The Optimal Monetary Policy Response to Belief Distortions: Model-Free Evidence*

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January 26, 2025

EARLY DRAFT

Abstract

Some inflation forecast errors are predictable. Economic theory predicts that these belief distortions affect the business cycle. How should monetary policy respond? We investigate this question with a model-free approach using high-frequency monetary policy shocks and a structural VAR method to identify the effects of shocks to belief distortions. The shocks are contractionary: when households become overly pessimistic about inflation, unemployment and deflation follow. Intuitively, the optimal policy response is to ease. This is most effective with short-term rates; we find that a 1 p.p. increase in the belief distortion is optimally offset by a 0.85 p.p. surprise interest rate decrease. Monetary policy targeting longer term rates are less effective but also useful.

JEL-Codes: E52, E30, D84, E70

Keywords: Monetary Policy, Belief Distortions, Optimal Policy, Policy Counterfactuals, Shocks

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1 Introduction

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People are poor forecasters. When their forecasts are inconsistent with the full information rational expectation, this is referred to as a belief distortion. In the aggregate, belief distortions are large and vary over time.[1] Theoretical and applied work conclude that shocks driving belief distortions can affect the business cycle. How should monetary policy respond?

In this paper, we evaluate how belief distortions affect the macroeconomy, and calculate the optimal monetary policy response from aggregate time series, without assuming any particular theoretical model. To do so, we apply the McKay and Wolf (2023) method, which allows for the calculation of counterfactual policies from macroeconomic time series in a way that is not subject to the Lucas critique. The method can be implemented using estimated impulse response functions (IRFs) to monetary policy shocks and to belief distortions. For monetary policy, we borrow from Swanson (2023), which uses high frequency data around Fed events to identify shocks corresponding to three policy instruments: the target rate, forward guidance, and large scale asset purchases. For belief distortions, we identify shocks in two ways. First, we apply the semi-structural approach developed in Adams and Barrett (2024), which decomposes VAR innovations to find the component that causes empirical forecasts to deviate from the rational expectation. Second, we construct a reduced form shock, which is simply the statistical innovation to an estimated belief distortion.

Specifically, we study belief distortions over inflation. Household inflation expectations are well known to violate full information rational expectations (FIRE), and the difference varies over the business cycle.[2] The theoretical literature suggests that belief distortion dynamics can be driven by structural shocks and have large effects on the economy.[3] Recent empirical evidence in (Ascari et al., 2023; Adams and Barrett, 2024) finds that these shocks have large, robust effects: inflation belief distortions are *contractionary*. When households' forecasts rise too high, inflation and real activity fall. This evidence is at odds with standard theory, which is one reason model-free evidence is so valuable for addressing this policy question.

We conclude that monetary policy should ease after a pessimist belief distortion shock. A shock that causes households to make inflation forecasts that are higher than the FIRE forecast depresses real activity, so intuitively, the optimal monetary policy response is stim-

[1] Some examples of recent evidence include Bianchi et al. (2022), Bianchi et al. (2024), and Farmer et al. (2024).

[2] See D'Acunto et al. (2023) or D'Acunto et al. (2024) for recent surveys.

[3] Some examples focusing on belief distortions over different variables include Ascari et al. (2023) (inflation), Candia et al. (2023) (exchange rates), Bhandari et al. (2024) (unemployment), and Maenhout et al. (2025) (GDP growth)

ulative. There are multiple tools that central banks have at their disposal to respond; we find that traditional interest rate policy is most effective. Specifically, we estimate that interest rates should fall roughly one-to-one in response to a structural shock that raises the inflation belief distortion. The optimal response is also negative – but half as much – for the reduced form shock. And while the target rate is the most effective single tool, incorporating stimulative forward guidance and asset purchases can do even better.

Our results contribute to a largely theoretical literature studying optimal monetary policy without FIRE. Adams (2024) proves that the belief distortion is a sufficient statistic for the the optimal policy response to deviations from FIRE; in a New Keynesian model, the policy rule for interest rates is increasing in inflation and income belief distortions. Focusing on belief distortions to guide policy is valuable for central banks, because it provides guidance without needing to specify the precise mechanism by which FIRE fails. Many such mechanisms abound, and give conflicting policy prescriptions.[4] A few recent papers explicitly study monetary policy with exogenous shocks to expectations, which most closely matches our structural approach; examples include Ascari et al. (2023) and Neri (2023), whose models predict that the real economy contracts after shocks to inflation expectations over the short and long run, respectively.

2 Model-Free Method

Our empirical strategy consists of four steps: measuring belief distortions, estimating the impulse response functions (IRFs) to belief distortion shocks, estimating the IRFs to monetary policy shocks, and calculating the optimal policy counterfactual. In this section, we outline each step in detail.

2.1 Measuring Belief Distortions

The "belief distortion" is the difference between average expectations in the economy, and the appropriate full information rational expectation. Define the belief distortion $d_t^{y,h}$ over quantity y at horizon h by

$$d_t^{y,h} \equiv f_t^{y,h} - re_t^{y,h} \tag{1}$$

where $f_t^{y,h}$ denotes the time t average forecast of y_{t+h}, and $re_t^{y,h}$ denotes the corresponding rational expectation. For some information set Ω_t, the rational expectation is the condi-

[4] Some examples of optimal monetary policy papers where FIRE fails for different reasons include: Woodford (2010) (sticky information), Di Bartolomeo et al. (2016) (heterogeneous expectations), Hommes et al. (2019) (heuristics), Gabaix (2020) (cognitive discounting), Iovino and Sergeyev (2023) (level-k thinking), and following Lucas (1972) many models of information frictions including Adam (2007), Nimark (2008), Lorenzoni (2010), Baeriswyl and Cornand (2010), Paciello and Wiederholt (2014), Angeletos and La'O (2019), Benhima and Blengini (2020), and Angeletos et al. (2020).

tional expectation of y_{t+h} given Ω_t:

$$re_t^{y,h} = \mathbb{E}_t[y_{t+h}|\Omega_t]$$

The average forecast $f_t^{y,h}$ is taken directly from survey data, but the rational expectation $re_t^{y,h}$ must be estimated. We estimate $re_t^{y,h}$ by selecting an information set Ω_t, and projecting y_{t+h} on the variables in Ω_t. Specifically, we estimate a regression of the following form:

$$y_{t+h} = \sum_{j=0}^{J} \left(\alpha_j f_{t-j}^{y,h} + \beta_j x_{t-j} \right) + \upsilon_{t+h} \tag{2}$$

where x_t is a vector of macroeconomic variables, and υ_{t+h} is the forecast error. Including $f_t^{y,h}$ and its lags guarantees that our rational expectation includes all information contained in the surveyed forecasts. We take the predicted value from regression (2) as the rational expectation $re_t^{y,h}$.

2.2 Identification of Shocks to Belief Distortions

In order to study the optimal monetary policy response to belief distortions, we need to estimate the IRFs to belief distortion shocks. To do so, we consider two methods: a *structural shock*, and a *reduced form shock*.

The *structural shock* is an identification strategy motivated by theory. We follow the approach of Adams and Barrett (2024); the identifying assumption is that structural belief shocks are the only shocks to cause belief distortions to depart from rational expectations. For example, if individuals respond to productivity shocks with rational expectations, but also independently exhibit stochastic belief distortions, this method correctly identifies the latter shock from the former.

To identify the effects of structural shocks, we estimate an n-dimensional VAR including the surveyed forecast $f^{y,h}$:

$$\begin{pmatrix} f_{t+1}^{y,h} \\ x_{t+1} \end{pmatrix} = \sum_{j=0}^{J} B_j^s \begin{pmatrix} f_t^{y,h} \\ x_t \end{pmatrix} + w_t^s \tag{3}$$

again, w_t^s are reduced form innovations related to structural shocks ε_t^s by

$$w_t^s = A^s \varepsilon_t^s$$

We encode the structural belief shock in the first entry of the shock vector ε_t^s, so this method identifies $A_{1\cdot}^s$, the first column of the impact matrix A^s.[5]

[5]This method contrasts with a traditional approach that orders expectation shocks first in a Cholesky

The alternative *reduced form shock* is the statistical innovation in the belief distortion. In other words, it is the residual in a regression of belief distortion $d_t^{y,h}$ on its lags and other controls; it appears in the first row dimension in the estimated innovation w_t. This shock is reduced form: it may be driven by any number of structural shocks at time t that cause measured expectations to depart from the rational expectation.

In this second case, we estimate another n-dimensional VAR, replacing the surveyed forecast from equation (3) with the belief distortion $d_t^{y,h}$:

$$\begin{pmatrix} d_{t+1}^{y,h} \\ x_{t+1} \end{pmatrix} = \sum_{j=0}^{J} B_j^r \begin{pmatrix} d_t^{y,h} \\ x_t \end{pmatrix} + w_t^r \tag{4}$$

where w_t^r are reduced form innovations related to structural shocks ε_t^r by

$$w_t^r = A^r \varepsilon_t^r$$

for some matrix A^r.

The reduced form shock is also not an *uncorrelated* shock: it covaries with the other residuals, to which monetary policy may already be responding. But the belief distortion shock is linearly independent from other dimensions of the VAR. If monetary policymakers ignore belief distortions, then they ignore a business cycle driver that demands a policy response. Studying the reduced form shock captures the additional policy response required by this typically overlooked dimension.

2.3 Calculating Optimal Policy

To calculate optimal monetary policy, we follow the method pioneered in McKay and Wolf (2023). To do so, we require 3 ingredients: IRFs to belief distortions, IRFs to monetary policy shocks, and a welfare criterion.

The IRFs to belief distortion shocks are given by the $n \times 1$ impulse response functions $\phi_r(k)$ and $\phi_s(k)$. The IRFs to n_m monetary policy shocks are given by the $n \times n_m$ impulse response function $\phi_m(k)$. We keep these functions abstract for the moment, but Section 3 will describe how we estimate them.

The welfare criterion is a function of the IRFs to the various shocks. In our baseline approach, we use a welfare criterion that depends on the unconditional variance of unem-

decomposition; the assumption is that the only shock can affect forecasts contemporaneously is an exogenous expectation shock. Estimation with this method typically finds that inflation expectation shocks are expansionary (Leduc et al., 2007; Clark and Davig, 2011).

ployment u_t and inflation π_t, with weighting parameter λ:

$$\mathcal{W}_w = \lambda V_w^u(\infty) + (1-\lambda) V_w^\pi(\infty) \tag{5}$$

where $V_w^u(h)$ and $V_w^\pi(h)$ denote the variance of unemployment and inflation, respectively, that is due to all w shocks at horizons no more than h.

The variance of quantity x that is due to all w shocks at horizons no more than h is

$$V_w^x(h) = \sum_{k=0}^{h} Var(x_{t+k}|w_t) = \sum_{k=0}^{h} (\phi_w^x(k))^2 \, Var(w_t)$$

where $\phi_w^x(k)$ is the IRF of x to the shock w at horizon k. Therefore, it is possible to express the welfare criterion as a function of the IRFs to the various shocks. For example, the welfare loss due to a shock w at all horizons is

$$\mathcal{W}_w = \sum_{k=0}^{\infty} \left(\lambda \left(e_u \phi_w(k) \right)^2 + (1-\lambda) \left(e_\pi \phi_w(k) \right)^2 \right) \tag{6}$$

where e_u and e_π are the basis vectors selecting unemployment and inflation from the vector $\phi_w(k)$.

Counterfactual policies can be studied by constructing alternative impulse response functions to minimize the welfare loss (6). The central insight from McKay and Wolf (2023) is that this can be done in a way that satisfies the Lucas critique by manipulating only the covariance between the shock w and monetary policy shocks. Specifically, this is done by constructing a counterfactual rule for monetary policy shock m:

$$m_t = \psi w_t$$

where ψ is an $n_m \times 1$ vector. Therefore we can construct the alternative impulse response function $\phi_\psi(k)$ by adding the IRF to the monetary policy shock to the IRF to the belief distortion shock:

$$\phi_\psi(k) = \phi_w(k) + \phi_m(k)\psi$$

With this approach, the welfare loss to shock w can be written as a function of the policy vector ψ by

$$\mathcal{W}_w(\psi) = \sum_{k=0}^{\infty} \left(\lambda e_u \left(\phi_w(k) + \phi_m(k)\psi \right) \right)^2 + \left((1-\lambda) e_\pi \left(\phi_w(k) + \phi_m(k)\psi \right) \right)^2$$

and the optimal policy vector ψ minimizes $\mathcal{W}_w(\psi)$.[6]

SEGUE HERE TO IMPLEMENTATION

3 Data

Our baseline VAR model is as standard as possible. As in Gertler and Karadi (2015), Bauer and Swanson (2023), and others, we include the following monthly series: the log consumer price index (CPI), the log of industrial production (IP), the Gilchrist and Zakrajšek (2012) excess bond premium (EBP), and the 2 year treasury yield (TREAS). We also include two additional variables necessary for our procedure: the unemployment rate (UNEMP), and the log of the 1-year-ahead implied CPI forecast or belief distortion, depending on whether we are estimating model (4) or (3). We apply the Akaike information criterion (AIC), which selects 7 monthly lags for our baseline model. We consider alternative lag lengths in Section 5.

When measuring belief distortions, we must construct a CPI forecast from surveyed forecasts of inflation, and also estimate the rational expectation. To calculate implied CPI forecasts we use the monthly median household forecast from the Michigan Survey of Consumers. The survey reports an inflation forecast in percentage points, so we construct a 1-year-ahead CPI forecast by:

$$f_t^{CPI,12} = (1 + f_t^{\pi,12}) \times CPI_t$$

The Michigan Survey data begin in 1978 which restricts our dataset for the main VARs to January 1978 – May 2024.[7]

When estimating the rational expectation, we estimate a regression specified by equation (2). On the right-hand side, we include contemporaneous and up to four lags of the variables from our baseline VAR (CPI, industrial production, unemployment, 2-year yield, excess bond premium, and the 1-year-ahead implied CPI forecast) as well as the PPI commodity index, 10-year treasury yield, and the Wu and Xia (2016) shadow fed funds rate. The adjusted R-squared is 0.9989 implying the estimated rational expectation is an accurate predictor.

We source the monetary policy shocks from Swanson (2023), which uses high-frequency data around Federal Reserve events to derive three distinct shocks: the target rate (FFR), forward guidance (FG), and large-scale asset purchases (LSAP). These shocks are identified

[6] *Here point to appendix describing calculation in detail*

[7] While the Survey has included a question on expected inflation since 1948, the question had a qualitative format where respondents needed to report if they expect inflation to go up, down, or stay the same. The survey revised this question in 1966 in bins format.

from their effects on short, medium, and long-term rates, respectively. Notice that while monetary policy shocks dataset does not start until 1988, this does not restrict the data used to estimate the VAR model. The effects of monetary policy shocks are estimated from their impact on the reduced-form residuals of the model.[8]

Describe briefly in one sentence, how are standard errors calculated. Again, point to Appendix for more details. We use bootstrapping standard errors from 1,000 iterations where we create synthetic response variables from randomly selected reduced-form residuals while keeping the order of the regressors fixed to maintain the properties of the VAR model. this needs a citation, and more elaboration in the appendix

4 Results

This section describes the results of our optimal policy analysis. We first present the impulse response functions to belief distortion shocks and to monetary policy shocks. Then using these estimates, we calculate optimal policy responses.

4.1 Impulse Response Functions

Figure 1 reports estimated IRFs for the two variables that are relevant to the optimal policy analysis – CPI and unemployment – as well as the belief distortion itself. Both the reduced-form and structural shock are scaled to causes 1-year-ahead CPI forecasts to depart from their rational expectations by 1 percentage point.[9] For example, if the average inflation forecast is 3%, and the rational expectation is 2%, this would be a one percentage point belief distortion.

Both belief distortion shocks are contractionary. Unemployment rises and prices fall. Unreported in the figure, output and the bond premium also rise, and treasury yields fall. Unreported in the figure, industrial production declines, and Treasury yields fall as the Fed responds to the contractionary shock by lowering interest rates. These patterns are consistent with those estimated by Adams and Barrett (2024).

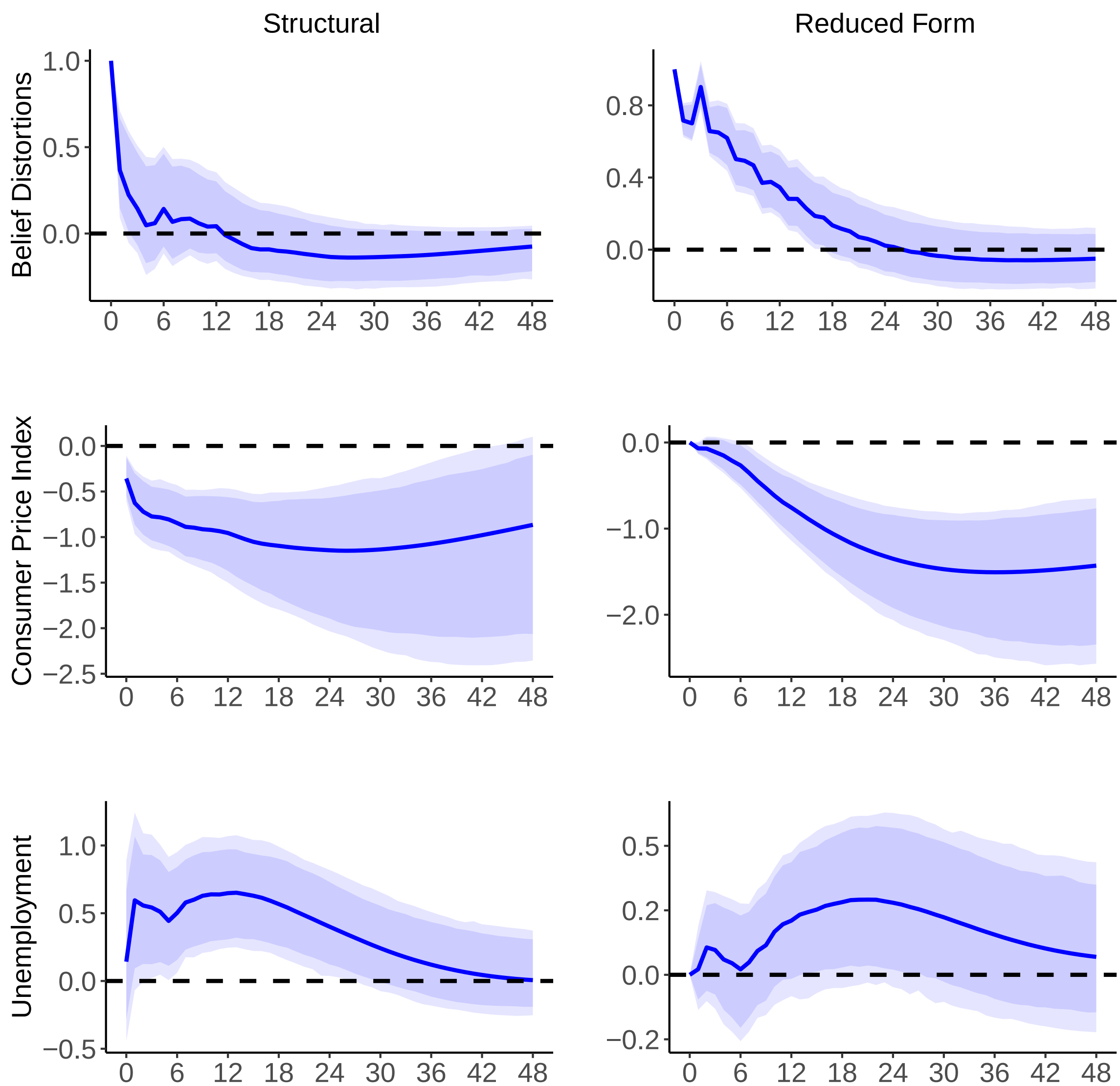
i need to revisit

The two shocks also differ in noticeable ways. The reduced form shock has a long, persistent effect on the belief distortion, which takes two years to decay, while after a structural shock, the belief distortion disappears within half a year. The reduced form shock also has delayed effects on the economy: prices take several months to fall and unemployment takes at least half a year to rise. In contrast, the structural shock causes a

[8] Furthermore, we follow Swanson's baseline approach and truncate the series after February 2020 to exclude the Covid-19 pandemic.

[9] To be more precise, the shock increases the implied belief distortion on log CPI by 0.01, which we refer to as 1 percentage point.

contraction almost immediately. Finally, the structural shock has a much larger effect on the real economy, causing a 0.6 p.p. peak increase in unemployment, twice as large as that of the reduced form shock. Given that these shocks have different effects, they will require different policy responses.



Notes: The figure shows the impulse responses to a positive belief distortion shock that causes 1-year-ahead CPI reported forecasts to depart from their rational expectations by 1 percentage point. Bootstrapped confidence intervals on 1,000 bootstrap runs are reported at the 95% and 90% level.

Figure 1: Impulse Responses to Belief Distortion Shocks

To conduct the optimal policy calculation, we also need to estimate the IRFs for the monetary policy shocks.[10] Figure 2 plots the impulse responses to the target rate, forward guidance, and large-scale asset purchase shocks. Each shock is normalized such that the 2-year yield increases by 1 percentage point. The first row includes the yield to illustrate how differently identified shocks have different effects on interest rate policy over the medium run. Otherwise, the figure plots the IRFs to inflation and unemployment, which are the only functions needed for the optimal policy calculation.

The policy shocks have distinct effects, reflecting the different dimensions of monetary policy that they capture. The Target Rate (FFR) shock resembles a textbook interest rate shock: it raises yields in the short run, reducing inflation and real activity. The Forward Guidance shock also raises yields, but over a long horizon. In the short run it is inflationary and expansionary, but in the medium run the effects reverse. The LSAP shock increases yields in the first year, but then reduces them in the following years; the effect is strongly contractionary.

4.2 Optimal Monetary Policy Response

We apply the methodology outlined in Section 2 to the data described in Section 3 in several ways. We calculate how optimal monetary policy responds to belief distortion shocks using both reduced-form and structural methodologies. We first calculate the optimal response for each type of monetary policy shock (FFR, FG, LSAP) separately. Then, we consider the optimal policy response using pairs of these policies, and finally using all three together. Table 1 presents these results, displaying the appropriate entries in the estimated optimal policy vector ψ.

The optimal policy rules are largely intuitive, and in most cases are qualitatively consistent between the reduced form and structural methodologies. Therefore we discuss the structural shocks first, and return to the differences with reduced form shocks below.

The optimal target rate response to a structural belief distortion shock is as expected: the shock is contractionary, so the optimal response is to loosen monetary policy. The coefficient -0.845 implies that if the shock increases the inflation belief distortion by 1 percentage point, then interest rates should be reduced enough to lower the two year yield by 0.845 percentage points. This policy is nearly one-to-one, because a target rate shock increases unemployment by roughly the same as a belief distortion shock. So offsetting the belief distortion with monetary policy mostly negates the distortion to the real economy.

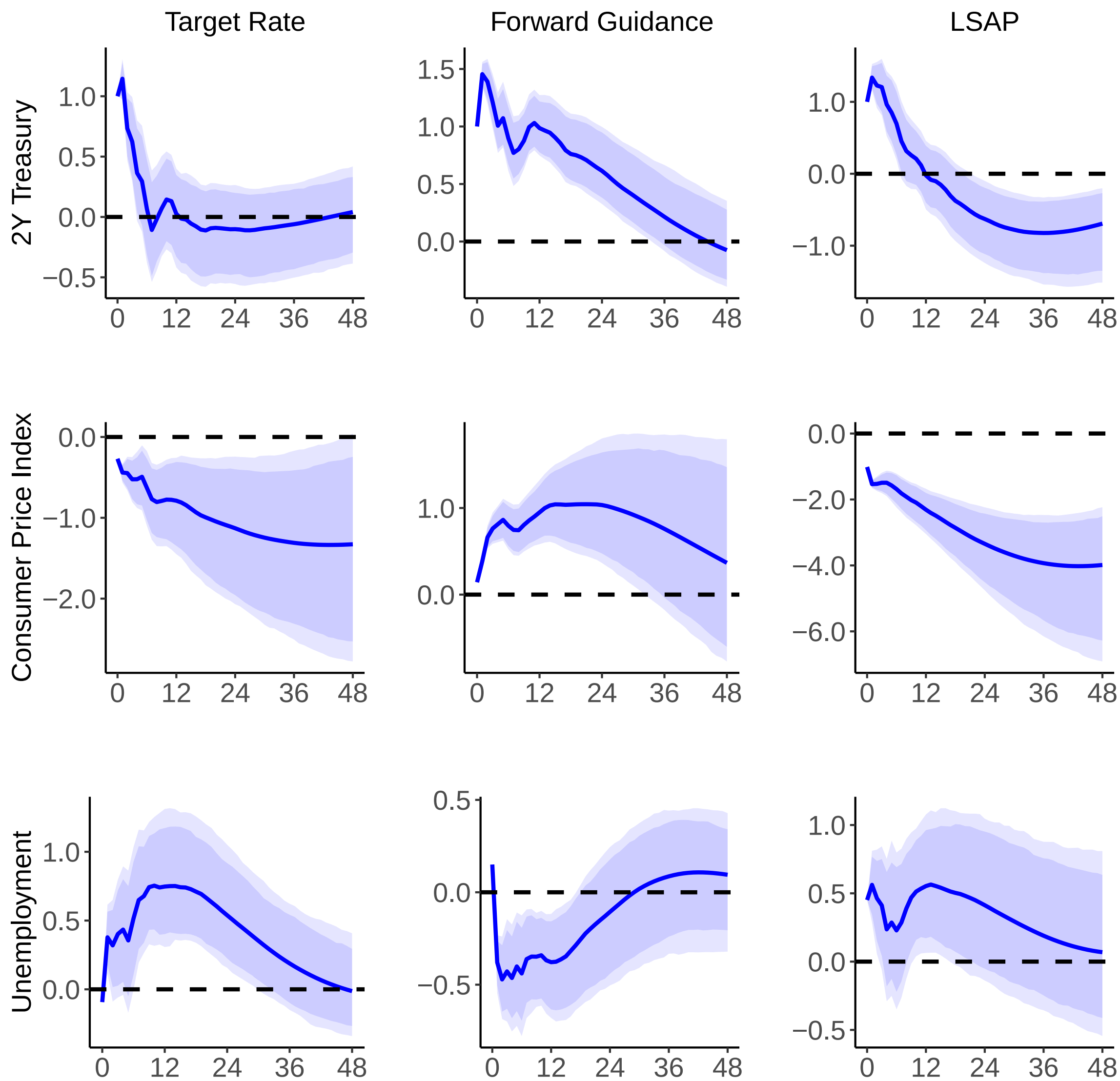
This offsetting policy rule can be seen clearly in Figure 3, which plots the IRFs to the belief distortion shocks under three scenarios: the baseline result, a counterfactual under the optimal policy that uses the target rate alone, and a counterfactual under the optimal policy using all three monetary tools. In the lower right plot, the belief distortion shock

[10] Here explain briefly how you do so, with an appropriate citation

	Structural Methodology				Reduced-form Methodology			
	Target	FG	LSAP	R^2	Target	FG	LSAP	R^2
Independent Tools	-0.845			0.948	-0.435			0.806
	(0.301)				(0.344)			
		1.417		0.731		0.24		0.082
		(0.463)				(0.404)		
			-0.949	0.847			-0.415	0.322
			(0.354)				(0.338)	
Pairwise Tools	-0.678	0.411		0.973	-0.518	-0.26		0.872
	(0.25)	(0.34)			(0.478)	(0.479)		
		0.71	-0.65	0.946		0.202	-0.401	0.38
		(0.429)	(0.256)			(0.348)	(0.309)	
	-0.649		-0.262	0.962	-0.418		-0.042	0.808
	(0.283)		(0.233)		(0.315)		(0.247)	
All Tools	-0.459	0.429	-0.283	0.988	-0.544	-0.28	0.05	0.875
	(0.208)	(0.27)	(0.179)		(0.443)	(0.442)	(0.212)	

Notes: Columns 1-4 (5-8) report estimations from the structural (reduced-form) methodology. Target stands for Target Rate tool, FG for Forward Guidance, and LSAP for Large Scale Asset Purchases. Regression R-squared are reported in columns 4 and 8 respectively. Bootstrapped standard errors from 1,000 iterations are reported in parentheses. Regressions in Panel A use single monetary tools, while in Panel B and Panel C use tools in pairs and a combination of all three tools, respectively.

Table 1: Optimal Monetary Policy Response

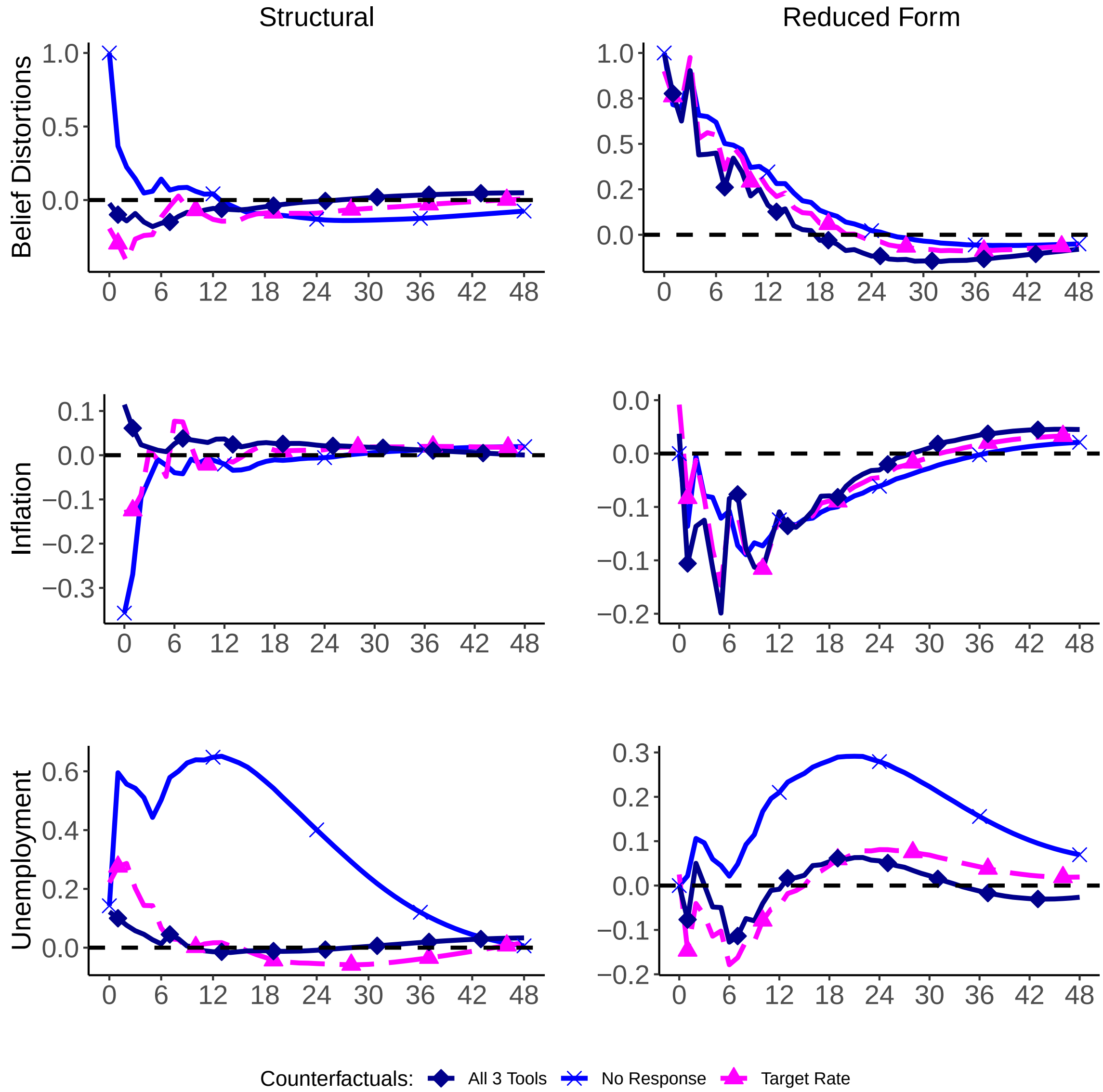


Notes: The figure shows the impulse responses to a monetary policy shock that raises the 2Y Treasury Yields by 100 basis points on impact. Bootstrapped confidence intervals on 1,000 bootstrap runs are reported at the 95% and 90% level. Each column reports impulse responses of different shock type (target rate, forward guidance, and large-scale asset purchases).

Figure 2: Impulse responses to monetary policy shocks

would ordinarily increase unemployment by more than 5% for many months, but when the target rate policy is used, then unemployment increases by half as much and returns rapidly.

The deflationary impact is also moderated, reducing the magnitude of the impact by more than half. This can be seen in the second row of Figure 3, which plots the inflation IRFs under the three scenarios. We plot inflation instead of the CPI here because inflation



Notes: The figure shows the effects of belief distortion shocks under three scenarios: no monetary policy reaction, optimal reaction with target rate policy, and optimal reaction when all three monetary tools are available (target rate, forward guidance and large scale asset purchases). The first column reports estimated responses from the reduced-form methodology and the second column reports estimated responses from the structural methodology.

Figure 3: Monetary Policy Response to Belief Distortions

is the object that enters the welfare criterion (5). The inflation IRF is simply the first difference of the CPI IRF plotted in Figure 1.

The target rate is close to the perfect policy instrument to address the structural belief distortion shock; Figure 3 shows that it eliminates most of the resulting variation in unemployment and inflation. We summarize this effectiveness with the R^2 statistic in Table 1. That the $R^2 = 0.948$ indicates that the optimal target rate policy eliminates $\sim 95\%$ of the welfare loss due to the shock. The FG and LSAP policies are less successful, but still effective with R^2 statistics of 0.731 and 0.847 respectively.[11] The reason that the target rate is the most effective policy is because its CPI and unemployment IRFs (Figure 2) most closely match the shape of the belief distortion IRFs (Figure 1). For example, forward guidance is less effective at moderating the unemployment response because the effect of FG on unemployment reverses sign after 2 years, and LSAP is less effective because its impact on unemployment decays almost immediately.

A policy response with multiple instruments is always more effective than using any of the instruments individually. Table 1 also reports these results, showing the optimal rule for pairwise combinations in the second block of rows, and the rule for all three in the final block of rows. In all cases, the coefficients on the contractionary instruments (target rate and LSAP) are negative, while the coefficient on the expansionary forward guidance instrument is positive. Figure 3 shows that the use of all three instruments is even more effective, almost entirely eliminating the response of inflation and unemployment to structural belief distortion shock.

The first row of Figure 3 provides another useful lesson: this negation of the effects of the belief distortion shock is achieved by directly negating the belief distortion entirely. Under the optimal policy counterfactual, the structural shock has almost no effect on the inflation belief distortion. This is not a necessary result of the method. Rather, it occurs because expansionary monetary policy reduces belief distortions. This is mainly because forecasts are inelastic: a sudden interest rate reduction increases future inflation, but expectations do not move one-for-one.

i need to write a brief paragraph about generalizations related to the structural shock

The optimal policy response to the reduced form shock is similar, but with quantitative differences. After a reduced form shock, the optimal policy is also to tighten the target rate, but by less (-0.435) because the effect on unemployment is roughly half that of the structural shock (Figure 1). And across the board, policy is less effective at moderating the reduced form shock: the R^2 statistics in Table 1 are smaller, and the counterfactual impulse response functions in Figure 3 are further from zero. This is because the shapes of the IRFs to the reduced form shock are not as closely spanned by the monetary policy IRFs as in the case of the structural shock. Specifically, the long delay from the reduced form shock's impact to the peak unemployment response is difficult to replicate with any combination of the unemployment IRFs in Figure 2. The FG and LSAP instruments are

[11] This result is thematically similar to evidence from **?**, who find that forward guidance is relatively ineffective at managing household inflation expectations.

particularly bad at matching this shape; thus in Table 1 their coefficients are small when all tools are included, and when the target rate is excluded the R^2 statistics are especially small. Still, the main conclusion remains: expansionary interest rate policy is an effective response to the belief distortion shock

5 Robustness Checks

This section presents a variety of robustness checks. In each case, we calculate the optimal policy response of the target rate shock (FFR) to a reduced-form or structural belief distortion shock.

Our first check is to consider alternative policy objectives. In the baseline, monetary policy has a dual mandate: minimize both unemployment and inflation volatility. When the welfare criterion places all the weight on unemployment ($\lambda = 1$) the results are nearly unchanged from the baseline, but when all the weight is on inflation ($\lambda = 0$) the optimal response to a structural shock is even more aggressive, but the response to a reduced form shock is near zero. This is because the reduced form belief distortion shock creates a very persistent deflation, which is nearly orthogonal to the inflationary effect of the target rate shock.

In the baseline we used the AIC to select a 7-lag structure for the VARs. We also consider 1 quarter and 1 year worth of lags. In general, shorter lag lengths yield more aggressive monetary policy responses. We also consider an alternative estimation of the rational expectation component of the belief distortion with additional lags; the results are mostly unchanged from the baseline.

Many other choices related to timing had limited effects on our conclusions. Changing the maximum IRF horizon from the 60 month? baseline to 24 or 120 months had little effect on the optimal policy. This was also true when we ended the sample on December 2019 before Covid-19. In the baseline we followed convention and estimated our VAR with untransformed data; when we detrend, the optimal response is smaller, mainly because the structural shock has a smaller effect on unemployment, consistent with the estimates in Adams and Barrett (2024) where the data are also detrended.

Finally, we considered an entirely different type of monetary policy shock. The Swanson (2023) shocks are identified from high-frequency data around monetary policy events. We also employed a narrative approach: the Aruoba and Drechsel (2022) shocks are identified... i need to fill in. The narrative shocks have smaller effects on the real economy, so the optimal policy coefficients are larger in magnitude.

	Structural		Reduced-form	
	Target	R^2	Target	R^2
Baseline Model	-0.845	0.948	-0.435	0.806
	(0.301)		(0.344)	
Inflation Targeting ($\lambda = 0$)	-1.006	0.718	0.075	0.004
	(0.263)		(0.284)	
Employment Targeting ($\lambda = 1$)	-0.843	0.955	-0.439	0.857
	(0.339)		(0.354)	
VAR with 3 lags	-1.046	0.892	-0.524	0.894
	(0.456)		(0.341)	
VAR with 12 lags	-0.599	0.861	-0.186	0.275
	(0.262)		(0.229)	
Belief Distortion estimation	-	-	-0.378	0.773
with 12 lags	-		(0.237)	
Detrended data	-0.271	0.866	-0.475	0.805
	(0.154)		(0.306)	
Excl. Covid-19 Era	-0.828	0.736	-0.677	0.793
	(0.239)		(0.475)	
24-Month Truncation of	-0.874	0.954	-0.378	0.764
Welfare Objective	(0.366)		(0.291)	
120-Month Truncation of	-0.822	0.913	-0.397	0.596
Welfare Objective	(0.303)		(0.377)	
Small VAR	1.861	0.745	0.504	0.317
	(0.926)		(0.408)	
Aruoba-Dreschel Monetary	-2.092	0.705	-0.912	0.845
Policy Shock	(1.059)		(0.942)	

Notes: table notes go here! Simon's Comment: I updated this table but I need to add Local Projections. 1,000 Bootstrap runs. About 30 mins to run.

Table 2: Robustness Tests

6 Conclusion

i need to write the conclusion!

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A Implementation Details

In this appendix we need to give all the details omitted in the main draft for computing/estimating everything. It should be sufficiently detailed for a PhD student to replicate without looking at our code.

A.1 Estimating IRFs

This should include the details for VAR estimation, as well as the formulae for the impulse response functions. Also need to explain that we back out the inflation IRF from the CPI IRF.

A.2 Calculating Optimal Policy

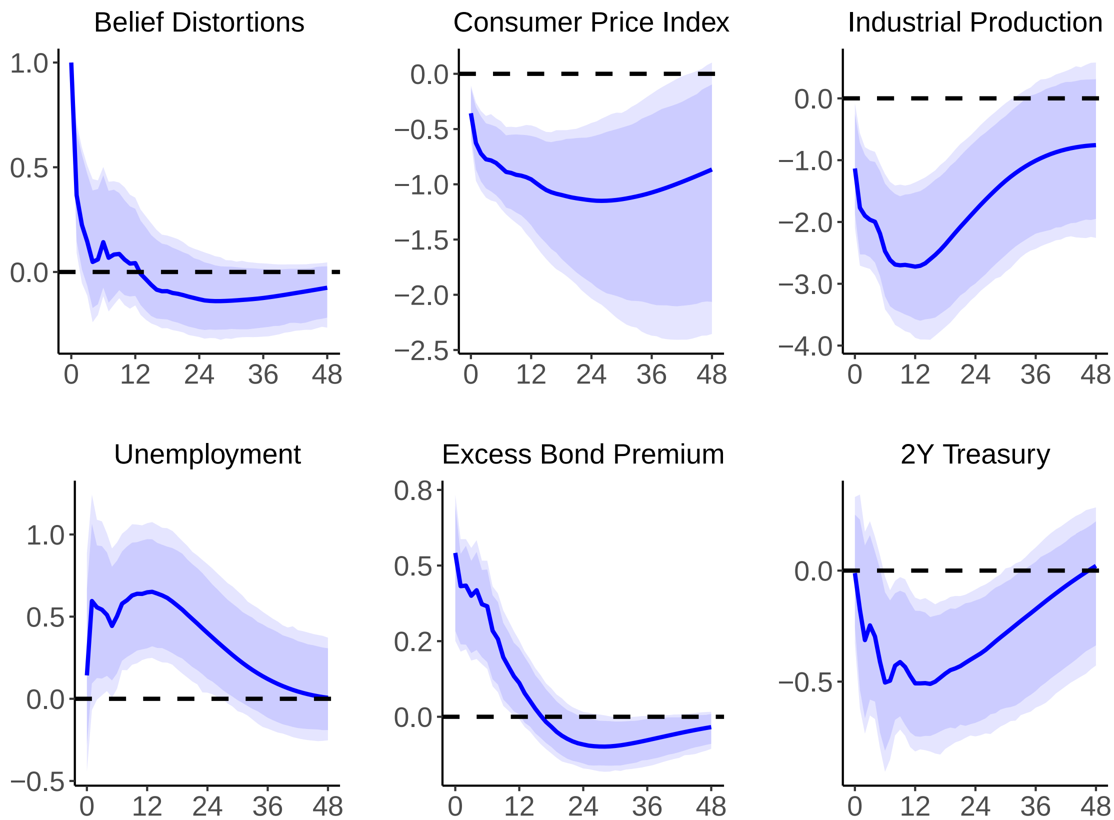
This should include the details for constructing the psi estimates from the estimated phi's. Adams can write this.

B Additional Plots (for Online Appendix)

Simon, please put any additional figures you care about in this section

Notes: The figure shows all impulse responses to a positive reduced-form belief distortion shock that causes 1-year-ahead CPI reported forecasts to depart from their rational expectations by 1 percentage point. Bootstrapped confidence intervals on 1,000 bootstrap runs are reported at the 95% and 90% level.

Figure 4: All Impulse Responses to Reduced-form Belief Distortion Shocks



Notes: The figure shows all impulse responses to a positive structural belief distortion shock that causes 1-year-ahead CPI reported forecasts to depart from their rational expectations by 1 percentage point. Bootstrapped confidence intervals on 1,000 bootstrap runs are reported at the 95% and 90% level.

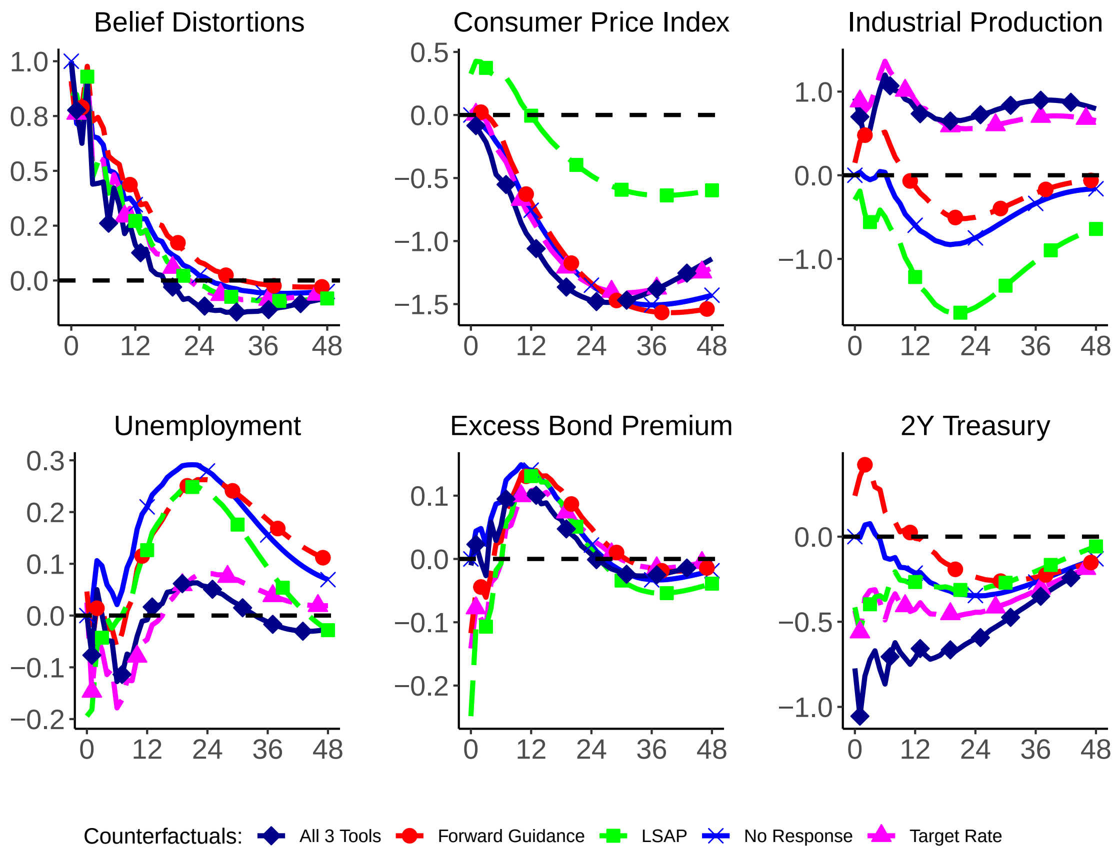
Figure 5: All Impulse Responses to Structural Belief Distortion Shocks

THIS IS ALL MISC STORAGE BELOW HERE

C Proofs

D Replicating Bauer-Swanson

We start with a baseline model of five endogenous variables as shown in Bauer and Swanson (2023). These variables are the log of industrial production, the log of CPI index, the unemployment rate, the excess bond premium from Gilchrist and Zakrajsek (2012) and the treasury yields with 2 years maturity. Although in Figure 3 of Bauer and Swanson (2023), the unemployment rate does not enter the VAR model, we show that its inclusion gives consistent results. The external instrument is the monetary policy shock that is orthogonal to macroeconomic variables. These monthly series are available at Bauer's website and cover the period between January 1973 to February 2020, with the exception of the monetary policy shock series which covers the period from February 1988 to December 2019.



Notes: The figure shows the effects of reduced-form belief distortion shocks under different scenarios of monetary policy responses.

Figure 6: Monetary Policy Response to Reduced-form Belief Distortions

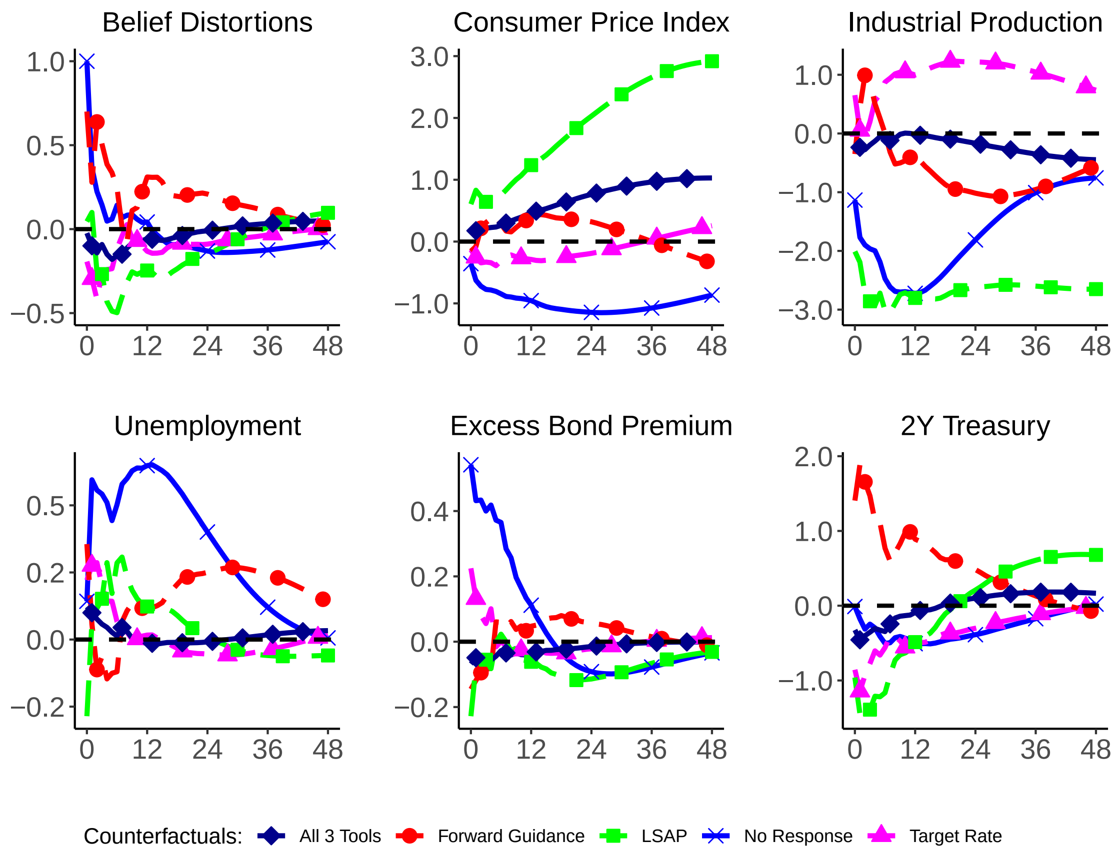
As with Bauer and Swanson (2023), we start by running a VAR model of the five endogenous variables with 12 lags and obtain the coefficient estimates, $B_j \ \forall j \in \{1, ..., 12\}$ and the reduced-form residuals, w_t. We stack the variables in the following order:

$$y_t = \begin{bmatrix} IP \\ CPI \\ UNEMP \\ EBP \\ TREAS \end{bmatrix}$$

The VAR system is:

$$\mathbf{y_t} = \mathbf{c} + \sum_{j}^{12} \mathbf{B_j} \cdot \mathbf{y_{t-j}} + \mathbf{w_t}$$

To estimate the impact effect of a monetary policy shock on the variables we run a TSLS regression. On the first stage, we regress the residuals of treasury yields (that is the fifth



Notes: The figure shows the effects of structural belief distortion shocks under different scenarios of monetary policy responses.

Figure 7: Monetary Policy Response to Structural Belief Distortions

element of the w_t column matrix) on the orthogonal monetary policy shocks, Z_t, and get the predicted values. Let $\mathbf{e_i}$ be a basis column vector of 5 elements that selects the variable positioned in the $i-th$ row of the VAR system. For example, $\mathbf{e_5}$ selects the treasury yields:

$$\mathbf{e_5} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Then the first stage involves the following regression:

$$\mathbf{e_5}' \cdot \mathbf{w_t} = \alpha_0 + \beta_0 \cdot Z_t + \zeta_t$$

For simplicity, we denote the predicted values by $\hat{w}_{5,t}$ and proceed to the second stage where

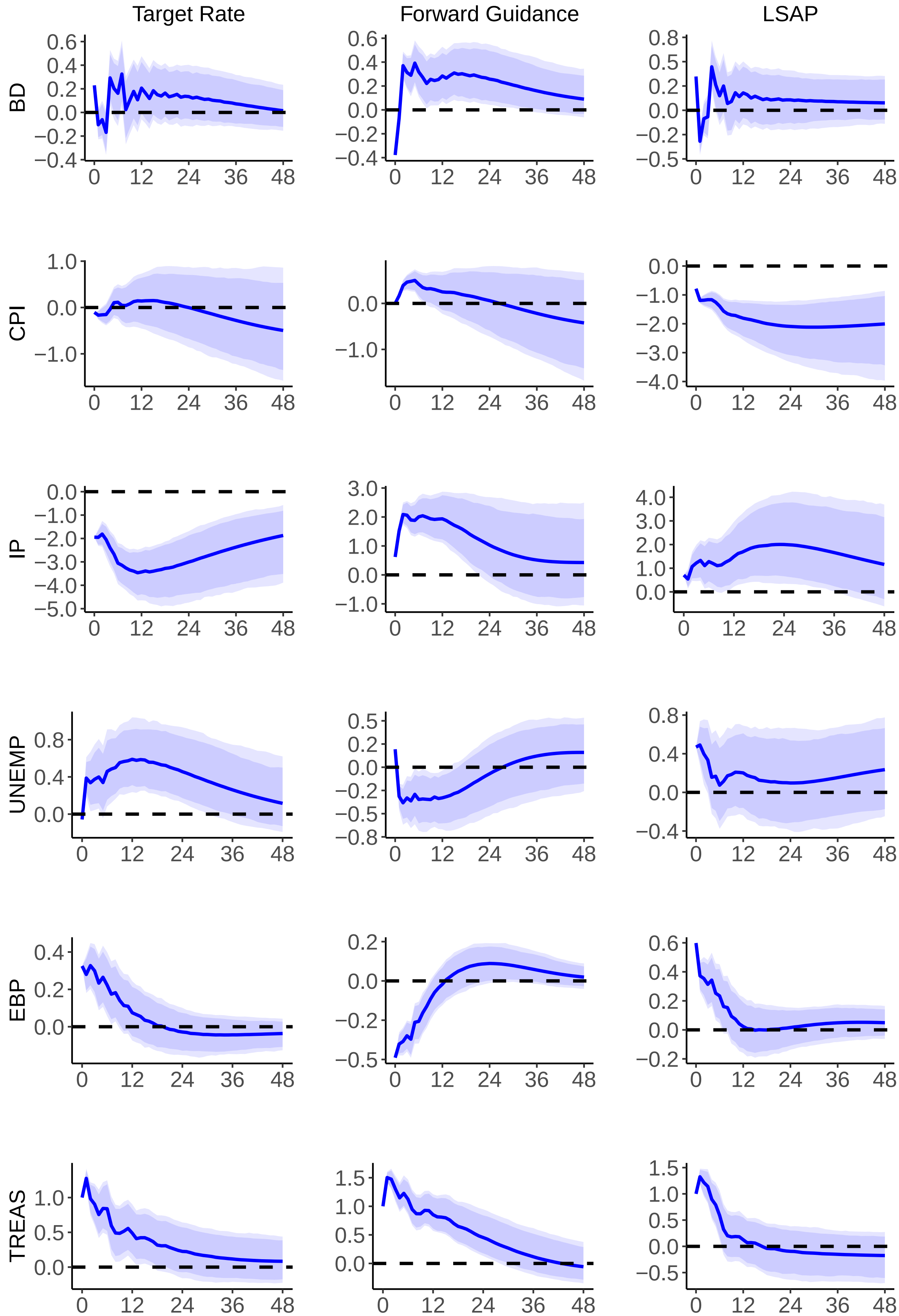


Figure 8: IRF of Monetary Policy Shocks in the Reduced-form Model (Detailed) - 1000 bootstrap runs.

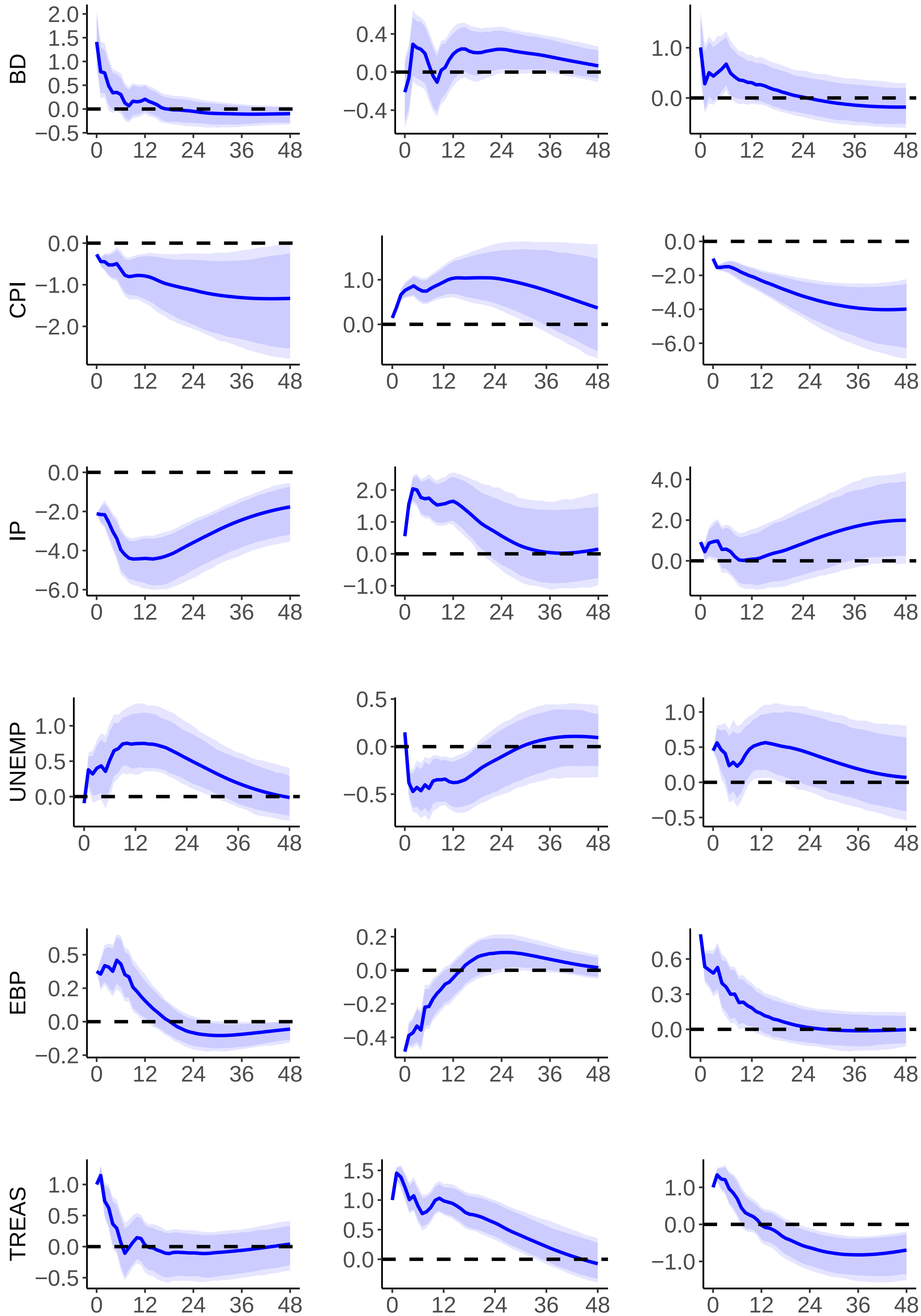


Figure 9: IRF of Monetary Policy Shocks in the Structural Model (Detailed) - 1000 bootstrap runs.

we run, for each variable $i \in \{1, ..., 5\}$:

$$\mathbf{e_i}' \cdot \mathbf{w_t} = \alpha_i + s_i \cdot \hat{w}_{5,t} + \eta_{i,t} \tag{7}$$

The slope coefficients are elements of the structural impact effect of a 1-unit positive monetary policy shock on the variables. In essence,

$$\mathbf{S} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ 1 \end{bmatrix}$$

The impulse responses at horizon k of a monetary policy shock that increases the 2Y treasury yields by 25 bps on impact are estimated as:

$$\phi(k) = \Sigma_{j=1}^{12} \mathbf{B_j} \cdot \phi(k-j)$$

where,

$$\phi(0) = \mathbf{S} \cdot 0.25$$

Figure **??** shows the impulse responses in the absence of belief distortion. As can be seen, the results are consistent with Bauer and Swanson (2023). The next step is to introduce belief distortion in the VAR system. Figure **??** shows how these impulse responses change when belief distortion is added in the VAR model.

The aforementioned approach allows us to estimate the impulse response functions of a monetary policy shock (as in Bauer and Swanson (2023)) but cannot identify responses to other types of shocks in the system. To study the effect of belief distortion shocks, we need an estimate of the impact effect of these shocks. We estimate this impact effect, by stacking this variable first in the VAR and Cholesky decomposing the variance matrix of the residuals from the second stage (see Equation 7). Cholesky decomposition gives us matrix $\mathbf{A}$ such that:

$$\mathbf{\Sigma} \equiv cov(\eta_t) = \mathbf{A} \cdot \mathbf{A}'$$

The impact effect of belief distortion shocks to the variables in the system are therefore given by the first column of $\mathbf{A}$.