Incomplete Information and Irreversible Investment*

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Abstract

How does incomplete information affect capital when investments are irreversible? We use a continuous-time model to analytically characterize how these frictions distort firms' decisions and the distribution of capital. Irreversibility causes misallocation and inaction; incomplete information amplifies the friction, increasing aggregate capital and allocating it less efficiently. Counterintuitively, firms with worse information are less likely to be inactive; they are more willing to invest and do so inefficiently. But their investment is also less responsive to macroeconomic shocks. We test and quantify these predictions using Japanese administrative data that match firms' forecasts with their balance sheets, incomes, and expenditures. In the data, firms underreact to news as if they face information frictions; those with more extreme underreaction are less inactive, as predicted.

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1 Introduction

Incomplete information distorts the allocation of capital: sometimes firms build too much, sometimes too little. The effects are well-understood in an idealized world, but in reality investment is frictional. This is why there is substantial investment inaction in the data, for which a central cause is investment irreversibility. Incomplete information and irreversibility are both crucial frictions for describing investment behavior, and yet they have mainly been studied independently. So a natural question is: how does incomplete information affect capital when investment is irreversible? And do the interactions matter for the aggregate economy?

We approach these questions in two steps, beginning with theory. We develop a tractable continuous-time model of heterogeneous firms that face two frictions that are widely recognized to distort firms' decisions. The first friction is that *capital investment is irreversible*. This implies that firms will sometimes choose not to invest, generating "inaction regions" where capital only depreciates. This non-convex adjustment cost has empirically relevant consequences for firm and macroeconomic dynamics (Baley and Blanco, 2024) because many forms of capital are highly illiquid or firm-specific.[1] The second friction is that *firms do not observe their productivity directly*. Instead, they must rely on noisy signals and only learn the truth with a lag, for example, after sales are realized. This delay between investment decisions and feedback on productivity is common in practice and is especially pronounced in capital-intensive industries. These two frictions have been extensively studied separately, but we show they interact in surprising and economically important ways. This is possible because of the tractability of the continuous-time model; we begin by proving analytically the micro implications, and then turn to the macro implications.

Our first microeconomic finding is the main interaction between the two frictions: the information friction reduces a firm's inaction region. All else equal, uncertainty about a firm's current productivity makes the firm more willing to invest. At first glance, this might be surprising given the well-documented relationship between investment and uncertainty over future productivity. When *future* productivity is more uncertain, firms are less willing to invest because uncertainty increases the option value of delaying irreversible investment, also known as the wait-and-see effect (Leahy and Whited, 1996; Hassler, 1996; Bloom, 2009; Bloom et al., 2018). However, when *current* productivity is uncertain, firms prefer to invest more because the marginal value of capital is convex in log productivity, so Jensen's inequality makes firms act as if they are risk-loving over productivity: the Oi-Hartman-Abel effect.[2] This effect also holds in traditional models of

[1] Ramey and Shapiro (2001) find that upon sale, aerospace plants recover less than 30% of the replacement cost of capital; selling imposes additional wind-down costs and takes years to implement. Kermani and Ma (2023) document that the liquidation value of capital for non-financial firms is only 35% of the net book value.

[2] Strictly speaking, this is only thematically related to the original Oi-Hartman-Abel effects (Oi, 1961; Hartman, 1972; Abel, 1983) because in our simple model, firms' only input is capital which it uses to produce with diminishing

future uncertainty, but is always dominated by the wait-and-see effect.

Our second microeconomic finding is the main motivation for combining the two frictions in practice: incomplete information reduces the short-run elasticity of investment to shocks. This is a common property of incomplete information, which tends to dampen the effects of shocks by reducing how informative a shock is for forecasting future fundamentals. When firms receive a productivity shock, their noisy signal increases one-for-one, but firms do not know whether this change was due to productivity or noise, so their productivity nowcast increases less than one-for-one. This attenuation effect is particularly valuable for describing aggregate investment behavior. Koby and Wolf (2020) argue that firm-level investment is relatively inelastic to aggregate shocks, which is necessary to match the empirical aggregate investment dynamics over the business cycle or responses to monetary policy shocks (Fang, 2020; Winberry, 2021). These heterogeneous firm models require high fixed capital adjustment costs to deliver this low elasticity but generate unrealistic non-smooth investment distribution at the firm level; this is a challenge for the literature. In sum, a model needs to generate a realistically low investment elasticity to aggregate shocks and realistic investment inaction simultaneously. But non-convex adjustment costs cannot deliver this result; House (2014) argues that firms must face other frictions in order to have a realistic investment elasticity. We show that information frictions can help deliver the required low investment elasticity to aggregate shocks, while simultaneously delivering realistic inaction rates and spike rates.

Turning to the macro implications: when investment is irreversible, information frictions amplify the capital distortions, causing overaccumulation and misallocation. This amplification is clear in two essential aggregate moments. First, greater information frictions increase the aggregate capital stock for the same reason that they decrease inaction: uncertainty about current productivity increases firms' willingness to invest. Second, investment irreversibility introduces capital misallocation, and we show analytically that the information friction increases this misallocation, as measured by the variance of the marginal product of capital. This effect occurs as firms make investment mistakes when their *expected* productivity is higher or lower than the actual level.

Finally, we also find that introducing information frictions affects the conventional relationship between volatility and investment inaction. In full information investment models, the option value effect implies that firms facing higher productivity volatility will be more inactive *ceteris paribus*. However, more severe information frictions dampen this relationship. If information frictions are severe, then raising productivity volatility has only small effects on investment

returns. However, the term is now commonly used to describe cases in which the marginal value of capital is convex in a random variable. See for example the decomposition in Senga (2025).

inaction. This is because increasing productivity volatility raises the variance of nowcast errors, leading to more severe information frictions and shrinking the inaction region. In total, this information effect counteracts the classical option value effect, attenuating the relationship between volatility and investment inaction.

In the second step, we test our central theoretical and quantitative predictions using Japanese administrative data. We construct a merged firm-level dataset combining the Business Outlook Survey (BOS) and the Financial Statements Statistics of Corporations (FSS), conducted by the Ministry of Finance and the Cabinet Office of Japan. The BOS provides both realized and forecasted sales for the past and upcoming semi-years, allowing us to estimate industry-level information frictions from the predictability of sales forecast errors. Additionally, firms report investment and investment plans in the BOS, while the FSS provides detailed data on capital stock, employment, and costs, enabling the construction of investment- and productivity-related variables.

We begin our empirical analysis by measuring the severity of firms' information frictions. First, firms are grouped into industries using the industry codes provided by the Ministry of Finance. For each industry, we estimate friction severity by regressing sales forecast errors on past productivity growth. Under full information, this coefficient would be zero, as past productivity would be immediately incorporated into current expectations. In contrast, our incomplete information model predicts a positive coefficient, reflecting delayed and partial incorporation of past information. Consistent with this prediction, the data show that the "underreaction coefficient" is positive across all industries, although it exhibits substantial heterogeneity.

Next, we examine how information frictions affect firms' investment behavior. Consistent with our theory, firms facing more severe information frictions are less likely to remain inactive, conditional on firm-level productivity and size. We test this prediction by regressing a binary investment inaction variable on the industry-level underreaction coefficients. The estimated effect is negative, confirming the model's prediction. A one standard deviation increase in the coefficient reduces the average inaction rate at the industry level by over five percentage points, a quantitatively large effect.[3] Next, we test the predicted attenuated response of investment to productivity shocks by including an interaction between industry-level underreaction and changes in firm-level productivity. This regression allows us to include appropriate fixed effects in order to account for all industry-level time-varying and firm-level time-invariant factors affecting investment. Consistent with the model, firms in industries with more severe information frictions are less responsive to productivity shocks.

Across all tests, we directly compare our regression results with analogous outcomes generated from simulated data using our model. Despite its stylized nature, the model provides clear

[3]The average inaction rate is approximately 36% at the semiannual frequency.

predictions for the signs of key coefficients. Our firm-level empirical results are broadly consistent with the model's analytical and quantitative predictions.

Literature: First, our theoretical work is closely related to a small but growing literature on incomplete information in continuous time models featuring inaction. In this existing literature, inaction is due to fixed costs; to the best of our knowledge, we are the first to study the interaction between incomplete information and irreversibility as the source of inaction. Verona (2014) studies inattentive firms who pay fixed costs to update information (as in Reis, 2006), which leads to periodic large investment spikes. Alvarez et al. (2011), Alvarez et al. (2016), and Stevens (2020) study price-setting by firms facing high fixed costs to both changing prices and observing fundamentals. Baley and Blanco (2019) consider a model of menu costs where firms observe noisy signals of their productivity; they predict that firms with higher uncertainty change prices more often and learn more quickly. We also join broader literatures studying how investment dynamics are affected by irreversibility and information frictions.[4] Of these, one closely related paper is Senga (2025), which studies how uncertainty over future productivity affects investment behavior (without investment frictions) in a quantitative business cycle model.

Second, our theoretical work joins a broad literature on irreversibility and investment inaction. Early work (Pindyck, 1991; Bertola and Caballero, 1994; Abel and Eberly, 1996; Abel et al., 1996) established the option value of irreversibility, and built the standard theoretical framework for analyzing it. More recent work has developed our understanding of the macroeconomic implications, particularly over the business cycle (Lanteri, 2018; Baley and Blanco, 2024). And new empirical findings have better quantified this friction, suggesting that its contribution to misallocation and productivity are substantial (Caunedo and Keller, 2021; Kermani and Ma, 2023).

Third, our empirical work connects to a burgeoning literature studying predictable errors in firms' forecasts. Most closely related is a set of papers studying whether managers overreact or underreact to news when forecasting their own firms' outcomes. The evidence is mixed. Ma et al. (2020) use Italian sales forecasts from a representative survey run by the Bank of Italy, and find evidence of underreaction, similar to our estimates in the Japanese data. In contrast, Barrero (2022) finds that in the Survey of Business Uncertainty, US CEOs and CFOs overreact to news. Bordalo et al. (2021) estimate overreaction in managers' earnings forecasts in the IBES guidance

[4]Some canonical and more recent examples studying irreversibility include Pindyck (1991), Bertola and Caballero (1994), Abel and Eberly (1996), Veracierto (2002), Ottonello (2017), and Baley and Blanco (2024). Stokey (2008) provides a textbook treatment. Papers studying business cycle models of capital investment with information frictions include Townsend (1983), Angeletos and Pavan (2004), Graham and Wright (2010), Angeletos et al. (2018), and Atolia and Chahrour (2020), among many more. In particular, Adams (2023) studies a model with investment frictions that does not induce inaction; instead, firms face investment adjustment costs in the style of Christiano et al. (2005). Instead of firms and investment, some recent papers study how information frictions affect savings heterogeneous agent models, including Adams and Rojas (2024) and Broer et al. (2021).

database. Many factors may contribute to these differences; forecast horizons may matter, the incentives may be different on earnings calls versus official central bank surveys, and there may be cultural differences between US, Italian, and Japanese managers.[5]

Finally, our paper is linked to the literature that utilizes firm-level survey data to document how micro, industry, and macroeconomic shocks affect firms' expectations formation. Andrade et al. (2022) show that industry-level inflation predicts forecast errors about firms' own prices in a survey of French manufacturers. Massenot and Pettinicchi (2018) and Born et al. (2022) use a survey of German manufacturing firms to document how business conditions and news, respectively, predict forecast errors. Other related papers include Bachmann et al. (2013), Bachmann and Elstner (2015), Bachmann et al. (2021), Born et al. (2023a). Born et al. (2023b) survey additional work in this field, while Candia et al. (2023) survey the larger literature studying biases in firms' expectations of the macroeconomy.

Layout. The remainder of the paper is organized as follows. Section 2 lays out the stylized firm's investment model, and Section 3 characterizes the aggregate economy. Section 4 validates the model's micro implications using Japanese firm-level data. Section 5 provides a quantitative analysis of the model's macro implications. Finally, Section 6 concludes.

2 A Stylized Investment Model

This section describes the economic environment, investment decisions, and information friction. We derive the value function and optimal decisions and demonstrate analytically how investment decisions depend on different parameters, including those controlling the information friction.

2.1 Firm's Problem

Environment There is a unit measure of atomistic competitive firms. Firms produce using capital K, modified by productivity A. Their production function is $F(A, K) = A^{1-\alpha}K^{\alpha}$, where $\alpha \in (0, 1)$. Investment I is irreversible. If firms invest, they do so at cost ψ. Accordingly, their

[5]This specific literature is also a part of a broader one studying firms' forecast errors of micro and aggregate variables more generally. Seminal work done by Coibion et al. (2018), Tanaka et al. (2020), and Candia et al. (2024) present stylized facts concerning firm-level expectations. Several studies also use data on Japanese firm-level expectations. Using a dataset of multinational firms, Chen et al. (2023b) document heterogeneity in the information frictions firms face that varies by firm size and age. Second, Chen et al. (2023a) document that the degree of information rigidity firms face is higher for aggregate inflation than for firm-specific outcomes. Finally, Charoenwong et al. (2024) show how capital budgeting can alleviate distortions originating from investment frictions and thus improve productivity. Other papers that use Japanese firm-level expectations include Charoenwong et al. (2020), Chen et al. (2020), and Chen et al. (2022).

instantaneous profit is $\pi = A^{1-\alpha}K^{\alpha} - \psi I$. Lowercase letters denote logs of variables, e.g., $a = \ln A$. Log productivity follows a random walk $da = \sigma_a dW^a$ where W^a is a Wiener process. The law of motion for capital is $dK = I - \delta K dt$ where δ is the depreciation rate.

Optimal firm behavior for this type of problem is characterized by an inaction region: above some level of capital (that depends on other state variables), firms choose not to invest. Firms discount the future at a constant rate r, so inside the inaction region, a firm's Hamilton-Jacobi-Bellman (HJB) equation is

$$rV(K,A) = A^{1-\alpha}K^{\alpha} - \delta K V_K(K,A) + \frac{\sigma_a^2 A^2}{2} V_{AA}(K,A) \tag{1}$$

This is the *full information* HJB. Of course, firms will not have full information when forecasting. However, in the inaction region, the firm's true value still follows this PDE. The wrinkle to this model is that when firms *do* make an action, they will not know A exactly.

Information Structure Firms do not know their productivity A exactly. Instead, they receive a noisy signal ds of productivity growth:

$$ds = da + dn \tag{2}$$

where

$$dn = \sigma_n dW^n \tag{3}$$

and the Wiener process W^n is independent of W^a. In discrete time, ds would be analogous to a signal of a productivity shock with iid noise. s and n represent the accumulated signals and noise respectively; both follow a random walk.

Additionally, we assume that after τ time, the productivity level is revealed to the firm, i.e., at time t, firms learn the productivity that they had at time $t - \tau$. This structure represents the notion that decision-makers do not know exactly how productive their firm is at any moment but learn ex-post after an accounting period is completed.

Expectation Formation To characterize how firms form expectations, it is useful to temporarily introduce time subscripts, which we have suppressed so far. Productivity growth over τ time is distributed $(a_t - a_{t-\tau}) \sim N(0, \tau\sigma_a^2)$, while $(s_t - s_{t-\tau})$ is distributed $N(0, \tau(\sigma_a^2 + \sigma_n^2))$ due to the independent Wiener processes W^a and W^n.

Lemma 1. *For a firm with information set $\Omega_t = \{a_{j-\tau}, s_j\}_{j \le t}$, productivity is conditionally distributed*

$$a_t|\Omega_t \sim N\left(a_{t-\tau} + \gamma\left(s_t - s_{t-\tau}\right), v\right)$$

where

$$\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2} \qquad \nu \equiv \frac{\tau \sigma_a^2 \sigma_n^2}{\sigma_a^2 + \sigma_n^2}$$

therefore the firm's expected productivity $\hat{a} \equiv \mathbb{E}[a|\Omega]$ *and nowcast error* u *follow the diffusions*

$$d\hat{a} = \sigma_a dW^{\hat{a}} \qquad du = \sigma_u dW^u$$

where

$$dW_t^{\hat{a}} = (1-\gamma) dW_{t-\tau}^a + \gamma dW_t^a + \gamma \frac{\sigma_n}{\sigma_a}(dW_t^n - dW_{t-\tau}^n)$$

$$dW_t^u = (1-\gamma)\frac{\sigma_a}{\sigma_u}(dW_t^a - dW_{t-\tau}^a) + \gamma \frac{\sigma_n}{\sigma_u}(dW_t^n - dW_{t-\tau}^n)$$

$$\sigma_u^2 = 2\frac{\sigma_n^2 \sigma_a^2}{\sigma_a^2 + \sigma_n^2}$$

Proof: Appendix A.2.1

Lemma 1 describes how firms form expectations of their productivity under incomplete information. Two parameters are worth explaining further. First, the signal coefficient $\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}$ measures how elastic firms' expectations are to the noisy signals. Second, the nowcast error variance $\nu \equiv \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ depends on the information delay τ, the noise volatility σ_n, and productivity volatility σ_a. With longer delays and larger noise, firms make larger nowcast errors.

2.2 Solving the Firm's Problem

The full information HJB equation (1) is homogeneous of degree 1 in (K, A) (Stokey, 2008, Ch. 11) so it is possible to rewrite in terms of a single variable $X \equiv \frac{K}{A}$, which we call "normalized capital":

$$rV(X) = X^\alpha - \delta X V'(X) + \frac{\sigma_a^2 X^2}{2} V''(X) \tag{4}$$

Moreover, it is convenient to express the HJB in terms of log normalized capital $x = k - a$:

$$rv(x) = e^{\alpha x} - \mu v'(x) + \frac{\sigma_a^2}{2} v''(x) \tag{5}$$

where $\mu \equiv \delta + \frac{\sigma_a^2}{2}$. This conversion to writing the HJB as a function of $x = \log(X)$ follows from $v(x) = V(X)$, $v'(x) = V'(X)X$, and $v''(x) = V''(X)X^2 + V'(X)X$.

These are the usual full information HJB equations. How does incomplete information affect the firm's problem if it does not change the HJB equation? It changes the *boundary conditions*, which are the equations characterizing optimal action. The HJB has many solutions; the correct

solution is determined by the appropriate boundary conditions.

Information is incomplete, so x is unknown to firms when making investment decisions. The usual optimality conditions of $V(X)$ cannot be applied in this case. Instead, firms have uncertainty; their expected value of $V(X)$ is given by

$$\mathbb{E}[V(X)|\Omega] = \mathbb{E}[V(X)|\hat{X}] \equiv \hat{V}(\hat{X})$$

Because conditional expectations are normally distributed with constant variance (Lemma 1), *expected log normalized capital* $\hat{x} \equiv \mathbb{E}[x|\Omega]$ is a summary statistic for firms' expectations, as is $\hat{X} \equiv e^{\hat{x}}$, which represents the firm's MLE nowcast of X. The firm's goal is thus to maximize $\mathbb{E}[V(X)|\hat{X}]$, which we write as the *expected value function* $\hat{V}(\hat{X})$.[6]

Optimal investment behavior is a threshold strategy, as in the case of full information. Except now, a firm invests only if its expected log normalized capital $\hat{x}$ is less than some boundary $\hat{b}$. Solving the firm's problem involves finding the optimal choice of $\hat{B} \equiv e^{\hat{b}}$. Lemma 2 reports the boundary conditions associated with the optimum. They are analogous to the full information case.

Lemma 2. *Under incomplete information, the boundary conditions consist of two value-matching conditions:*

$$\hat{V}'(\hat{B}) = \psi \qquad \lim_{\hat{X}\to\infty} \hat{V}'(\hat{X}) = 0$$

and two super contact conditions:

$$\hat{V}''(\hat{B}) = 0 \qquad \lim_{\hat{X}\to\infty} \hat{V}''(\hat{X}) = 0$$

Proof: Appendix A.2.2

The proof is standard and follows closely the arguments in Dumas (1991). We include the proof in order to show that we can apply the usual full information optimality conditions to the expected value function $\hat{V}(\hat{X})$.

Lemma 3 below characterizes the solution to the firm's problem. The critical value $\hat{b}$ depends on several parameters: the variance of nowcast errors v, the returns to scale α, the cost of investment ψ, as well as ϱ and m defined as:

$$\varrho \equiv \frac{\mu - \sqrt{\mu^2 + 2\sigma_a^2 r}}{\sigma_a^2} \qquad m \equiv \frac{1}{r + \mu\alpha - \frac{\sigma_a^2}{2}\alpha^2}$$

[6] Appendix A.1 describes the general solution to the HJB equation.

Lemma 3. *The critical value of expected normalized capital is*

$$\hat{b} = b^{FI} + \frac{\alpha^2 v}{2(1-\alpha)}$$

where b^{FI} is the full information critical value given by:

$$b^{FI} = \frac{1}{(1-\alpha)} \log\left(\frac{m\alpha(\alpha-\varrho)}{\psi(1-\varrho)}\right)$$

Proof: Appendix A.2.3

Lemma 3 demonstrates how the information friction affects the firm's optimal investment decisions. Conveniently, most of these terms affect the critical values in the same way as in the full information model. The proposition shows that the difference between full and incomplete information critical values depends only on the variance of nowcast errors v, adjusted by the returns to scale α.[7]

2.3 Micro Implications of Incomplete Information

We can now analytically characterize how the investment and information frictions interact to change firms' investment behavior. First, incomplete information reduces the inaction region. Second, incomplete information attenuates the investment response to productivity shocks.

2.3.1 Investment Inaction

The information friction has a clear effect on the firm's optimal behavior: stronger information frictions *shift* the inaction region to the *right*. This is because the optimal boundaries increase as the variance of the nowcast errors v gets larger. Proposition 1 formalizes this result.

Proposition 1. *The inaction region bound is increasing in both the noisy signal variance σ_n^2 and the revelation delay τ.*

Proof. The nowcast error variance $v = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ is increasing in both τ and σ_n^2. Lemma 3 implies that $\hat{b}$ is increasing in v. □

Why does the information friction shift the inaction region rightwards? The most intuitive answer is because of an Oi-Hartman-Abel effect: the marginal value of capital is convex in log

[7] This conclusion is not unique to fully irreversible investment. In Appendix B.1, we extend the model to allow for partial irreversibility, and come to the same conclusion: incomplete information increases inaction region boundaries relative to the full information case.

productivity a. This is true whether information is incomplete or not. The effect of productivity is asymmetric; for the marginal value of capital, the upside of an improvement to a outweighs the downside of a symmetric decrease. Thus a mean-preserving spread in a increases the expected marginal value of capital. And a mean-preserving spread in a is equivalent to a mean-preserving spread in x. This makes firms risk-loving over normalized capital: if they do not know the true value, the expected marginal value exceeds the certainty equivalent, i.e.

$$\mathbb{E}[V(\exp(x))|\Omega] > V(\exp(\mathbb{E}[x|\Omega]))$$

The expected instantaneous return to an additional unit of marginal capital is larger when firms are uncertain about the value of their X. This uncertainty raises a firm's incentive to invest, thus they are willing to do so at higher levels of expected X, raising the lower bound on their inaction region.

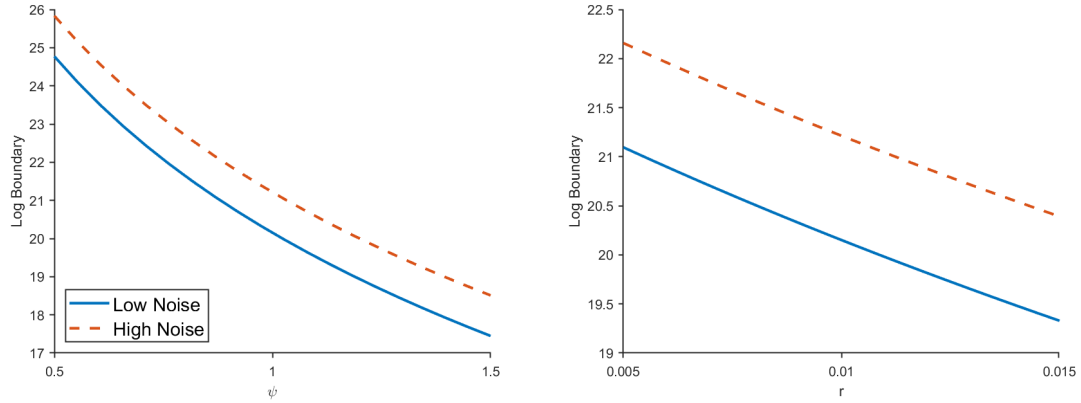
Figure 1 illustrates how different parameters affect firms' decisions. In all cases, the solid blue line plots how $\hat{b}$ depends on the parameter of interest when firms' signals are not particularly noisy ($\sigma_n = 0.25\sigma_a$). The dashed red line plots the $\hat{b}$ sensitivity for a noisier calibration ($\sigma_n = \sigma_a$). In all cases, the dashed red line is above the solid blue line: when firms have *worse* information, they are *more* willing to invest, as implied by Proposition 1.

The first two panels show that investment responds in the usual way to standard parameters. Panel 1a shows that $\hat{b}$ is decreasing in the investment cost ψ: when capital is more expensive, firms are less willing to invest and do so only when their normalized capital is lower. Similarly, 1b shows that $\hat{b}$ is decreasing in the interest rate r: when firms discount the future at a higher rate, they are less willing to invest. The information friction raises the inaction boundary for all values of ψ and r, but does not fundamentally change how inaction depends on these parameters.

The next two panels show how signal noise interacts with parameters. Panel 1c demonstrates the other way that the information friction increases willingness to invest. $\hat{b}$ increases in τ, the amount of time before productivity is revealed to the firm. Larger values of τ make firms less certain about their current productivity level, increasing their nowcast error variance (Lemma 1). When τ is larger, the information friction is worse. Like σ_n, exacerbating the information friction increases the incentive to invest and raises $\hat{b}$. Moreover, when the delay is larger, the signal noise volatility has even stronger effects, because firms accumulate disproportionately more noise in their signals. This is why the curves diverge in Panel 1c.

The information friction has an unusual interaction with productivity volatility. To show this, Panel 1d plots how $\hat{b}$ depends on the productivity standard deviation σ_a. The standard result is that higher productivity volatility should make firms less willing to invest because it increases

Figure 1: How the Boundary $\hat{b}$ Depends on Various Parameters



(a) Investment Cost

(b) Interest Rate

(c) Revelation Delay

(d) Productivity Volatility

Notes: Each plot displays how the optimal inaction boundary $\hat{b}$ depends on a parameter value. The information processes are parameterized in two ways: the blue line is less noisy with $\sigma_n = 0.25\sigma_a$, while the red dashed line is more noisy, with $\sigma_n = \sigma_a$. In both cases $\sigma_a = 0.5$ and $\tau = 4$ so that information is revealed after 4 quarters. The remaining parameters are set as in Table 3a.

the option value of waiting (Leahy and Whited, 1996; Hassler, 1996). This is the case in our model, too: raising volatility reduces the boundary, i.e., making firms less willing to invest. However, volatility also plays a role in information friction. When volatility increases, firms' nowcasts are less accurate (ν decreases in σ_a). This information effect *attenuates* the standard option-value effect of volatility on inaction. And the attenuating information effect is stronger when signals are noisier. Panel 1d makes this clear: if signals are relatively precise (solid blue line), then volatility sharply reduces the boundary, but if signals are relatively noisy (dashed red line), then volatility has less effect on inaction.

This result demonstrates a new channel by which "uncertainty" affects capital investment. The traditional *wait-and-see channel* is that uncertainty over future productivity reduces the incentive to invest by increasing the option value of delaying new capital. In recent work, uncertainty has been documented to be a major driver of business cycles, and this investment wait-and-see channel is considered a central mechanism (Bloom, 2009; Bloom et al., 2018). The new *information channel* is that uncertainty over current productivity increases the incentive to invest. Moreover, these two channels interact as shown by Panel 1d. When information is incomplete, higher productivity volatility still increases the inaction region of investment. However, information frictions change this relationship. If information frictions are severe, then raising productivity volatility has only small effects on investment inaction. This is because increasing productivity volatility raises the variance of nowcast errors, shrinking the inaction region. This information effect counteracts the classical option value effect, attenuating the relationship between volatility and investment inaction.

2.3.2 Investment Sensitivity

The information friction attenuates the short-run impact of shocks by reducing the pass-through from productivity shocks to firms' productivity expectations. With full information, productivity shocks affect forecasts one-for-one because productivity follows a random walk. This is not the case when information is incomplete.

Lemma 4 shows that the short-run attenuation depends on $\gamma = \frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}$, as defined in Lemma 1. When the noise variance σ_n^2 is large, then signals are noisy, and productivity shocks $da = \sigma_a dW^a$ have little effect on firms' immediate nowcasts. But as $\sigma_n^2 \to 0$, firms more accurately nowcast their productivity and approach the full information case.

Lemma 4.

$$\frac{d}{da_{t-h}}\mathbb{E}[a_t|\Omega_t] = \begin{cases} \gamma & 0 \leq h < \tau \\ 1 & h \geq \tau \end{cases}$$

Proof: Appendix A.2.4

Lemma 4 above implies that the immediate pass-through of a productivity shock to the firm's expected log normalized capital ($\hat{x} = k - \hat{a}$) is $-\gamma$. Thus, any quantity that depends on $\hat{x}$ will be similarly attenuated in the short run, e.g., the average time until leaving the inaction region, the expected investment over a time period, and so forth. However, because the actual productivity level is eventually revealed to firms, Lemma 4 also says that the long-run pass-through from productivity to nowcasts and forecasts is one-for-one.

What does this result imply for tests of incomplete information? In Section 4.2, we estimate a standard *underreaction coefficient* (Kohlhas and Walther, 2021), which measures how future forecast errors responds to current information. Proposition 2 derives how this coefficient – the effect of productivity shock $da_t = \sigma_a dW_t^a$ on forecast error $(a_{t+h} - \mathbb{E}[a_{t+h}|\Omega_t])$ – depends on the information friction parameters in the model.

Proposition 2. *A firm's underreaction coefficient is*

$$\frac{d(a_{t+h} - \mathbb{E}[a_{t+h}|\Omega_t])}{da_t} = 1 - \gamma$$

Proof. Log productivity a_t follows a random walk, which implies

$$\frac{d(a_{t+h} - \mathbb{E}[a_{t+h}|\Omega_t])}{da_t} = \frac{da_{t+h}}{da_t} - \frac{d\mathbb{E}[a_t|\Omega_t]}{da_t} = 1 - \frac{d\mathbb{E}[a_t|\Omega_t]}{da_t}$$

with $\frac{d\mathbb{E}[a_t|\Omega_t]}{da_t} = \gamma$ by Lemma 4. □

The coefficient γ controls how sensitive firms' decisions are to productivity shocks. But it is even more informative than this: the next section shows that γ also controls the sensitivity of the macroeconomic response to aggregate shocks.

3 The Aggregate Economy

To characterize the aggregate economy, we first must make several assumptions about the distribution of firms. We derive the partial differential equations governing the dynamics of firm distributions and solve explicitly for the stationary distribution. Then, we show analytically how the information friction amplifies capital misallocation, creates overinvestment, and dampens aggregate dynamics.

3.1 Firm Entry and Exit

We assume that a measure η of firms enter the economy at every moment. Entering firms do not know their productivity. They are as uncertain as existing firms, i.e., their Bayesian prior is that their log productivity is normally distributed with variance ν.

Across entering firms, expected log productivity $\hat{a}$ is distributed by $\hat{a}_{enter} \sim N(0, \varsigma)$. Thus, the entering distribution of actual log productivity is $a_{enter} \sim N(0, \varsigma + \nu)$. Firms' log expected normalized capital $\hat{x}$ enters at the critical value $\hat{b}$: $\hat{x}_{enter} = \hat{b}$. This is a natural assumption for

the entering distribution of capital: firms are born with some unknown productivity level and acquire capital until they are no longer willing to invest. Firm exit is random, independent of other variables. We assume that when firms exit, their value is returned to shareholders, so the exit risk does not change the firm's HJB equation. For the probability distribution to integrate into one, firms must exit at the same rate they enter: η.

3.2 The Stationary Distribution of Normalized Capital

Lemma 1 implies that expected normalized capital $\hat{x} = k - \hat{a}$ follows the diffusion

$$\hat{x} = -\delta dt - \sigma_a dW^{\hat{a}}$$

Firms exit at rate η, so the Kolmogorov Forward Equation (KFE) for the distribution $h(\hat{x}, t)$ of expected normalized capital in the inaction region is

$$\partial_t h(\hat{x}, t) = \delta \partial_{\hat{x}} h(\hat{x}, t) + D\partial^2_{\hat{x}} h(\hat{x}, t) - \eta h(\hat{x}, t) \tag{6}$$

where $D \equiv \frac{\sigma_a^2}{2}$. PDFs written without time arguments denote stationary distributions. The KFE for the stationary distribution of expected normalized capital is thus

$$0 = \delta h'(\hat{x}) + Dh''(\hat{x}) - \eta h(\hat{x}) \tag{7}$$

The boundary condition is that $h(\hat{x})$ must integrate to one on the interval $[\hat{b}, \infty)$. Lemma 5 below gives the solution.

Lemma 5. *The stationary distribution of expected normalized capital $h(\hat{x})$ for $\hat{x} \geq \hat{b}$ is*

$$h(\hat{x}) = \rho e^{-\rho(\hat{x} - \hat{b})}, \quad \textit{where} \quad \rho \equiv \frac{\delta}{\sigma_a^2} + \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$$

and the stationary distribution of log-normalized capital is

$$f_x(x) = h(x) e^{\frac{\nu\rho^2}{2}} \Phi\left(\frac{x - (\hat{b} + \nu\rho)}{\sqrt{\nu}}\right)$$

where $\Phi(\cdot)$ is the standard normal CDF.

Proof: Appendix A.2.5

Lemma 5 shows that the information friction does not affect the shape of the stationary distribution $h(\hat{x})$. The root ρ is determined from purely economic fundamentals: depreciation δ, productivity volatility σ_a, and exit risk η; ν never appears. The information friction only shifts the stationary distribution left or right by determining the lower bound $\hat{b}$.

The joint distribution $f_{\hat{x},u}(\hat{x}, u)$ of expected normalized capital $\hat{x}$ and productivity nowcast errors $u = a - \hat{a}$ follow straightaway from Lemma 5, because nowcast errors must be independent of nowcasts. Thus, their joint distribution is the product of their marginal distributions:

$$f_{\hat{x},u}(\hat{x}, u) = h(\hat{x})\phi(\frac{u}{\sqrt{\nu}}) \tag{8}$$

where $\phi(\cdot)$ is the standard normal pdf. From the joint distribution, it is straightforward to calculate the marginal distribution $f_x(x)$ of actual normalized capital $x = \hat{x} - u$.

We illustrate the differences between the two distributions. Figure 2 plots how the stationary distribution of *realized* normalized capital $f_x(x)$ compares to the distribution $h(\hat{x})$ of *expected* normalized capital. $h(\hat{x})$ is monotonic and has a discrete lower bound at the barrier $\hat{x}$. In contrast, normalized capital $x = \hat{x} - u$ is smoothed out because it subtracts an independent Gaussian. Instead of featuring a lower bound, it has an infinite domain. And Lemma 5 implies that the larger the nowcast error variance ν is, the smoother the distribution.

What is the firm size distribution? We have characterized the distribution of normalized capital; to answer this question, we need to decompose normalized capital into its capital and productivity components. To do this, we need to solve the KFE in multiple dimensions. In the inaction region, the KFE for the distribution of capital and expected productivity $g(k, \hat{a}, t)$ is

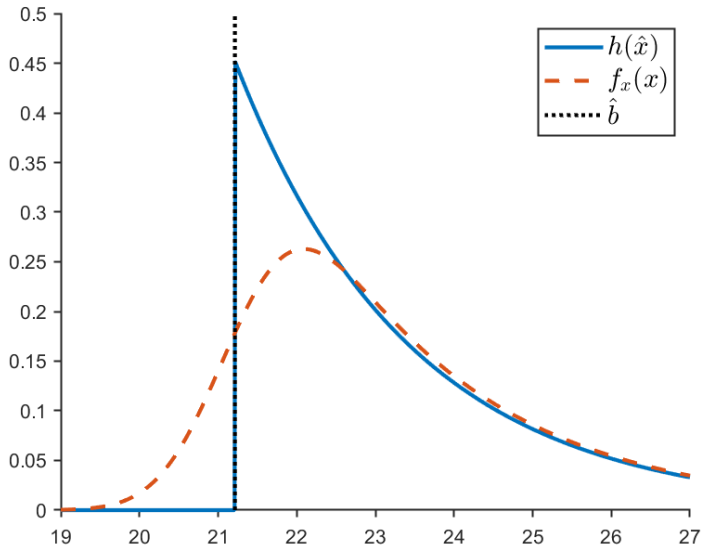
$$\partial_t g(k, \hat{a}, t) = \delta \partial_k g(k, \hat{a}, t) + D\partial^2_{\hat{a}} g(k, \hat{a}, t) - \eta g(k, \hat{a}, t) \tag{9}$$

With the stationary distribution satisfying the partial differential equation

$$0 = \delta \partial_k g(k, \hat{a}) + D\partial^2_{\hat{a}} g(k, \hat{a}) - \eta g(k, \hat{a}) \tag{10}$$

This distribution is more challenging to characterize analytically than the univariate normalized capital distribution, because it requires solving a PDE with an unusual boundary condition. Appendix A.3 does so. After calibrating the model in Section 5, Figure 5 plots the joint distribution.

Figure 2: Stationary Distributions for Expected and Realized Normalized Capital



Notes: The plot compares the marginal distributions of expected log normalized capital $\hat{x}$ and actual log normalized capital x. The vertical line denotes the boundary of the inaction region $\hat{b}$. The parameters match the "High Noise" case from Figure 1: $\sigma_a = 0.5$, $\sigma_n = \sigma_a$, and $\tau = 4$.

3.3 Macro Implications of Incomplete Information

Having a closed-form solution for the steady-state distribution of expected normalized capital is valuable because it allows us to characterize in closed form how various macroeconomic aggregates depend on the parameters of the information friction. In this section, we show that information frictions increase both capital misallocation and average normalized capital, and attenuate the short-run effects of aggregate shocks.

3.3.1 Capital Accumulation

Aggregate normalized capital $\bar{X}$ in the economy is given by

$$\bar{X} \equiv \int_{-\infty}^{\infty} e^x f_x(x) dx$$

where $f_x(x)$ is the stationary distribution of log normalized capital as defined in Proposition 5.

We documented in Section 2.2 that the information friction increases firms' willingness to build capital. Noisier information raised the lower bound on firms' inaction region. This effect

increases aggregate normalized capital, as Proposition 3 demonstrates.

Proposition 3. *If $\rho > 1$, then steady state normalized capital is finite and increasing in both the noisy signal variance σ_n^2 and the revelation delay τ.*

Proof: Appendix A.2.6

3.3.2 Capital Misallocation

We measure misallocation as the variance of the log marginal product of capital:

$$\text{Var}\left[\log \frac{\partial F(A,K)}{\partial K}\right] = (1-\alpha)^2 \text{Var}[x]$$

The information friction increases the above capital misallocation in a straightforward manner. Misallocation can be decomposed into two components: dispersion in expected capital $\text{Var}[\hat{x}]$ and dispersion in nowcast errors v. The former is due to endogenous decisions, while the latter is due to mistakes made by firms that do not know their productivity precisely. Proposition 4 shows that the information friction increases misallocation entirely due to mistakes.

Proposition 4. *Steady-state capital misallocation is increasing in both the variance of the noisy signal σ_n^2 and the revelation delay τ.*

Proof. Normalized capital is decomposed into nowcast errors by $x = \hat{x} - u$. $\hat{x}$ and u are independent, so

$$\text{Var}[x] = \text{Var}[\hat{x}] + v$$

where $v = \frac{\tau \sigma_a^2 \sigma_n^2}{\sigma_a^2 + \sigma_n^2}$ is the nowcast error variance. Lemma 5 implies that $\text{Var}[\hat{x}]$ is independent of the information friction parameters. Thus misallocation is increasing in v, which is increasing in σ_n^2 and τ. □

3.3.3 Aggregate Shock Attenuation

A typical effect of information frictions is to attenuate aggregate shocks. This model is no different. Firms are slow to invest after productivity improvements because they do not know the improvement with certainty. This dampens the endogenous capital response to the shock, at least until information is revealed.

In this section, we describe the attenuation result theoretically. The endogenous response of capital is *exactly* attenuated by the nowcasting coefficient $\gamma < 1$ described in Lemma 4. This result

holds to first order after a permanent productivity shock. We numerically explore the attenuation further in the quantitative Section 5.

Consider an aggregate productivity shock to an economy in the steady state. This shock permanently raises the log productivity a of all firms by ε at time $t = 0$. Accordingly, this shock lowers log normalized capital x, shifting the distribution $h(x, 0)$ to the left by ε. Any firms that are pushed outside of their inaction region immediately invest up to the boundary. However, Lemma 4 implies that the information friction causes the distribution of expected normalized capital $\hat{x}$ to only shift left by $\gamma\varepsilon$, with $\gamma < 1$. This is how the information friction attenuates the effects of aggregate shocks.

The model's tractability allows us to characterize precisely how the economy-wide impulse response function (IRF) is attenuated. To do so, let $f_x(x, t, \varepsilon)$ denote the dynamic distribution of log normalized capital x given the ε shock at time $t = 0$. Define the IRF of average log normalized capital relative to the steady state by

$$IRF_x(t, \varepsilon) \equiv \int_x x f_x(x, t, \varepsilon) dx - \bar{x}$$

where $\bar{x}$ denotes the steady state averages. Following from $k - a = x$, the IRF for average log capital to the permanent ε shock is

$$IRF_k(t, \varepsilon) = IRF_x(t, \varepsilon) + \varepsilon$$

The IRF is nonlinear in ε, so to make progress analytically we consider the marginal effect of a small productivity shock, following Borovička et al. (2014) and Alvarez and Lippi (2022). Let $\widehat{IRF}_k(t)$ denote the *marginal IRF* of average log capital k, i.e.

$$\widehat{IRF}_k(t) \equiv \frac{\partial}{\partial \varepsilon} IRF_k(t, \varepsilon)\Big|_{\varepsilon=0}$$

and let $\widehat{IRF}_k^{FI}(t)$ denote the counterfactual marginal IRF under full information.

Proposition 5. *The marginal IRF of average log capital k is attenuated relative to the full information marginal IRF by*

$$\widehat{IRF}_k(t) = \gamma \widehat{IRF}_k^{FI}(t)$$

for time $t < \tau$.

Proof: Appendix A.2.8

Proposition 5 proves that the information friction attenuates the endogenous response of

capital in the short run. Combined with Proposition 2, this result also reveals that firms' underreaction coefficient $1-\gamma$ is a *sufficient statistic* for the aggregate attenuation. If underreaction is large, as Ma et al. (2020) find in Italian data, then the information friction can severely distort the dynamic effects of aggregate shocks. The attenuation result above delivers one of the main motivations for the model: combining irreversibility and information frictions can deliver realistic investment behavior in the cross-section while also delivering the requisite low elasticity to macro shocks. To this end, the next section shows that this stylized model delivers firm-level investment behavior that resembles the data. And more generally, we empirically test the main mechanisms using microdata.

4 Validation with Microdata

In this section, we use Japanese firm-level data to test our key theoretical predictions at the micro level, specifically that incomplete information reduces investment inaction and attenuates the effects of productivity shocks. We proceed in three steps. First, we estimate the extent of information incompleteness by measuring the industry-specific underreaction coefficient in the data. Second, we numerically simulate a model economy of firms following the optimal decision-making derived in Section 2 and match the estimated underreaction coefficient. Third, we run the same regressions on both the microdata and the simulated data, comparing the results to illustrate the model's micro-level implications. This comparison shows that several key coefficients estimated from the firm-level data align with those from the simulated data.

4.1 Japanese Firm-level Data

We utilize the BOS (*Business Outlook Survey*, 2004-2018) and the FSS (*Financial Statements Statistics of Corporations*, 2004-2018), conducted by the Ministry of Finance and the Cabinet Office of Japan, which span approximately 15 years from 2004 to 2018. These two quarterly datasets include all large firms and a representative sample of small and medium-sized firms across both manufacturing and non-manufacturing sectors. Large firms are surveyed every quarter, while medium-sized and small firms are sampled on a rotating basis.[8]

[8]In the BOS, all firms with registered capital above 2 billion JPY are surveyed every quarter. For firms with registered capital between 0.5 billion and 2 billion JPY, 50% are randomly sampled each quarter. For firms with registered capital between 0.1 billion and 0.5 billion JPY, 10% are randomly sampled each quarter. For firms with registered capital below 0.1 billion JPY, 1% are randomly sampled each quarter. The random sample is redrawn at the beginning of each fiscal year, so once a firm is selected, it appears in all four quarters of that fiscal year (if it answers the survey every time). In the FSS, all firms with registered capital above 5 billion JPY are surveyed quarterly, while for firms with registered capital between 1 billion and 5 billion JPY, 50% are randomly sampled each quarter.

Sample of Firms The sample size of the BOS is about 11,000 with a response rate of more than 75%, and the sample size of the FSS is about 21,000 with a response rate of about 70%. The FSS covers basic financial statement information in the balance sheet and profit and loss account. At the same time, the BOS contains firm-level forecasts of sales and profit at the semiannual frequency, as well as firms' investment and investment plans at the quarterly frequency. Fortunately, both datasets share standard time-invariant firm identifiers for firms with registered capital exceeding 0.1 billion JPY, which is approximately 0.7 million US dollars. As a result, we merge the two datasets and construct a panel dataset that only contains firms with registered capital exceeding 1 billion JPY for the period 2004-2018.

A unique feature of the BOS is that it provides quantitative forecasts of sales and profits, which allows us to compute forecast errors. Reporting of both realized and expected sales and operating profits occurs on a semiannual basis, whereas investment plans are reported quarterly.[9] Because the forecasting targets (i.e., sales and profits) are semiannual, we define related variables (e.g. sales, investment, usage of intermediate goods, and productivity) at the semiannual frequency in our analysis. Furthermore, the BOS directly collects firms' investment data, enabling us to measure investment directly rather than inferring it indirectly from capital stock. Because the variables in our regressions are defined at the semiannual frequency, we use firm-level semiannual observations. Table 1a reports the summary statistics for observations in our merged dataset and the original FSS dataset. First, the merged dataset contains about 6,500 firms per quarter, compared with roughly 21,000 in the original FSS. Second, firms in both datasets are, on average, relatively large: average employment exceeds 490 workers, and average sales are above 8.5 billion JPY (about 7 million USD) per quarter.

Calculating Variables of Interest We construct firm-level variables of interest in a manner closely aligned with our model. First, we compute the forecast errors using the forecasts reported at the beginning of each semiannual period. Second, we calculate the firm-level investment rate at the quarterly level, defined as the ratio of fixed investment to the fixed capital stock, and use it to identify investment inaction and spikes.[10] Similarly, we compute the semiannual investment rate by summing investment expenditures over two consecutive quarters. Third, we aggregate firms into 30 industries based on their reported industry affiliations (originally 47 industries), since some industries contain too few observations in a given quarter. Fourth, we measure labor productivity using revenue per worker as our baseline productivity metric.[11] The dataset provides

[9] For example, in the October survey, a firm reports its realized sales from April to September and its projected sales for the upcoming October–March period. In the same survey, the firm also provides investment plans for October–December and January–March of the following year.

[10] An investment spike is defined as an investment rate greater than 20%, while investment inaction is defined as an investment rate of 1% or less.

[11] We use labor productivity instead of TFP in the baseline analysis for two reasons. First, estimating TFP requires

Table 1: Summary Statistics of Japanese Firm-level Data

(a) Sample Comparison at Quarterly Frequency

Moments	Merged Dataset	Entire Sample (FSS)
Number of obs. (Non-missing sales)	392,158	1,260,836
Average employment	1040.582	491.6123
Average sales (million JPY)	19991.75	8541.767
Average fixed capital stock	59919.34	24842.79

(b) Investment Moments Using Fixed Capital at Both Frequencies

Frequency	Exit Rate	Agg. Inv. Rate	Inv. Rate Mean	Inv. Rate S.D.	Inaction Rate	Spike Rate
Quarterly	2.00%	1.23%	2.27%	6.10%	60.00%	0.90%
Semiannual	3.96%	2.64%	4.00%	8.3%	36.6%	2.45%

(c) Untrimmed Forecast Errors at Semiannual Frequency

Variable	Obs.	Mean	Median	S.D.	Min.	Max.
Log forecast error of sales	119,335	-.0106	-.0005	0.199	-8.472	5.759
Percentage forecast error of sales	119,359	.0198	-.0005	1.556	-1	316

Notes: The sample period covers 2004–2018 (15 years, or 29 semiannual observations). In Sub-table (b), all statistics are based on variables defined at the quarterly frequency. Investment refers to the total of expenditures on equipment, machinery, and land, while capital denotes the stock of fixed capital. An investment spike is defined as an investment rate greater than 20%, and investment inaction as an investment rate of 1% or less. In Sub-table (c), the log forecast error is defined as the log deviation of realized sales in period t from the sales forecast made in period $t-1$. The percentage forecast error is defined as the percentage deviation of realized sales from the forecasted sales reported at the beginning of each semiannual period.

information on total sales, cost of goods sold (including capital appreciation and labor costs), capital depreciation, and labor costs (wages, salaries, and benefits). Using these components, we infer the purchase of intermediate goods, which serves as an input in our TFP estimation.

Next, we describe how we construct industry-level variables, which are used as independent variables in our regressions. First, we define firm-level capital intensity as the ratio of capital stock to the sum of capital stock, labor costs, and purchases of intermediate goods. We then average these firm-level measures within each industry to create the industry-level capital intensity.[12] Next, we compute the semiannual sales growth rate of each firm.[13] We then compute industry-

assumptions about the production function. Second, because estimated production-function coefficients carry confidence intervals, the resulting TFP measure would be imprecisely estimated rather than directly observed.

[12]We also constructed an alternative industry-level measure using a weighted average of firm capital intensity. The results are largely unchanged when using this measure.

[13]To account for firm entry and exit, we follow Davis et al. (1998) and use the mid-point growth rate: $sg_{it} = \frac{sales_{i,t}-sales_{i,t-1}}{(sales_{i,t}+sales_{i,t-1})/2}$, where i indexes firms and t refers to time. The denominator is the average sales in semi-years t and $t-1$. Growth rates are bounded between -200% and 200%, with firm entry (exit) corresponding to 200% (-200%) growth. Using a growth rate that ignores entry/exit yields similar empirical results. We do not have sufficient observations to reliably calculate growth volatility at finer industry-region or industry-semiannual levels.

level sales growth volatility and average sales growth using all firm-semiannual observations within the same industry.

Tables 1b and 1c report summary statistics for our constructed investment-related variables (based on fixed capital) and for our untrimmed forecast errors, respectively. Table 1b shows that firm investment in the merged dataset is highly lumpy: both the average (and aggregate) investment rate and the average investment spike rate are low, while the average inaction rate is substantial. Table 1c indicates that although the median of the two forecast errors is close to zero, their standard deviations are large, reflecting substantial heterogeneity across firms and over time, as well as the potential presence of outliers. To address this, we trim the top and bottom 1% of sales forecast errors in our empirical regressions.

4.2 Estimating Information Incompleteness in the Data

Pooled Estimates The first step is to quantify how the severity of information frictions affects firms' investment decisions. To do so, we first estimate the severity of information frictions for the entire economy using the following equation:

$$e_{it+1} = \bar{\xi} w_{it} + \Gamma z_{it} + \gamma_{gt} + \gamma_r + \gamma_{rt} + \gamma_s + \gamma_{st} + \gamma_i + \epsilon_{it+1} \tag{11}$$

Here, $e_{it+1} = y_{it+1} - \widehat{y}_{it+1}$ denotes the firm's forecast error, calculated as the difference between the logarithm of realized sales (y_{it+1}) at time $t+1$ and the logarithm of forecasted sales at time $t+1$ ($\widehat{y}_{it+1}$) made at time t. The key dependent variable $w_{it} = a_{it} - a_{it-1}$ represents the productivity shock at time t, where a_{it} and a_{it-1} are the logarithms of the firm's measured productivity at times t and $t-1$, respectively. Our coefficient of interest is the economy-wide underreaction coefficient, $\bar{\xi}$, which is estimated for the economy as a whole. We include lagged productivity, a_{it-1}, and some other variables in z_{it} as firm-level controls. Additionally, we control for a variation of fixed effects for robustness concerns, including size-time (γ_{gt}), region (γ_r), region-time (γ_{rt}), industry (γ_s), industry-time (γ_{st}), and firm (γ_i).[14]

Table 2 presents the estimation results of the economy-wide underreaction coefficient in equation (11). For concreteness, we use either the logarithm of the forecast error in Table 2, or the percentage forecast error in Table 7 in Appendix C.2, as the dependent variable and gradually add firm-level control variables, such as lagged capital stock and employment (in log terms), into the regressions. We also include various sets of fixed effects. Across specifications, both tables consistently show that the estimated $\bar{\xi}$ is positively significant. More importantly, this positive

[14]Regions are defined by the survey agency. In total, there are 53 survey regions, roughly corresponding to Japan's 47 prefectures. For large prefectures, such as Tokyo or Hokkaido, multiple survey regions exist.

significance persists even when firm fixed effects are included in the last three columns.

Table 2: Degree of Information Frictions for the Entire Economy

	e_{it+1} : $FE^{\log}_{t,t+1} \equiv \log(sales_{t+1}) - \log(\mathbb{E}[sales_{t+1}\|\Omega_t])$								
$w_{i,t}$	0.022***	0.021***	0.021***	0.020***	0.020***	0.019***	0.010***	0.010***	0.006***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
$a_{i,t-1}$	0.005***	0.005***	0.004***	0.005***	0.005***	0.004***	-0.012***	-0.012***	-0.027***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)
$k_{i,t-1}$		0.001***	0.002***		0.001**	0.002***		-0.009***	-0.003*
		(0.000)	(0.001)		(0.000)	(0.001)		(0.002)	(0.002)
$\log(emp)_{i,t-1}$			-0.002***			-0.002**			-0.027***
			(0.001)			(0.001)			(0.002)
Size-time FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Industry FE	Y	Y	Y	N	N	N	N	N	N
Industry-time FE	N	N	N	Y	Y	Y	Y	Y	Y
Region FE	Y	Y	Y	N	N	N	N	N	N
Region-time FE	N	N	N	Y	Y	Y	Y	Y	Y
Firm FE	N	N	N	N	N	N	Y	Y	Y
N	83219	81122	80335	83212	81115	80328	82004	79932	79166
adj. R^2	0.043	0.043	0.043	0.070	0.070	0.071	0.179	0.180	0.183

Notes: Standard errors are clustered at the firm level. Significance levels are indicated as follows: * p < 0.10, ** p < 0.05, *** p < 0.01. The degree of information friction, ξ, is estimated for the entire economy. The top and bottom 1% of productivity observations and forecast errors are winsorized. *emp* denotes employment. The regressions are conducted at the semiannual frequency.

Industry-specific Estimates Next, we estimate the industry-specific underreaction coefficient ξ_s using forecast errors and lagged productivity shocks calculated from our data:

$$e_{it+1} = \xi_s w_{it} + \Gamma z_{it} + \gamma_{gt} + \gamma_{rt} + \gamma_{st} + \epsilon_{it+1} \tag{12}$$

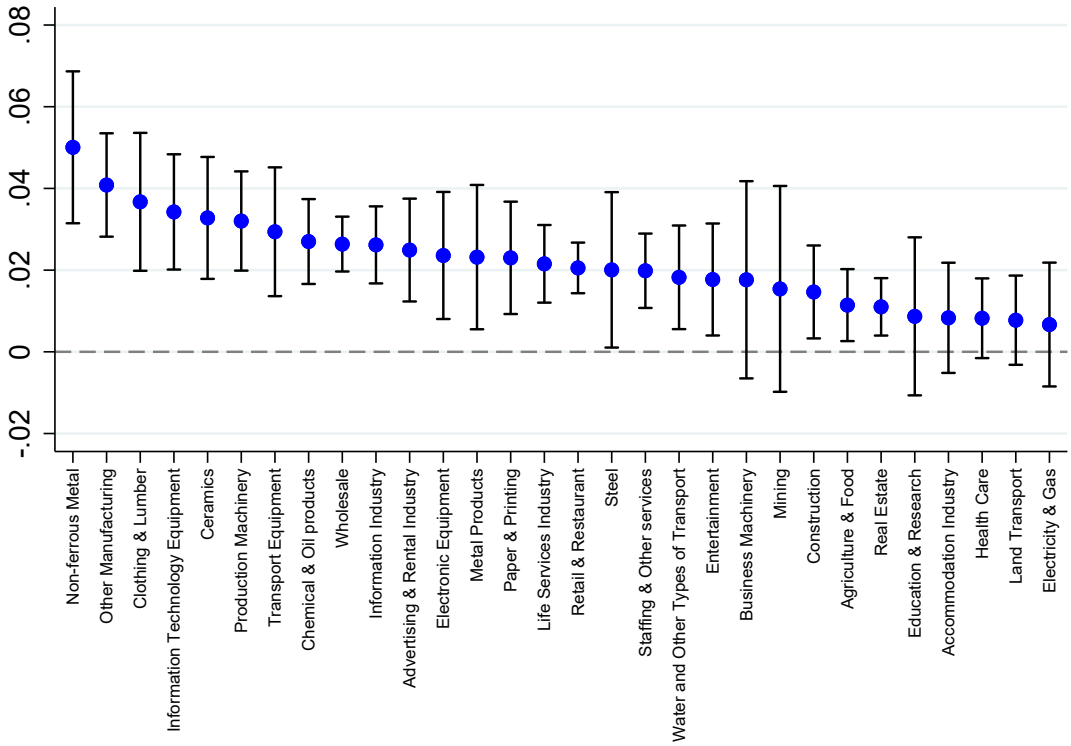
where the coefficient of interest is the industry-specific underreaction coefficient, ξ_s, estimated for 30 industries, we include lagged productivity, a_{it-1}, in z_{it} as the firm-level control to remain consistent with the model regressions below. We control for fixed effects including size-time (γ_{gt}), region-time (γ_{rt}), and industry-time (γ_{st}), and firm (γ_i).

Figure 3 presents the estimated underreaction coefficients and their 95% confidence intervals. The figure shows that the estimated industry-specific coefficient, ξ_s, is positive and statistically significant for most industries, indicating the presence of information frictions even among large and mature firms.[15] The coefficient varies across industries, from near zero to about 0.05, with manufacturing industries exhibiting higher frictions, likely due to exposure to domestic and international shocks. At the industry level, information frictions are negatively correlated with capital intensity and growth volatility, consistent with the idea that firms in more volatile or

[15]See Chen et al. (2023a) for evidence that small and young firms face even higher degrees of information frictions.

capital-intensive environments are more sensitive to frictions and may act to mitigate them.

Figure 3: Estimated Underreaction Coefficients across Industries: Labor Productivity



Notes: This figure illustrates how the coefficient governs the impulse response of the (log) sales forecast error in period $t+1$ to realized (log) productivity shocks in period t. Each dot represents an industry estimate, with 95% confidence intervals, across all 30 industries. The top and bottom 1% of observations are trimmed to remove outliers. The data are semiannual, and productivity is measured using labor productivity.

For robustness checks, we re-estimate equation (12) using the percentage forecast error as the dependent variable and, alternatively, the lagged value and the innovation in log productivity, log sales, or log value added as dependent variables. Figures 10 to 13 in Appendix C.2 consistently show that the estimated industry-specific coefficient, ξ_s, remains positive and statistically significant for most industries. Overall, the results indicate the presence of information frictions, as evidenced by the positive correlation between the innovation in several firm-level variables and future forecast errors.

We also estimated the industry-level underreaction coefficients while including firm fixed effects; the results are presented in Figure 9 in the appendix. Most estimated coefficients remain positive and are highly correlated with those obtained without firm fixed effects (correlation coefficient: 0.71). However, including firm fixed effects can introduce downward bias in the presence

of measurement errors (Appendix C.1).

Moreover, firm-level time-invariant characteristics do not appear to be significantly correlated with firm-level forecast errors. Specifically, we compute the average forecast error for each firm and three sets of time-invariant firm characteristics: average size (measured by capital stock or employment), capital-to-sales ratio, and the share of short-term debt in total debt (as a measure of financial constraint). We do not use average sales as a measure of firm size because sales are used directly in the calculation of forecast errors. Correlating the firm-level forecast errors with these characteristics reveals no statistically significant relationships, as reported in Table 14 (Appendix C.5). For these reasons, we prefer the estimates obtained without firm fixed effects.

4.3 Model-based Simulations

In the second step, to compare the model qualification to the data, we calibrate and simulate the model for 30 different industries, which yields a similar theoretical industry-specific underreaction coefficient ξ_s as the estimated ones from equation (12) and presented in Figure 3. Since the model is stylized, we fix most parameters to match corresponding moments directly, but we leave the standard deviation of noise, σ_n^s, to vary across industries to generate the corresponding industry-specific underreaction coefficient ξ_s.

Table 3: Parametrization and Investment Moments at Quarterly Frequency

(a) Parametrization of the Stylized Model

Parameter	r	α	τ	ψ	η	ς	δ	σ_a	σ_n^0	σ_n^{30}	$\Delta\sigma_n$
Value	1%	0.85	1	1	2%	0	1.23%	0.15	0.00	$0.75\sigma_a$	$0.025\sigma_a$

(b) Average Investment Moments

σ_n	ξ_s	Inv. Rate Mean	Inv. Rate S.D.	Inaction Rate	Spike Rate
$0.375\sigma_a$	0.018	2.29%	6.1%	79.8%	3.3%

Notes: The investment rate is defined as the ratio of investment to the capital stock. The inaction rate is defined as the frequency at which the investment rate is below 1%. The spike rate is defined as the frequency at which the investment rate exceeds 20%.

Parametrization and Simulation We calibrate our model quantitatively in Table 3a to roughly match our Japanese firm-level data, which is presented quarterly in Table 1b. We first fix the quarterly interest rate r to be 1% to match an annual rate of 4%, returns to scale $\alpha = 0.85$ as in Winberry (2021), and a one-period revelation delay $\tau = 1$ before past productivity is revealed, consistent with quarterly accounting reports. We normalize the investment cost $\psi = 1$ and then choose the exit rate η of 2% to match the quarterly exit rate and choose a depreciation rate of$\delta = 1.23\%$ to

match the aggregate quarterly investment rate. We choose the entry productivity volatility $\varsigma = 0$ for simplicity. We then select the standard deviation of the productivity process, $\sigma_a = 0.15$, to roughly match the average standard deviation of the investment rate, which is 6.1%.

To illustrate the effects of incomplete information on various moments of investment dynamics, and to test our empirical methods, we simulate 31 industries with different degrees of information frictions, $\{\sigma_n^s\}_{s=0}^{30}$. The industries include a 31st artificial industry ($s = 30$) that has full information $\{\sigma_n^{30} = 0, \xi_{30} = 0\}$ for comparison. Each industry has 10,000 firms for 50 quarters. For each industry, we calculate the corresponding ξ_s from the regression equation (12) and make sure our range of $\{\sigma_n^s\}_{s=1}^{30}$ delivers the range of $\{\xi_s\}_{s=1}^{30}$ from $\xi_0 = 0.055$ to $\xi_{30} = 0$. Our simulation is quarterly. As usual in discrete time, we assume capital is built one period in advance. This allows us to calculate the ξ_s coefficients directly from the model parametrization as follows:

Corollary 1. *In the discrete time simulation, the industry-specific ξ_s underreaction coefficient for output forecast errors is given by*

$$\xi_s = (1-\alpha)(1-\gamma_s)$$

Proof. The underreaction coefficient is

$$\xi_s = \frac{d(y_{t+1} - \mathbb{E}[y_{t+1}|\Omega_t])}{da_t} = (1-\alpha)\frac{d(a_{t+1} - \mathbb{E}[a_{t+1}|\Omega_t])}{da_t} + \alpha\frac{d(k_{t+1} - \mathbb{E}[k_{t+1}|\Omega_t])}{da_t}$$

because $y_t = (1-\alpha)a_t + \alpha k_t$. In discrete time k_{t+1} is known at time t, so $\frac{d(k_{t+1}-\mathbb{E}[k_{t+1}|\Omega_t])}{da_t} = 0$, and Proposition 2 implies $\frac{d(a_{t+1}-\mathbb{E}[a_{t+1}|\Omega_t])}{da_t} = 1 - \gamma_s$. □

From Corollary 1, we directly calculate $\xi_s = (1-\alpha)(1-\gamma_s)$ for each industry, using $\gamma_s = \sigma_a^2/(\sigma_a^2 + (\sigma_n^s)^2)$. Thus the noisiest industry in our sample has $\sigma_n^0 \equiv \sigma_a\sqrt{\xi_0/(1-\alpha-\xi_0)} \approx 0.75\sigma_a$. As a result, we choose $\{\sigma_n^s\}_{s=0}^{30} = \{\sigma_n^0, 0.675\sigma_a, 0.650\sigma_a, ..., 0\}$ with a step size $\Delta\sigma_n = 0.025\sigma_a$ to match a corresponding empirical underreaction coefficient $\{\xi_s\}_{s=0}^{30} = \{0.055, ..., 0\}$ estimated from the regression equation (12) and as presented in Figure 3. Table 3b reports the simulated investment moments in our median noise industry.

Table 4 presents the estimation results of the economy-wide underreaction coefficient in equation (11) from our simulated firm sample (analogous to the empirical Table 2). We use the same logarithm of the forecast error as the dependent variable and gradually add firm-level control variables, such as lagged capital stock, into the regressions. We also include various sets of fixed effects. Across specifications, both tables consistently show that the estimated $\bar{\xi}$ is positively significant. More importantly, the coefficients are consistent between the estimation in the data in Table 2 and the model in Table 4. Without controlling for firm fixed effects, the coefficients

remain stable at around 0.02. After controlling for firm fixed effects, the coefficients decrease to approximately 0.01, both in the data and in the model.

Table 4: Degree of Information Frictions for the Entire Economy: Simulation

	$e_{it+1} : FE^{\log}_{t,t+1} \equiv \log(sales_{t+1}) - \log(\mathbb{E}[sales_{t+1}\|\Omega_t])$					
$w_{i,t}$	0.021	0.021	0.021	0.021	0.010	0.009
$a_{i,t-1}$	0.000	0.004	0.000	0.004	-0.012	-0.010
$k_{i,t-1}$		-0.007		-0.007		-0.005
Time FE	Y	Y	N	N	N	N
Industry FE	Y	Y	N	N	N	N
Industry-time FE	N	N	Y	Y	Y	Y
Firm FE	N	N	N	N	Y	Y
N	13714883	13714883	13714883	13714883	13697896	13697896
adj. R^2	0.018	0.023	0.018	0.023	0.150	0.151

Notes: This table shows the pooled estimates of $\bar{\xi}$ in the regression equation (11) from the model simulated sample to be compared to the empirical results in Table 2. Across specifications, both tables consistently show that the estimated $\bar{\xi}$ is positively significant.

4.4 Validation of the Micro Implications

In the third step, we run the same regressions using the microdata and simulated data and compare the results directly to illustrate the micro implications.

Investment Inaction First, we run regressions to examine whether the industry-level coefficient ξ_s negatively affects the probability of firm-level investment inaction using semiannual observations:

$$\text{inaction}_{it} = \beta\xi_s + \Gamma z_{it} + \Lambda\gamma_s + \gamma_t + \epsilon_{it} \tag{13}$$

where inaction_{it} is a binary variable equal to one if the investment rate is $\leq 1\%$ and zero otherwise. z_{it} includes firm-level controls such as lagged log capital stock (k_{it-1}), log labor productivity (a_{it}), and intermediate goods usage per worker (m_{it}). γ_s captures industry-level controls, including capital intensity and growth volatility, while γ_t represents time (semiannual) fixed effects. We standardize ξ_s (mean zero, standard deviation one) to facilitate interpretation. Since the attenuation coefficient varies only at the industry level, firm- or industry-fixed effects cannot be included. Standard errors are clustered at the industry level. We also replicate the regression using the simulated firm sample of 31 industries.

Table 5 presents the estimation results. Specifically, the first six columns show the regression results using our firm-level data, while the last two present the results using the simulated

Table 5: Incomplete Information and Investment Inaction

	inaction = 1							
	Data						Model	
ξ_s	-0.076**	-0.079***	-0.054**	-0.069**	-0.039*	-0.051**	-0.013	-0.011
	(0.028)	(0.026)	(0.025)	(0.026)	(0.020)	(0.021)	(——)	(——)
$a_{i,t}$	0.039	0.059*	0.104***	0.113***	0.091**	0.099***	-0.206	-0.298
	(0.034)	(0.031)	(0.038)	(0.033)	(0.033)	(0.032)	(——)	(——)
$k_{i,t-1}$		-0.050***	-0.049***	-0.044***	-0.041***	-0.039***		-0.458
		(0.009)	(0.009)	(0.007)	(0.008)	(0.007)		(——)
$m_{i,t}$			-0.026	-0.045***	-0.015	-0.030**		
			(0.021)	(0.016)	(0.019)	(0.014)		
cap share$_s$				-0.549*		-0.366		
				(0.314)		(0.304)		
growth vol$_s$					1.016***	0.870***		
					(0.279)	(0.278)		
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	99027	99027	86294	86294	86294	86294	14291997	14291997
adj. R^2	0.038	0.069	0.063	0.089	0.078	0.095	0.116	0.180

Notes: Standard errors are clustered at the industry level. Significance levels: * 0.10 ** 0.05 *** 0.01. The degree of information friction, ξ_s, is estimated at the industry level and normalized to mean zero and standard deviation one, both in the model and the data. The top and bottom 1% of productivity observations are winsorized.

data. Consistent with the theory's predictions, the industry-specific attenuation coefficient has a negative impact on investment inaction at the firm level. Moreover, one standard deviation increase in this measure reduces the inaction probability by 5.1%. This is a substantial change: the average inaction rate is 36.6% at the semiannual frequency and 20.7% annually. The inclusion of industry-level confounding factors barely changes the estimated coefficient of α, as revealed in columns four to six. Finally, the industry-level volatility is estimated to increase the firm-level inaction rate, consistent with the findings from the previously discussed literature on uncertainty shocks. Although we have controlled for two important industry-level confounding factors in the above regression, other factors at the industry level can potentially affect the average inaction rate. Since ξ_s is measured at the industry level, we cannot include industry fixed effects in the regression equation (13). To address this concern, in our next test, we estimate the firm-level effects of the industry-wide information friction.

Investment Sensitivity In order to account for industry and firm-level fixed effects, we estimate interaction regressions to examine how a realized and unexpected productivity shock, w_{it}, affects a firm's investment inaction differently across industries with varying degrees of information frictions. The key variable of interest is the interaction term, $w_{it} \times \xi_s$. Specifically, we run

the following regression:

$$\text{inaction}_{it} = \beta(w_{it} \times \xi_s) + \gamma w_{it} + \Gamma z_{it} + \gamma_i + \gamma_t + \gamma_{st} + \epsilon_{it} \tag{14}$$

where inaction_{it} is a binary variable equal to one if the investment rate is $\leq$ 1% and zero otherwise. z_{it} includes firm-level controls such as lagged log capital stock (k_{it-1}), lagged log productivity (a_{it-1}), and intermediate goods usage per worker (m_{it}). γ_i represents firm fixed effects, γ_t represents time (semiannual) fixed effects, and γ_{st} represents industry-time fixed effects. The industry-level coefficient, ξ_s, is standardized for interpretability. The advantage of this interaction regression is that including firm-level fixed effects allows us to control for firm-level time-invariant factors affecting investment. We also replicate the regression using the same simulated firm sample.

Table 6: Incomplete Information and Investment Sensitivity

	inaction = 1					
	Data				Model	
$\xi_s \times w_{i,t}$	0.010**	0.011**	0.011**	0.010**	0.012	0.013
	(0.005)	(0.005)	(0.005)	(0.005)	(——)	(——)
w_{it}	-0.036	-0.030	-0.036	-0.029	-0.188	-0.188
	(0.031)	(0.031)	(0.032)	(0.032)	(——)	(——)
a_{it-1}	-0.028**	-0.015	-0.029**	-0.016	-0.670	-0.670
	(0.012)	(0.012)	(0.011)	(0.011)	(——)	(——)
Productivity	Rand. Walk	Rand. Walk	Auto. Reg(1)	Auto. Reg(1)	Rand. Walk	Rand. Walk
Firm FE	Y	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y	Y
Industry-Time FE	N	Y	N	Y	N	Y
N	84656	84656	84313	84313	14274640	14274640
adj. R^2	0.446	0.451	0.446	0.451	0.450	0.450

Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. The degree of information friction, ξ_s, is estimated at the industry level and standardized to have a mean of zero and a standard deviation of one in both the model and the data. The top and bottom 1% of productivity observations are winsorized.

The interaction regression confirms that firms in industries with more severe information frictions are more likely to remain inactive following a positive productivity shock, compared with firms in sectors with lower frictions. Table 6 reports the regression results.[16] The first four columns use firm-level data, while the last two columns present results from the simulated data. Our estimation allows for flexible inclusion of various fixed effects, and productivity innovations are computed both from a random walk process (consistent with the model) and from an AR(1) process. Across specifications, the coefficient β is positive and statistically significant (or very

[16]Tables 8 and 9 in Appendix C.3 present detailed estimation results (columns 2 and 4 of the two tables).

nearly so) for investment inaction in the first four columns, matching the model qualitatively and quantitatively. The magnitude of the productivity coefficients is smaller in magnitude than in the model. Still, they have the correct sign when estimated using the firm-level interaction regression, which was not the case with the industry-level regression.

Robustness We implement a variety of robustness checks for our empirical analysis. Appendix C reports details and results. Specifically, we first estimate the underreaction coefficients in Section C.1 using alternative specifications that include firm-level fixed effects, and find that they are similar to our baseline estimates reported in Figure 3. The coefficients are generally attenuated, but still show variation across industries and consistent evidence of underreaction, as in Ma et al. (2020). Appendix C.3 presents the full details of our main regressions using labor productivity. Finally, we revisit all of our main regressions in Section C.4 using estimated total factor productivity instead of the simple productivity measures. With this approach, our key estimates are quantitatively similar to the main results we presented in Figure 3, and Tables 5 and 6. Across all these alternatives, we conclude that the central conclusions above are robust.

5 Quantification of the Macro Implications

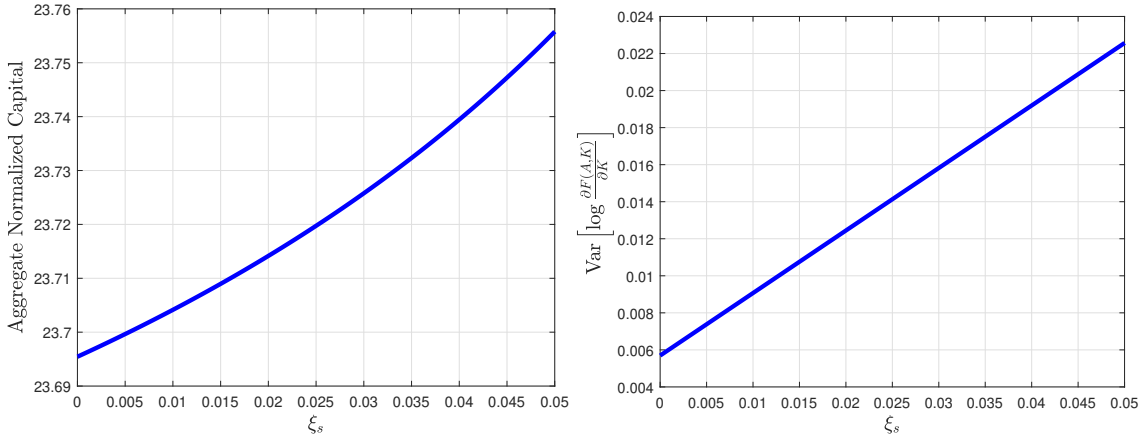
Having estimated the information frictions in the data and validated the micro implications, we now quantify the macroeconomic implications of incomplete information when investment is irreversible. This section evaluates the theoretical predictions presented in Section 3.3, regarding capital accumulation, capital misallocation, and aggregate shock attenuation.

5.1 Capital Accumulation and Misallocation

The first macro implication of the information friction is that steady-state aggregate normalized capital should increase with the noisy-signal variance σ_n^2 (Proposition 3). Intuitively, noisier information raises the lower bound of firms' inaction region, leading to higher steady-state capital holdings. We document this prediction in our calibrated model by varying the underreaction coefficient, $\check{\xi}_s$, from 0 (full information) to 0.055 (highest noise). For each case, we compute the steady-state level of normalized capital. The quantitative results are reported in Figure 4.

Panel (a) of Figure 4 shows that for larger underreaction coefficients (i.e., larger variance of the noisy signal), the steady state aggregate normalized capital increases. This result is mainly driven by the fact that the boundary is increasing in $\check{\xi}_s$, which ultimately induces firms to accumulate more capital. Thus, the simulated model aligns with the theoretical prediction that steady-state normalized capital increases with the severity of the information friction.

Figure 4: Aggregate Capital Distortions from Information Incompleteness



(a) Aggregate Normalized Capital

(b) Capital Misallocation

Notes: These figures illustrate how information incompleteness, as measured by the underreaction coefficient ξ_s, affects steady state aggregate variables. The underreaction coefficient $\xi_s = (1-\alpha)\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ ranges from 0 (full information) to 0.055 (noisiest empirical measure). Panel (a) plots aggregate normalized capital X, and Panel (b) plots misallocation $\mathrm{Var}\left[\log \frac{\partial F(A,K)}{\partial K}\right]$.

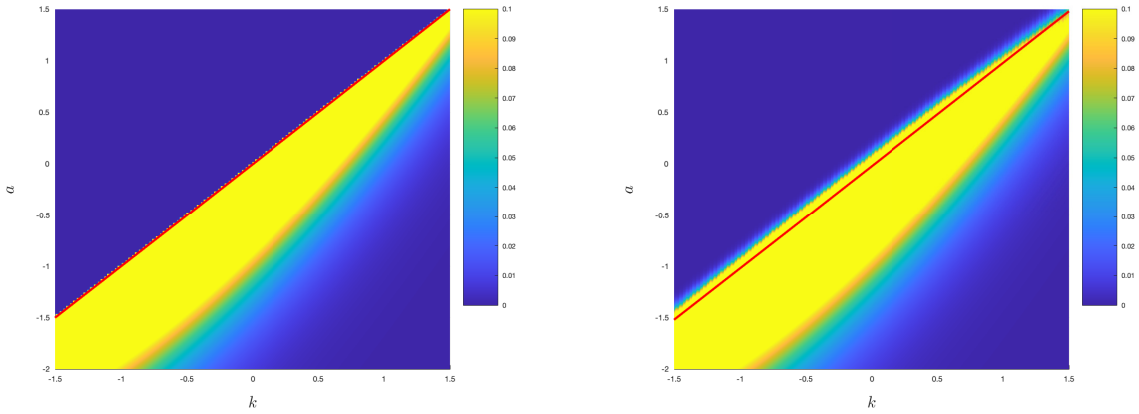
The second macro implication of the information friction is that steady-state capital misallocation should increase with the noisy-signal variance σ_n^2 (Proposition 4). Intuitively, noisier information increases the nowcast errors of firms, leading to greater mistakes and higher steady-state capital misallocation. Again, we document this prediction in our calibrated model by varying the underreaction coefficient, ξ_s, from 0 (full information) to 0.055 (highest noise). We compute the steady-state level of capital misallocation as the variance of log MPK.

The misallocatione ffects are large. Panel (b) in Figure 4 shows the quantitative results of increments in capital misallocation due to information incompleteness. Specifically, we plot the changes in the variance of the log marginal product of capital $\mathrm{Var}\left[\log \frac{\partial F(A,K)}{\partial K}\right] = (1-\alpha)^2\mathrm{Var}[x]$ relative to the full information case. According to Proposition 4, such changes equal to $(1-\alpha)^2(\mathrm{Var}[x] - \mathrm{Var}[\hat{x}]) \equiv (1-\alpha)^2\nu$, which is solely governed by the nowcast error $\nu = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$. The figure illustrates that a higher noise level leads to increased capital misallocation. From the full information sector to the noisiest sector, the variance of the log marginal product of capital increases from 0.006 to over 0.022.

To further visualize how incomplete information amplifies irreversibility's capital misallocation, we also present the joint stationary densities for log capital and log productivity in Figure 5 for two cases.[17] In each panel, the red line represents the expected log productivity $\hat{a}$ such that, for a given log capital k, the firm's expected normalized capital is exactly at the boundary

[17] Appendix A.3 solves the Kolmogorov PDE to derive these densities analytically.

Figure 5: Information Incompleteness and Joint Stationary Densities



(a) Full Information ($\xi_0 = 0$)

(b) Highest Noise ($\xi_{30} = 0.055$)

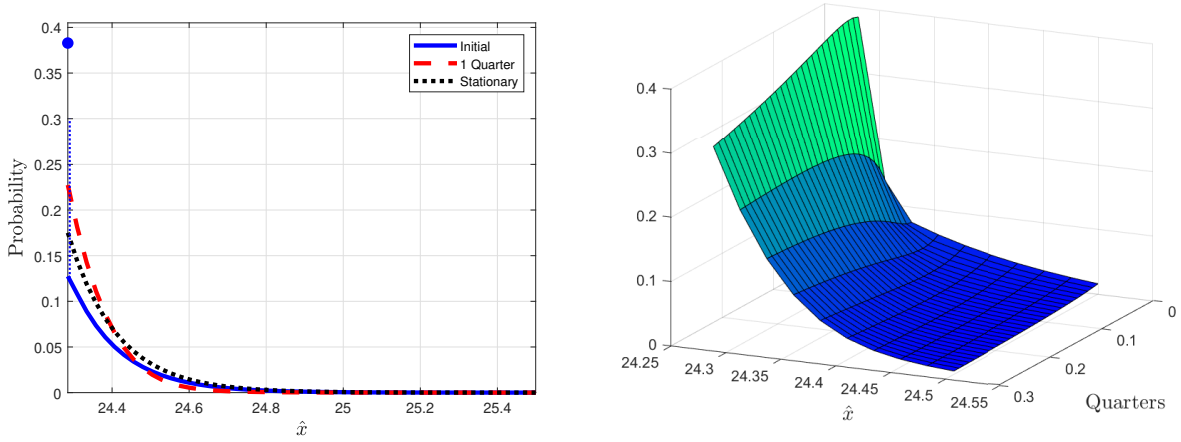
Notes: The red solid line denotes the inaction boundary for expected log productivity $\hat{a} = k - \hat{b}$. Grids for log capital (k) and log productivity (a) are normalized around 0. To zoom-in on detail, the figure plots a subset of the full distribution, which extends out of frame. See Appendix A.3 for the analytical derivation of the joint stationary densities.

$\hat{b}$. Panel (a) presents the full information case: investment irreversibility induces misallocation by leaving firms in the interior of the inaction region (i.e. below the red line) where they hold too much capital but cannot divest it. Panel (b) illustrates how the information friction increases capital misallocation by smearing out the distribution of firms relative to panel (a). This panel shows the incomplete information case with the highest noise ($\xi_{30} = 0.055$). These plots illustrate the role that information frictions play in distorting the distribution of capital. Specifically, we observe that firms tend to remain closer to the boundary when there is incomplete information, compared to when there is complete information. Moreover, when information becomes incomplete, firms make mistakes: there is a relatively greater mass to the left of the boundary. The two-dimensional distributions validate Proposition 4, which states that more severe information frictions lead to greater capital misallocation.

5.2 Aggregate Shock Attenuation

Finally, we demonstrate how aggregate productivity shocks impact the expected log-normalized capital $\hat{x}$. Under incomplete information, an aggregate shock to expected productivity differs from one to *actual* productivity. We define an aggregate shock as an exogenous increase of one quarterly standard deviation σ_a in the log productivity of all firms. The aggregate shock induces a shift in the distribution of normalized capital, which is illustrated in Figure 6.

Figure 6: Response of the Normalized Capital Distribution to an Aggregate Productivity Shock

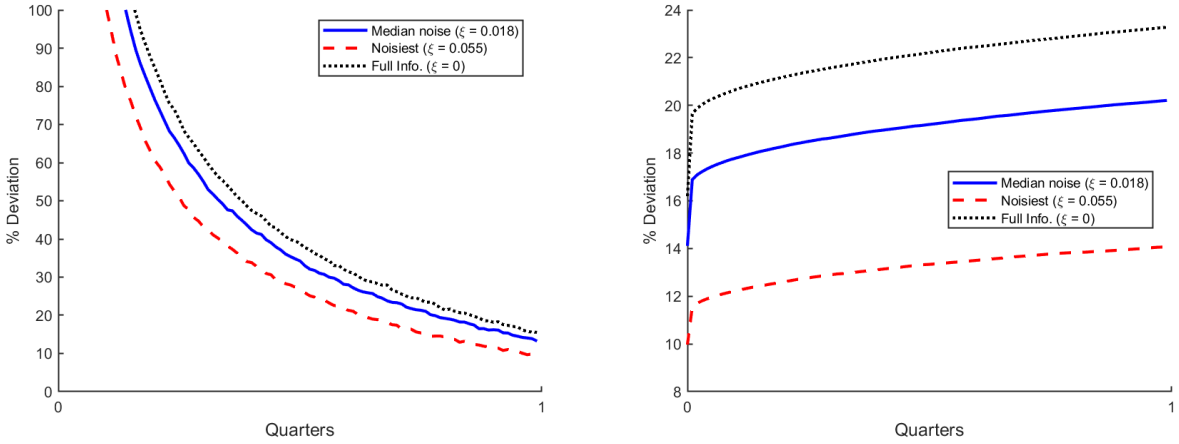


(a) Distribution on Impact and After One Quarter

(b) Distribution Over Time

Notes: These figures show how the normalized capital distribution evolves after the economy experiences an aggregate productivity shock. Panel (a) presents the distribution on impact ("Initial"), one quarter after the shock ("1 Quarter"), and the stationary distribution ("Stationary"). Panel (b) shows how the distribution is within the first quarter after the shock.

Figure 7: Incomplete Information Attenuates Aggregate Impulse Responses



(a) Investment

(b) Output

Notes: This figure presents the impulse response functions of aggregate investment and output to an aggregate shock increasing $da = \sigma_a$ for all firms. Each line corresponds to a different parameterization of the signal noise. The IRFs are calculated by perturbing the stationary distribution and simulating 10 million firms. IRFs are defined as in Section 3.3.3.

Figure 6 shows that the aggregate shock immediately pushes many firms to the boundary. If the aggregate shock pushes a firm's normalized capital past the boundary $\hat{b}$, the firm immedi-

ately invests in capital to remain at the barrier. Then after the shock, firms start adjusting their expected log normalized capital levels in several directions: we observe firms moving *away* from the boundary where there is excess mass, but we also observe firms moving *toward* the boundary, as the mass on higher values of normalized capital is lost relative to the initial equilibrium.

To analyze how the productivity shock affects the aggregate economy, we again perturb the stationary distribution by increasing all firms' log TFP by σ_a. Unlike the regression Monte Carlo, these IRFs are calculated from a short-run high-fidelity particle simulation: we simulate 10 million firms with a discrete but small dt. Unlike normalized capital (which is one-dimensional), GDP depends on the joint distribution of productivity and capital, and while we have analytically solved the steady state Kolmogorov PDE (Appendix A.3), the dynamic PDE is even more challenging, so a simulation is useful. Figure 7 plots the IRFs of aggregate investment and output to the productivity shock. The blue line corresponds to our estimate of the average industry information friction. Firms have become more productive (although they do not know precisely how much more) so they immediately invest and GDP rises. In the short run, firms remain close to the inaction boundary, so investment remains elevated and GDP grows quickly. Over time, firms diffuse, and investment falls.

Proposition 5 says that the information friction attenuates IRFs. To illustrate this effect, we simulate two additional exercises. First, we consider a noisier signal, setting a large σ_n to approximate $\xi_s = 0.055$ (the highest level of noise in our data).[18] Second, we calculate the full information IRFs. Figure 7 plots these cases with dashed red lines and dotted black lines, respectively. As expected, the IRFs are ordered by noise: full information causes a rapid response; modest noise is attenuated, and the noisiest calibration is attenuated further.

6 Conclusion

This paper presents a framework for a better understanding of the roles of two key frictions that impact investment dynamics: irreversibility and information frictions. Our framework is tractable and generates testable theoretical predictions, many of which can be derived analytically. This stylized model can serve as a building block for further studies, incorporating additional features, enhanced realism, and more complex information problems.

We first learned that investment irreversibility and information frictions interact in meaningful ways in a stylized continuous time model. Two results stand out. First, information frictions introduce a new type of uncertainty that raises firms' willingness to invest, in contrast to the

[18]The mapping between σ_n and ξ_s is not straightforward in continuous time, but in the discrete time approximation, it is analytical (Corollary 1).

current effects of uncertainty in the literature; this effect reduces inaction and increases capital. Second, information frictions reduce the elasticity of investment to aggregate shocks, a valuable property for investment frictions that have macroeconomic effects in larger models.

Finally, we disciplined our stylized model with rich Japanese firm-level data. Sectoral heterogeneity in information frictions allowed us to test the model's predictions. We found that firms in sectors that face worse information frictions are less inactive and less elastic in their responses to productivity shocks. This confirms the theory's characteristic prediction.

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Appendix A Supplements to Theoretical Analysis

A.1 The General Solution to the HJB

This intermediate result is used in multiple proofs that follow.

Lemma 6. *The normalized HJB* (5) *is solved by*

$$v(x) = me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}$$

for some c_1 *and* c_2.

Proof. The normalized HJB (5) has a particular solution

$$v_p(x) = me^{\alpha x}$$

with m solved by

$$rme^{\alpha x} = e^{\alpha x} - \mu\alpha me^{\alpha x} + \frac{\sigma^2}{2}\alpha^2 me^{\alpha x}$$

$$rm = 1 - \mu\alpha m + \frac{\sigma^2}{2}\alpha^2 m$$

$$m = \frac{1}{r + \mu\alpha - \frac{\sigma^2}{2}\alpha^2}$$

The homogeneous solution is

$$v_h(x) = c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}$$

where ϱ_1 and ϱ_2 are roots of the polynomial $-\frac{\sigma^2}{2}\varrho_j^2 + \mu\varrho_j + r = 0$. Thus the value function is

$$v(x) = me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}$$

□

One result of Lemma 6 is that the expected value function $\hat{v}(\hat{x})$ has a similar form.

Corollary 2. *The expected value function* $\hat{v}(\hat{x})$ *satisfies*

$$\hat{v}(\hat{x}) = me^{\alpha\hat{x}} e^{\frac{\alpha^2\nu}{2}} + c_1 e^{\varrho_1\hat{x}} e^{\frac{\varrho_1^2\nu}{2}} + c_2 e^{\varrho_2\hat{x}} e^{\frac{\varrho_2^2\nu}{2}} \tag{15}$$

for some c.

Proof. The firm's expectation of the value function derived in Lemma 6 is

$$\hat{v}(\hat{x}) = E[v(x)|\hat{x}] = E[me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x}|\hat{x}]$$

The firm's conditional expectation of x is $x \sim N(\hat{x}, v)$:

$$= \int_{-\infty}^{\infty} (me^{\alpha x} + c_1 e^{\varrho_1 x} + c_2 e^{\varrho_2 x})\, \phi(\frac{x-\hat{x}}{\sqrt{v}})dx$$

$$= \int_{-\infty}^{\infty} \left(me^{\alpha(x-\hat{x})}e^{\alpha\hat{x}} + c_1 e^{\varrho_1(x-\hat{x})}e^{\varrho_1\hat{x}} + c_2 e^{\varrho_2(x-\hat{x})}e^{\varrho_2\hat{x}}\right) \phi(\frac{x-\hat{x}}{\sqrt{v}})dx$$

Then use that $e^{\alpha(x-\hat{x})}$, $e^{\varrho_1(x-\hat{x})}$ and $e^{\varrho_2(x-\hat{x})}$ are log-normal, where the associated normal distributions have zero mean and variance $\alpha^2 v$, $\varrho_1^2 v$ and $\varrho_2^2 v$ respectively:

$$= me^{\alpha\hat{x}}e^{\frac{\alpha^2 v}{2}} + c_1 e^{\varrho_1\hat{x}}e^{\frac{\varrho_1^2 v}{2}} + c_2 e^{\varrho_2\hat{x}}e^{\frac{\varrho_2^2 v}{2}}$$

□

A.2 Proofs

A.2.1 Proof of Lemma 1

Proof. The firm's conditional expectation of a_t is

$$E[a_t|\Omega_t] = a_{t-\tau} + E[a_t - a_{t-\tau}|\Omega_t]$$

From the firm's perspective, $s_t - s_{t-\tau}$ is a noisy signal of $a_t - a_{t-\tau}$:

$$s_t - s_{t-\tau} = a_t - a_{t-\tau} + n_t - n_{t-\tau}$$

the noise $n_t - n_{t-\tau}$ is independent of productivity and distributed $N(0, \tau\sigma_n^2)$, while $a_t - a_{t-\tau}$ is distributed $N(0, \tau\sigma_a^2)$. Therefore:

$$E[a_t - a_{t-\tau}|\Omega_t] = \frac{\text{Cov}(a_t - a_{t-\tau}, s_t - s_{t-\tau})}{\text{Var}(s_t - s_{t-\tau})}(s_t - s_{t-\tau}) = \frac{\tau\sigma_a^2}{\tau\sigma_a^2 + \tau\sigma_n^2}(s_t - s_{t-\tau})$$

and the definition $\gamma = \frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}$ implies

$$E[a_t|\Omega_t] = a_{t-\tau} + \gamma\,(s_t - s_{t-\tau})$$

The nowcast errors $u_t = a_t - E[a_t|\Omega_t]$ are normally distributed and have variance

$$\text{Var}\left(a_t - E[a_t|\Omega_t]\right) = \text{Var}\left(a_t - a_{t-\tau} - \gamma\left(s_t - s_{t-\tau}\right)\right) = \text{Var}\left((1-\gamma)(a_t - a_{t-\tau}) - \gamma\left(n_t - n_{t-\tau}\right)\right)$$

$$= \text{Var}\left((1-\gamma)(a_t - a_{t-\tau})\right) + \text{Var}\left(\gamma\left(n_t - n_{t-\tau}\right)\right) = (1-\gamma)^2\tau\sigma_a^2 + \gamma^2\tau\sigma_n^2$$

$$= \left(\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}\right)^2 \tau\sigma_a^2 + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \tau\sigma_n^2 = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2} = \nu$$

The conditional distribution implies that the diffusion for $\hat{a}_t$ is given by

$$d\hat{a}_t = da_{t-\tau} + \gamma(ds_t - ds_{t-\tau}) = (1-\gamma)da_{t-\tau} + \gamma da_t + \gamma dn_t - \gamma dn_{t-\tau}$$

$$= (1-\gamma)\sigma_a dW_{t-\tau}^a + \gamma\sigma_a dW_t^a + \gamma\sigma_n dW_t^n - \gamma\sigma_n dW_{t-\tau}^n$$

The right-hand side is the sum of independent innovations, so they can be recomposed as innovations to a single Wiener process:

$$d\hat{a}_t = \sigma_{\hat{a}} dW^{\hat{a}}$$

It remains to show that $\sigma_{\hat{a}} = \sigma_A$. The independence of the innovations imply

$$\sigma_{\hat{a}}^2 = (1-\gamma)^2\sigma_a^2 + \gamma^2\sigma_a^2 + 2\gamma^2\sigma_n^2$$

$$= \left(\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_a^2 + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_a^2 + 2\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_n^2$$

$$= \frac{\sigma_n^2\sigma_a^2}{\sigma_a^2+\sigma_n^2} + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_a^2 + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2 \sigma_n^2$$

$$= \frac{\sigma_n^2\sigma_a^2}{\sigma_a^2+\sigma_n^2} + \frac{\sigma_a^4}{\sigma_a^2+\sigma_n^2} = \sigma_a^2$$

As a result, innovations to nowcast errors follow

$$du_t = da_t - d\hat{a}_t = \sigma_a dW_t^a - ((1-\gamma)\sigma_a dW_{t-\tau}^a + \gamma\sigma_a dW_t^a + \gamma\sigma_n dW_t^n - \gamma\sigma_n dW_{t-\tau}^n)$$

$$= (1-\gamma)(\sigma_a dW_t^a - \sigma_a dW_{t-\tau}^a) - \gamma(\sigma_n dW_t^n - \sigma_n dW_{t-\tau}^n) = \sigma_u dW_t^u$$

Again, independence of the innovations implies

$$\sigma_u^2 = 2(1-\gamma)^2\sigma_a^2 + 2\gamma^2\sigma_n^2$$

$$= 2\left(\frac{\sigma_n^2}{\sigma_a^2+\sigma_n^2}\right)^2\sigma_a^2 + 2\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_n^2}\right)^2\sigma_n^2 = 2\frac{\sigma_n^2\sigma_a^2}{\sigma_a^2+\sigma_n^2}$$

□

A.2.2 Proof of Lemma 2

Proof. The value-matching and super contact conditions at infinity are standard.

Corollary 2 gives the firm's expectation of the value function in terms of two roots ϱ_1 and ϱ_2. The conditions at infinity imply that the coefficient on the positive root is zero. We write the remaining negative root ϱ and coefficient c without subscripts; the expected value function becomes

$$\hat{v}(\hat{x}) = me^{\alpha\hat{x}}e^{\frac{\alpha^2 v}{2}} + ce^{\varrho\hat{x}}e^{\frac{\varrho^2 v}{2}}$$

which in levels is

$$\hat{V}(\hat{X};\hat{B}) = m\hat{X}^{\alpha}e^{\frac{\alpha^2 v}{2}} + c(\hat{B})\hat{X}^{\varrho}e^{\frac{\varrho^2 v}{2}} \tag{16}$$

This solution is written as a function of the boundary $\hat{B}$, to be clear about how the choice of $\hat{B}$ determines which solution to the HJB is the true value function.

To derive the value-matching condition at the boundary, use that firms are indifferent between applying the infinitesimal regulator dI at the boundary $\hat{B}$:

$$\hat{V}(\hat{B}) = \hat{V}(\hat{B}+dI) - \psi dI$$

$$\hat{V}(\hat{B}) = \hat{V}(\hat{B}) + \hat{V}'(\hat{B})dI - \psi dI$$

$$\implies \psi = \hat{V}'(\hat{B})$$

To derive the super-contact condition at the boundary, first consider the problem of a firm: their only decision is to select the critical value $\hat{B}$ that maximizes their value (16). The first order condition for this problem is

$$c'(\hat{B})\hat{X}^{\varrho}e^{\frac{v}{2}} = 0 \tag{17}$$

Next, take the derivative of the value matching condition $\psi = \hat{V}'(\hat{B})$ with respect to $\hat{B}$:

$$0 = m\alpha\hat{B}^{\alpha-1}e^{\frac{\alpha^2 v}{2}} + c(\hat{B})\varrho\hat{B}^{\varrho-1}e^{\frac{\rho^2 v}{2}} + c'(\hat{B})\hat{B}^{\varrho}e^{\frac{\varrho^2 v}{2}}$$

then substitute using (17) to find the super contact condition:

$$0 = m\alpha\hat{B}^{\alpha-1}e^{\frac{\alpha^2 v}{2}} + c(\hat{B})\varrho\hat{B}^{\varrho-1}e^{\frac{\varrho^2 v}{2}} = \hat{V}''(\hat{B})$$

□

A.2.3 Proof of Lemma 3

Proof. Per Corollary 2 , the first derivative of the value function in expected log normalized capital $\hat{v}(\hat{x})$ is

$$\hat{v}'(\hat{x}) = m\alpha e^{\alpha\hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho e^{\varrho\hat{x}} e^{\frac{\varrho^2 \nu}{2}}$$

Apply this to the value-matching condition from Lemma 2 (using $\hat{V}'(\hat{X}) = \frac{d\hat{V}(\hat{X})}{d\hat{X}} = \frac{d\hat{V}(\hat{X})}{d\hat{x}} \frac{d\hat{x}}{d\hat{X}} = \hat{v}'(\hat{x}) e^{-\hat{x}}$):

$$\psi = \hat{v}'(\hat{b}) e^{-\hat{b}} = m\alpha e^{(\alpha-1)\hat{b}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho e^{(\varrho-1)\hat{b}} e^{\frac{\varrho^2 \nu}{2}} \tag{18}$$

Before evaluating the super contact condition, it is helpful to rewrite $\hat{V}''(\hat{X})$ in terms of $\hat{x}$:

$$\hat{V}''(\hat{X}) = \frac{d\hat{V}'(\hat{X})}{d\hat{X}} = \frac{d\hat{V}'(\hat{X})}{d\hat{x}} \frac{d\hat{x}}{d\hat{X}} = \frac{d\hat{v}'(\hat{x}) e^{-\hat{x}}}{d\hat{x}} \frac{1}{\hat{X}} = (\hat{v}''(\hat{x}) e^{-\hat{x}} - \hat{v}'(\hat{x}) e^{-\hat{x}}) \frac{1}{\hat{X}}$$

$$= (\hat{v}''(\hat{x}) - \hat{v}'(\hat{x})) e^{-2\hat{x}} = m\alpha(\alpha-1) e^{(\alpha-2)\hat{x}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho(\varrho-1) e^{(\varrho-2)\hat{x}} e^{\frac{\varrho^2 \nu}{2}}$$

The super contact condition from Lemma 2 becomes

$$0 = \hat{V}''(\hat{b}) = m\alpha(\alpha-1) e^{(\alpha-2)\hat{b}} e^{\frac{\alpha^2 \nu}{2}} + c\varrho(\varrho-1) e^{(\varrho-2)\hat{b}} e^{\frac{\varrho^2 \nu}{2}} \tag{19}$$

Equations (18) and (19) imply

$$\psi(1-\varrho) = m\alpha(\alpha-\varrho) e^{(\alpha-1)\hat{b}} e^{\frac{\alpha^2 \nu}{2}}$$

$$\implies \hat{b} = \frac{1}{1-\alpha} \log\left(\frac{m\alpha(\alpha-\varrho)}{\psi(1-\varrho)}\right) + \frac{\alpha^2 \nu}{2(1-\alpha)}$$

□

A.2.4 Proof of Lemma 4

Proof. Per Lemma 1:

$$\frac{d}{da_{t-h}} \mathbb{E}[a_t | \Omega_t] = \frac{d}{da_{t-h}} \left(a_{t-\tau} + \gamma \left(s_t - s_{t-\tau}\right)\right)$$

There are two cases. In both, $\frac{ds_t}{da_{t-h}} = 1$. But if $0 \leq h < \tau$, then $\frac{da_{t-\tau}}{da_{t-h}} = \frac{ds_{t-\tau}}{da_{t-h}} = 0$:

$$[0 \leq h < \tau] : \qquad \frac{d}{da_{t-h}} \mathbb{E}[a_t|\Omega_t] = \gamma \frac{d}{da_{t-h}} s_t = \gamma$$

If $h \geq \tau$, then $\frac{da_{t-\tau}}{da_{t-h}} = \frac{ds_{t-\tau}}{da_{t-h}} = 1$:

$$[h \geq \tau] : \qquad \frac{d}{da_{t-h}} \mathbb{E}[a_t|\Omega_t] = 1 + \gamma - \gamma = 1$$

□

A.2.5 Proof of Lemma 5

Proof. The general solution to the ODE (7) is

$$h(\hat{x}) = c_{h1} e^{-\rho_1 \hat{x}} + c_{h2} e^{-\rho_2 \hat{x}}$$

where ρ_1 and ρ_2 are roots of the equation

$$0 = D\rho_j^2 - \delta\rho_j - \eta$$

Using $D = \frac{\sigma_a^2}{2}$, the roots are given by

$$\rho_j = \frac{\delta}{\sigma_a^2} \pm \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$$

To satisfy the boundary condition at infinity, only the positive root can have a non-zero coefficient. Therefore, the solution simplifies to

$$h(\hat{x}) = c_h e^{-\rho \hat{x}}$$

where ρ (without subscript) denotes the positive root $\frac{\delta}{\sigma_a^2} + \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$. The coefficient c_h is yet to be found.

The remaining boundary condition is that $h(\hat{x})$ integrates to one:

$$1 = \int_{\hat{b}}^{\infty} c_h e^{-\rho \hat{x}} d\hat{x} = \frac{c_h}{\rho} e^{-\rho \hat{b}}$$

which implies

$$c_h = \rho e^{\rho \hat{b}}$$

The joint distribution $f_{\hat{x},u}(\hat{x}, u)$ given by equation (8) implies

$$f_x(x) = h(x) * \phi(-\frac{x}{\sqrt{\nu}})$$

$$= \int_{-\infty}^{\infty} h(\hat{x})\phi(\frac{\hat{x} - x}{\sqrt{\nu}})d\hat{x}$$

$h(\hat{x}) = 0$ for $\hat{x} < \hat{b}$, so the convolution becomes

$$= \int_{\hat{b}}^{\infty} h(\hat{x})\phi(\frac{\hat{x} - x}{\sqrt{\nu}})d\hat{x} = \int_{\hat{b}}^{\infty} \rho e^{-\rho(\hat{x}-\hat{b})} \frac{1}{\sqrt{2\pi\nu}} e^{-\frac{(\hat{x}-x)^2}{2\nu}} d\hat{x}$$

$$= e^{-\frac{x^2}{2\nu}} \int_{\hat{b}}^{\infty} \rho e^{\rho\hat{b}} \frac{1}{\sqrt{2\pi\nu}} e^{-\frac{\hat{x}^2 - 2(x-\nu\rho)\hat{x}}{2\nu}} d\hat{x} = e^{-\frac{x^2-(x-\nu\rho)^2}{2\nu}} \int_{\hat{b}}^{\infty} \rho e^{\rho\hat{b}} \frac{1}{\sqrt{2\pi\nu}} e^{-\frac{\hat{x}^2 - 2(x-\nu\rho)\hat{x} + (x-\nu\rho)^2}{2\nu}} d\hat{x}$$

$$= \rho e^{-\rho(x-\hat{b})} e^{\frac{\nu\rho^2}{2}} \int_{\hat{b}}^{\infty} \frac{1}{\sqrt{2\pi\nu}} e^{-\frac{(\hat{x}-(x-\nu\rho))^2}{2\nu}} d\hat{x} = \rho e^{-\rho(x-\hat{b})} e^{\frac{\nu\rho^2}{2}} \int_{\hat{b}}^{\infty} \phi(\frac{\hat{x} - (x - \nu\rho)}{\sqrt{\nu}})d\hat{x}$$

$$= \rho e^{-\rho(x-\hat{b})} e^{\frac{\nu\rho^2}{2}} \left(1 - \Phi\left(\frac{\hat{b} + \nu\rho - x}{\sqrt{\nu}}\right)\right) = h(x) e^{\frac{\nu\rho^2}{2}} \Phi\left(\frac{x - (\hat{b} + \nu\rho)}{\sqrt{\nu}}\right)$$

□

A.2.6 Proof of Proposition 3

Proof. Decompose normalized capital into the independent nowcasts and errors by $x = \hat{x} - u$:

$$\int_{-\infty}^{\infty} e^x f_x(x)dx = \int_{\hat{b}}^{\infty} \int_{-\infty}^{\infty} e^{\hat{x}-u} f_{\hat{x},u}(\hat{x}, u)dud\hat{x}$$

$$= \int_{\hat{b}}^{\infty} \int_{-\infty}^{\infty} e^{\hat{x}-u} h(\hat{x})\phi(\frac{u}{\sqrt{\nu}})dud\hat{x} = \int_{\hat{b}}^{\infty} e^{\hat{x}} h(\hat{x}) \int_{-\infty}^{\infty} e^{-u}\phi(\frac{u}{\sqrt{\nu}})dud\hat{x}$$

Use that $\int_{-\infty}^{\infty} e^{-u}\phi(\frac{u}{\sqrt{\nu}})du = e^{\frac{\nu}{2}}$ is the mean of a log-normal distribution:

$$= e^{\frac{\nu}{2}} \int_{\hat{b}}^{\infty} e^{\hat{x}} h(\hat{x})d\hat{x} = e^{\frac{\nu}{2}} \int_{\hat{b}}^{\infty} e^{\hat{x}} \rho e^{-\rho(\hat{x}-\hat{b})} d\hat{x}$$

$$= \frac{e^{\frac{\nu}{2}+\hat{b}}\rho}{\rho - 1}$$

which is increasing in ν, per Proposition 1, and $\nu = \frac{\tau\sigma_a^2\sigma_n^2}{\sigma_a^2+\sigma_n^2}$ is increasing in σ_n^2 and τ. □

A.2.7 Proof of Proposition 11

Proof. Per Corollary 2, the firm's expectation of the value function derived in Lemma 6 is

$$\hat{v}(\hat{x}) = me^{\alpha\hat{x}}e^{\frac{\alpha^2\nu}{2}} + c_1 e^{\varrho_1\hat{x}}e^{\frac{\varrho_1^2\nu}{2}} + c_2 e^{\varrho_2\hat{x}}e^{\frac{\varrho_2^2\nu}{2}}$$

which in levels is

$$\hat{V}(\hat{X};\hat{B}) = m\hat{X}^{\alpha}e^{\frac{\alpha^2\nu}{2}} + c_1(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + c_2(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_2}e^{\frac{\varrho_2^2\nu}{2}} \tag{20}$$

This solution is written as a function of the boundaries $(\hat{B}_L,\hat{B}_U)$, to be clear about how the boundary choice determines which solution to the HJB is the true value function.

To derive the value-matching condition at the lower boundary, use that firms are indifferent between applying the infinitesimal regulator dI at the boundary $\hat{B}_L$:

$$\hat{V}(\hat{B}_L) = \hat{V}(\hat{B}_L + dI) - \psi_+ dI$$

$$\hat{V}(\hat{B}_L) = \hat{V}(\hat{B}_L) + \hat{V}'(\hat{B}_L)dI - \psi_+ dI$$

$$\implies \psi_+ = \hat{V}'(\hat{B}_L)$$

and a similar argument gives the value-matching condition at the upper boundary:

$$\psi_- = \hat{V}'(\hat{B}_U)$$

To derive the super-contact condition at the boundary, first consider the problem of a firm: their only decision is to select the critical values $\hat{B}_L$ and $\hat{B}_U$ that maximize their value (16). The first order conditions for this problem are

$$\partial_{\hat{B}_L}c_1(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + \partial_{\hat{B}_L}c_2(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_2}e^{\frac{\varrho_2^2\nu}{2}} = 0 \tag{21}$$

$$\partial_{\hat{B}_U}c_1(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + \partial_{\hat{B}_U}c_2(\hat{B}_L,\hat{B}_U)\hat{X}^{\varrho_2}e^{\frac{\varrho_2^2\nu}{2}} = 0 \tag{22}$$

Next, take the derivative of the value matching condition $\psi_+ = \hat{V}'(\hat{B}_L)$ with respect to $\hat{B}_L$:

$$0 = m\alpha\hat{B}_L^{\alpha-1}e^{\frac{\alpha^2\nu}{2}} + c_1(\hat{B}_L,\hat{B}_U)\varrho_1\hat{B}_L^{\varrho_1-1}e^{\frac{\varrho_1^2\nu}{2}} + c_2(\hat{B}_L,\hat{B}_U)\varrho_2\hat{B}_L^{\varrho_2-1}e^{\frac{\varrho_2^2\nu}{2}} + \partial_{\hat{B}_L}c_1(\hat{B}_L,\hat{B}_U)\hat{B}_L^{\varrho_1}e^{\frac{\varrho_1^2\nu}{2}} + \partial_{\hat{B}_L}c_2(\hat{B}_L,\hat{B}_U)\hat{B}_L^{\varrho_1}e^{\frac{\varrho_2^2\nu}{2}}$$

then substitute using (21) to find the super contact condition:

$$0 = m\alpha \hat{B}_L^{\alpha-1} e^{\frac{\alpha^2 \nu}{2}} + c_1(\hat{B}_L, \hat{B}_U)\varrho_1 \hat{B}_L^{\varrho_1 - 1} e^{\frac{\varrho_1^2 \nu}{2}} + c_2(\hat{B}_L, \hat{B}_U)\varrho_2 \hat{B}_L^{\varrho_2 - 1} e^{\frac{\varrho_2^2 \nu}{2}} = \hat{V}''(\hat{B}_L)$$

Again, a similar argument taking the derivative of the value matching condition $\psi_- = \hat{V}'(\hat{B}_U)$ with respect to $\hat{B}_U$ gives the super contact condition at the upper boundary:

$$0 = \hat{V}''(\hat{B}_U)$$

□

A.2.8 Proof of Proposition 5

Before proving the proposition, some notation and a lemma are needed.

Just as $f_x(x, t, \varepsilon)$ denotes the dynamic distribution of log normalized capital x after a time 0 permanent productivity shock of size the ε, let $f_x^{FI}(x, t, \varepsilon)$ denote the distribution of x under the counterfactual economy if firms were to have full information. Define the IRF of average log normalized capital relative to the steady state in each scenario by

$$IRF_x(t, \varepsilon) \equiv \int_x f_x(x, t, \varepsilon)dx - \bar{x} \qquad IRF_x^{FI}(t, \varepsilon) \equiv \int_x f_x^{FI}(x, t, \varepsilon)dx - \bar{x}^{FI}$$

where $\bar{x}$ and $\bar{x}^{FI}$ denote the steady state averages under incomplete and full information respectively.

Lemma 7.

$$IRF_x(t, \varepsilon) = IRF_x^{FI}(t, \gamma\varepsilon) - (1 - \gamma)\varepsilon$$

Proof. A $\gamma\varepsilon$ shock to the counterfactual full information economy shifts $f^{FI}(x, t, \varepsilon)$ left by $\gamma\varepsilon$, the same amount as the ε shock shifts $h(\hat{x}, t, \varepsilon)$. The dynamic KFE (6) is the same regardless of the severity of the information friction; the information friction only affects the boundary condition. Therefore (for $t < \tau$) the distribution of $\hat{x} - \hat{b}$ responds to a ε shock the same as the distribution of $x - b^{FI}$ responds to a $\gamma\varepsilon$ shock in the full information counterfactual:

$$h(\hat{x}, t, \varepsilon) = f^{FI}(\hat{x} - \hat{b} + b^{FI}, t, \gamma\varepsilon) \qquad t < \tau \tag{23}$$

where b^{FI} denotes the full information boundary.

Log normalized capital $x = \hat{x} - u$ has PDF

$$f_x(\hat{x} - u, t, \varepsilon) = h(\hat{x}, t, \varepsilon) f_u(u, t, \varepsilon)$$

where $f_u(u, t, \varepsilon)$ is the marginal distribution of the productivity nowcast error u, because u and $\hat{x}$ are independent. Ordinarily this distribution is mean zero, but after the aggregate productivity shock, all firms' nowcast errors increase by $(1 - \gamma)\varepsilon$:

$$f_u(u, t, \varepsilon) = \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) \qquad t < \tau$$

Therefore, for $t < \tau$, the impulse response function of average log normalized capital $x = \hat{x} - u$ is

$$IRF_x(t, \varepsilon) = \int_{\hat{x}} \int_u (\hat{x} - u) h(\hat{x}, t, \varepsilon) \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) du d\hat{x}$$

$$= \int_{\hat{x}} \int_u \hat{x} h(\hat{x}, t, \varepsilon) \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) du d\hat{x} - \int_{\hat{x}} \int_u u h(\hat{x}, t, \varepsilon) \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) du d\hat{x}$$

$$= \int_{\hat{x}} \hat{x} h(\hat{x}, t, \varepsilon) d\hat{x} - \int_u u \phi(\frac{u - (1 - \gamma)\varepsilon}{\sqrt{\nu}}) du = IRF_{\hat{x}}(t, \varepsilon) - (1 - \gamma)\varepsilon$$

where $IRF_{\hat{x}}(t, \varepsilon) \equiv \int_{\hat{x}} \hat{x} h(\hat{x}, t, \varepsilon) d\hat{x}$ denotes the IRF of average expected log normalized capital. We can relate this term back to the full information counterfactual by equation (23):

$$IRF_{\hat{x}}(t, \varepsilon) = \int_{\hat{b}}^{\infty} \hat{x} f^{FI}(\hat{x} - \hat{b} + b^{FI}, t, \gamma\varepsilon) d\hat{x}$$

apply the change of variable $x^{FI} = \hat{x} - \hat{b} + b^{FI}$:

$$= \int_{b^{FI}}^{\infty} (x^{FI} + \hat{b} - b^{FI}) f^{FI}(x^{FI}, t, \gamma\varepsilon) dx^{FI} = IRF_x^{FI}(t, \gamma\varepsilon) + \hat{b} - b^{FI}$$

Reintroduce the $(1 - \gamma)\varepsilon$ term to express the IRF as

$$IRF_x(t, \varepsilon) = IRF_x^{FI}(t, \gamma\varepsilon) + \hat{b} - b^{FI} - (1 - \gamma)\varepsilon$$

and $\bar{x} - \bar{x}^{FI} = \hat{b} - b^{FI}$ implies the desired expression. □

Proof. Because $x = k - a$, and the aggregate change to a is simply ε, the IRF for capital is

$$IRF_k(t, \varepsilon) = IRF_x(t, \varepsilon) + \varepsilon = IRF_x^{FI}(t, \gamma\varepsilon) + \gamma\varepsilon$$

for $t < \tau$ by Lemma 7. To find the marginal IRF, take the derivative with respect to ε under both

incomplete and full information:

$$\frac{\partial}{\partial \varepsilon} IRF_k(t,\varepsilon) = \frac{\partial}{\partial \varepsilon} IRF_x^{FI}(t,\gamma\varepsilon) + \gamma \qquad \frac{\partial}{\partial \varepsilon} IRF_k^{FI}(t,\varepsilon) = \frac{\partial}{\partial \varepsilon} IRF_x^{FI}(t,\varepsilon)\gamma + 1$$

then evaluate at $\varepsilon = 0$

$$\widehat{IRF}_k(t) = \gamma \widehat{IRF}_x^{FI}(t) + \gamma \qquad \widehat{IRF}_k^{FI}(t) = \widehat{IRF}_x^{FI}(t) + 1$$

Combine these equations to yield

$$\widehat{IRF}_k(t) = \gamma \widehat{IRF}_k^{FI}(t)$$

□

A.2.9 Proof of Lemma 7

Before the proof, we prove a lemma that is independently useful for computing the model:

Lemma 8. *The difference between the upper and lower log bounds of the inaction region $\Delta \equiv \hat{b}_H - \hat{b}_L$ solves the implicit equation*

$$\frac{\psi_- e^{(1-\varrho_2)\Delta} - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+}{\psi_- e^{(1-\varrho_2)\Delta} - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+} = \frac{\left(e^{(\alpha-\varrho_2)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)}{\left(\alpha e^{(\alpha-\varrho_2)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)} \tag{24}$$

and the lower bound $\hat{b}_L$ is given in terms of Δ by

$$\hat{b}_L = \frac{\alpha^2\nu}{2(1-\alpha)} - \frac{1}{1-\alpha}\log\left(\frac{1}{m\alpha}\frac{\psi_- - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\varrho_2-1)\Delta}}\right) \tag{25}$$

Proof. Per Corollary 2, the first derivative of the value function in expected log normalized capital $\hat{v}(\hat{x})$ is

$$\hat{v}'(\hat{x}) = m\alpha e^{\alpha\hat{x}} e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1 e^{\varrho_1\hat{x}} e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{x}} e^{\frac{\varrho_2^2\nu}{2}}$$

Apply this to the value-matching conditions (using $\hat{V}'(\hat{X}) = \frac{d\hat{V}(\hat{X})}{d\hat{X}} = \frac{d\hat{V}(\hat{X})}{d\hat{x}}\frac{d\hat{x}}{d\hat{X}} = \hat{v}'(\hat{x})e^{-\hat{x}}$):

$$\psi_+ e^{\hat{b}_L} = \hat{v}'(\hat{b}_L) = m\alpha e^{\alpha\hat{b}_L} e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1 e^{\varrho_1\hat{b}_L} e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_L} e^{\frac{\varrho_2^2\nu}{2}} \tag{26}$$

$$\psi_- e^{\hat{b}_H} = \hat{v}'(\hat{b}_H) = m\alpha e^{\alpha\hat{b}_H} e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1 e^{\varrho_1\hat{b}_H} e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_H} e^{\frac{\varrho_2^2\nu}{2}} \tag{27}$$

Before evaluating the super contact conditions, it is helpful to rewrite $\hat{V}''(\hat{X})$ in terms of $\hat{x}$:

$$\hat{V}''(\hat{X}) = \frac{d\hat{V}'(\hat{X})}{d\hat{X}} = \frac{d\hat{V}'(\hat{X})}{d\hat{x}}\frac{d\hat{x}}{d\hat{X}} = \frac{d\hat{v}'(\hat{x})e^{-\hat{x}}}{d\hat{x}}\frac{1}{\hat{X}} = (\hat{v}''(\hat{x})e^{-\hat{x}} - \hat{v}'(\hat{x})e^{-\hat{x}})\frac{1}{\hat{X}}$$

$$= (\hat{v}''(\hat{x}) - \hat{v}'(\hat{x}))e^{-2\hat{x}}$$

Thus the super contact conditions $0 = \hat{V}''(\hat{B}_L)$ and $0 = \hat{V}''(\hat{B}_H)$ imply $\hat{v}''(\hat{b}_L) = \hat{v}'(\hat{b}_L)$ and $\hat{v}''(\hat{b}_H) = \hat{v}'(\hat{b}_H)$ respectively. These conditions become

$$\psi_+ e^{\hat{b}_L} = \hat{v}''(\hat{b}_L) = m\alpha e^{\alpha\hat{b}_L}e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1 e^{\varrho_1\hat{b}_L}e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_L}e^{\frac{\varrho_2^2\nu}{2}} \tag{28}$$

$$\psi_- e^{\hat{b}_H} = \hat{v}''(\hat{b}_H) = m\alpha^2 e^{\alpha\hat{b}_H}e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1^2 e^{\varrho_1\hat{b}_H}e^{\frac{\varrho_1^2\nu}{2}} + c_2\varrho_2^2 e^{\varrho_2\hat{b}_H}e^{\frac{\varrho_2^2\nu}{2}} \tag{29}$$

Combining the b_L value-matching condition (26) and super contact condition (28) can be used to solve for c_1 and c_2 in terms of b_L. First, difference out the c_2 term:

$$(1-\varrho_2)\psi_+ e^{\hat{b}_L} = m\alpha(\alpha-\varrho_2)e^{\alpha\hat{b}_L}e^{\frac{\alpha^2\nu}{2}} + c_1\varrho_1(\varrho_1-\varrho_2)e^{\varrho_1\hat{b}_L}e^{\frac{\varrho_1^2\nu}{2}}$$

$$\implies c_1\varrho_1 e^{\frac{\varrho_1^2\nu}{2}} = \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_1)\hat{b}_L} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_1)\hat{b}_L}e^{\frac{\alpha^2\nu}{2}}$$

Plug back into the value matching condition (26):

$$\psi_+ e^{\hat{b}_L} = m\alpha e^{\alpha\hat{b}_L}e^{\frac{\alpha^2\nu}{2}} + \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{\hat{b}_L} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{\alpha\hat{b}_L}e^{\frac{\alpha^2\nu}{2}} + c_2\varrho_2 e^{\varrho_2\hat{b}_L}e^{\frac{\varrho_2^2\nu}{2}}$$

$$\implies c_2\varrho_2 e^{\frac{\varrho_2^2\nu}{2}} = \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_2)\hat{b}_L} + m\alpha\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_2)\hat{b}_L}e^{\frac{\alpha^2\nu}{2}}$$

Use these expressions to substitute for c_1 and c_2 in the b_H value matching condition (27):

$$\begin{aligned}\psi_- e^{\hat{b}_H} = {} & m\alpha e^{\alpha\hat{b}_H}e^{\frac{\alpha^2\nu}{2}} \\ & + \left(\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_1)\hat{b}_L} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_1)\hat{b}_L}e^{\frac{\alpha^2\nu}{2}}\right)e^{\varrho_1\hat{b}_H} \\ & + \left(\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(1-\varrho_2)\hat{b}_L} + m\alpha\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\alpha-\varrho_2)\hat{b}_L}e^{\frac{\alpha^2\nu}{2}}\right)e^{\varrho_2\hat{b}_H}\end{aligned}$$

Express in terms of the difference $\Delta \equiv \hat{b}_H - \hat{b}_L$:

$$\psi_- = m\alpha e^{(\alpha-1)(\hat{b}_L+\Delta)} e^{\frac{\alpha^2 v}{2}} + \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - m\alpha\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_1-1)\Delta} e^{\frac{\alpha^2 v}{2}} + \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta} + m\alpha\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_2-1)\Delta} e^{\frac{\alpha^2 v}{2}} \quad (30)$$

and do the same for the super contact condition (29):

$$\psi_- = m\alpha^2 e^{(\alpha-1)(\hat{b}_L+\Delta)} e^{\frac{\alpha^2 v}{2}} + \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - m\alpha\varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_1-1)\Delta} e^{\frac{\alpha^2 v}{2}} + \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta} + m\alpha\varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\alpha-1)\hat{b}_L} e^{(\varrho_2-1)\Delta} e^{\frac{\alpha^2 v}{2}} \quad (31)$$

Collect terms in equation (30):

$$\psi_- = m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 v}{2}} \left(e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right) + \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} + \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}$$

and equation (31):

$$\psi_- = m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 v}{2}} \left(\alpha e^{(\alpha-1)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right) + \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta} + \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta}$$

Rearrange both to isolate $\hat{b}_L$:

$$m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 v}{2}} = \frac{\psi_- - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{\left(e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right)}$$

$$m\alpha e^{(\alpha-1)\hat{b}_L} e^{\frac{\alpha^2 v}{2}} = \frac{\psi_- - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{\left(\alpha e^{(\alpha-1)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)} e^{(\varrho_1-1)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)} e^{(\varrho_2-1)\Delta} \right)}$$

either of which give $\hat{b}_L$ in terms of Δ.

Combining the two equations yields an implicit equation determining Δ:

$$\frac{\psi_- - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}}{\psi_- - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-1)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_2-1)\Delta}} = \frac{\left(e^{(\alpha-1)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-1)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\varrho_2-1)\Delta}\right)}{\left(\alpha e^{(\alpha-1)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-1)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}e^{(\varrho_2-1)\Delta}\right)}$$

which can be rearranged as

$$\frac{\psi_- e^{(1-\varrho_2)\Delta} - \frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+}{\psi_- e^{(1-\varrho_2)\Delta} - \varrho_1\frac{(1-\varrho_2)}{(\varrho_1-\varrho_2)}\psi_+ e^{(\varrho_1-\varrho_2)\Delta} - \varrho_2\frac{(\varrho_1-1)}{(\varrho_1-\varrho_2)}\psi_+} = \frac{\left(e^{(\alpha-\varrho_2)\Delta} - \frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)}{\left(\alpha e^{(\alpha-\varrho_2)\Delta} - \varrho_1\frac{(\alpha-\varrho_2)}{(\varrho_1-\varrho_2)}e^{(\varrho_1-\varrho_2)\Delta} + \varrho_2\frac{(\alpha-\varrho_1)}{(\varrho_1-\varrho_2)}\right)}$$

□

With Lemma 8 in hand, proving Lemma 7 is straightforward:

Proof of Lemma 7. ν does not appear in equation (24), so Δ is unaffected by the information friction. b_L^{FI} denotes the solution for $\nu = 0$. Equation (25) implies

$$\hat{b}_L = b_L^{FI} + \frac{\alpha^2\nu}{2(1-\alpha)}$$

Finally, $\Delta = \hat{b}_H - \hat{b}_L = b_H^{FI} - b_L^{FI}$ implies

$$\hat{b}_H = b_H^{FI} + \frac{\alpha^2\nu}{2(1-\alpha)}$$

□

A.3 The Joint Distribution

This appendix derives the analytical joint distribution of capital and productivity by solving a PDE: the multi-dimensional Kolmogorov Forward Equation.

A.3.1 Transformation and Solution of the Kolmogorov Forward Equation

The stationary joint distribution of capital and expected productivity $g(k, \hat{a})$ is determined by the stationary KFE (10). To solve for this distribution, it is convenient to first find the joint distribution of k and the deviation of expected normalized capital $\hat{x}$:

$$f_{k,\hat{x}}(k, \hat{x}) \equiv g(k, k - \hat{x})$$

with $f_{k,\hat{x}}(k,\hat{x}) = 0$ for $\hat{x} < \hat{b}$.

Lemma 9. *The stationary distribution $f_{k,\hat{x}}(k,\hat{x})$ satisfies the stationary KFE*

$$D\partial^2_{\hat{x}} f_{k,\hat{x}} + \delta \partial_{\hat{x}} f_{k,\hat{x}} = -\delta \partial_k f_{k,\hat{x}} + \eta f_{k,\hat{x}}$$

Proof. The joint distributions are related by $f_{k,\hat{x}}(k,\hat{x}) = g(k,\hat{a})$ with $\hat{a} = k - \hat{x}$. By the chain rule:

$$\begin{aligned}
\partial_{\hat{x}} f_{k,\hat{x}}(k,\hat{x}) &= -\partial_{\hat{a}} g(k,\hat{a}) \\
\partial^2_{\hat{x}} f_{k,\hat{x}}(k,\hat{x}) &= \partial^2_{\hat{a}} g(k,\hat{a}) \\
\partial_k f_{k,\hat{x}}(k,\hat{x}) &= \partial_k g(k,\hat{a}) + \partial_{\hat{a}} g(k,\hat{a})
\end{aligned}$$

Substitute into the KFE (10):

$$0 = \delta\left(\partial_k f_{k,\hat{x}}(k,\hat{x}) - \partial_{\hat{a}} g(k,\hat{a})\right) + D\partial^2_{\hat{a}} g(k,\hat{a}) - \eta g(k,\hat{a})$$

$$0 = \delta \partial_k f_{k,\hat{x}}(k,\hat{x}) + \delta \partial_{\hat{x}} f_{k,\hat{x}}(k,\hat{x}) + D\partial^2_{\hat{x}} f_{k,\hat{x}}(k,\hat{x}) - \eta g(k,\hat{a})$$

Rearranging gives the solution. □

The flow of firms outwards across the barrier depends on the capital level:

$$Flux(k) = D\partial_{\hat{x}} f_{k,\hat{x}}(k,\hat{b}) + \delta f_{k,\hat{x}}(k,\hat{b})$$

Recall that the distribution of expected productivity among entering firms is $\hat{a}_{enter} \sim N(0,\varsigma)$. Thus, conditional on k, this is also the distribution of expected normalized capital. The measure of entering firms is η, so the flux at the boundary must be

$$-\eta\phi\left(\frac{k}{\varsigma}\right) = D\partial_{\hat{x}} f_{k,\hat{x}}(k,\hat{b}) + \delta f_{k,\hat{x}}(k,\hat{b}) \tag{32}$$

where $\phi(\cdot)$ is the standard normal pdf. This is the first boundary condition. The solution remains indeterminate, so we also must impose that $f_{k,\hat{x}}$ is a density, i.e.:

$$1 = \iint f_{k,\hat{x}}(k,\hat{b})\,dk\,d\hat{x} \tag{33}$$

and

$$f_{k,\hat{x}}(k,\hat{b}) \geq 0 \;\forall k,\; \hat{x} \geq \hat{b}$$

A.3.2 Solving for the Transformed Distribution

Lemma 10. *The stationary joint distribution of capital and expected normalized capital $f_{k,\hat{x}}(k,\hat{x})$ is*

$$f_{k,\hat{x}}(k,\hat{x}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{-\mathcal{N}(\xi)}{D\lambda_-(\xi)+\delta} e^{i\xi k+\lambda_-(\xi)(\hat{x}-\hat{b})} d\xi$$

where $\mathcal{N}(\xi)$ denotes the Fourier transform of $\eta\phi\left(\frac{k}{\varsigma}\right)$ and $\lambda_-(\xi) \equiv \frac{-\delta-\sqrt{\delta^2+4D(i\delta\xi+\eta)}}{2D}$.

Proof. Apply the Fourier transform in k:

$$\tilde{f}(\hat{x},\xi) = \int_{-\infty}^{\infty} f_{k,\hat{x}}(k,\hat{x})\, e^{-i\xi k}\, dk$$

which converts the PDE into an ODE in $\hat{x}$:

$$D\,\partial_{\hat{x}}^2\tilde{f} + \delta\,\partial_{\hat{x}}\tilde{f} = (i\delta\xi+\eta)\,\tilde{f}$$

The general solution is

$$\tilde{f}(\hat{x},\xi) = A(\xi)\, e^{\lambda_+(\xi)(\hat{x}-\hat{b})} + B(\xi)\, e^{\lambda_-(\xi)(\hat{x}-\hat{b})} \tag{34}$$

with characteristic roots

$$\lambda_\pm(\xi) = \frac{-\delta \pm \sqrt{\delta^2+4D(i\delta\xi+\eta)}}{2D}$$

The flux boundary condition determines a linear relationship between $A(\xi)$ and $B(\xi)$. Take the Fourier transform of equation (32):

$$-\mathcal{N}(\xi) = D\left(\lambda_+(\xi)A(\xi)+\lambda_-(\xi)B(\xi)\right) + \delta\left(A(\xi)+B(\xi)\right) \tag{35}$$

This gives a linear relationship between $B(\xi)$ and $A(\xi)$.

The integrating constraint pins down $A(0)$:

$$1 = \int_{\hat{x}}\int_k f_{k,\hat{x}}(k,\hat{x})\, dk\, d\hat{x} = \frac{1}{2\pi}\int_{\hat{x}}\int_k\int_\xi \tilde{f}(\hat{x},\xi)e^{i\xi k}\, d\xi\, dk\, d\hat{x}$$

$$= \frac{1}{2\pi}\int_{\hat{x}}\int_\xi \tilde{f}(\hat{x},\xi)\left(\int_k e^{i\xi k}\, dk\right) d\xi\, d\hat{x}$$

Use $\int_k e^{i\xi k}\, dk = 2\pi\delta(\xi)$ where $\delta(\xi)$ denotes the Dirac delta:

$$= \int_{\hat{x}} \int_{\xi} \tilde{f}(\hat{x}, \xi)\delta(\xi) d\xi \, d\hat{x} = \int_{\hat{x}} \tilde{f}(\hat{x}, 0) d\hat{x}$$

$$= \int_{\hat{x}} \left(A(0)e^{\lambda_+(0)(\hat{x}-\hat{b})} + B(0)e^{\lambda_-(0)(\hat{x}-\hat{b})} \right) d\hat{x}$$

For the integral to remain finite, it must be that $A(0) = 0$.

For non-zero $A(\xi)$, the non-negativity constraint requires $A(\xi) = 0$, else equation (34) implies explosive oscillations as $\hat{x}$ becomes large. Combining the $A(\xi) = 0$ constraint with equation (35) gives

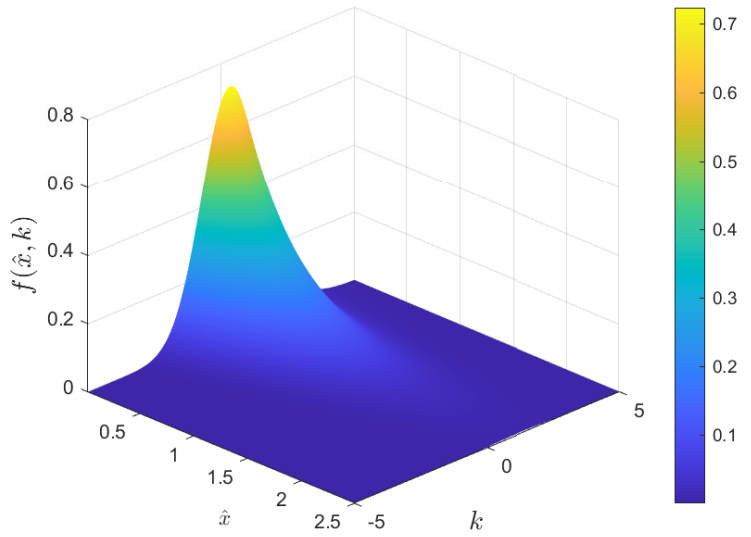
$$B(\xi) = \frac{-\mathcal{N}(\xi)}{D\lambda_-(\xi) + \delta}$$

Plug this into the general solution (34) and invert the Fourier transform:

$$f_{k,\hat{x}}(k, \hat{x}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\hat{x}, \xi) e^{i\xi k} d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-\mathcal{N}(\xi)}{D\lambda_-(\xi) + \delta} e^{i\xi k + \lambda_-(\xi)(\hat{x}-\hat{b})} d\xi$$

□

Figure 8: Joint Stationary Density Function $f(\hat{x}, k)$



Notes: $f(\hat{x}, k)$ measures the joint density for log capital k and expected log normalized capital $\hat{x}$. The grid for k is normalized around 0, while the grid for $\hat{x}$ is normalized with boundary 0.

Figure 8 presents the joint stationary density function $f(\hat{x}, k)$ given our parametrization and a level of noise $\xi_{30} = 0.055$.

A.3.3 Recovering the Original Distribution

The original object of interest, $g(k, \hat{a})$ is then reconstructed by a change of variable.

Proposition 6. *The stationary joint distribution of capital and expected productivity is*

$$g(k, \hat{a}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-\mathcal{N}(\xi)}{D\lambda_{-}(\xi) + \delta} e^{i\xi k + \lambda_{-}(\xi)(k - \hat{a} - \hat{b})} d\xi$$

for $k \geq \hat{a} + \hat{b}$, and zero otherwise.

Proof. The distribution $g(k, \hat{a})$, is constructed by a change of variable using $\hat{x} = k - \hat{a}$:

$$g(k, \hat{a}) = \begin{cases} f_{k,\hat{x}}(k, k - \hat{a}) & \text{if } \hat{b} \leq k - \hat{a} \\ 0 & \text{otherwise} \end{cases}$$

and $f_{k,\hat{x}}(k, k - \hat{a})$ is given by Lemma 10. □

As in the univariate case, the joint distribution with nowcast errors is simply

$$f_{k,\hat{a},u}(k, \hat{a}, u) = g(k, \hat{a})\phi(\frac{u}{\sqrt{\nu}}) \tag{36}$$

because u is independent of k and $\hat{a}$ and normally distributed.

Corollary 3. *The joint distribution of capital and productivity is*

$$f_{k,a}(k, a) = \int_{\hat{a}} g(k, \hat{a})\phi(\frac{a - \hat{a}}{\sqrt{\nu}}) d\hat{a}$$

Proof. Apply the change of variable $u = a - \hat{a}$ to equation (36) to get the joint distribution

$$f_{k,\hat{a},a}(k, \hat{a}, a) = g(k, \hat{a})\phi(\frac{a - \hat{a}}{\sqrt{\nu}})$$

and integrate over $\hat{a}$ to yield

$$f_{k,a}(k, a) = \int_{\hat{a}} f_{k,\hat{a},a}(k, \hat{a}, a) d\hat{a} = \int_{\hat{a}} g(k, \hat{a})\phi(\frac{a - \hat{a}}{\sqrt{\nu}}) d\hat{a}$$

□

Online Appendix

Appendix B Irreversibility Alternatives

In the main text, investment is fully irreversible. This appendix derives results under partial irreversibility and full reversibility.

B.1 Partial Irreversibility

In this appendix, we modify the baseline model to relax the assumption of full irreversibility.

Investment I is now partially irreversible. If firms invest, they do so at a cost $\Psi(I)$:

$$\Psi(I) = \begin{cases} \psi_+ I & I \geq 0 \\ \psi_- I & I < 0 \end{cases}$$

with $\psi_+ > \psi_- > 0$. Accordingly, their instantaneous profit is $\pi = A^{1-\alpha}K^\alpha - \Psi(I)$.

Optimal firm behavior for this type of problem is characterized by an inaction region: for a range of capital values (that depends on other state variables), firms choose to neither invest nor divest. Firms with partial irreversibility face the usual full information HJB equation (1) in the inaction region.

Optimal investment behavior is a threshold strategy, as in the full information case. Except now, a firm invests only if its expected normalized capital $\hat{X}$ is less than than some critical lower value $\hat{B}_L$, and divests only if above some upper value $\hat{B}_U$. So solving the firm's problem comes down to finding the optimal choice of $\hat{B}_L$ and $\hat{B}_U$. Lemma 2 reports the boundary conditions associated with the optimum. They are analogous to the full information case.

Lemma 11. *Under incomplete information, the boundary conditions consist of two value-matching conditions:*

$$\hat{V}'(\hat{B}_L) = \psi_+ \qquad \hat{V}'(\hat{B}_U) = \psi_-$$

and two super contact conditions:

$$\hat{V}''(\hat{B}_L) = 0 \qquad \hat{V}''(\hat{B}_U) = 0$$

Proof: Appendix A.2.7

Lemma 7 summarizes the solution to the firm's problem. The log critical values $\hat{b}_L \equiv \log \hat{B}_L$

and $\hat{b}_U \equiv \log \hat{B}_U$ depend on several parameters: the interest rate r, depreciation rate δ, time series properties of the productivity process, the investment and divestment costs ψ_+ and ψ_-, and so forth. But conveniently, most of these terms affect the critical values in the same way that they would in the full information model. The proposition shows that the difference between full and incomplete information critical values depend only on the variance of nowcast errors v, and the returns to scale α.

Proposition 7. *The critical values of expected normalized capital are*

$$\hat{b}_L = b_L^{FI} + \frac{\alpha^2 v}{2(1-\alpha)} \qquad \hat{b}_H = b_H^{FI} + \frac{\alpha^2 v}{2(1-\alpha)}$$

where b_L^{FI} and b_H^{FI} denote the full information solutions such that $v = 0$.

Proof: Appendix A.2.9

B.2 Full Reversibility

This appendix derives capital demand when investment is fully reversible, under both full and incomplete information.

B.2.1 Full Reversibility with Full Information

This section quickly derives the (well-known) capital demand without either friction. It follows directly from the property that firms set the marginal product of capital equal to the user cost.

Proposition 8. *Under full reversibility and full information, firms' demand for capital is*

$$K = \left(\frac{\alpha}{(r+\delta)\psi}\right)^{\frac{1}{1-\alpha}} A$$

Proof. With reversibility, firms' HJB equation (1) becomes

$$rV(K,A) = \max_I A^{1-\alpha}K^\alpha - \psi I + (I - \delta K)V_K(K,A) + \frac{\sigma_a^2 A^2}{2} V_{AA}(K,A) \tag{37}$$

which has first order condition

$$\psi = V_K(K,A)$$

The envelope condition is

$$rV_K(K,A) = \alpha A^{1-\alpha}K^{\alpha-1} - \delta V_K(K,A) + (I-\delta K)V_{KK}(K,A) + \frac{\sigma_a^2 A^2}{2} V_{KAA}(K,A)$$

This simplifies because the first order condition implies $0 = V_{KK}(K, A) = V_{KAA}(K, A)$. Rearrange:

$$(r + \delta)V_K(K, A) = \alpha A^{1-\alpha} K^{\alpha-1}$$

then substitute with ψ:

$$(r + \delta)\psi = \alpha A^{1-\alpha} K^{\alpha-1}$$

and rearranging provides the desired expression. □

B.2.2 Full Reversibility with Incomplete Information

This section derives the capital demand by firms that face only the information friction. In this case, firms set the user cost equal to their *expected* marginal product of capital.

With incomplete information, the firm's state variables are K and $\hat{A}$, where $\log \hat{A} = \hat{a} = \mathbb{E}[a]$. Because a is normal with constant variance, $\hat{A}$ is a sufficient state variable for firms' information. Proposition 9 gives the firms' demand for capital in terms of $\hat{A}$.

Proposition 9. *Under full reversibility and incomplete information, firms' demand for capital is*

$$K = \left(\frac{\alpha}{(r + \delta)\psi}\right)^{\frac{1}{1-\alpha}} e^{\frac{(1-\alpha)v}{2}} \hat{A}$$

Proof. Under incomplete information, the firm maximizes its expected present value of future profits. The HJB becomes

$$rV(K, \hat{A})dt = \max_I \mathbb{E}\left[A^{1-\alpha}K^{\alpha}dt - \psi I + V_K(K, \hat{A})dK + V_{\hat{A}}(K, \hat{A})d\hat{A}\right]$$

Apply Itô's lemma using Lemma 1 to get the usual form:

$$rV(K, \hat{A}) = \max_I \mathbb{E}\left[A^{1-\alpha}K^{\alpha}\right] - \psi I + (I - \delta K)V_K(K, \hat{A}) + \frac{\sigma_a^2 \hat{A}^2}{2} V_{\hat{A}\hat{A}}(K, \hat{A})$$

Recall $\hat{A} = e^{\hat{a}} = e^{\mathbb{E}[a]}$, so

$$\mathbb{E}\left[A^{1-\alpha}K^{\alpha}\right] = \mathbb{E}\left[e^{(1-\alpha)a}\right] K^{\alpha} = e^{(1-\alpha)\hat{a} + \frac{(1-\alpha)^2 v}{2}} K^{\alpha}$$

by Lemma 1 and the log-normality of $e^{(1-\alpha)a}$. Therefore the HJB becomes

$$rV(K, \hat{A}) = \max_I e^{\frac{(1-\alpha)^2 v}{2}} \hat{A}^{1-\alpha}K^{\alpha} - \psi I + (I - \delta K)V_K(K, \hat{A}) + \frac{\sigma_a^2 \hat{A}^2}{2} V_{\hat{A}\hat{A}}(K, \hat{A})$$

which is similar to the full information HJB in equation (37), albeit with the revenue scaled by $e^{\frac{(1-\alpha)^2 v}{2}}$. Following the same logic as in the proof of Proposition 8,

$$(r+\delta)\psi = \alpha \hat{A}^{1-\alpha} K^{\alpha-1} e^{\frac{(1-\alpha)^2 v}{2}}$$

thus the user cost $(r+\delta)\psi$ is equal to the expected marginal product of capital. Substituting with $\hat{X} = \frac{K}{\hat{A}}$ gives

$$(r+\delta)\psi = \alpha \hat{X}^{\alpha-1} e^{\frac{(1-\alpha)^2 v}{2}}$$

and rearranging provides the desired expression. □

Proposition 9 clarifies how incomplete information increases capital demand even without the investment friction. When firms have incomplete information, they invest until their *expected* MPK is equal to the user cost. Log productivity is uncertain and normally distributed, so Jensen's inequality raises the expected MPK relative to the median MPK. The higher the uncertainty (larger v), the larger the effect.

Appendix C Supplements to Empirical Analysis

C.1 Alternative Estimation of the Underreaction Coefficients

Figure 9 shows the estimated underreaction coefficients when firm fixed effects are included. Including firm fixed effects can mechanically produce negative attenuation coefficients. The intuition is as follows. We regress $realized\ sales_{t+1} - forecasted\ sales_{t,t+1}$ on $productivity\ shock_t \equiv productivity_t - productivity_{t-1}$. Running a first-difference regression, which is analogous to including fixed effects, leads to

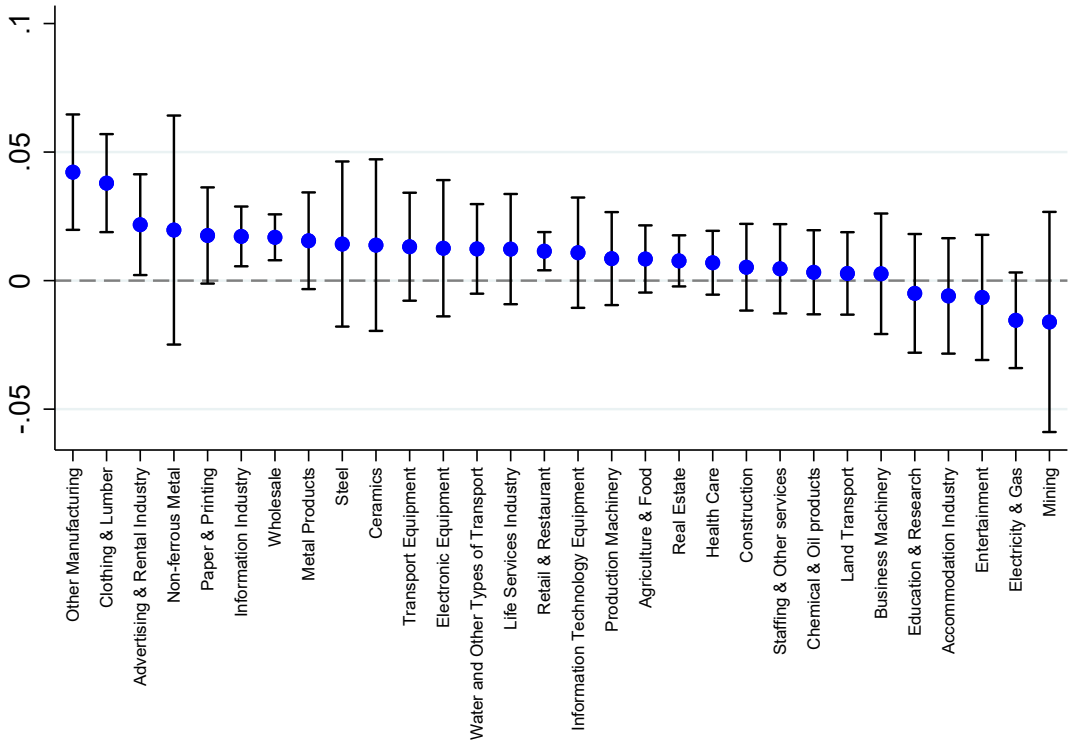
$$realized\ sales_{t+1} - forecasted\ sales_{t,t+1} - realized\ sales_t + forecasted\ sales_{t-1,t}$$

on the left-hand side (LHS) and

$$productivity_t - 2 * productivity_{t-1} + productivity_{t-2}$$

on the right-hand side (RHS). Importantly, $realized\ sales_t$ appears negatively on the LHS and is highly positively correlated with $productivity_t$ on the RHS. This creates a mechanically negative correlation between the LHS and RHS. Because measurement errors are inevitable, including firm fixed effects can induce a downward bias in the estimated attenuation coefficients.

Figure 9: Estimated Underreaction Coefficients Across Industries with Firm FE: Labor Productivity



Notes: This figure shows how the coefficient governs the impulse response of the log sales forecast error made in period $t+1$ with respect to the realized log productivity innovation in period t. We estimate the coefficients by including the firm fixed effects. Each dot denotes the estimate for an industry (with the 95% confidence interval), and there are 30 industries in total. Top and bottom 1% observations are winsorized. The data frequency is semi-annual, and the productivity measure is labor productivity.

C.2 Additional Forecast Error Regressions

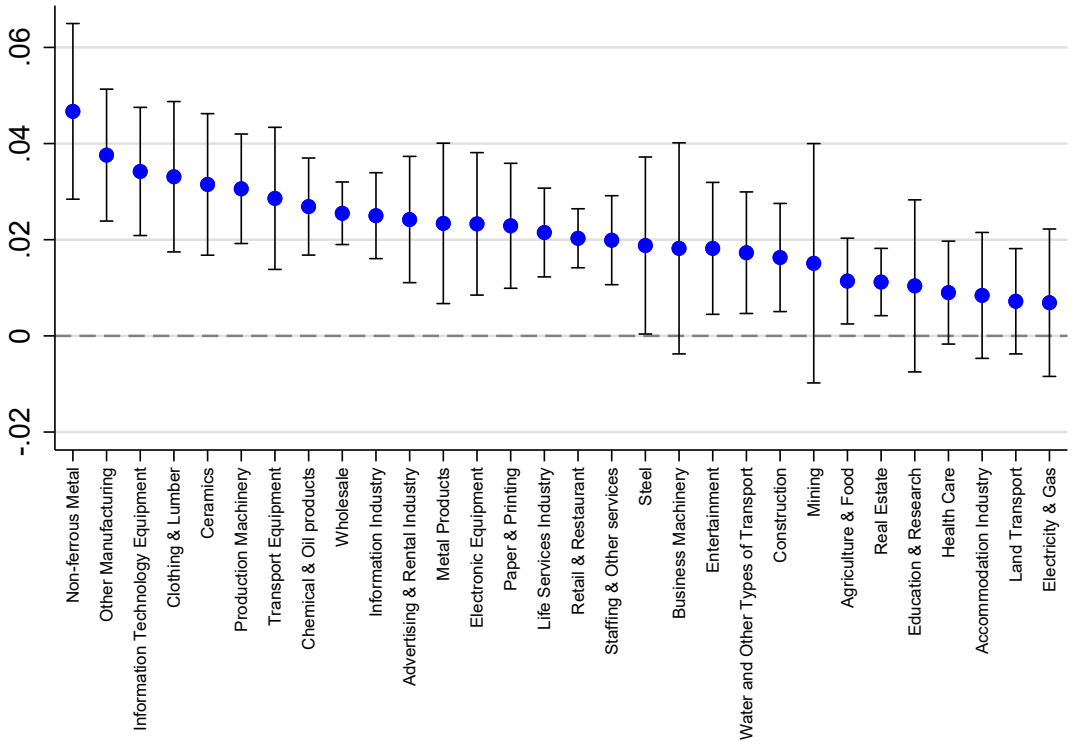
Table 7 presents the regression result for equation (11) using the percentage forecast error of sales as the dependent variable. Figures 10-13 present regression results using the log/percentage forecast error as the dependent variable and the lagged value and the innovation in log productivity (or log sales or log value added) as the dependent variables.

Table 7: Degree of Information Frictions for the Entire Economy: Percentage Forecast Error

	$FE^{pct}_{t,t+1} \equiv \frac{(sales_{t+1} - \mathbb{E}[sales_{t+1}\|\Omega_t])}{\mathbb{E}[a_{t+1}\|\Omega_t])}$								
$w_{i,t}$	0.022***	0.021***	0.021***	0.020***	0.020***	0.019***	0.010***	0.010***	0.006***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
$a_{i,t-1}$	0.005***	0.005***	0.004***	0.005***	0.005***	0.004***	-0.012***	-0.012***	-0.027***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)
$k_{i,t-1}$		0.001***	0.002***		0.001**	0.002***		-0.009***	-0.003*
		(0.000)	(0.001)		(0.000)	(0.001)		(0.002)	(0.002)
$\log(emp)_{i,t-1}$			-0.002***			-0.002**			-0.027***
			(0.001)			(0.001)			(0.002)
Size-year FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Region FE	Y	Y	Y	N	N	N	N	N	N
Industry FE	Y	Y	Y	N	N	N	N	N	N
Region-year FE	N	N	N	Y	Y	Y	Y	Y	Y
Industry-year FE	N	N	N	Y	Y	Y	Y	Y	Y
Firm FE	N	N	N	N	N	N	Y	Y	Y
N	83201	81104	80317	83194	81097	80310	81986	79914	79148
adj. R^2	0.046	0.047	0.047	0.074	0.075	0.075	0.184	0.185	0.188

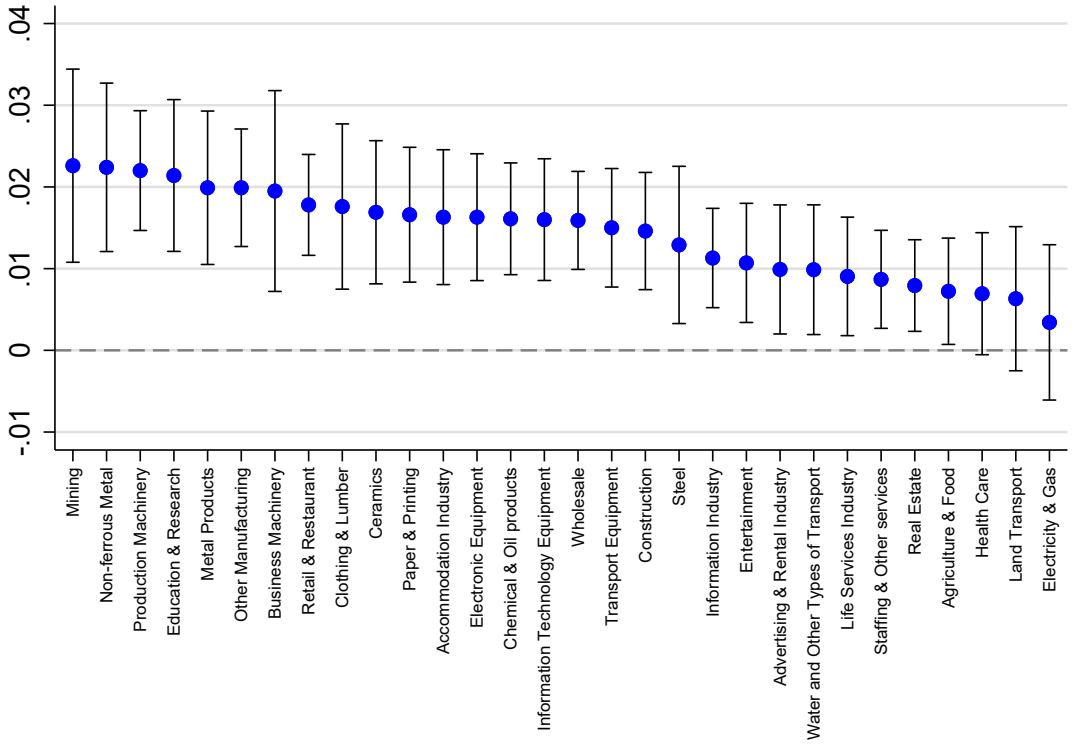
Notes: Standard errors are clustered at the firm level. Significance levels are indicated as follows: * 0.10, ** 0.05, *** 0.01. The degree of information friction, ξ, is estimated for the entire economy. The top and bottom 1% of productivity observations and forecast errors are winsorized. *emp* denotes employment. The regressions are conducted at the semiannual frequency.

Figure 10: Industry-Specific Underreaction Coefficients:
Percentage Forecast Error to Productivity Innovation



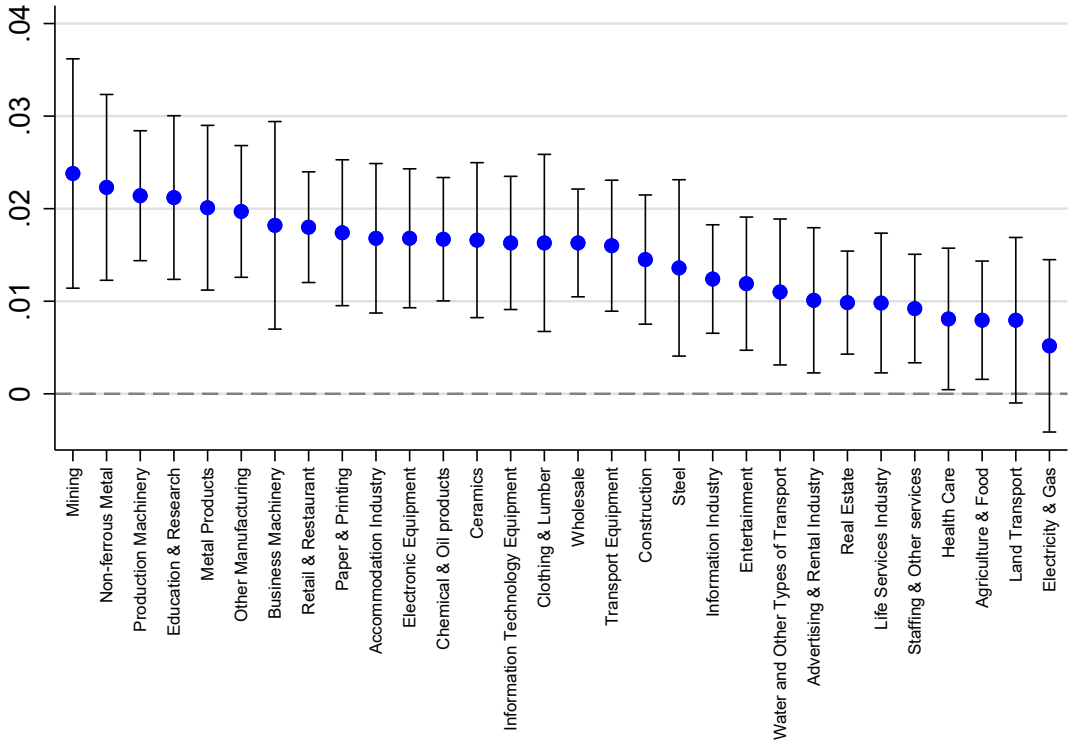
Notes: This figure illustrates how the coefficient governs the impulse response of the percentage sales forecast error in period $t + 1$ to realized (log) productivity shocks in period t. Each dot represents an industry estimate, with 95% confidence intervals, across all 30 industries. The top and bottom 1% of observations are trimmed to remove outliers. The data are semiannual, and productivity is measured in terms of labor productivity.

Figure 11: Industry-Specific Underreaction Coefficients:
Log Forecast Error to Sales Innovation



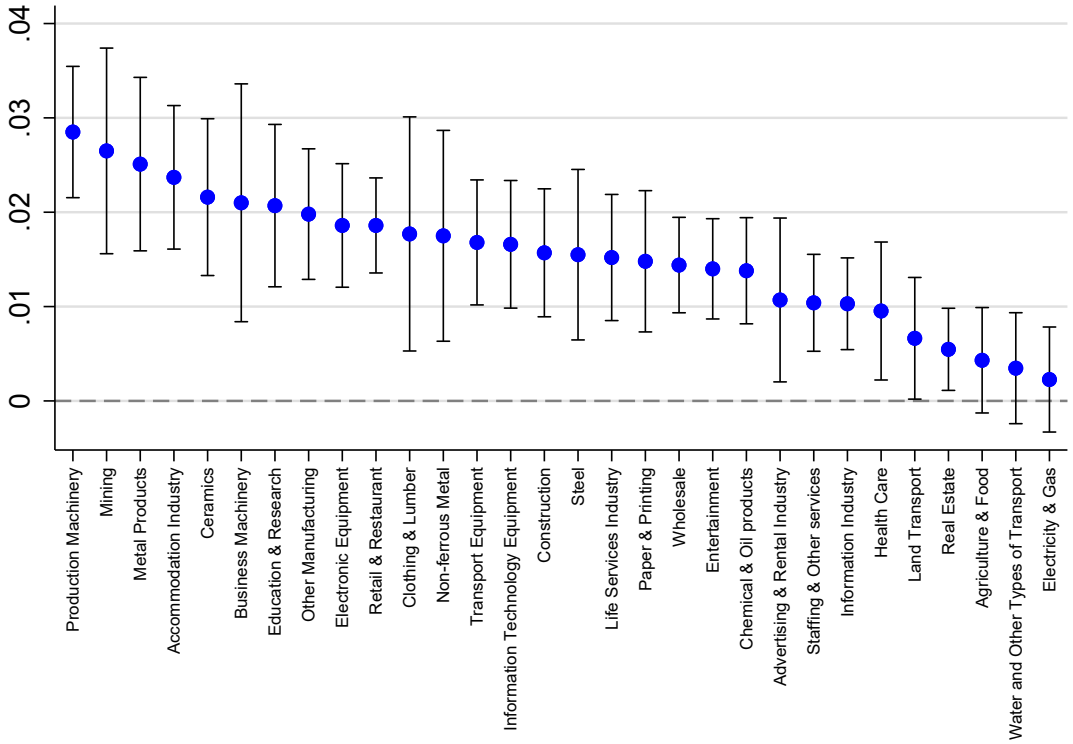
Notes: This figure illustrates how the coefficient governs the impulse response of the log sales forecast error in period $t + 1$ to realized (log) sales shocks in period t. Each dot represents an industry estimate, with 95% confidence intervals, across all 30 industries. The top and bottom 1% of observations are trimmed to remove outliers. The data are semiannual, and productivity is measured in terms of labor productivity.

Figure 12: Industry-Specific Underreaction Coefficients:
Percentage Forecast Error to Sales Innovation



Notes: This figure illustrates how the coefficient governs the impulse response of the percentage sales forecast error in period $t+1$ to realized (log) sales shocks in period t. Each dot represents an industry estimate, with 95% confidence intervals, across all 30 industries. The top and bottom 1% of observations are trimmed to remove outliers. The data are semiannual, and productivity is measured using labor productivity.

Figure 13: Industry-Specific Underreaction Coefficients:
Log Forecast Error to Value-Added Innovation



Notes: This figure illustrates how the coefficient governs the impulse response of the log sales forecast error in period $t+1$ to realized (log) value-added shocks in period t. Each dot represents an industry estimate, with 95% confidence intervals, across all 30 industries. The top and bottom 1% of observations are trimmed to remove outliers. The data are semiannual, and productivity is measured in terms of labor productivity.

C.3 Regression Results Using Labor Productivity

We present the full regression results for Table 6 in this subsection of the appendix. Specifically, Table 8 reports the results under the random-walk specification of the (log) productivity process, while Table 9 reports the results under the AR(1) specification of the (log) productivity process.

Table 8: Incomplete Information and Investment Sensitivity: Labor Productivity

	(1)	(2)	(3)	(4)
	inv. inaction = 1			
$\xi_s \times w_{i,t}$	0.009*	0.010**	0.010**	0.011**
	(0.005)	(0.005)	(0.005)	(0.005)
$w_{i,t}$	-0.003	-0.036	0.003	-0.030
	(0.009)	(0.031)	(0.010)	(0.031)
$a_{i,t-1}$	-0.028**	-0.028**	-0.015	-0.015
	(0.012)	(0.012)	(0.012)	(0.012)
$m_{i,t}$	-0.005	-0.005	-0.006	-0.006
	(0.006)	(0.006)	(0.005)	(0.005)
$k_{i,t-1}$	0.077***	0.077***	0.082***	0.082***
	(0.009)	(0.009)	(0.008)	(0.008)
$cap\ share_s \times w_{i,t}$		0.048		0.048
		(0.043)		(0.043)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes
N	84656	84656	84656	84656
adj. R^2	0.446	0.446	0.451	0.451

Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are winsorized.

Table 9: Incomplete Information and Investment Sensitivity:
AR(1) Process of Labor Productivity

	(1)	(2)	(3)	(4)
		inaction= 1		
$\xi_s \times w_{i,t}$	0.010*	0.011**	0.009*	0.010**
	(0.005)	(0.005)	(0.005)	(0.005)
$w_{i,t}$	-0.005	-0.036	0.002	-0.029
	(0.009)	(0.032)	(0.009)	(0.032)
$a_{i,t-1}$	-0.029**	-0.029**	-0.016	-0.016
	(0.011)	(0.011)	(0.011)	(0.011)
$m_{i,t}$	-0.005	-0.005	-0.006	-0.006
	(0.005)	(0.005)	(0.005)	(0.005)
$k_{i,t-1}$	0.076***	0.076***	0.082***	0.082***
	(0.009)	(0.009)	(0.008)	(0.008)
$cap\ share_s \times w_{i,t}$		0.046		0.046
		(0.045)		(0.045)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes
N	84313	84313	84313	84313
adj. R^2	0.446	0.446	0.451	0.451

Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are winsorized. Labor productivity is assumed to follow an AR(1) process in this specification.

C.4 Robustness Checks Using Total Factor Productivity

So far, we have used labor productivity (revenue per worker) and its innovations. While transparent, this measure does not isolate the effects of other inputs, such as capital and intermediate goods usage, on firm-level productivity. Total factor productivity (TFP) is a commonly used alternative in the literature. In this subsection, we conduct robustness checks using TFP and find that all previously documented empirical results remain qualitatively unchanged.

We construct our TFP measure following the standard approach in the IO literature (e.g., Olley and Pakes (1996); Levinsohn and Petrin (2003)). Specifically, we use the Olley and Pakes (1996) method, employing investment as a proxy to invert for TFP conditional on capital stock. This approach is suitable for our analysis for two reasons. First, our sample consists of large firms, so the usual zero-investment issue is less of a concern. The investment inaction rate is 36%, substantially higher than the share of zero investment (15%) in our data, as inaction includes investment rates $\leq 1\%$. Second, the usual collinearity problem between intermediate goods usage and labor is mitigated because we use firm investment as the proxy.

We estimate a firm-level Cobb-Douglas production function with labor, capital, and intermediate goods as inputs. To address sample size constraints, we group firms into four broad sectors for sector-specific production function estimation: (1) light manufacturing (e.g., food and beverages, textiles, clothing, footwear, printing, lumber), (2) heavy manufacturing (e.g., construction, chemicals, metals, steel, oil-related products), (3) machinery (e.g., production machinery, business and electrical equipment, telecommunications, transportation equipment), and (4) services (e.g., real estate, finance, accommodation, catering, healthcare, transportation, entertainment, rental and leasing). The estimated production function coefficients are reported in Table 10.

Table 10: Estimated Production Function

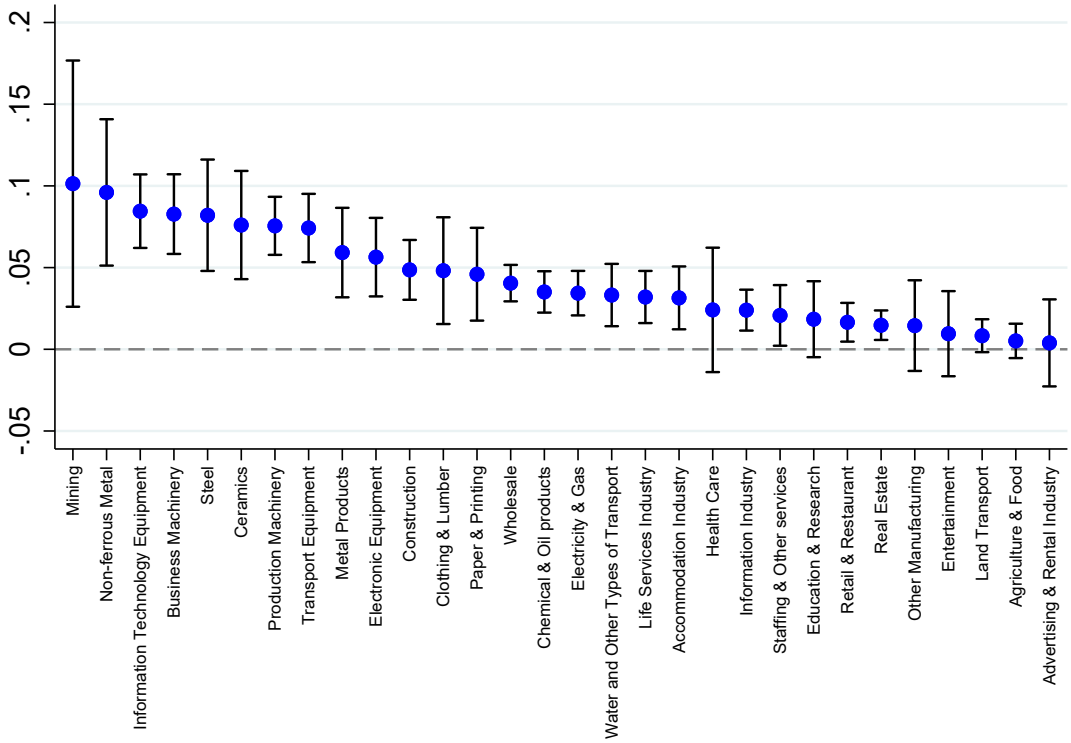
Sector	coef. of labor	coef. of intermediate goods	coef. of capital
Light Manufacturing	0.240	0.630	0.162
Heavy Manufacturing	0.212	0.578	0.207
Machinery	0.269	0.585	0.160
Service	0.231	0.554	0.163

Notes: The period is 2004-2018. All the estimated coefficients are highly significant.

As expected, the estimated production function shows higher capital intensity in the heavy manufacturing sector and higher labor intensity in the other three sectors. Across all sectors, technology exhibits near-constant returns to scale. We then re-estimate the industry-specific coefficient governing the impulse response of (log) sales forecast errors in period $t+1$ to realized (log) productivity innovations in period t for the 30 industries. The attenuation coefficients

estimated using labor productivity and TFP are positively correlated, with a correlation of 0.44.

Figure 14: Estimated Underreaction Coefficients Across Industries:
Using Estimated TFP



Notes: This figure shows how the coefficient governs the impulse response of the (log) sales forecast error made in period $t+1$ with respect to the realized (log) productivity innovation in period t. Each dot denotes the estimate for an industry (with the 95% confidence interval), and there are 30 industries in total. Top and bottom 1% observations are winsorized.

We rerun the main regressions as in equations (13) and (14) using firm-level TFP and the corresponding attenuation coefficients. Table 11 shows that results for equation (13) are largely unchanged from Table 5. Table 12 presents results for equation (14); although the coefficient of interest ($\xi_s \times w_{i,t}$) is slightly smaller, it remains positive and statistically significant, consistent with findings using labor productivity. Using an AR(1) process to estimate productivity innovations yields qualitatively similar results in Table 13. Overall, our empirical findings are robust to using TFP to construct the attenuation coefficients.

Table 11: Incomplete Information and Investment Inaction: Using Estimated TFP

	inv. inaction = 1			
ξ_s	-0.037	-0.029	-0.046*	-0.035*
	(0.025)	(0.019)	(0.026)	(0.019)
$a_{i,t}$	0.072	0.039	0.056	0.032
	(0.053)	(0.042)	(0.045)	(0.0356)
$k_{i,t-1}$	-0.040***	-0.034***	-0.037***	-0.033***
	(0.007)	(0.007)	(0.008)	(0.008)
$m_{i,t}$	0.042**	0.046**	0.033**	0.040***
	(0.019)	(0.017)	(0.016)	(0.014)
$growth\ vol_s$		1.044***		0.971***
		(0.275)		(0.285)
$cap\ share_s$			-0.384	-0.230
			(0.362)	(0.328)
Time FE	Yes	Yes	Yes	Yes
N	84941	84941	84941	84941
adj. R^2	0.057	0.083	0.064	0.086

Notes: Standard errors are clustered at the industry level. * 0.10 ** 0.05 *** 0.01. The degree of information friction is estimated at the industry level. Top and bottom 1% productivity observations are winsorized.

Table 12: Incomplete Information and Investment Sensitivity: Using Estimated TFP

	(1)	(2)	(3)	(4)
	inv. inaction = 1			
$\xi_s \times w_{i,t}$	0.008**	0.008**	0.007*	0.007**
	(0.004)	(0.004)	(0.004)	(0.004)
$w_{i,t}$	-0.107***	-0.112***	-0.109***	-0.123***
	(0.031)	(0.043)	(0.031)	(0.043)
$a_{i,t-1}$	-0.088***	-0.088***	-0.079***	-0.079***
	(0.013)	(0.013)	(0.013)	(0.013)
$m_{i,t}$	-0.003	-0.003	0.0003	0.0005
	(0.006)	(0.006)	(0.006)	(0.006)
$k_{i,t-1}$	0.068***	0.068***	0.076***	0.077***
	(0.009)	(0.009)	(0.009)	(0.009)
$vol_s \times w_{i,t}$	0.137***	0.138***	0.147***	0.153***
	(0.050)	(0.051)	(0.050)	(0.051)
$cap\ share_s \times w_{i,t}$		0.005		0.015
		(0.036)		(0.036)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes
N	80305	80305	80305	80305
adj. R^2	0.447	0.447	0.452	0.452

Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are winsorized.

Table 13: Incomplete Information and Investment Sensitivity: AR(1) Process of Estimated TFP

	(1)	(2)	(3)	(4)
	inv. inaction = 1			
$\xi_s \times w_{i,t}$	0.007	0.007	0.006	0.007
	(0.005)	(0.005)	(0.005)	(0.005)
$w_{i,t}$	-0.129***	-0.143**	-0.122***	-0.141**
	(0.042)	(0.058)	(0.042)	(0.058)
$a_{i,t-1}$	-0.073***	-0.073***	-0.066***	-0.066***
	(0.010)	(0.010)	(0.010)	(0.010)
$m_{i,t}$	-0.002	-0.002	0.001	0.001
	(0.006)	(0.006)	(0.006)	(0.006)
$k_{i,t-1}$	0.068***	0.068***	0.077***	0.077***
	(0.009)	(0.009)	(0.009)	(0.009)
$vol_s \times w_{i,t}$	0.175**	0.181***	0.173**	0.181***
	(0.069)	(0.069)	(0.068)	(0.069)
$cap\ share_s \times w_{i,t}$		0.015		0.020
		(0.049)		(0.048)
Firm FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes
N	80301	80301	80301	80301
adj. R^2	0.447	0.447	0.452	0.452

Notes: Standard errors are clustered at the firm level. Significance levels: * 0.10 ** 0.05 *** 0.01. Top and bottom 1% productivity observations are winsorized.

C.5 Correlations Between Forecast Errors and Firm Characteristics

Table 14: Unconditional Correlation between
Firm Characteristics and Firm-level Average Forecast Errors

Firm Characteristics	FE^{log}_{firm}	FE^{pct}_{firm}
Capital	0.0139	0.00641
Employment	0.00582	-0.00765
Capital/Sales	0.00108	-0.000219
Short-term Debt/Total Debt	0.0175	0.0260

Notes: Significance levels: * 0.05, ** 0.01, *** 0.001. All variables are firm-level averages over the relevant periods. The top and bottom 1% of each variable are winsorized.