{\frac{\varrho_2^2\nu}{2}} \tag{26}$$

Before evaluating the super contact conditions, it is helpful to rewrite $\hat{V}''(\hat{X})$ in terms of $\hat{x}$:

$$\hat{V}''(\hat{X}) = \frac{d\hat{V}'(\hat{X})}{d\hat{X}} = \frac{d\hat{V}'(\hat{X})}{d\hat{x}}\frac{d\hat{x}}{d\hat{X}} = \frac{d\hat{v}'(\hat{x})e^{-\hat{x}}}{d\hat{x}}\frac{1}{\hat{X}} = (\hat{v}''(\hat{x})e^{-\hat{x}} - \hat{v}'(\hat{x})e^{-\hat{x}})\frac{1}{\hat{X}}$$

$$= (\hat{v}''(\hat{x}) - \hat{v}'(\hat{x}))e^{-2\hat{x}}$$

Thus the super contact conditions $0 = \hat{V}''(\hat{B}_L)$ and $0 = \hat{V}''(\hat{B}_H)$ imply $\hat{v}''(\hat{b}_L) = \hat{v}'(\hat{b}_L)$ and $\hat{v}''(\hat{b}_H) = \hat{v}'(\hat{b}_H)$ respectively. These conditions become

$$\psi_+ e^{\hat{b}_L} = \hat{v}''(\hat{b}_L) = m\alpha e^{\alpha\hat{b}_L} e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1 e^{\varrho_1\hat{b}_L} e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_L} e^{\frac{\varrho_2^2\nu}{2}} \tag{27}$$

$$\psi_- e^{\hat{b}_H} = \hat{v}''(\hat{b}_H) = m\alpha^2 e^{\alpha\hat{b}_H} e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1^2 e^{\varrho_1\hat{b}_H} e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2^2 e^{\varrho_2\hat{b}_H} e^{\frac{\varrho_2^2\nu}{2}} \tag{28}$$

Combining the b_L value-matching condition (25) and super contact condition (27) can be used to solve for c_1 and c_2 in terms of b_L. First, difference out the c_2 term:

$$(1-\varrho_2)\psi_+ e^{\hat{b}_L} = m\alpha(\alpha-\varrho_2)e^{\alpha\hat{b}_L} e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1(\varrho_1-\varrho_2)e^{\varrho_1\hat{b}_L} e^{\frac{\varrho_1^2\nu}{2}}$$

$$\implies c_1\varrho_1 e^{\frac{\varrho_1^2\nu}{2}} = \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_1)\hat{b}_L} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_1)\hat{b}_L} e^{\frac{\alpha^2\nu}{2}}$$

Plug back into the value matching condition (25):

$$\psi_+ e^{\hat{b}_L} = m\alpha e^{\alpha\hat{b}_L} e^{\frac{\alpha^2\nu}{2}} + \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{\hat{b}_L} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{\alpha\hat{b}_L} e^{\frac{\alpha^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_L} e^{\frac{\varrho_2^2\nu}{2}}$$

$$\implies c_2\varrho_2 e^{\frac{\varrho_2^2\nu}{2}} = \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_2)\hat{b}_L} + m\alpha\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_2)\hat{b}_L} e^{\frac{\alpha^2\nu}{2}}$$

Use these expressions to substitute for c_1 and c_2 in the b_H value matching condition (26):

$$\begin{aligned}\psi_- e^{\hat{b}_H} = m\alpha e^{\alpha\hat{b}_H} e^{\frac{\alpha^2\nu}{2}} \\ + \left(\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_1)\hat{b}_L} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_1)\hat{b}_L} e^{\frac{\alpha^2\nu}{2}}\right) e^{\varrho_1\hat{b}_H} \\ + \left(\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_2)\hat{b}_L} + m\alpha\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_2)\hat{b}_L} e^{\frac{\alpha^2\nu}{2}}\right) e^{\varrho_2\hat{b}_H}\end{aligned}$$

Express in terms of the difference $\Delta \equiv \hat{b}_H - \hat{b}_L$:

$$\psi_- = m\alpha e^{(\alpha-1)(\hat{b}_L+\Delta)} e^{\frac{\alpha^2 v}{2}} + \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_1-1)\Delta} e^{\frac{\alpha^2 v}{2}}$$
$$+ \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta} + m\alpha\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_2-1)\Delta} e^{\frac{\alpha^2 v}{2}} \tag{29}$$

and do the same for the super contact condition (28):

$$\psi_- = m\alpha^2 e^{(\alpha-1)(\hat{b}_L+\Delta)} e^{\frac{\alpha^2 v}{2}} + \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - m\alpha\varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_1-1)\Delta} e^{\frac{\alpha^2 v}{2}}$$
$$+ \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta} + m\alpha\varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_2-1)\Delta} e^{\frac{\alpha^2 v}{2}} \tag{30}$$

Collect terms in equation (29):

$$\psi_- = m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 v}{2}} \left(e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right)$$
$$+ \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} + \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}$$

and equation (30):

$$\psi_- = m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 v}{2}} \left(\alpha e^{(\alpha-1)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right)$$
$$+ \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta} + \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta}$$

Rearrange both to isolate $\hat{b}_L$:

$$m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 v}{2}} = \frac{\psi_- - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{\left(e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right)}$$

$$m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 v}{2}} = \frac{\psi_- - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{\left(\alpha e^{(\alpha-1)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right)}$$

either of which give $\hat{b}_L$ in terms of Δ.

Combining the two equations yields an implicit equation determining Δ:

$$\frac{\psi_- - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{\psi_- - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}} = \frac{\left(e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\varrho_2-1)\Delta}\right)}{\left(\alpha e^{(\alpha-1)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-1)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\varrho_2-1)\Delta}\right)}$$

which can be rearranged as

$$\frac{\psi_- e^{(1-\varrho_2)\Delta} - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+}{\psi_- e^{(1-\varrho_2)\Delta} - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+} = \frac{\left(e^{(\alpha-\varrho_2)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)}{\left(\alpha e^{(\alpha-\varrho_2)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)}$$

$\square$

With Lemma 8 in hand, proving Lemma 7 is straightforward:

Proof of Lemma 7. ν does not appear in equation (23), so Δ is unaffected by the information friction. b_L^{FI} denotes the solution for $\nu = 0$. Equation (24) implies

$$\hat{b}_L = b_L^{FI} + \frac{\alpha^2 \nu}{2(1-\alpha)}$$

Finally, $\Delta = \hat{b}_H - \hat{b}_L = b_H^{FI} - b_L^{FI}$ implies

$$\hat{b}_H = b_H^{FI} + \frac{\alpha^2 \nu}{2(1-\alpha)}$$

$\square$

Appendix C The Joint Distribution

This appendix derives the analytical joint distribution of capital and productivity by solving a PDE: the multi-dimensional Kolmogorov Forward Equation.

C.1 Transformation and Solution of the Kolmogorov Forward Equation

The stationary joint distribution of capital and expected productivity $g(k, \hat{a})$ is determined by the stationary KFE (10). To solve for this distribution, it is convenient to first find the joint distribution of k and the deviation of expected normalized capital $\hat{x}$:

$$f_{k,\hat{x}}(k,\hat{x}) \equiv g(k,k-\hat{x}).$$

with $f_{k,\hat{x}}(k,\hat{x})=0$ for $\hat{x}<\hat{b}$.

Lemma 9. *The stationary distribution $f_{k,\hat{x}}(k,\hat{x})$ satisfies the stationary KFE*

$$D\,\partial^2_{\hat{x}} f_{k,\hat{x}} + \delta\,\partial_{\hat{x}} f_{k,\hat{x}} = -\delta\,\partial_k f_{k,\hat{x}} + \eta\, f_{k,\hat{x}},$$

Proof. The joint distributions are related by $f_{k,\hat{x}}(k,\hat{x})=g(k,\hat{a})$ with $\hat{a}=k-\hat{x}$. By the chain rule:

$$\begin{aligned}
\partial_{\hat{x}} f_{k,\hat{x}}(k,\hat{x}) &= -\partial_{\hat{a}} g(k,\hat{a})\\
\partial^2_{\hat{x}} f_{k,\hat{x}}(k,\hat{x}) &= \partial^2_{\hat{a}} g(k,\hat{a})\\
\partial_k f_{k,\hat{x}}(k,\hat{x}) &= \partial_k g(k,\hat{a}) + \partial_{\hat{a}} g(k,\hat{a})
\end{aligned}$$

Substitute into the KFE (10):

$$0=\delta\left(\partial_k f_{k,\hat{x}}(k,\hat{x})-\partial_{\hat{a}} g(k,\hat{a})\right)+D\partial^2_{\hat{a}} g(k,\hat{a})-\eta g(k,\hat{a})$$

$$0=\delta\partial_k f_{k,\hat{x}}(k,\hat{x})+\delta\partial_{\hat{x}} f_{k,\hat{x}}(k,\hat{x})+D\partial^2_{\hat{x}} f_{k,\hat{x}}(k,\hat{x})-\eta g(k,\hat{a})$$

Rearranging gives the solution. □

The flow of firms outwards across the barrier depends on the capital level:

$$Flux(k)=D\partial_{\hat{x}} f_{k,\hat{x}}(k,\hat{b})+\delta f_{k,\hat{x}}(k,\hat{b})$$

Recall that the distribution of expected productivity among entering firms is $\hat{a}_{enter}\sim N(0,\varsigma)$. Thus, conditional on k, this is also the distribution of expected normalized capital. The measure of entering firms is η, so the flux at the boundary must be

$$-\eta\phi\left(\frac{k}{\varsigma}\right)=D\partial_{\hat{x}} f_{k,\hat{x}}(k,\hat{b})+\delta f_{k,\hat{x}}(k,\hat{b}) \tag{31}$$

where $\phi(\cdot)$ is the standard normal pdf. This is the first boundary condition. The solution remains indeterminate, so we also must impose that $f_{k,\hat{x}}$ is a density, i.e.:

$$1=\iint f_{k,\hat{x}}(k,\hat{b})dkd\hat{x} \tag{32}$$

and

$$f_{k,\hat{x}}(k,\hat{b}) \geq 0 \ \forall k,\ \hat{x} \geq \hat{b}$$

C.2 Solving for the Transformed Distribution

Lemma 10. *The stationary joint distribution of capital and expected normalized capital $f_{k,\hat{x}}(k,\hat{b})$ is*

$$f_{k,\hat{x}}(k,\hat{x}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{-\mathcal{N}(\xi)}{D\lambda_{-}(\xi)+\delta} e^{i\xi k+\lambda_{-}(\xi)(\hat{x}-\hat{b})} d\xi$$

where $\mathcal{N}(\xi)$ denotes the Fourier transform of $\eta\phi\left(\frac{k}{\varsigma}\right)$ and $\lambda_{-}(\xi) \equiv \frac{-\delta-\sqrt{\delta^2+4D(i\delta\xi+\eta)}}{2D}$.

Proof. Apply the Fourier transform in k:

$$\tilde{f}(\hat{x},\xi) = \int_{-\infty}^{\infty} f_{k,\hat{x}}(k,\hat{x})\, e^{-i\xi k}\, dk$$

which converts the PDE into an ODE in $\hat{x}$:

$$D\,\partial_{\hat{x}}^2 \tilde{f} + \delta\,\partial_{\hat{x}}\tilde{f} = (i\delta\xi+\eta)\,\tilde{f}$$

The general solution is

$$\tilde{f}(\hat{x},\xi) = A(\xi)\, e^{\lambda_{+}(\xi)(\hat{x}-\hat{b})} + B(\xi)\, e^{\lambda_{-}(\xi)(\hat{x}-\hat{b})} \tag{33}$$

with characteristic roots

$$\lambda_{\pm}(\xi) = \frac{-\delta \pm \sqrt{\delta^2+4D(i\delta\xi+\eta)}}{2D}$$

The flux boundary condition determines a linear relationship between $A(\xi)$ and $B(\xi)$. Take the Fourier transform of equation (31):

$$-\mathcal{N}(\xi) = D\left(\lambda_{+}(\xi)A(\xi)+\lambda_{-}(\xi)B(\xi)\right) + \delta\left(A(\xi)+B(\xi)\right) \tag{34}$$

This gives a linear relationship between $B(\xi)$ and $A(\xi)$.

The integrating constraint pins down $A(0)$:

$$1 = \int_{\hat{x}}\int_{k} f_{k,\hat{x}}(k,\hat{x})\, dk\, d\hat{x} = \frac{1}{2\pi}\int_{\hat{x}}\int_{k}\int_{\xi} \tilde{f}(\hat{x},\xi) e^{i\xi k}\, d\xi\, dk\, d\hat{x}$$

$$= \frac{1}{2\pi} \int_{\hat{x}} \int_{\xi} \tilde{f}(\hat{x}, \xi) \left(\int_k e^{i\xi k}\, dk \right) d\xi\, d\hat{x}$$

Use $\int_k e^{i\xi k}\, dk = 2\pi\delta(\xi)$ where $\delta(\xi)$ denotes the Dirac delta:

$$= \int_{\hat{x}} \int_{\xi} \tilde{f}(\hat{x}, \xi)\delta(\xi) d\xi\, d\hat{x} = \int_{\hat{x}} \tilde{f}(\hat{x}, 0) d\hat{x}$$

$$= \int_{\hat{x}} \left(A(0) e^{\lambda_+(0)(\hat{x}-\hat{b})} + B(0) e^{\lambda_-(0)(\hat{x}-\hat{b})} \right) d\hat{x}$$

For the integral to remain finite, it must be that $A(0) = 0$.

For non-zero $A(\xi)$, the non-negativity constraint requires $A(\xi) = 0$, else equation (33) implies explosive oscillations as $\hat{x}$ becomes large. Combining the $A(\xi) = 0$ constraint with equation (34) gives

$$B(\xi) = \frac{-\mathcal{N}(\xi)}{D\lambda_-(\xi) + \delta}$$

Plug this into the general solution (33) and invert the Fourier transform:

$$f_{k,\hat{x}}(k, \hat{x}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\hat{x}, \xi) e^{i\xi k} d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-\mathcal{N}(\xi)}{D\lambda_-(\xi) + \delta} e^{i\xi k + \lambda_-(\xi)(\hat{x}-\hat{b})} d\xi$$

□

Figure 9 presents the joint stationary density function $f(\hat{x}, k)$ given our parametrization and a level of noise $\xi_{30} = 0.055$.

C.3 Recovering the Original Distribution

The original object of interest, $g(k, \hat{a})$ is then reconstructed by a change of variable.

Proposition 6. *The stationary joint distribution of capital and expected productivity is*

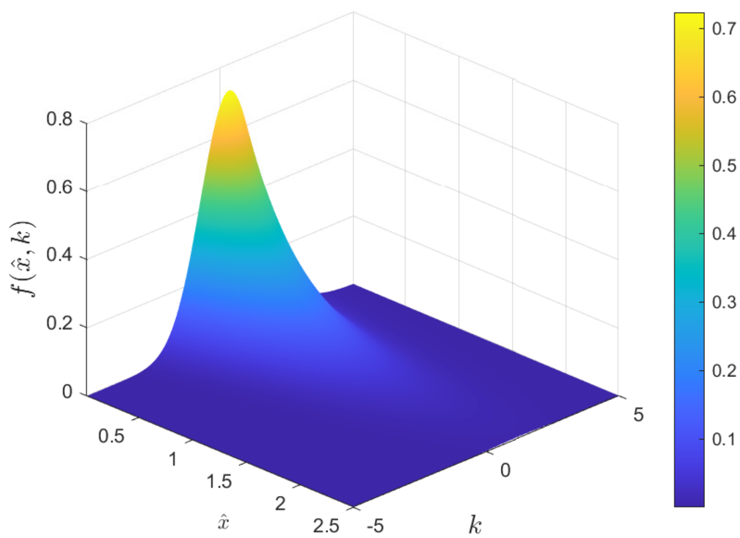
$$g(k, \hat{a}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-\mathcal{N}(\xi)}{D\lambda_-(\xi) + \delta} e^{i\xi k + \lambda_-(\xi)(k-\hat{a}-\hat{b})} d\xi$$

for $k \geq \hat{a} + \hat{b}$, and zero otherwise.

Proof. The distribution $g(k, \hat{a})$, is constructed by a change of variable using $\hat{x} = k - \hat{a}$:

$$g(k, \hat{a}) = \begin{cases} f_{k,\hat{x}}(k, k - \hat{a}) & \text{if } \hat{b} \leq k - \hat{a} \\ 0 & \text{otherwise} \end{cases}$$

Figure 9: Joint Stationary Density Function $f(\hat{x}, k)$



Notes: $f(\hat{x}, k)$ measures the joint density for log capital k and expected log normalized capital $\hat{x}$. The grid for k is normalized around 0, while the grid for $\hat{x}$ is normalized with boundary 0.

and $f_{k,\hat{x}}(k, k - \hat{a})$ is given by Lemma 10. □

As in the univariate case, the joint distribution with nowcast errors is simply

$$f_{k,\hat{a},u}(k, \hat{a}, u) = g(k, \hat{a})\phi(\frac{u}{\sqrt{v}}) \tag{35}$$

because u is independent of k and $\hat{a}$ and normally distributed.

Corollary 3. *The joint distribution of capital and productivity is*

$$f_{k,a}(k, a) = \int_{\hat{a}} g(k, \hat{a})\phi(\frac{a - \hat{a}}{\sqrt{v}})d\hat{a}$$

Proof. Apply the change of variable $u = a - \hat{a}$ to equation (35) to get the joint distribution

$$f_{k,\hat{a},a}(k, \hat{a}, a) = g(k, \hat{a})\phi(\frac{a - \hat{a}}{\sqrt{v}})$$

and integrate over $\hat{a}$ to yield

$$f_{k,a}(k, a) = \int_{\hat{a}} f_{k,\hat{a},a}(k, \hat{a}, a)d\hat{a} = \int_{\hat{a}} g(k, \hat{a})\phi(\frac{a - \hat{a}}{\sqrt{v}})d\hat{a}$$

□

Appendix D Partial Irreversibility

In this appendix, we modify the baseline model to relax the assumption of full irreversibility.

Investment I is now partially irreversible. If firms invest, they do so at cost $\Psi(I)$:

$$\Psi(I) = \begin{cases} \psi_+ I & I \geq 0 \\ \psi_- I & I < 0 \end{cases}$$

with $\psi_+ > \psi_- > 0$. Accordingly, their instantaneous profit is $\pi = A^{1-\alpha}K^{\alpha} - \Psi(I)$.

Optimal firm behavior for this type of problem is characterized by an inaction region: for a range of capital values (that depends on other state variables), firms choose to neither invest nor divest. Firms with partial irreversibility face the usual full information HJB equation (1) in the inaction region.

Optimal investment behavior is a threshold strategy, as in the full information case. Except now, a firm invests only if its expected normalized capital $\hat{X}$ is less than than some critical lower value $\hat{B}_L$, and divests only if above some upper value $\hat{B}_U$. So solving the firm's problem comes down to finding the optimal choice of $\hat{B}_L$ and $\hat{B}_U$. Lemma 2 reports the boundary conditions associated with the optimum. They are analogous to the full information case.

Lemma 11. *Under incomplete information, the boundary conditions consist of two value-matching conditions:*

$$\hat{V}'(\hat{B}_L) = \psi_+ \qquad \hat{V}'(\hat{B}_U) = \psi_-$$

and two super contact conditions:

$$\hat{V}''(\hat{B}_L) = 0 \qquad \hat{V}''(\hat{B}_U) = 0$$

Proof: Appendix B.7

Lemma 7 summarizes the solution to the firm's problem. The log critical values $\hat{b}_L \equiv \log \hat{B}_L$ and $\hat{b}_U \equiv \log \hat{B}_U$ depend on several parameters: the interest rate r, depreciation rate δ, time series properties of the productivity process, the investment and divestment costs ψ_+ and ψ_-, and so forth. But conveniently, most of these terms affect the critical values in the same way that they would in the full information model. The proposition shows that the difference between full and incomplete information critical values depend only on the variance of nowcast errors ν, and the returns to scale α.

Proposition 7. *The critical values of expected normalized capital are*

$$\hat{b}_L = b_L^{FI} + \frac{\alpha^2 \nu}{2(1-\alpha)} \qquad\qquad \hat{b}_H = b_H^{FI} + \frac{\alpha^2 \nu}{2(1-\alpha)}$$

where b_L^{FI} and b_H^{FI} denote the full information solutions such that $\nu = 0$.

Proof: Appendix B.9

Appendix E Full Reversibility

This appendix derives capital demand when investment is fully reversible, under both full and incomplete information.

E.1 Full Reversibility with Full Information

This section quickly derives the (well-known) capital demand without either friction. It follows directly from the property that firms set the marginal product of capital equal to the user cost.

Proposition 8. *Under full reversibility and full information, firms' demand for capital is*

$$K = \left(\frac{\alpha}{(r+\delta)\psi}\right)^{\frac{1}{1-\alpha}} A$$

Proof. With reversibility, firms' HJB equation (1) becomes

$$rV(K,A) = \max_I A^{1-\alpha}K^{\alpha} - \psi I + (I - \delta K)V_K(K,A) + \frac{\sigma_a^2 A^2}{2} V_{AA}(K,A) \tag{36}$$

which has first order condition

$$\psi = V_K(K,A)$$

The envelope condition is

$$rV_K(K,A) = \alpha A^{1-\alpha}K^{\alpha-1} - \delta V_K(K,A) + (I-\delta K)V_{KK}(K,A) + \frac{\sigma_a^2 A^2}{2} V_{KAA}(K,A)$$

This simplifies because the first order condition implies $0 = V_{KK}(K,A) = V_{KAA}(K,A)$. Rearrange:

$$(r+\delta)V_K(K,A) = \alpha A^{1-\alpha}K^{\alpha-1}$$

then substitute with ψ:

$$(r+\delta)\psi = \alpha A^{1-\alpha}K^{\alpha-1}$$

and rearranging provides the desired expression. $\square$

E.2 Full Reversibility with Incomplete Information

This section derives the capital demand by firms that face only the information friction. In this case, firms set the user cost equal to their *expected* marginal product of capital.

With incomplete information, the firm's state variables are K and $\hat{A}$, where $\log \hat{A} = \hat{a} = \mathbb{E}[a]$. Because a is normal with constant variance, $\hat{A}$ is a sufficient state variable for firms' information. Proposition 9 gives the firms' demand for capital in terms of $\hat{A}$.

Proposition 9. *Under full reversibility and incomplete information, firms' demand for capital is*

$$K = \left(\frac{\alpha}{(r+\delta)\psi}\right)^{\frac{1}{1-\alpha}} e^{\frac{(1-\alpha)v}{2}} \hat{A}$$

Proof. Under incomplete information, the firm maximizes its expected present value of future profits. The HJB becomes

$$rV(K,\hat{A})dt = \max_I \mathbb{E}\left[A^{1-\alpha}K^\alpha dt - \psi I + V_K(K,\hat{A})dK + V_{\hat{A}}(K,\hat{A})d\hat{A}\right]$$

Apply Itô's lemma using Lemma 1 to get the usual form:

$$rV(K,\hat{A}) = \max_I \mathbb{E}\left[A^{1-\alpha}K^\alpha\right] - \psi I + (I-\delta K)V_K(K,\hat{A}) + \frac{\sigma_a^2\hat{A}^2}{2}V_{\hat{A}\hat{A}}(K,\hat{A})$$

Recall $\hat{A} = e^{\hat{a}} = e^{\mathbb{E}[a]}$, so

$$\mathbb{E}\left[A^{1-\alpha}K^\alpha\right] = \mathbb{E}\left[e^{(1-\alpha)a}\right]K^\alpha = e^{(1-\alpha)\hat{a}+\frac{(1-\alpha)^2 v}{2}}K^\alpha$$

by Lemma 1 and the log-normality of $e^{(1-\alpha)a}$. Therefore the HJB becomes

$$rV(K,\hat{A}) = \max_I e^{\frac{(1-\alpha)^2 v}{2}}\hat{A}^{1-\alpha}K^\alpha - \psi I + (I-\delta K)V_K(K,\hat{A}) + \frac{\sigma_a^2\hat{A}^2}{2}V_{\hat{A}\hat{A}}(K,\hat{A})$$

which is similar to the full information HJB in equation (36), albeit with the revenue scaled by $e^{\frac{(1-\alpha)^2 v}{2}}$. Following the same logic as in the proof of Proposition 8,

$$(r+\delta)\psi = \alpha\hat{A}^{1-\alpha}K^{\alpha-1}e^{\frac{(1-\alpha)^2 v}{2}}$$

thus the user cost $(r+\delta)\psi$ is equal to the expected marginal product of capital. Substituting with $\hat{X} = \frac{K}{\hat{A}}$ gives

$$(r+\delta)\psi = \alpha \hat{X}^{\alpha-1} e^{\frac{(1-\alpha)^2 \nu}{2}}$$

and rearranging provides the desired expression. □

Proposition 9 clarifies how incomplete information increases capital demand even without the investment friction. When firms have incomplete information, they invest until their *expected* MPK is equal to the user cost. Log productivity is uncertain and normally distributed, so Jensen's inequality raises the expected MPK relative to the median MPK. The higher the uncertainty (larger ν), the larger the effect.

Appendix F Supplements to Empirical Validation

F.1 Alternative Estimation of Attenuation Coefficients

Figure 10 shows the estimated attenuation coefficients when firm fixed effects are included. Including firm fixed effects can mechanically produce negative attenuation coefficients. The intuition is as follows. We regress $realized\ sales_{t+1} - forecasted\ sales_{t,t+1}$ on $productivity\ shock_t \equiv productivity_t - productivity_{t-1}$. Running a first-difference regression, which is analogous to including fixed effects, leads to

$$realized\ sales_{t+1} - forecasted\ sales_{t,t+1} - realized\ sales_t + forecasted\ sales_{t-1,t}$$

on the left-hand side (LHS) and

$$productivity_t - 2 * productivity_{t-1} + productivity_{t-2}$$

on the right-hand side (RHS). Importantly, $realized\ sales_t$ appears negatively on the LHS and is highly positively correlated with $productivity_t$ on the RHS. This creates a mechanically negative correlation between the LHS and RHS. Because measurement errors are inevitable, including firm fixed effects can induce a downward bias in the estimated attenuation coefficients.

F.2 Regression Results Using Labor Productivity

We present the full regression results for Table 4 in this subsection of the appendix. See Tables 5 and 6 below for more details.

Table 5: Incomplete Information and Investment Sensitivity: Labor Productivity

	(1)	(2)	(3)	(4)
	inv. inaction = 1			
$\xi_s \times w_{i,t}$	0.009*	0.010**	0.010**	0.011**
	(0.005)	(0.005)	(0.005)	(0.005)
$w_{i,t}$	-0.003	-0.036	0.003	-0.030
	(0.009)	(0.031)	(0.010)	(0.031)
$a_{i,t-1}$	-0.028**	-0.028**	-0.015	-0.015
	(0.012)	(0.012)	(0.012)	(0.012)
$m_{i,t}$	-0.005	-0.005	-0.006	-0.006
	(0.006)	(0.006)	(0.005)	(0.005)
$k_{i,t-1}$	0.077***	0.077***	0.082***	0.082***
	(0.009)	(0.009)	(0.008)	(0.008)
$cap\ share_s \times w_{i,t}$		0.048		0.048
		(0.043)		(0.043)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes
N	84656	84656	84656	84656
adj. R^2	0.446	0.446	0.451	0.451

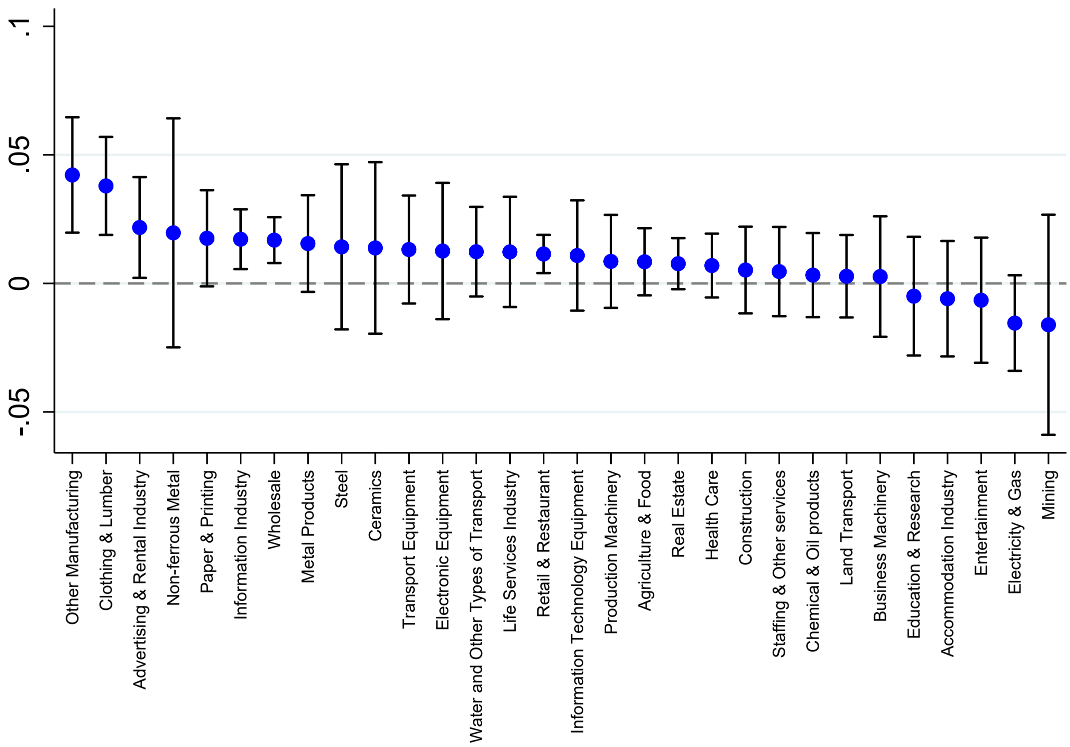
Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are winsorized.

Table 6: Incomplete Information and Investment Sensitivity: AR(1) Process of Labor Productivity

	(1)	(2)	(3)	(4)
	inaction= 1			
$\xi_s \times w_{i,t}$	0.010*	0.011**	0.009*	0.010**
	(0.005)	(0.005)	(0.005)	(0.005)
$w_{i,t}$	-0.005	-0.036	0.002	-0.029
	(0.009)	(0.032)	(0.009)	(0.032)
$a_{i,t-1}$	-0.029**	-0.029**	-0.016	-0.016
	(0.011)	(0.011)	(0.011)	(0.011)
$m_{i,t}$	-0.005	-0.005	-0.006	-0.006
	(0.005)	(0.005)	(0.005)	(0.005)
$k_{i,t-1}$	0.076***	0.076***	0.082***	0.082***
	(0.009)	(0.009)	(0.008)	(0.008)
$cap\ share_s \times w_{i,t}$		0.046		0.046
		(0.045)		(0.045)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes
N	84313	84313	84313	84313
adj. R^2	0.446	0.446	0.451	0.451

Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are winsorized. Labor productivity is assumed to follow an AR(1) process in this specification.

Figure 10: Estimated Attenuation Coefficients Across Industries with Firm FE: Labor Productivity



Notes: This figure shows how the coefficient governs the impulse response of the log sales forecast error made in period $t+1$ with respect to the realized log productivity innovation in period t. We estimate the coefficients by including the firm fixed effects. Each dot denotes the estimate for an industry (with the 95% confidence interval), and there are 30 industries in total. Top and bottom 1% observations are winsorized. The data frequency is semi-annual, and the productivity measure is labor productivity.

F.3 Robustness Checks Using Total Factor Productivity

So far, we have used labor productivity (revenue per worker) and its innovations. While transparent, this measure does not isolate the effects of other inputs, such as capital and intermediate goods usage, on firm-level productivity. Total factor productivity (TFP) is a commonly used alternative in the literature. In this subsection, we conduct robustness checks using TFP and find that all previously documented empirical results remain qualitatively unchanged.

We construct our TFP measure following the standard approach in the IO literature (e.g., Olley and Pakes (1996); Levinsohn and Petrin (2003)). Specifically, we use the Olley and Pakes (1996) method, employing investment as a proxy to invert for TFP conditional on capital stock. This approach is suitable for our analysis for two reasons. First, our sample consists of large firms, so the usual zero-investment issue is less of a concern. The investment inaction rate is 36%, substantially higher than the share of zero investment (15%) in our data, as inaction includes investment rates $\leq 1\%$. Second, the usual collinearity problem between intermediate goods usage and labor is mitigated because we use firm investment as the proxy.

We estimate a firm-level Cobb-Douglas production function with labor, capital, and intermediate goods as inputs. To address sample size constraints, we group firms into four broad sectors for sector-specific production function estimation: (1) light manufacturing (e.g., food and beverages, textiles, clothing, footwear, printing, lumber), (2) heavy manufacturing (e.g., construction, chemicals, metals, steel, oil-related products), (3) machinery (e.g., production machinery, business and electrical equipment, telecommunications, transportation equipment), and (4) services (e.g., real estate, finance, accommodation, catering, healthcare, transportation, entertainment, rental and leasing). The estimated production function coefficients are reported in Table 7.

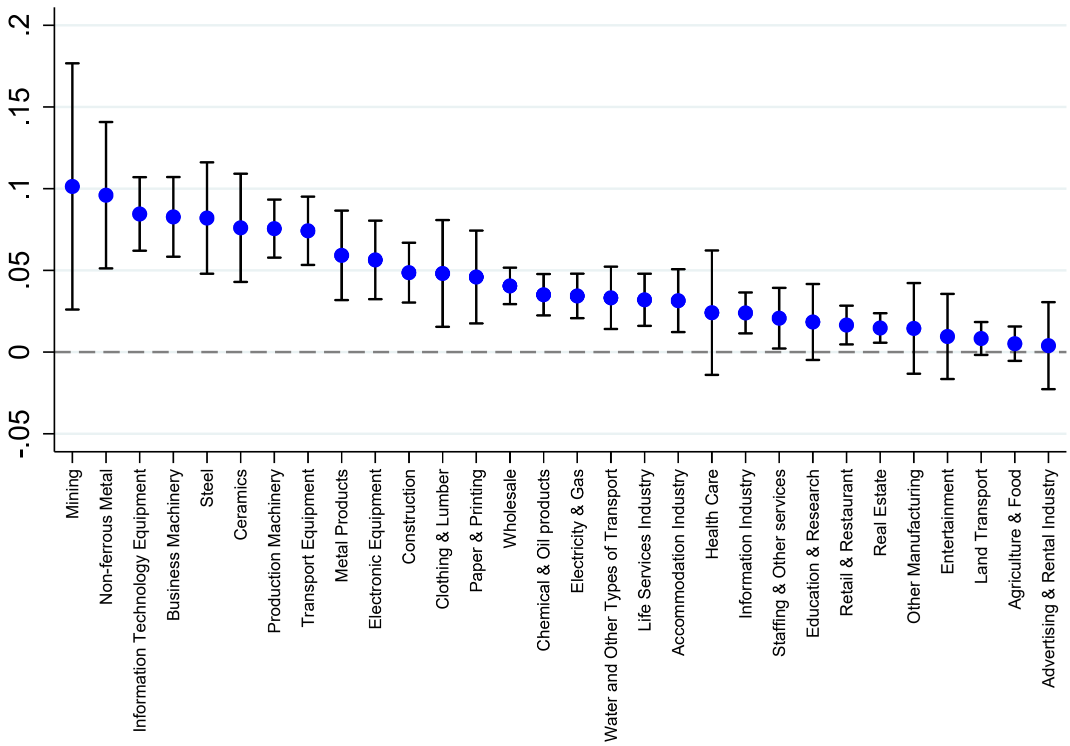
Table 7: Estimated Production Function

Sector	coef. of labor	coef. of intermediate goods	coef. of capital
Light manufacturing	0.240	0.630	0.162
Heavy manufacturing	0.212	0.578	0.207
Machinery	0.269	0.585	0.160
Service	0.231	0.554	0.163

Notes: The period is 2004-2018 (15 years and 29 semi-years). All the estimated coefficients are highly significant.

As expected, the estimated production function shows higher capital intensity in the heavy manufacturing sector and higher labor intensity in the other three sectors. Across all sectors, technology exhibits near-constant returns to scale. We then re-estimate the industry-specific coefficient governing the impulse response of (log) sales forecast errors in period $t + 1$ to realized (log) productivity innovations in period t for the 30 industries. The attenuation coefficients estimated using labor productivity and TFP are positively correlated, with a correlation of 0.44.

Figure 11: Estimated Attenuation Coefficients Across Industries: Using Estimated TFP



Notes: This figure shows how the coefficient governs the impulse response of the (log) sales forecast error made in period $t + 1$ with respect to the realized (log) productivity innovation in period t. Each dot denotes the estimate for an industry (with the 95% confidence interval), and there are 30 industries in total. Top and bottom 1% observations are winsorized.

We rerun the main regressions as in equations (12) and (13) using firm-level TFP and the corresponding attenuation coefficients. Table 8 shows that results for equation (12) are largely unchanged from Table 3. Table 9 presents results for equation (13); although the coefficient of interest ($\xi_s \times w_{i,t}$) is slightly smaller, it remains positive and statistically significant, consistent with findings using labor productivity. Using an AR(1) process to estimate productivity innovations yields qualitatively similar results in Table 10. Overall, our empirical findings are robust to using TFP to construct the attenuation coefficients.

Table 8: Incomplete Information and Investment Inaction: Using Estimated TFP

	inv. inaction = 1			
ξ_s	-0.037	-0.029	-0.046*	-0.035*
	(0.025)	(0.019)	(0.026)	(0.019)
$a_{i,t}$	0.072	0.039	0.056	0.032
	(0.053)	(0.042)	(0.045)	(0.0356)
$k_{i,t-1}$	-0.040***	-0.034***	-0.037***	-0.033***
	(0.007)	(0.007)	(0.008)	(0.008)
$m_{i,t}$	0.042**	0.046**	0.033**	0.040***
	(0.019)	(0.017)	(0.016)	(0.014)
$growth\ vol_s$		1.044***		0.971***
		(0.275)		(0.285)
$cap\ share_s$			-0.384	-0.230
			(0.362)	(0.328)
Time FE	Yes	Yes	Yes	Yes
N	84941	84941	84941	84941
adj. R^2	0.057	0.083	0.064	0.086

Notes: Standard errors are clustered at the industry level. * 0.10 ** 0.05 *** 0.01. The degree of information friction is estimated at the industry level. Top and bottom 1% productivity observations are winsorized.

Table 9: Incomplete Information and Investment Sensitivity: Using Estimated TFP

	(1)	(2)	(3)	(4)
	inv. inaction = 1			
$\xi_s \times w_{i,t}$	0.008**	0.008**	0.007*	0.007**
	(0.004)	(0.004)	(0.004)	(0.004)
$w_{i,t}$	-0.107***	-0.112***	-0.109***	-0.123***
	(0.031)	(0.043)	(0.031)	(0.043)
$a_{i,t-1}$	-0.088***	-0.088***	-0.079***	-0.079***
	(0.013)	(0.013)	(0.013)	(0.013)
$m_{i,t}$	-0.003	-0.003	0.0003	0.0005
	(0.006)	(0.006)	(0.006)	(0.006)
$k_{i,t-1}$	0.068***	0.068***	0.076***	0.077***
	(0.009)	(0.009)	(0.009)	(0.009)
$vol_s \times w_{i,t}$	0.137***	0.138***	0.147***	0.153***
	(0.050)	(0.051)	(0.050)	(0.051)
$cap\ share_s \times w_{i,t}$		0.005		0.015
		(0.036)		(0.036)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes
N	80305	80305	80305	80305
adj. R^2	0.447	0.447	0.452	0.452

Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are winsorized.

Table 10: Incomplete Information and Investment Sensitivity: AR(1) Process of Estimated TFP

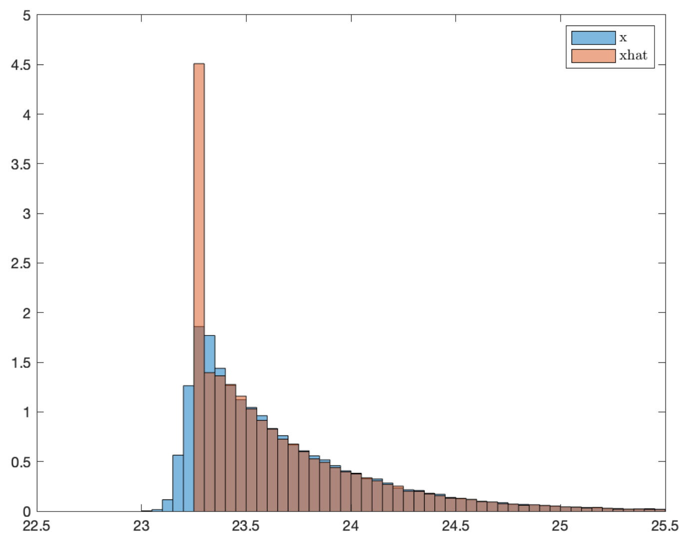
	(1)	(2)	(3)	(4)
		inv. inaction =1		
$\xi_s \times w_{i,t}$	0.007	0.007	0.006	0.007
	(0.005)	(0.005)	(0.005)	(0.005)
$w_{i,t}$	-0.129***	-0.143**	-0.122***	-0.141**
	(0.042)	(0.058)	(0.042)	(0.058)
$a_{i,t-1}$	-0.073***	-0.073***	-0.066***	-0.066***
	(0.010)	(0.010)	(0.010)	(0.010)
$m_{i,t}$	-0.002	-0.002	0.001	0.001
	(0.006)	(0.006)	(0.006)	(0.006)
$k_{i,t-1}$	0.068***	0.068***	0.077***	0.077***
	(0.009)	(0.009)	(0.009)	(0.009)
$vol_s \times w_{i,t}$	0.175**	0.181***	0.173**	0.181***
	(0.069)	(0.069)	(0.068)	(0.069)
$cap\ share_s \times w_{i,t}$		0.015		0.020
		(0.049)		(0.048)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes
N	80301	80301	80301	80301
adj. R^2	0.447	0.447	0.452	0.452

Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are winsorized.

Appendix G Supplements to Quantitative Simulation

For the distributions in Figure 6, we simulate 500,000 firms over 500 quarters. For industry-level correlations, we simulate 10,000 firms over 50 quarters per industry. Although the simulation uses discrete time (quarterly periods), Figure 12 validates that shrinking the interval produces results consistent with the continuous-time model. The simulated distributions of x and $\hat{x}$ align with theoretical predictions illustrated in Figure 2.

Figure 12: Simulated Stationary Distribution for Expected and Realized Normalized Capital



Notes: These figures show the calibrated, simulated analog of the theoretical normalized capital distribution in Figure 2. The blue histogram is the distribution of x, and the orange histogram is the $\hat{x}$ distribution. We exclude the large fixed mess of entry firms at the boundary, so the simulated stationary distribution is consistent with the theoretical one.