

Incomplete Information and Investment Inaction

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 - Firms' forecast errors are serially correlated, predictable; firm-level data imply this lowers aggregate productivity (Chen et al 2023)
- Both frictions are important, but studied individually. Do they interact?

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- Firms with worse information behave as predicted by model

Theory

Firms' Problem

- Atomistic firms face simple investment problem
- Produce using capital K and stochastic productivity A by

$$F(A, K) = A^{1-\alpha} K^\alpha$$

- Log productivity a follows a random walk:

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- Investment I is irreversible. Conditional on investing, profits are

$$\pi = A^{1-\alpha} K^\alpha - \psi I$$

- The law of motion for capital is

$$dK = I - \delta K dt$$

Firms' Behavior: Investment Inaction Region

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- Effect of incomplete information? *It determines the inaction region*

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- \hat{a} follows a random walk with the same properties as a

► Nowcast Behavior

Solving the Firm's Problem: Inaction Boundary

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- We show that the optimum is characterized by usual value-matching and super contact conditions, except applied to \hat{V} :

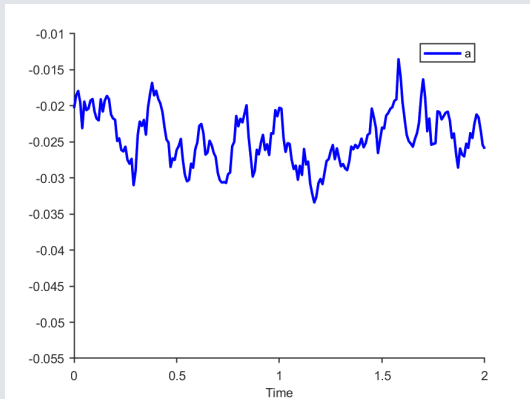
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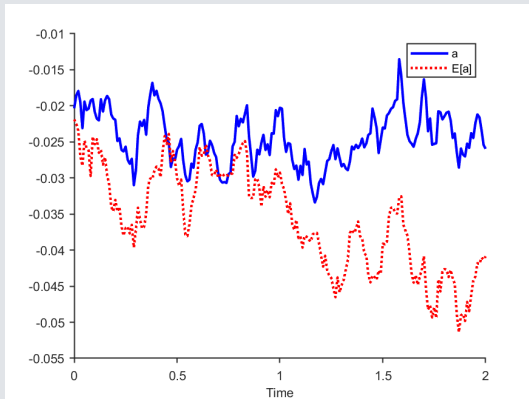
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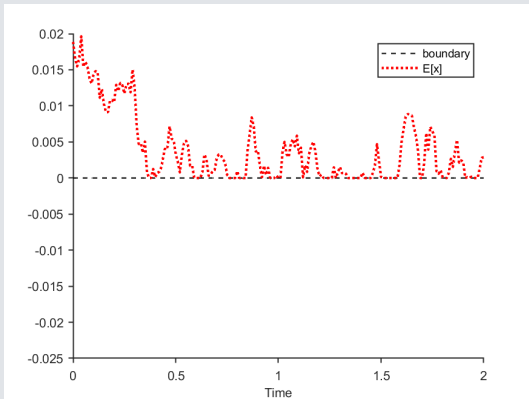
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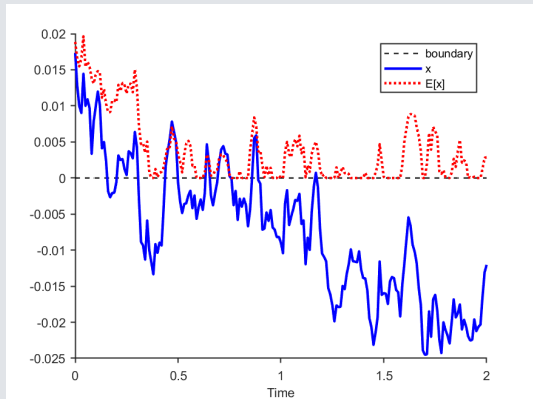
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- Actual norm. capital x follows $x = k - a = \hat{x} - \hat{a} + a$

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$$\frac{d}{dW_{t-h}^a} \mathbb{E}[a_t | \Omega_t] = \begin{cases} \gamma = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2} < 1 & 0 \leq h < \tau \\ 1 & h \geq \tau \end{cases}$$

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Predictions for later: worse information associated with

- **Lower inaction rate**, conditional on firm size
- **Lower sensitivity of investment to productivity shocks**

(Macro) Investment Behavior Under Incomplete Information

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$$\hat{b} = b^{FI} + \frac{\alpha^2}{2(1-\alpha)} \underbrace{\frac{\tau \sigma_a^2 \sigma_n^2}{\sigma_a^2 + \sigma_n^2}}_{\text{Var}[u]}$$

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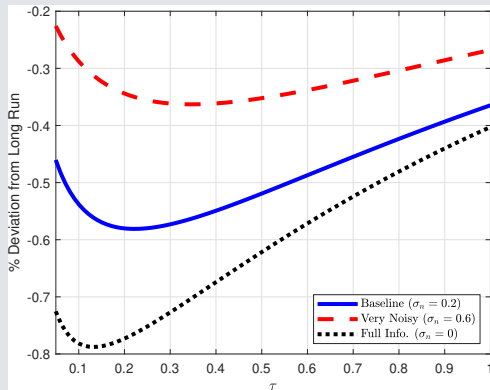
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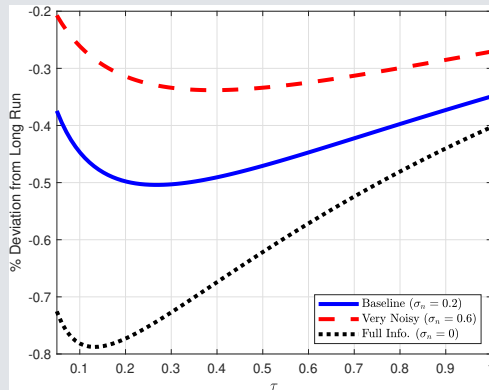
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4. Information friction **attenuates** aggregate responses to productivity shocks

Aggregate Response of $\hat{x} = k - \hat{a}$ to a Productivity Shock



Quick Revelation $\tau = 1$



Slow Revelation $\tau = 4$

Information friction **attenuates** aggregate response

Testing Theoretical Predictions

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 - Merged dataset contains firms with at least 1 billion JPY in registered capital

► Descriptive Statistics

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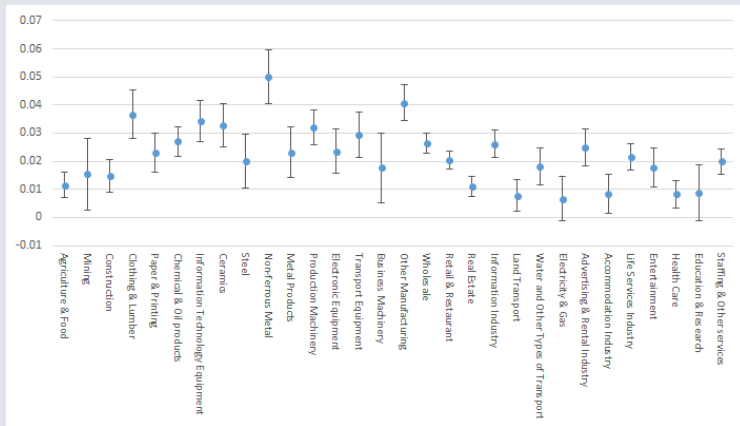
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 - Industry-time, region-time, size-time fixed effects
- Positive $\xi_s \implies$ forecast *underreaction*

Attenuation Coefficients across Industries



- Positive & statistically significant coefficients. Larger for manufacturing industries

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- γ_t is the time (semi-year) fixed effect
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Empirical Exercise 1: Information Frictions & Inv. Inaction

- Do we observe more investment inaction for firms in industries with more severe information frictions?
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- α is the coefficient of interest
- We calibrate & simulate our model and do a similar analysis for comparison.

Empirical Exercise 1: Information Frictions & Inv. Inaction

	Data			Model	
	<i>investment inaction = 1</i>			<i>investment inaction = 1</i>	
ξ_s	-0.076** (0.028)	-0.079*** (0.026)	-0.0544** (0.025)	-0.085	-0.087
$a_{i,t}$	0.039 (0.034)	0.059* (0.031)	0.104*** (0.038)	-0.028	-0.035
$k_{i,t-1}$		-0.050*** (0.009)	-0.049*** (0.008)		-0.007
$m_{i,t}$			-0.026 (0.021)		
Time FE	Yes	Yes	Yes	Yes	Yes
N	99027	99027	86294		
adj. R^2	0.038	0.069	0.063	0.052	0.053

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- More severe information frictions \Rightarrow more inaction
- 1 SD in $\xi_s \Rightarrow 5.44$ p.p. (15%) less inaction

Empirical Exercise 2: Information Frictions & Inv. Sensitivity

- Do we see lower investment sensitivity to productivity shocks in industries with stronger information frictions?

Empirical Exercise 2: Information Frictions & Inv. Sensitivity

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- We estimate

$$\mathbb{1}(\text{inaction})_{it} = \beta(w_{it} \times \xi_s) + \gamma w_{it} + \Gamma z_{it} + \gamma_s + \gamma_t + \epsilon_{it}$$

- w_{it} : productivity shock (random walk or AR(1))
- z_{it} : firm-level controls
- γ_s is the firm fixed effect
- γ_t is the time (semi year) fixed effect
- Standardize ξ_s

Empirical Exercise 2: Information Frictions & Inv. Sensitivity

	Data				Model
	inaction (random walk)	inaction (AR(1))	inaction (AR(1))	inaction	inaction
$\xi_s \times w_{i,t}$	0.009* (0.005)	0.010** (0.005)	0.010** (0.005)	0.009* (0.005)	0.136
w_{it}	-0.003 (0.009)	0.003 (0.009)	-0.005 (0.009)	0.002 (0.009)	-0.212
a_{it-1}	-0.028** (0.012)	-0.015 (0.012)	-0.029** (0.011)	-0.016 (0.011)	-0.022
Firm FE	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y
Industry-year FE	N	Y	N	Y	-
N	84656	84656	84313	84313	
adj. R^2	0.446	0.451	0.446	0.451	0.240

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Firm FE	Y	Y	Y	Y	Y
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N	84656	84656	84313	84313	
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- Dampened* inaction responses to prod. shocks in industries with higher ξ

Conclusions

- We show that information and investment frictions interact in rich ways
- Parsimonious model delivers testable predictions, consistent with the data
- Information frictions are easily incorporated into continuous time inaction models (there are many applications beyond investment)
- An alternative structure for investment frictions:
 - Old paradigm: convex fixed costs to get inaction, + convex adjustment costs to get attenuation
 - New paradigm: *irreversibility* to get inaction, + *information frictions* to get attenuation
 - Plenty of micro evidence!

Appendix

Selected Literature

1. **Partial Irreversibility: Theory** Pindyck (1991), Bertola and Caballero (1994), Abel and Eberly (1996), Veracierto (2002), Stokey (2008), Ottonello (2017), and Baley and Blanco (2022)
2. **Incomplete Information and Inaction in Continuous Time**
 - Price-setting: Alvarez, Lippi, and Paciello (2011) Alvarez, Lippi, and Paciello (2016), Baley and Blanco (2019)
 - Attention fixed costs and investment: Verona (2014)
3. **Firms in the Data: Systematic Errors in Expectations** Massenot and Pettinicchi (2018), Born et al. (2022), Andrade et al (2022) Chen et al (2023), Chen, Hattori, and Luo (2023)

How Do Firms Nowcast?

Proposition (1)

For a firm with information set $\Omega(t)$, productivity is conditionally distributed

$$a(t)|\Omega(t) \sim N(a(t-\tau) + \gamma(s(t) - s(t-\tau)), \nu)$$

where

$$\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2} \quad \nu \equiv \frac{\tau \sigma_a^2 \sigma_n^2}{\sigma_a^2 + \sigma_n^2}$$

How Do Nowcasts Behave?

Proposition (2)

A firm's expected productivity $\hat{a} \equiv \mathbb{E}[a|\Omega]$ and nowcast error u follow the diffusions

$$d\hat{a} = \sigma_a dW^{\hat{a}} \quad du = \sigma_u dW^u$$

where

$$dW_t^{\hat{a}} = (1 - \gamma)dW_{t-\tau}^A + \gamma dW_t^A + \gamma \frac{\sigma_n}{\sigma_a}(dW_t^n - dW_{t-\tau}^n)$$

$$dW_t^u = (1 - \gamma)\frac{\sigma_a}{\sigma_u}(dW_t^A - dW_{t-\tau}^A) + \gamma \frac{\sigma_n}{\sigma_u}(dW_t^n - dW_{t-\tau}^n)$$

$$\sigma_u^2 = 2 \frac{\sigma_n^2 \sigma_a^2}{\sigma_a^2 + \sigma_n^2}$$

Boundary Solution

The critical value \hat{b} depends on: the variance of nowcast errors ν , the capital share α , the cost of investment ψ , as well as ϱ and m defined as:

$$\varrho \equiv \frac{\mu - \sqrt{\mu^2 + 2\sigma_a^2 r}}{\sigma_a^2} \quad m \equiv \frac{1}{r + \mu\alpha - \frac{\sigma_a^2}{2}\alpha^2}$$

Proposition (4)

The critical value of expected normalized capital is

$$\hat{b} = \frac{1}{1-\alpha} \log \left(\frac{m\alpha}{\psi} \left(e^{\nu \frac{(1-\alpha)^2}{2}} - \frac{1-\alpha}{1-\varrho} e^{\nu \left(\frac{(2-\alpha)^2}{2} - \frac{(2-\varrho)^2}{2} + \frac{(1-\varrho)^2}{2} \right)} \right) \right)$$

Solving the Firm's Problem: Normalization

- Standard approach: define log **normalized capital**

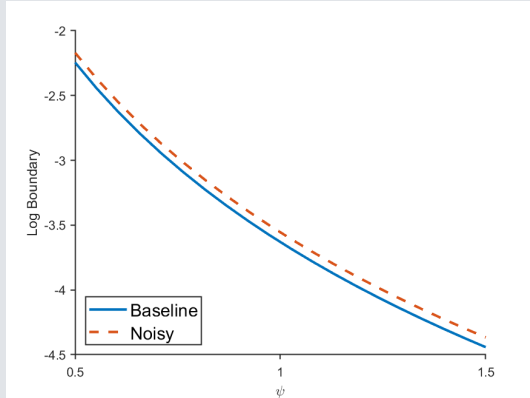
$$x \equiv k - a$$

- HJB is simpler in one dimension:

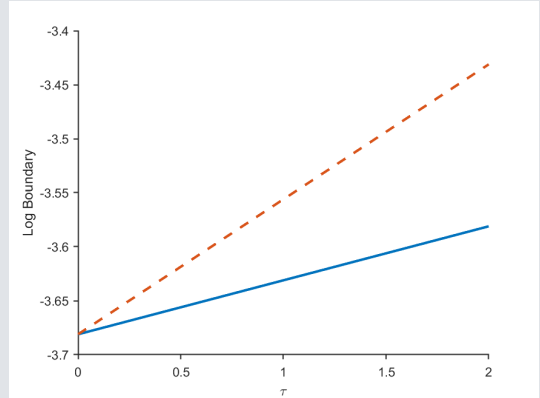
$$rv(x) = e^{\alpha x} - \mu v'(x) + \frac{\sigma_a^2}{2} v''(x)$$

where $\mu \equiv \delta + \frac{\sigma_a^2}{2}$

How the Boundary \hat{b} Depends on the Information Friction

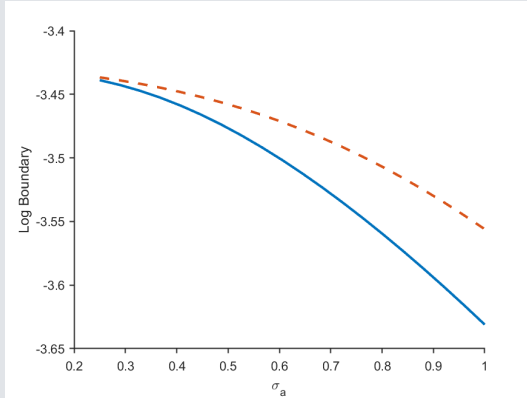


Investment Cost ψ



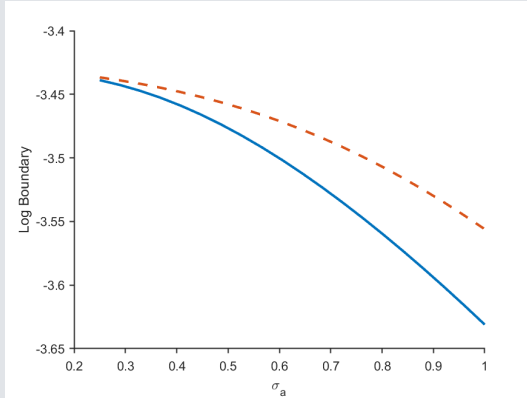
Revelation Delay τ

How the Boundary \hat{b} Depends on “Uncertainty”



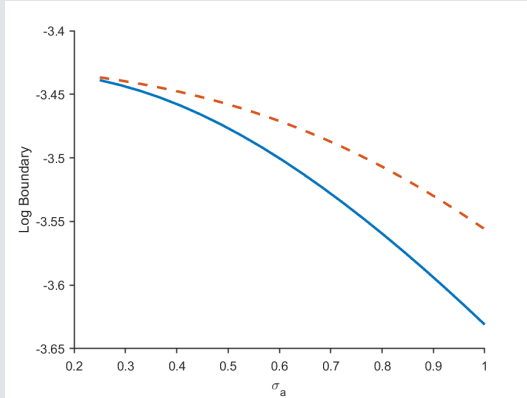
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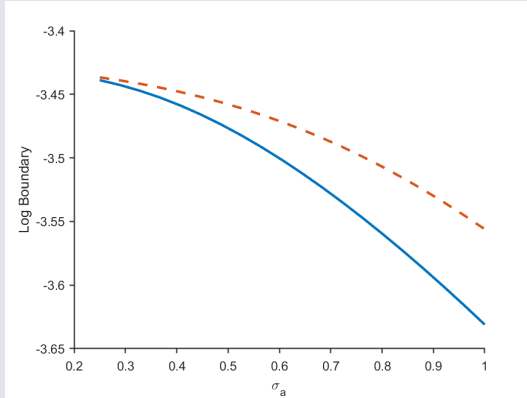
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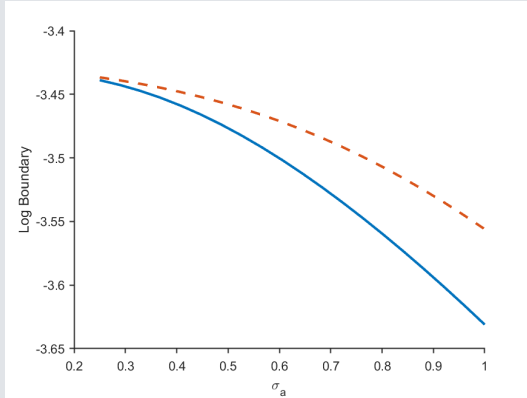
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► Back

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Information Friction's Micro Effects: Inelastic Forecasts

- Recall from Proposition (1):

$$\hat{a}(t) = a(t - \tau) + \gamma(s(t) - s(t - \tau))$$

where $\gamma = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2} < 1$ and $s = a + n$

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- Increased noise lowers the elasticity of forecasts to productivity γ :

$$\frac{d}{dW_{t-h}^a} \mathbb{E}[a_t | \Omega_t] = \begin{cases} \gamma & 0 \leq h < \tau \\ 1 & h \geq \tau \end{cases}$$

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- Prediction for later: worse information reduces γ

The Macroeconomy

- Firm entry/exit keeps the size distribution non-degenerate

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- Firms exit randomly at rate η , with value returned to owners. Measure η of firms enter at every moment

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 - Their expected normalized capital \hat{x} enters at the critical value \hat{b}

The Stationary Distribution of Expected Normalized Capital

- The Kolmogorov Forward equation (KFE) for the distribution of expected normalized capital $h(\hat{x}, t)$:

$$\partial_t h(\hat{x}, t) = \delta \partial_{\hat{x}} h(\hat{x}, t) + \frac{\sigma_a^2}{2} \partial_{\hat{x}}^2 h(\hat{x}, t) - \eta h(\hat{x}, t)$$

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- Boundary condition: $h(\hat{x})$ must integrate to one over $[\hat{b}, \infty)$
- The stationary distribution of expected normalized capital $h(\hat{x})$ for $\hat{x} \geq \hat{b}$ is

$$h(\hat{x}) = \rho e^{-\rho(\hat{x}-\hat{b})}$$

where $\rho \equiv \frac{\delta}{\sigma_a^2} + \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$ (which is *unaffected* by the info. friction)

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- Integrate to calculate the marginal distribution $f_x(x)$ of actual normalized capital $x = \hat{x} - u$:

$$f_x(x) = h(x)e^{\frac{\nu\rho^2}{2}}\Phi\left(\frac{x - (\hat{b} + \nu\rho)}{\sqrt{\nu}}\right)$$

where $\Phi(\cdot)$ is the standard normal CDF.

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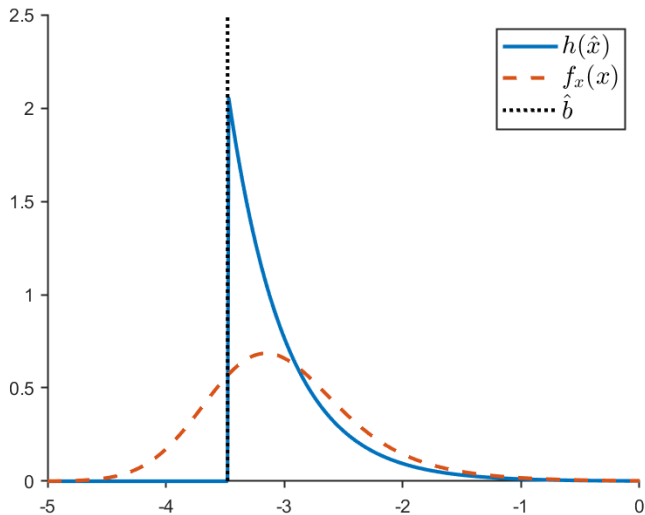
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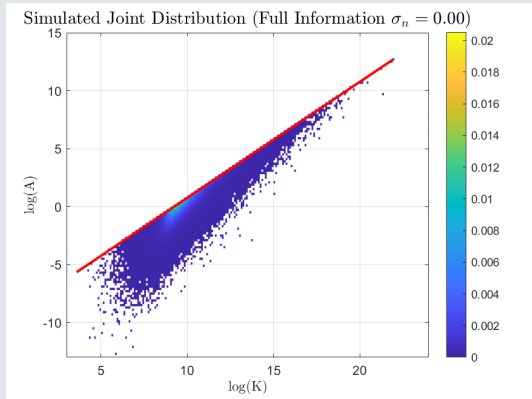
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- Error variance ν smooths out the distribution

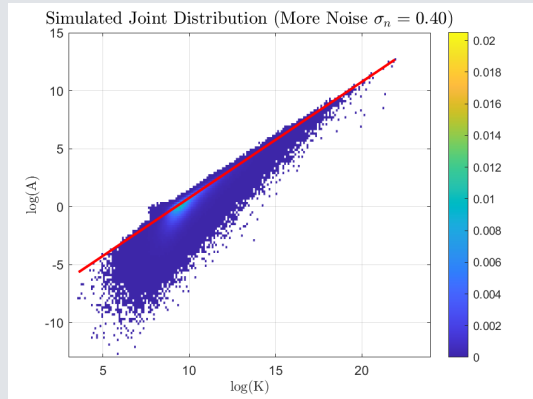
Stationary Distribution: Expected & Realized Normalized Capital



Joint Distribution for Capital and Productivity



Full Information $\sigma_n = 0$



Noisy Information $\sigma_n = 0.4$

Information Friction's Macro Effects: Greater Capital Misallocation

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- \implies noise does not affect $\text{Var}[\hat{x}]$, but does increase $\text{Var}[u]$, and thus misallocation.

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if $\rho > 1$ (otherwise infinite)

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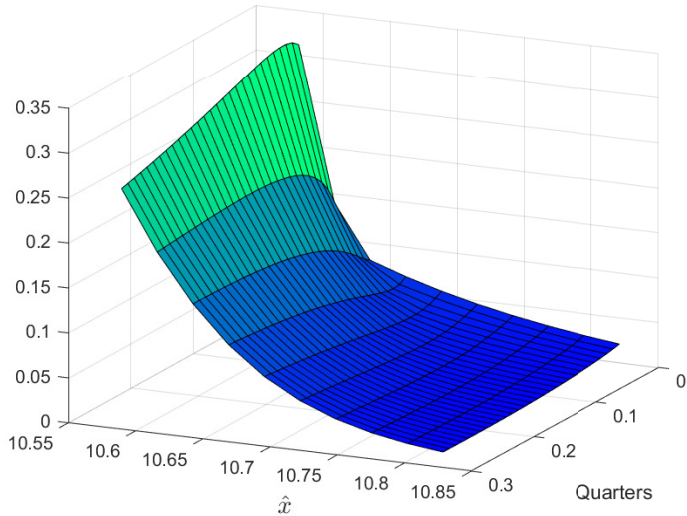
if $\rho > 1$ (otherwise infinite)

- Greater noise ($\sigma_n \uparrow$) or delay ($\tau \uparrow$) increase both the nowcast error variance ν and boundary \hat{b}

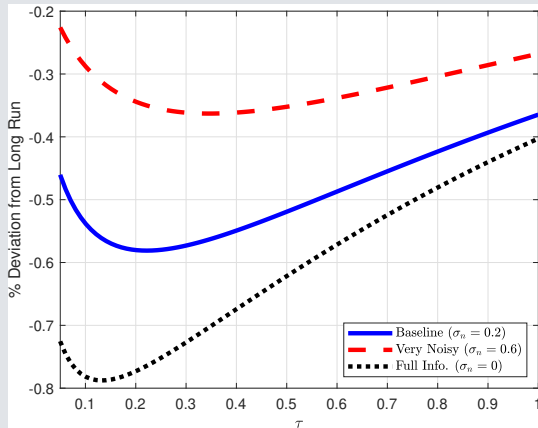
Dynamic Effects of Aggregate Productivity

- What happens if all firms receive a productivity increase da ?
- Expected productivity \hat{a} increases by γda (less than one-for-one!)
- Expected normalized capital $\hat{x} = k - \hat{a}$ decreases by γda
- ... so the entire distribution shifts left, with a mass point at the boundary \hat{b} .
- Then, the distribution evolves per the KFE.

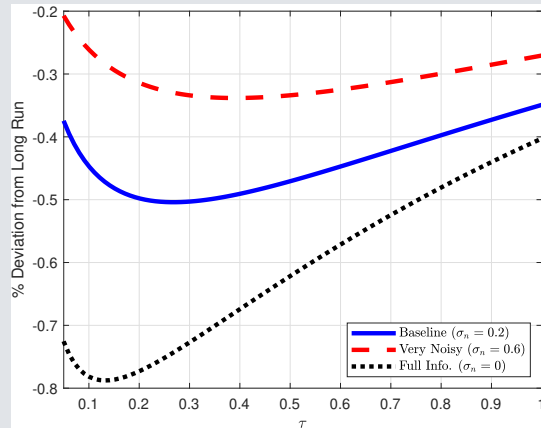
Distribution Across Time



Worse Information Attenuates the Aggregate Response



Quick Revelation $\tau = 1$



Slow Revelation $\tau = 4$

Empirical Evidence: Summary

- There is substantial heterogeneity in degree of information frictions across industries
- Information frictions reduce firm-level investment inaction
- Information frictions attenuate the firm-level investment response to firm-level productivity shocks

Datasets

- Two firm-level administrative data sets (2004-2018) from Japan:
 1. Business Outlook Survey (BOS)
 - Contains forecasts of sales, profit (semi-year frequency: Apr. to Sep. and Oct. to next Mar.) and firms' investment and investment plans (quarterly frequency).

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 2. Financial Statements Statistics of Corporations (FSS)
 - Contains financial statement information in balance sheet and profit and loss account at quarterly frequency (e.g., various assets, debt, equity, various types of capital etc.)

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- Two firm-level administrative data sets (2004-2018) from Japan:
 1. Business Outlook Survey (BOS)
 - Contains forecasts of sales, profit (semi-year frequency: Apr. to Sep. and Oct. to next Mar.) and firms' investment and investment plans (quarterly frequency).
 2. Financial Statements Statistics of Corporations (FSS)
 - Contains financial statement information in balance sheet and profit and loss account at quarterly frequency (e.g., various assets, debt, equity, various types of capital etc.)
- Basic features:
 - FSS: roughly 21,000 per quarter; BOS: roughly 11,000 per quarter
 - Cover all large firms and a representative and rotating sample of small and medium-sized firms
 - Both datasets have time-invariant common firm IDs for large firms → a merged dataset with firms that have at least 1 billion IPY in terms of

Empirical Test of Our Model

- Variables Used in Empirical Analysis
 - Forecast error (FE) = $\log \text{ realized value (e.g., sales) } - \log \text{ forecasted value}$

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 - Group firms into 30 industries: construction, metal, transportation equipment, wholesale etc.
 - Labor productivity = sales per worker
- Information friction estimated as forecast error response to productivity shocks.

Summary of the Datasets

Table 1: Sample Comparison (Quarterly)

Moments	Merged dataset	Entire Sample (FSS)
The number of obs. (non-missing sales)	392,158	1,260,836
Average employment	1040.58	491.61
Average sales (million JPY)	19991.75	8541.77
Average fixed capital stock	59919.34	24842.79

Notes: Notes: The time span is 2004-2018 (15 years and 60 quarters)

Summary Statistics of Merged Dataset (Semi-Year): Untrimmed Forecast Errors

Variable	Obs.	mean	median	standard deviation	min.	max.
log FE of sales	119,335	-.0106	-.0005	0.199	-8.472	5.759
percentage FE of sales	119,359	.0198	-.0005	1.556	1	316

Time span: 2004-2018 (29 semi-years). Forecast is made at the beginning of each semi-year.

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Model Calibration

Parameter	Interpretation	Value	Reference
r	Real interest rate	0.01	Annual rate of 4%
α	Capital share	0.67	Decreasing return to scale of 2/3
ψ	Investment cost	1.00	Normalization
δ	Depreciation rate	0.0136	Target average I/K in Japanese data
η	Exit risk	0.02	Annual exit rate of 8% in Japanese data
σ_A	S.D. of productivity process	0.20	Investment dynamics in Japanese data
σ_n	S.D. of noise process	0.20	Investment dynamics in Japanese data
τ	Revelation delay	1	Arbitrary

Standard deviations chosen to target investment moments

Investment Moments (Quarterly)

Moments	Data	Baseline Model	Full Info. ($\sigma_n = 0$)
Aggregate Investment Rate	1.36%	1.36%	1.36%
Investment Rate Mean	2.10%	2.63%	2.84%
Investment Rate S.D.	7.1%	7.1%	8.7%
Investment Rate Autocorrelation	0.70	0.51	0.25
Investment Inaction Rate	57.8%	79.7%	82.9%
Investment Spike Rate	1.4%	4.5%	5.4%

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	<i>investment inaction = 1</i>		
ξ_s	-0.0445*	-0.0401	-0.0461**
	(0.0245)	(0.0242)	(0.0231)
$a_{i,t}$	-0.0377	-0.00736	-0.0289
	(0.0683)	(0.0698)	(0.0386)
$k_{i,t-1}$		-0.0367***	-0.0421***
		(0.00836)	(0.00903)
$m_{i,t}$			0.0481*
			(0.0245)
Year \times quarter fixed effects	Yes	Yes	Yes
N	84987	84987	84987
adj. R^2	0.016	0.033	0.051

The degree of information friction is estimated at the industry level.

Standard errors are clustered at the industry level. * 0.10 ** 0.05 *** 0.01

Top and bottom 1% productivity obs. are trimmed out (i.e., outliers).

Robustness checks

- Alternative productivity measure: TFP using proxy estimator from Olley and Pakes (1996)
- Exercise 1: using TFP [▶ results](#)
- Exercise 2:
 - using TFP: [▶ results](#)
 - Investment rate (consistent with our prediction, but marginally insignificant p value: 1.1-1.3): [▶ results](#)

Sensitivity Analysis - TFP

► Go back

	(1)	(2)	(3)	(4)
	<i>inv. inaction</i> = 1		<i>inv. rate</i> (<i>inv. inaction</i> ≠ 1)	
$\xi_s \times w_{i,t}$	0.00581*	0.00600*	0.000342	0.000204
	(0.00340)	(0.00339)	(0.00105)	(0.00105)
$a_{i,t-1}$	-0.112***	-0.103***	0.0235***	0.0208***
	(0.0132)	(0.0134)	(0.00516)	(0.00527)
$w_{i,t}$	-0.0404***	-0.0350***	0.0117***	0.00976***
	(0.00848)	(0.00854)	(0.00282)	(0.00284)
$m_{i,t}$	-0.00566	-0.00418	0.00476***	0.00360*
	(0.00634)	(0.00637)	(0.00184)	(0.00193)
$k_{i,t-1}$		0.0727***		-0.0408***
		(0.00873)		(0.00498)
Firm FE	Y	Y	Y	Y
Time FE	Y	Y	Y	Y
N	80508	80508	54747	54747
adj. R^2	0.445	0.447	0.303	0.312

Sensitivity Analysis - Labor Productivity

► Go back

	(1)	(2)	(3)	(4)
	<i>inv. inaction = 1</i>		<i>inv. rate (inv. inaction \neq 1)</i>	
$\xi_s \times w_{i,t}$	0.00848* (0.00466)	0.00885* (0.00465)	-0.0400 (0.0386)	-0.0408 (0.0388)
$w_{i,t}$	0.00213 (0.00931)	-0.00344 (0.00931)	0.0170 (0.0179)	0.0325 (0.0279)
$a_{i,t}$	-0.0204* (0.0119)	-0.0281** (0.0120)	-0.0259 (0.0299)	-0.00409 (0.0158)
$m_{i,t}$	-0.00891 (0.00551)	-0.00523 (0.00552)	0.00593 (0.00411)	-0.00534 (0.00795)
$k_{i,t-1}$		0.0771*** (0.00857)		-0.152 (0.103)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
N	84656	84656	57143	57143
adj. R^2	0.444	0.446	0.045	0.059

Degree of information friction is estimated at the industry level.

Standard errors are clustered at the firm level. * 0.10 ** 0.05 *** 0.01

Top and bottom 1% productivity obs. are trimmed out (i.e., outliers).

Table 2: Incomplete Information and Investment Sensitivity: industry-year FEs

	(1)	(2)	(3)	(4)
	<i>inv. inaction = 1</i>		<i>inv. rate (inv. inaction \neq 1)</i>	
$\xi_s \times w_{i,t}$	0.00991** (0.00473)	0.01000** (0.00472)	-0.0424 (0.0406)	-0.0424 (0.0401)
$a_{i,t-1}$	-0.00857 (0.0120)	-0.0146 (0.0119)	-0.0313 (0.0326)	-0.0127 (0.0202)
$w_{i,t}$	0.00735 (0.00930)	0.00302 (0.00919)	0.0146 (0.0170)	0.0276 (0.0253)
$m_{i,t}$	-0.00879* (0.00524)	-0.00633 (0.00514)	0.00705 (0.00443)	-0.00133 (0.00585)
$k_{i,t-1}$		0.0824*** (0.00823)		-0.168 (0.114)
Industry-year FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Time (semi-year) FE	Yes	Yes	Yes	Yes
N	84656	84656	57137	57137
adj. R^2	0.448	0.451	0.044	0.059

Degree of information friction is estimated at the industry level.

Standard errors are clustered at the firm level. * 0.10 ** 0.05 *** 0.01

Top and bottom 1% productivity obs. are trimmed out (i.e., outliers).

Sensitivity Analysis - Labor Productivity AR(1) [Go back](#)

	(1)	(2)	(3)	(4)
	<i>inv. inaction = 1</i>		<i>inv. rate (inv. inaction \neq 1)</i>	
$\xi_s \times w_{i,t}$	0.00952* (0.00500)	0.00976** (0.00499)	-0.0408 (0.0396)	-0.0414 (0.0396)
$a_{i,t-1}$	-0.0218* (0.0112)	-0.0288** (0.0113)	-0.0266 (0.0305)	-0.00653 (0.0173)
$w_{i,t}$	0.000772 (0.00934)	-0.00456 (0.00935)	0.0182 (0.0189)	0.0334 (0.0287)
$m_{i,t}$	-0.00890 (0.00547)	-0.00536 (0.00549)	0.00548 (0.00403)	-0.00552 (0.00807)
$k_{i,t-1}$		0.0764*** (0.00861)		-0.153 (0.103)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
N	84313	84313	56911	56911
adj. R^2	0.444	0.446	0.045	0.059

Degree of information friction is estimated at the industry level.

Standard errors are clustered at the firm level. * 0.10 ** 0.05 *** 0.01

Top and bottom 1% productivity obs. are trimmed out (i.e., outliers).

Sensitivity Analysis - Labor Productivity AR(1)

► Go back

	(1)	(2)	(3)	(4)
	<i>inv. inaction</i> = 1		<i>inv. rate</i> (<i>inv. inaction</i> ≠ 1)	
$\xi_s \times w_{i,t}$	0.00905* (0.00498)	0.00922* (0.00496)	-0.0416 (0.0402)	-0.0419 (0.0399)
$a_{i,t-1}$	-0.0104 (0.0114)	-0.0158 (0.0113)	-0.0324 (0.0334)	-0.0157 (0.0222)
$w_{i,t}$	0.00561 (0.00931)	0.00161 (0.00921)	0.0148 (0.0173)	0.0270 (0.0250)
$m_{i,t}$	-0.00863 (0.00525)	-0.00631 (0.00515)	0.00704 (0.00442)	-0.000940 (0.00566)
$k_{i,t-1}$		0.0818*** (0.00827)		-0.169 (0.114)
Industry-year fixed effects	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes
Time (semi-year) fixed effects	Yes	Yes	Yes	Yes
N	84313	84313	56906	56906
adj. R^2	0.449	0.451	0.043	0.059

Degree of information friction is estimated at the industry level.

Standard errors are clustered at the firm level. * 0.10 ** 0.05 *** 0.01

Top and bottom 1% productivity obs. are trimmed out (i.e., outliers).

Empirical Exercise 3: Information Frictions & Volatility

- Do we see dampened effect of higher volatility on investment inaction in regions where information frictions are more severe?
- We estimate

$$\mathbb{1}(\text{inaction})_{it} = \beta(\text{vol}_r \times \xi_s) + \gamma_1 \xi_s + \gamma_2 \text{vol}_r + \Gamma z_{i,t} + \bar{s}g_r + \gamma_t + \epsilon_{it}$$

- $w_{it} : a_{it} - a_{it-1}$
- z_{it} : lagged (log) capital stock k_{it-1} , (log) labor productivity a_{it} , and intermediate goods per worker m_{it}
- $\bar{s}g_r$ and vol_r are mean and volatility of firm-level sales growth in region r
- γ_t is the semi year (i.e., time) fixed effects
- Standardize ξ_s , $\bar{s}g_r$ and vol_r

Empirical Exercise 3: Information Frictions & Volatility

[▶ full result](#)

	Data	Model
	inaction	
$vol_r \times \xi_s$	-0.00549** (0.00253)	-0.009 (0.001)
ξ_s	-0.0551** (0.0231)	-0.145 (0.001)
vol_r	0.00612 (0.00524)	0.041 (0.000)
Time FE	Y	Y
N	85920	4178503
adj. R^2	0.067	0.016

- Higher volatility of productivity leads to *dampened* increase in investment inaction when information friction is more severe

Table 3: Investment Inaction and Region-level Volatility

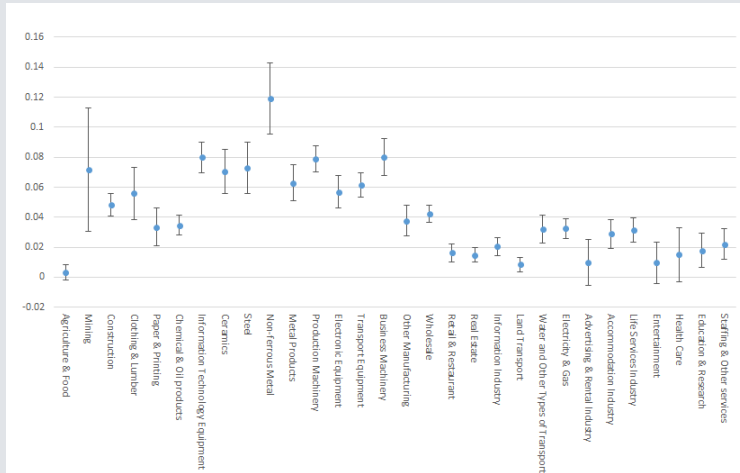
<i>inv. inaction = 1</i>			
$vol_r \times \xi_s$	-0.0113** (0.00434)	-0.00927** (0.00368)	-0.00549** (0.00253)
ξ_s	-0.0769*** (0.0260)	-0.0796*** (0.0238)	-0.0551** (0.0231)
vol_r	0.00684 (0.00513)	0.00636 (0.00529)	0.00612 (0.00524)
\tilde{g}_r	-0.0199** (0.00873)	-0.0318*** (0.0107)	-0.0365*** (0.00817)
$a_{i,t}$	0.0375 (0.0291)	0.0565** (0.0264)	0.101*** (0.0320)
$k_{i,t-1}$		-0.0512*** (0.00748)	-0.0507*** (0.00727)
$m_{i,t}$			-0.0249 (0.0195)
Time FE	Yes	Yes	Yes
N	98515	98515	85920
adj. R^2	0.039	0.072	0.067

Degree of information friction is estimated at the industry level.

Standard errors are clustered at the firm level. * 0.10 ** 0.05 *** 0.01

Top and bottom 1% productivity obs. are trimmed out (i.e., outliers).

Attenuation Coefficients across Industries - TFP



- Positive & statistically significant coefficients

Partial Irreversibility

- If firms invest, they do so at cost $\Psi(I)$:

$$\Psi(I) = \begin{cases} \psi_+ I & I \geq 0 \\ \psi_- I & I < 0 \end{cases}$$

with $\psi_+ > \psi_- > 0$

- Instantaneous profit is $\pi = A^{1-\alpha} K^\alpha - \Psi(I)$
- Optimal firm behavior: for a range of capital values, firms choose to neither invest nor divest. Usual HJB in the inaction region.
- Solving the firm's problem comes down to finding the optimal choice of \hat{B}_L and \hat{B}_U

Partial Irreversibility

Proposition

Under incomplete information, the boundary conditions consist of two value-matching conditions:

$$\hat{V}'(\hat{B}_L) = \psi_+ \qquad \hat{V}'(\hat{B}_U) = \psi_-$$

and two super contact conditions:

$$\hat{V}''(\hat{B}_L) = 0 \qquad \hat{V}''(\hat{B}_U) = 0$$

Partial Irreversibility

Proposition

The critical values of expected normalized capital are

$$\hat{b}_L = b_L^{FI} + \frac{\alpha^2 \nu}{2(1 - \alpha)} \qquad \hat{b}_H = b_H^{FI} + \frac{\alpha^2 \nu}{2(1 - \alpha)}$$

where b_L^{FI} and b_H^{FI} denote the full information solutions such that $\nu = 0$.