The Term Structure of Monetary Policy News*

Jonathan J. Adams† — University of Florida

Philip Barrett‡ — International Monetary Fund

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Abstract

Empirical monetary policy shocks (MPS) contain information about monetary policy both today and in future. We define the *term structure of monetary policy news* as the marginal impact of a MPS on the policy residual at each horizon. Policy news at different horizons has different effects, so knowing the term structure is necessary in order to use a MPS to evaluate theory. We develop an IV method to estimate this term structure. We find that most MPS in the literature convey more information about policy in future than in the present, but there is substantial heterogeneity. The estimated term structures can be used to construct synthetic MPS approximating any desired term structure. We construct synthetic forward guidance and surprise shocks, and estimate their macroeconomic effects. Monetary surprises are contractionary and deflationary, whereas news about policy hikes 3-9 months ahead increases activity and inflation. At longer horizons, the effects die out.

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†email: adamsjonathanj@gmail.com

‡email: pbarrett@imf.org

1 Introduction

How do central bank decisions affect the economy? Empirical answers to this question require cleanly identified monetary policy shocks (MPS). Reliable identification using high frequency data (Gürkaynak et al., 2005) or narrative methods (Romer and Romer, 2004) have yielded an array of high quality estimated MPS in recent years. Monetary theory gives sharp predictions about the effects of these shocks, but only if we know precisely what they represent.

So what do these empirical MPS represent? The premise of this paper is that many of these approaches correctly capture the nature of the shock – i.e. an exogenous perturbation to interest rate policy – but they may vary in their information about policy *timing*. Different MPS can have different weights on policy surprises versus policy news across many future horizons.[1] This fact poses a challenge when trying to confront theory with data, since models imply that shocks with news at different horizons should have different effects. Does the response to a given MPS tell use something about the macroeconomic effects of policy? Or just how news and surprise are combined in that particular shock?

Our first contribution is to resolve these questions by developing a method to estimate the *term structure of monetary policy news*. This decomposes an empirical MPS into news about monetary policy residuals at every horizon. The procedure utilizes plausibly exogenous macroeconomic shocks as instrumental variables in order to identify the monetary policy rule, following insights from Barnichon and Mesters (2020). The monetary policy residual is calculated from the estimated rule, and whitened to find the monetary policy innovations. Finally, the innovations are regressed on lags of the MPS to identify the term structure. The resulting estimator has a closed-form expression; we prove that it is unbiased, and derive asymptotic standard errors.

We demonstrate our method by estimating the term structure for several well-known narrative and high-frequency MPS. We find that empirical MPS are mostly driven by news, capturing forward guidance, rather than immediate policy surprises. However there is substantial heterogeneity across methods. For example, the modern

[1]This conundrum is well-known. Their creators emphasize that MPS are not textbook surprises; instead, they "include shocks to forward guidance" Gertler and Karadi (2015). Swanson (2021) describes the challenge thus: "identifying the effects of forward guidance and LSAP [large scale asset prices] is difficult, because many of the FOMC's announcements provide information about both types of policies simultaneously".

narrative-based MPS constructed by Aruoba and Drechsel (2024) is the closest to a true policy surprise, while the high-frequency-identified shocks constructed by Swanson (2023) are almost entirely driven by news about future interest rate decisions.

The method allows for a valuable application: it is possible to construct a *synthetic* monetary policy shock with a desired term structure. Once we have estimated the term structure for a variety of MPS, we show that a linear combination of MPS has a linear combination of the component term structures. As a result, we can construct a synthetic MPS that closely approximates a true policy surprise, news about a particular horizon, or any other pattern of forward guidance.

The synthetic monetary policy surprise is directly comparable to a textbook monetary policy shock. We estimate the effects of the synthetic surprise on the macroeconomy and find that it roughly resembles the textbook effects: after a sudden interest rate increase, prices fall quickly and the economy contracts. However, when we test the effects of synthetic news shocks, we find more surprising results. For example, a 6-month-ahead policy shock is immediately inflationary, and prices rise until the interest rate hike is realized.

We contribute the literature working to separately estimate the effects of forward guidance (news) versus policy surprises. Gürkaynak et al. (2005) decomposes high frequency MPS into a target factor that moves the current rate, and a path factor that only moves expected future rates. Swanson (2021) studies how monetary policy announcements affect multiple asset prices, and decomposes the effects into three shocks, based on how they move yields at different horizons. For example, we confirm in Section 4 that Swanson's "forward guidance" shock has a much longer news term structure than the short term "target rate" shock. Campbell et al. (2012) estimate a simple Taylor rule, and use forecasts to decompose the residual into components revealed when the rate is set versus in prior quarters. Hansen and McMahon (2016) use textual analysis to identify components of Fed announcements corresponding to current policy, views about the economy, and forward guidance. Many further papers apply these types of strategies to other settings.

The remainder of the paper is organized as follows. Section 2 contains a motivating example to demonstrate why knowing the term structure of a MPS is necessary to draw conclusions. Already motivated readers can skip to Section 3, which describes our method in detail. In Section 4 we apply it to estimate the term structures for many MPS. Section 5 describes and applies the process for constructing synthetic

MPS. Section 6 concludes.

2 A Motivating Example

Our motivation is clearly demonstrated with a concrete example. In this section, we show how the estimated IRF to a MPS can be rationalized by *some* term structure of monetary policy news, even if the it does not resemble the IRF to a true policy surprise.

The textbook New Keynesian model is given by

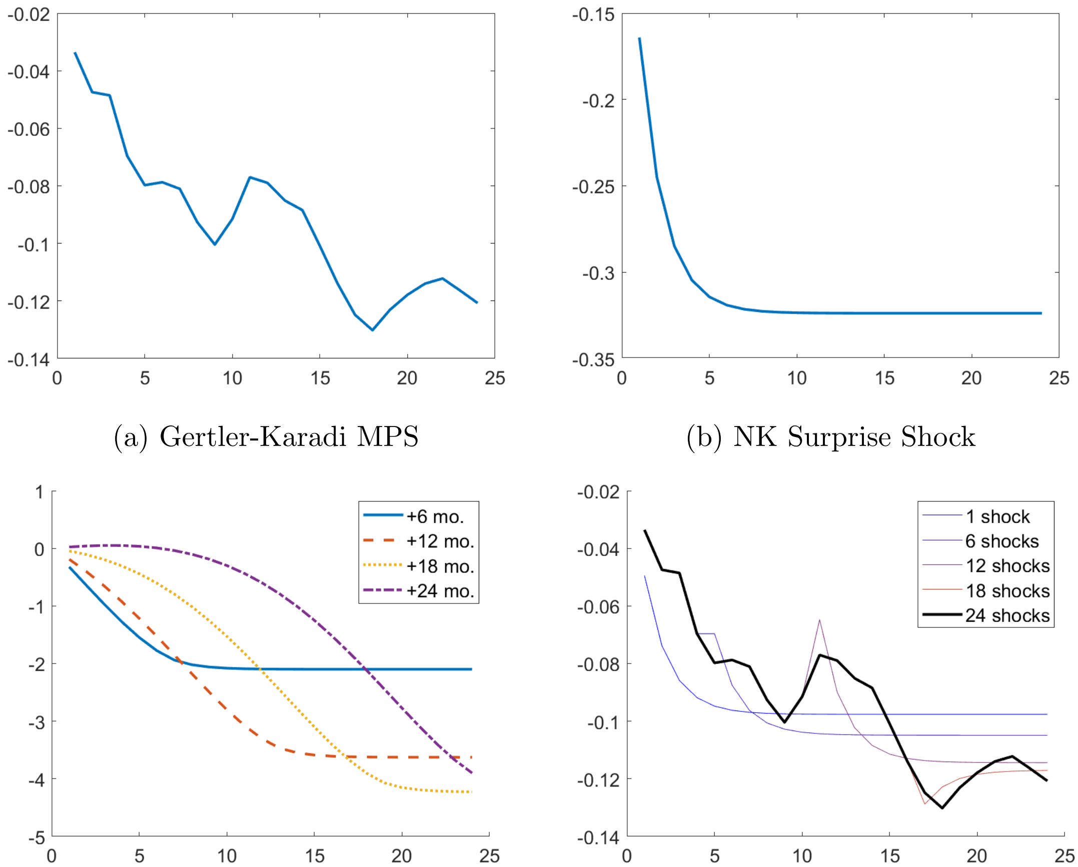
$$\begin{aligned} &\text{New Keynesian Phillips curve:} && \pi_t = \beta \mathbb{E}[\pi_{t+1}] + \kappa y_t \\ &\text{Euler equation:} && i_t = \mathbb{E}_t[\gamma(y_{t+1} - y_t) + \pi_{t+1}] \\ &\text{Taylor rule:} && i_t = \phi_y y_t + \phi_\pi \pi_t + \nu_t \end{aligned}$$

where π_t is inflation, y_t is the output gap, and i_t is the nominal interest rate. ν_t is exogenous and white noise. However, we introduce news to this model: ν_t is partially anticipated, given by

$$\nu_t = \nu_{0,t} + \nu_{1,t-1} + \nu_{2,t-2} + ...$$

where the component $\nu_{h,t-h}$ is learned at time $t-h$. The $\nu_{h,t}$ components are i.i.d. over time and independent of one another. $\nu_{h,t}$ represents a news shock at time t about monetary policy h periods into the future.

Figure 1 compares the price level IRFs from the New Keynesian model to that of a high frequency MPS. The effect of the Gertler and Karadi (2015) MPS on prices is plotted in Panel 1a. The shock causes a gradual deflation over 18 months. In contrast, Panel 1b plots the standard New Keynesian monetary policy surprise $\nu_{0,t}$. As usual, there is an immediate deflation, then prices rapidly stabilize.



(a) Gertler-Karadi MPS

(b) NK Surprise Shock

(c) NK News Shocks

(d) MPS Approximations

Figure 1: CPI Responses to Monetary Shocks

The MPS IRF is directly from Gertler and Karadi (2015). IRFs to surprise and news shocks are calculated from a standard calibration (Galí, 2008) of the textbook New Keynesian model. The MPS Approximation IRFs use the first n news shocks to find the linear combination that most closely matches (in terms of least squares) the MPS IRF.

But a surprise is not the only kind of monetary policy shock. A news shock $\nu_{h,t}$ has a different effect on prices for every horizon h: an anticipated future tightening causes a smooth deflation. Panel 1c demonstrates, plotting the deflationary effects of news at several semi-year horizons. Each looks different from a surprise shock, and different from one another. Indeed, they are linearly independent.

The IRF of the Gertler-Karadi shock is perfectly consistent with the New Keynesian model for *some* term structure. In other words, there is some linear combination of surprise and news that exactly replicates the empirical IRF. Panel 1d demonstrates, by approximating the Gertler-Karadi IRF as linear combinations of the first n news horizons. As n increases, the IRF is approximated more accurately. When 24 shocks are used, the Panel 1a is reproduced perfectly.

In this example, it is meaningless to use an empirical MPS to evaluate a model without knowing the news term structure associated with that MPS. Without further analysis, we cannot know to what extent impulse responses are informative about how true policy shocks transmit versus how they are combined in the individual MPS. The reverse is also true: MPS cannot be described as exhibiting a price "puzzle" with respect to some model on the basis of the IRF alone. Why? Because there is some combination of news shocks that can justify any inflation response. In order to evaluate a model based on empirical MPS, it is necessary to estimate its term structure of monetary policy news.

3 Methodology

This section describes the methodology used to estimate the term structure of monetary policy news. We outline the monetary policy framework, the estimation strategy, and the theoretical properties of the estimator.

3.1 Monetary Policy Framework

We model monetary policy as being determined by a Taylor-type rule:

$$y_t = x_t \phi + r_t \tag{1}$$

where y_t is the policy instrument (typically a short-term rate), r_t is the exogenous monetary policy residual (MPR), x_t is a vector of endogenous inputs to the policy rule, and ϕ is a vector of coefficients. Residuals r_t may be autocorrelated:

$$r_t = \sum_{\ell=1}^{L} \rho_{r,\ell} r_{t-\ell} + \nu_t \tag{2}$$

The monetary policy innovation (MPI) ν_t is white noise, but not necessarily unforecastable. We write the residual ν_t as a sum of news shocks at H_ν horizons:

$$\nu_t = \nu_{0,t} + \nu_{1,t-1} + \nu_{2,t-2} + ... + \nu_{H_\nu,t-H_\nu} \tag{3}$$

$\nu_{0,t}$ represents the surprise at time t, while $\nu_{h,t-h}$ represents the news component known at time $t-h$. This captures the idea that there may be information today about how policymakers intend to depart from their usual behavior in future. The news shocks are iid Gaussian, distributed $\nu_{h,t} \sim N(0, \sigma_h^2)$.[2]

We model MPSs as containing some (but not all) information about news shocks at multiple horizons. There may be many types of MPS, indexed by $j \in \mathcal{J}$. Each MPS w_t^j contains information about potentially many future residuals, as well as Gaussian error ξ_t:

$$w_t^j = \sum_{h=0}^{H_w} \beta_h^j \nu_{h,t} + \xi_t^j \tag{4}$$

where ξ_t is orthogonal to the MPI ν_{t+h} for all h. ξ_t could be measurement error, but it can also represent a central bank information effect. Equation (4) represents the data-generating process for a MPS. How does it relate to the term structure?

The *term stucture of MPS j* is the effect of the MPS w_t^j on expectations of the MPI over many horizons:

$$\gamma_h^j \equiv \frac{d\mathbb{E}[\nu_{t+h}|w_t^j]}{dw_t^j}$$

given the linear DGP in equation (4), the term structure can also be written as a linear relationship between MPS u_t^j and the MPI ν_t:

$$\nu_t = \sum_{h=0}^{H_w} \gamma_h^j w_{t-h}^j + u_t \tag{5}$$

where u_t is a residual. The β_h^j coefficients from equation (4) and γ_h^j coefficients are related by

$$\gamma_h^j = \beta_h^j \frac{Var(\nu_{h,t})}{Var(w_t^j)} \tag{6}$$

Equation (5) encodes the term structure, but cannot be directly estimated since the MPS u_t^j are data, but the MPI ν_t are not. The next section describes how to estimate the term structure using instrumental variables.

[2]We assume Gaussianity so that we can write linear projections as expectations. This assumption is not necessary for our results; without it, the OLS implementation would be unchanged.

3.2 Estimation Strategy

Estimating the γ_h^j coefficients from equation (5) faces several challenges: ν_t is unobserved, it is unknown how it relates to the MPR r_t, and the MPR is not orthogonal to the endogenous variables x_t. To resolve these challenges, our estimation takes a 4-stage approach:

1. Instrument for the endogenous variables x_t in the policy rule, using exogenous macroeconomic shocks z_t that are orthogonal to both u_t and the monetary policy shocks w_t^j.

2. Use the instrumented variables to estimate the policy rule coefficients $\hat{\phi}$ from equation (1). This is standard 2SLS estimation.

3. Calculate the implied residuals $\hat{r}_t$ using the estimated policy rule:

$$\hat{r}_t = y_t - x_t\hat{\phi} \tag{7}$$

   then whiten to find the estimated $\hat{\nu}_t$ innovations. In this step, we can project the residual $\hat{r}_t$ onto lagged values of i_t and x_t:[3]

$$\hat{r}_t = \sum_{\ell=1}^{L} i_{t-\ell}\varrho_{i,\ell} + x_{t-\ell}\varrho_{x,\ell} + \nu_t \tag{8}$$

4. Use the estimated $\hat{\nu}_t$ innovations to estimate the term structure of MPS γ_h^j from equation (5).

The 4-stage approach for estimating the γ_h^j coefficients is convenient because it is linear, and there is a closed form expression for the estimator. Proposition 1 gives the expression using the following notation. We stack lags of observables in the vector $\mathbf{x}_t \equiv \begin{pmatrix} y_{t-1} & x_{t-1} & \dots & y_{t-L} & x_{t-L} \end{pmatrix}$ which includes L lags of y and x. This allows us to write the whitening regression (8) as

$$\hat{R}_t = \mathbf{X}_t\varrho + \nu_t \tag{9}$$

[3]This is preferable to regressing on lags of $\hat{r}_t$ which include estimation error, and also do not give a nice closed form solution for the standard errors.

Simiarly, we stack lags of MPS in the vector $\mathbf{w}_t \equiv \begin{pmatrix} w_t^j & w_{t-1}^j & \dots & w_{t-H_w}^j \end{pmatrix}$ which allows us to write the fourth regression as

$$\hat{\nu}_t = \mathbf{w}_t \gamma + u_t \tag{10}$$

where we have suppressed the j superscript for readability. X, Z, and W are matrices of the endogenous variables, instruments, and MPSs, respectively. Each row corresponds to a time t observation. y and u are vectors of policy observations and equation (5) residuals, respectively. $\mathbf{X}$ denotes the matrix of $\mathbf{x}_t$ observations, and we write the residual projection matrix as $M_{\mathbf{X}} \equiv I - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. Lastly, $P_Z \equiv Z(Z'Z)^{-1}Z'$ denotes the matrix projecting onto the instruments.

Proposition 1 *The 4-stage estimator $\hat{\gamma}$ is given by*

$$\hat{\gamma} = (W'W)^{-1}W'M_{\mathbf{X}}(I - X(X'P_ZX)^{-1}X'P_Z)y$$

Proof: Appendix A

The $\hat{\gamma}$ coefficient vector can be estimated by four independent OLS regressions or in one step, following Proposition 1. Then the β_h^j coefficients can be calculated from the γ_h^j coefficients using equation (6).

The closed form expression is also useful because it allows for easy derivation of the estimation properties of our method.

3.3 Theoretical Properties

We prove that if the macroeconomic shocks are valid instruments, then the 4-stage estimation approach is unbiased. The key conditions are that the instruments are orthogonal to all terms on the right-hand side of equation (5): the w_t^j MPS and the u_t residuals. The first condition is easy to satisfied: z_t can always be orthogonalized with respect to the observed MPS. The second condition is theoretical: the macroeconomic shocks must not contain any information about the monetary policy residual. This is the typical exclusion restriction, and requires whichever shocks used as instruments to have been carefully identified.

Proposition 2 *If $Z'W = 0$ and $\mathbb{E}[Z'u] = 0$, then the 4-stage estimator is unbiased.*

Proof: Appendix A

The 4-stage estimator also has closed form standard errors. Proposition 3 gives the conditional variance of the estimator, if the same orthogonality assumptions hold for the instruments.

Proposition 3 *If $Z'W = 0$ and $\mathbb{E}[Z'u] = 0$, then the conditional variance of the $\hat{\gamma}$ estimator is*

$$Var\left(\hat{\gamma}|W, X, Z\right) =$$
$$(W'W)^{-1}W'M_{\mathbf{X}}\left(I - X(X'P_ZX)^{-1}X'P_Z\right)\Omega\left(I - X(X'P_ZX)^{-1}X'P_Z\right)'M_{\mathbf{X}}W(W'W)^{-1}$$

where $\Omega = \mathbb{E}[uu']$.

Proof: Appendix A

And an immediate corollary is

Corollary 1 *If $\hat{\Omega}$ is a consistent estimator of $\mathbb{E}[uu']$, and the Proposition 3 assumptions hold, then a consistent estimator of the conditional variance is*

$$Var\left(\hat{\gamma}|W, X, Z, \hat{\Omega}\right) =$$
$$(W'W)^{-1}W'M_{\mathbf{X}}\left(I - X(X'P_ZX)^{-1}X'P_Z\right)\hat{\Omega}\left(I - X(X'P_ZX)^{-1}X'P_Z\right)'M_{\mathbf{X}}W(W'W)^{-1}$$

To actually calculate the standard errors, an estimate of Ω is needed as usual. Because Proposition 1 ensures that γ is estimated consistently, this can be obtained using the estimated residuals $\hat{u}_t$ from equation (10), and then calculating the sample covariance matrix of the residuals with appropriate restrictions.

4 Estimated Term Structures

In this section, we estimate the term structures of popular MPS using our proposed methodology. We first describe the data used for the estimation, including the different MPS series and the macroeconomic instruments. Then we present the estimation results, highlighting the heterogeneity in the term structures of different MPS. Finally, we provide a summary statistic to represent the relative importance of news for each MPS and discuss the implications of our findings.

4.1 Data

Our method requires two types of data: monetary policy shocks from the literature, and other macroeconomic instruments and series used to estimate the policy rule.

4.1.1 Monetary Policy Shock Data

We estimate the term structure of monetary policy news for a variety of well-known MPS. They are summarized in Table 1.

Shock Source	Method	Notes	Range
Gertler and Karadi (2015)	HFI	30 min. window around FOMC decisions	1990:M1-2007:M12
Jarociński and Karadi (2020)	HFI	2 shocks: pure monetary and Fed information	1990:M1-2016:M12
Miranda-Agrippino and Ricco (2021)	HFI	Orthogonalized w.r.t. Greenbook forecasts	1991:M1-2009:M12
Bauer and Swanson (2023)	HFI	Includes Fed minutes and speeches	1988:M2-2023:M12
Swanson (2023)	HFI	Decomposed into 3 types of MPS	1988:M2-2023:M12
Romer and Romer (2004),	Narrative	Orthogonalized w.r.t. Greenbook forecasts	1983:M1-2007:M12
Aruoba and Drechsel (2024)	Narrative	Natural language processing of Fed docs	1982:M10-2008:M10

Table 1: Monetary Policy Shocks

Many shock series rely on intra-day data for identification, constructing instruments based on high-frequency changes in asset prices around FOMC announcements as a measure of monetary policy surprises. A classic example, Gertler and Karadi (2015) use 3-month-ahead federal funds futures rates. This horizon covers multiple FOMC meetings, and is interpreted as capturing both current rate decisions and forward guidance. Bauer and Swanson (2023) refines standard high-frequency methods by including additional policy events (e.g. speeches and press conferences) to the usual FOMC announcements to add observations, while also orthogonalizing with respect to high frequency data to ensure that the MPS series is unforecastable. Swanson (2023) applies these refinements to the Swanson (2021) methodology, which uses multiple asset prices to construct three distinct MPS (the “target rate”, “forward guidance” and “large-scale asset purchases” (LSAP)) that correspond roughly to effects at short, medium, and long-term yields.

One concern with high-frequency MPS is that it includes a “Fed information effect” (Romer and Romer, 2000; Nakamura and Steinsson, 2018) where the central bank reveals private information about the state of the economy, which is independent of its policy residuals. We include two MPS series that attempt to isolate the information effects from true policy shocks. Jarociński and Karadi (2020) measure high-frequency changes in interest rates and stock prices, and use sign-restrictions to isolate information from policy shocks, assuming that information moves rates and stock prices in the same direction, while policy has opposite effects. Miranda-Agrippino and Ricco (2021) identify a pure policy shock by orthogonalizing the MPS with respect to internal Fed forecasts.

We also use two shocks identified with narrative methods. The classic Romer and Romer (2004) shock (updated by Wieland and Yang (2020)) identifies policy actions motivated by the Fed's policy stance, rather than reactions to contemporaneous economic data, by orthogonalizing with respect to internal forecasts. In a modern refinement, Aruoba and Drechsel (2024) incorporate substantially more information, via natural language processing of internal Fed documents. Then they orthogonalize interest rate changes with respect to both forecasts and the text-based time series.

4.1.2 Data for Estimating the Monetary Policy Rule

In our baseline method, we specify the monetary policy rule (1) with the Effective Federal Funds rate as the policy variable, and with unemployment and CPI inflation on the right-hand side. The policy residual is allowed to be autocorrelated, so in effect interest rate decisions depend on lagged variables as well.

Shock Source	Method	Notes	Range
Government Spending Shocks			
Romer and Romer (2016)	Narrative	Social Security expansions	1951:M1-1991:M12
Fieldhouse et al. (2018)	Narrative	Government housing purchases	1952:M11-2014:M12
Oil Shocks			
Känzig (2021)	HFI	Oil supply news	1975:M1-2023:M6
Baumeister and Hamilton (2019)	SVAR	Oil supply, consumption/inventory demand	1975:M2-2024:M3
Other Shocks			
Kim et al. (2022)	External	ACI severe weather shocks	1964:M4-2019:M5
Adams and Barrett (2024)	SVAR	Shocks to inflation expectations	1979:M1-2024:M5

Table 2: Structural Shock Instruments

To address endogeneity concerns in estimating the Taylor rule, we employ instrumental variables (IVs) drawn from the literature. Over the last decade, the collection of well-identified macroeconomic shocks has expanded substantially. However, our options are limited because we require monthly series. Still, we were able to collect six monthly instruments that represent a diverse variety of shocks. They are summarized in Table 2.

Our first two instruments are related to government expenditures. We utilize the narrative measure of transfer payment shocks constructed by Romer and Romer (2016). This measure uses historical accounts of Social Security benefits to identify changes in transfer payments that are not a systematic response to macroeconomic conditions. To capture government spending shocks, we use the Fieldhouse et al. (2018) narrative instrument constructed from significant regulatory events impacting federal housing agency mortgage holdings. This series captures the ex ante impact of policy changes on the capacity of agencies to purchase mortgages. It focuses on non-cyclically motivated policy interventions by the federal government, excluding changes resulting from the agencies' regular response to market developments. These non-cyclically motivated policy shifts provide a source of exogenous variation in credit supply within the mortgage market.

Our next two instruments capture exogenous variations in the oil market. First, we use oil supply news shocks identified through high frequency changes in oil futures prices around OPEC production announcements (Känzig, 2021). Second, we employ structural oil shocks identified from a structural VAR by Baumeister and Hamilton (2019). This approach distinguishes contemporaneous shocks to oil supply and shocks to oil demand, and, unlike other methods, does not require that there is no short-run response of oil supply to the price.

We take severe weather shocks from the Actuaries Climate Index, a meteorological time series for severe weather. We take this series as exogenous, and use as shocks the statistical innovations calculated by Kim et al. (2022).

Finally, we use the Adams and Barrett (2024) inflation expectation shocks. This series is derived from a structural VAR that identifies exogenous shocks to inflation forecasts. To do so, the approach identifies the dimension of the VAR statistical innovation that causes survey forecasts to deviate from the rational expectation. In models where belief distortions are exogenous and stochastic, this method identifies the exogenous shock.

4.2 Estimation Results

In this section, we present the estimated terms structures of each MPS both numerically and graphically.

We start by reporting the estimated Taylor rule parameters from the second stage of our method, which are shown in Table 3. Generally, the coefficients on monthly inflation are close to satisfying the Taylor requirement, that $\phi_\pi > 1$, although the exact value varies with the activity. In our baseline results we use the Taylor rule in the first column, with unemployment as the activity indicator.

	(1)	(2)	(3)	(4)
CPI inflation	0.892	0.912	1.147	0.767
	(1.004)	(1.403)	(1.017)	(0.977)
Unemployment	-0.506			-0.488
	(0.501)			(0.425)
GDP, growth rate		0.105		
		(0.282)		
Industrial production, growth rate			0.015	
			(0.228)	
ρ	0.996	0.993	0.993	0.996
	(0.003)	(0.004)	(0.004)	(0.003)
Number of lags	6	6	6	3

Table 3: Estimated Taylor Rule Parameters

Table reports the estimated Taylor rule parameters from the second stage of the four-stage method using instrumental variables. Standard errors are reported in parentheses.

It is helpful to have a summary statistic to represent the relative importance of news for a given MPS. To do so, we use the R_k^2 statistic, which captures how much of the information in a MPS is due to news at horizon k.

Proposition 4 *The share of MPI variation R_k^2 that is explained by a MPS at horizon k news is*

$$R_k^2 \equiv \frac{Var(\nu_t | w_{t-k}^j)}{Var(\nu_t | \{w_{t-h}^j\}_{h=0}^{H_w})} = \frac{(\gamma_k^j)^2}{\sum_{h=0}^{H_w} (\gamma_h^j)^2}$$

Proof: Appendix A

Table 4 reports several of these statistics for each monetary policy shock. The first column is R_0^2, which is the share of the MPS that is due to an immediate change in the monetary policy innovation. There is substantial heterogeneity. The MPS that are most driven by surprises are the Gertler and Karadi (2015) HFI shock and the Aruoba and Drechsel (2024) narrative shock. But the values are still relatively low: roughly a third. Thus the majority of the variation in our MPS is driven by news.

Shock	R^2_0	$R^2_{1:3}$	$R^2_{4:11}$
Swanson (2023) HFI Forward Guidance	0.02	0.34	0.61
Swanson (2023) HFI Fed Funds	0.09	0.51	0.40
Jarociński and Karadi (2020) HFI pure monetary policy	0.21	0.45	0.34
Miranda-Agrippino and Ricco (2021) HRI pure monetary shock	0.11	0.39	0.47
Bauer and Swanson (2023) HFI MPS using Fed decisions and speeches	0.09	0.36	0.55
Gertler and Karadi (2015) HFI MPS, prepared by Ramey	0.27	0.39	0.33
Aruoba and Drechsel (2024) Narrative MPS, Natural Language Processing of Fed documents	0.24	0.50	0.25
Swanson (2023) HFI Large Scale Asset Purchases	0.09	0.58	0.32
Romer and Romer (2004) Narrative MPS, updated by Wieland and Yang (2020)	0.06	0.48	0.45

Table 4: Decomposition of term structure by horizon

Table reports the R^2_k measures in Proposition 4, summed over monthly horizons denoted in subscripts. For example, $R^2_{1:3}$ is the total variation in the Taylor residual attributable to 1- to 3-month news in a given identified monetary policy shock.

In contrast, the MPS that is least like a surprise is the Swanson (2023) forward guidance shock. Nearly all of its variation is due to news, which we break down into two components. The second column of Table 4 reports the sum of R^2_k for $k \in 1, 2, 3$. This is *short-run news*, which is realized in the quarter after the impact month. The remainder is reported in column 3 as *medium-run news*, which sums the R^2_k statistic for horizons 4 through 11.[4] The forward guidance shock is predominantly a medium-term news shock. Most MPS have a large share of their variation due to the medium-term horizons. The exception is the Aruoba-Drechsel shock, which is most dependent on the surprise and short-term news.

To get a clearer sense of how these shocks affect monetary policy, Figure 2 reports the IRFs of the MPS on the Taylor rule residuals. Recall that the residual r_t depends on the innovation ν_t by equation (2), and the innovation depends on MPS by equation (5).

The impulse response functions can tell a different story than the R^2_k statistics reported in Table 4. This is because the term structure described by the table captures how a MPS affects innovations at different horizons, but the transmission to actual policy residuals is transformed because of autocorrelation. For example, in both cases, the Aruoba-Drechsel and Gertler-Karadi MPS have large surprise effects. But

[4] We estimate 12 months of the term structure, so these three columns necessarily sum to 1, before rounding.

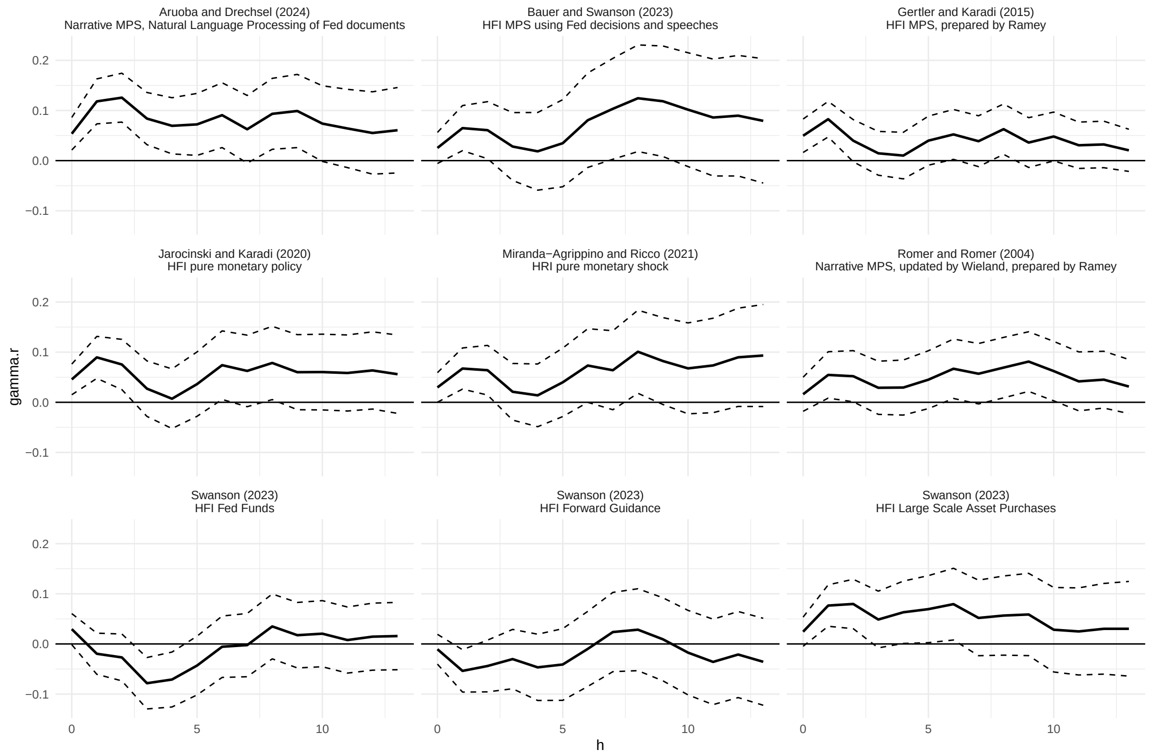


Figure 2: Estimated Term Structure: Taylor Rule Residuals

Figure shows the impact of each identified monetary policy shock h periods later on the Taylor rule residual $\hat{r}_{t+h}$.

the Swanson forward guidance shock does not have a large effect on medium-term residuals, even though its news is mostly at medium-term horizons. This is possible because the propagation of its short-term effect on news to medium-term residuals is mostly offset by its medium-term news, which can have the opposite sigh. Thus the MPS with the largest medium-term effect on residuals in Figure 2 is the Bauer-Swanson shock, even though it does not have the largest medium-term news share in Table 4.

5 Synthetic Monetary Policy Shocks

This section explains how to construct a synthetic monetary policy shock with a desired term structures, and then does so for several examples, including a synthetic surprise.

5.1 Method

The MPS that we estimate in the data have a variety of news term structures. Calculating these term structures is innately useful, because it allows us to interpret the shocks in standard DSGE models. However, we can also use the results from multiple MPSs to construct *synthetic* shocks with a new term structure. This allows us to study the effects of MPS of particular interest that are not directly estimated in the data.

Let $\vec{\gamma}^j$ denote the vector of *normalized* term structure coefficients for MPS j, estimated from Proposition 1, where the MPS has been normalized so that $Var(w_t^j) = 1$.

Proposition 5 *For a linear combination of MPS $w_t^c = \lambda_a w_t^a + \lambda_b w_t^b$, the resulting term structure of monetary policy news $\vec{\gamma}^c$ is proportional to the linear combination of term structures:*

$$\vec{\gamma}^c \propto \lambda_a \vec{\gamma}^a + \lambda_b \vec{\gamma}^b$$

Proof: Appendix A

Proposition 5 is useful because it allows us to construct a synthetic MPS with a desired term structure by finding the appropriate linear combination of existing MPS. The normalization involved in the term structure vector $\vec{\gamma}^j$ is inconsequential, because the scale of a MPS is typically arbitrary. The shape of the term structure is what matters. The proposition shows that linear combinations of MPSs can be used to construct a new MPS with a desired term structure shape.

The ability to construct a synthetic MPS was an arbitrary term structure is valuable. This property allows us to study specific types of monetary policy shocks that are relevant to theoretical models but not directly estimated in the data. For example, might be interested in studying the effects of a true monetary surprise, as in Figure 1b. But we learned in Section 4.2 that the empirical MPSs all feature news at multiple horizons. To estimate the effects of a surprise, we need to construct a synthetic MPS with a term structure $\vec{\gamma}^0 = \begin{pmatrix} 1 & 0 & 0 & ... \end{pmatrix}'$. Or, if we wanted to study a pure 1-period-ahead news shock, we would n construct a synthetic MPS with term structure $\vec{\gamma}^1 = \begin{pmatrix} 0 & 1 & 0 & ... \end{pmatrix}'$. Indeed, the term structure of any h-period-ahead news shock is simply the corresponding basis vector. Proposition 6 states when this is feasible.

Proposition 6 *MPS with normalized term structures in the set* $\mathcal{J} = \{\vec{\gamma}^j\}$ *can be used to construct any synthetic MPS s with term structure*

$$\vec{\gamma}^s \in span\left(\{\vec{\gamma}^j\}_{j\in\mathcal{J}}\right)$$

This property follows directly from Proposition 5. An immediate corollary is:

Corollary 2 *If* $\mathcal{J}$ *contains* $H_w + 1$ *MPS with linearly independent term structures, then a synthetic MPS can be constructed with any term structure of horizon length up to* H_w*.*

In practice, the number of linearly independent MPS may be less than the IRF horizon $H_w + 1$. In this case, the span of the term structures is a lower-dimensional vector space. The synthetic MPS can be constructed with any term structure in that space. If the term structure of interest (e.g. $\vec{\gamma}^0$) is not in the space, it must be approximated. The following Proposition explains how to do so.

Proposition 7 *Let* $\Gamma_{\mathcal{J}}$ *denote the matrix of normalized term structures for the linearly independent set* $\mathcal{J}$ *of observed MPS, and let* $\vec{\gamma}^i$ *denote the term structure of interest. The term structure of the synthetic MPS* $\vec{\gamma}^s$ *that is closest to* $\vec{\gamma}^i$ *(in the Euclidean norm) is given by*

$$\vec{\gamma}^s = \Gamma_{\mathcal{J}}(\Gamma'_{\mathcal{J}}\Gamma_{\mathcal{J}})^{-1}\Gamma'_{\mathcal{J}}\vec{\gamma}^i$$

Proof: Appendix A

5.2 Synthetic Surprise and News

To estimate synthetic MPS, we take two steps to improve parsimony. First, instead of using the four-stage linear estimates for the term structure vector $\vec{\gamma}^j$, we fit the estimated term structure to a polynomial. Second, many of the MPS are estimated in a similar way, and have relatively colinear term structures; Figure 3 presents their absolute correlations. Therefore, we selected the three most dissimilar[5] MPS to use for the synthetic MPS exercise: the Swanson forward guidance and Fed Fund shocks, and the Jarociński & Karadi shock.

[5] As measured by the average Euclidean distance to the other vectors $\vec{\gamma}^j$. This is the ordering of the variables in Figure 3, and is why the lower lines are the lightest colors

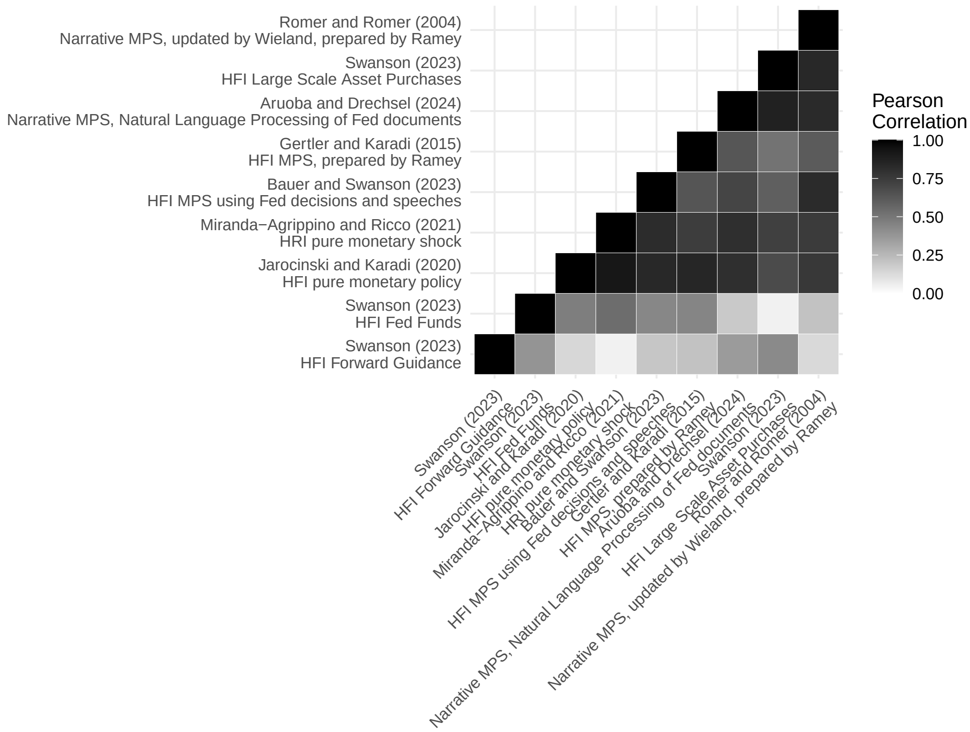


Figure 3: Absolute Term Structure Correlations

Figure shows absolute cross-correlations of the estimated term structures of candidate MPS, ordered from least to most dissimilar top to bottom.

Using the three shocks, we calculate synthetic MPS representing a pure surprise and pure news at each horizon. As described in Section 5.1, this implies that the target term structures are standard basis vectors. Table 5 presents the weights on the MPS components associated with each synthetic MPS, and the constructed series are plotted in Appendix B. With only three shocks, these synthetic MPS do not achieve the desired term structure exactly. Instead, they are the best approximation possible using the three components.

We estimate the effects of the synthetic MPS on the macroeconomy. To do so, we include each synthetic MPS in a VAR with log CPI, log real GDP, unemployment, and the effective federal funds rate (FFR).[6] The synthetic MPS is taken as exogenous, so we order it first in the VAR, and estimate IRFs by causal ordering.

Figure 4 presents the estimated IRFs. The first column corresponds to the syn-

[6]We estimate monthly GDP using a Kalman Filter which matches the quarterly NIPA data and monthly consumption series.

Horizon	0	1	2	3	6	9	12
Swanson (2023) HFI Forward Guidance	-0.27	0.04	0.12	0.15	0.12	-0.17	7.11
Swanson (2023) HFI Fed Funds	-0.23	0.15	0.28	0.35	0.52	0.88	-6.93
Jarociński and Karadi (2020) HFI pure monetary policy	1.50	0.81	0.60	0.50	0.36	0.29	0.82

Table 5: Synthetic Shock Weights

Table reports the weights on the MPS used to create the synthetic shocks.

thetic monetary policy surprise, i.e. the shock at $h = 0$. Other columns show the response to shocks at horizons $h > 0$. The surprise shock raises rates immediately, raises unemployment, and lowers prices with a brief delay. However, as the horizon of the news extends, the impact of the shock is much more stimulatory. Expected monetary tightening at horizons between about 3 and 9 months ahead lead to an increase in employment, output, and interest rates, and a mitigation (or in some cases reserving) of falling prices. However, at the longest horizons, the impact of the shock is generally reduced.

Figure 6 provides another cut of the impulse responses, showing the impact and 6-month impact by shock horizon. On impact, the overall impression is not always entirely obvious, but by the six-month mark the direction and significance of the response is clearer. In any case, these both show the same non-monotonic pattern, that short-horizon shocks are contractionary and deflationary, that at moderate horizons they are expansionary, and at very long horizons they have little effect. Figure 5 summarizes the impact on the Federal Fund Rate, plotting how the month of the peak response depends on the news horizon.

These results are not all that surprising, given the inputs to exercise: the Jarociński-Karadi shock closely approximates a true surprise, and has traditional contractionary effects. In contrast, the Swanson shocks have larger news components, and qualitatively different effects on the macroeconomy. In particular, the forward guidance shock is known to be inflationary and expansionary.

These effects are not entirely at odds with the New Keynesian model, despite the well-known forward guidance puzzle. To understand why, we return to the motivating example from Section 2. News shocks about policy far in the future can have very large effects as the horizon increases. But those effects are not necessarily large on when the news is revealed to agents (Figure 1c). To see if this is consistent with the evidence from the synthetic shocks, we compute plots from the textbook model that are analogous to our estimated results in Figure 6. These model analogs are

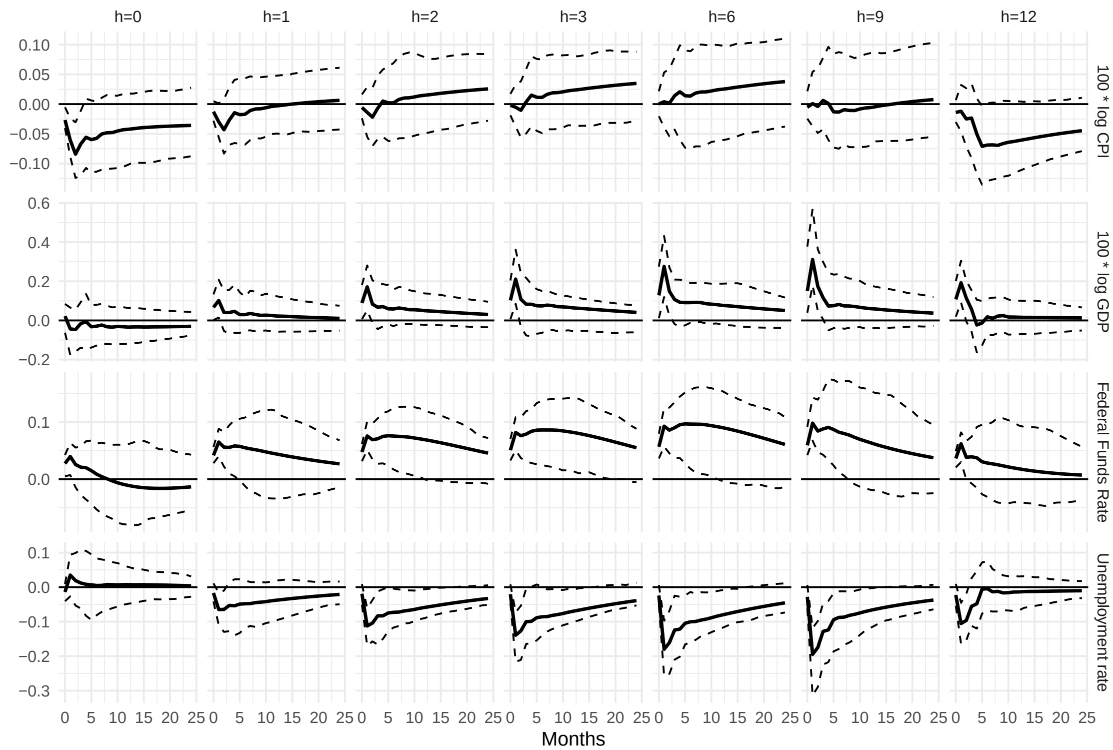


Figure 4: Impulse Responses to Synthetic Shocks

Figure shows impulse responses from a VAR to synthetic monetary policy shocks. Each column shows the response at a different news horizons $h = 0$ through $h = 12$. Lag length chosen by AIC.

displayed in Figure 7. The signs and magnitudes do not match the empirical results at all. But Figure 7 shows that the empirical estimates do not completely fail a theory-motivated sanity check. Specifically, it reveals that far-in-the-future forward guidance is consistent with quantitatively small immediate impacts, as in Panel 6a. It also shows that relationship between the 6-month impacts and the news horizon can be non-monotonic, as in Panel 6b.

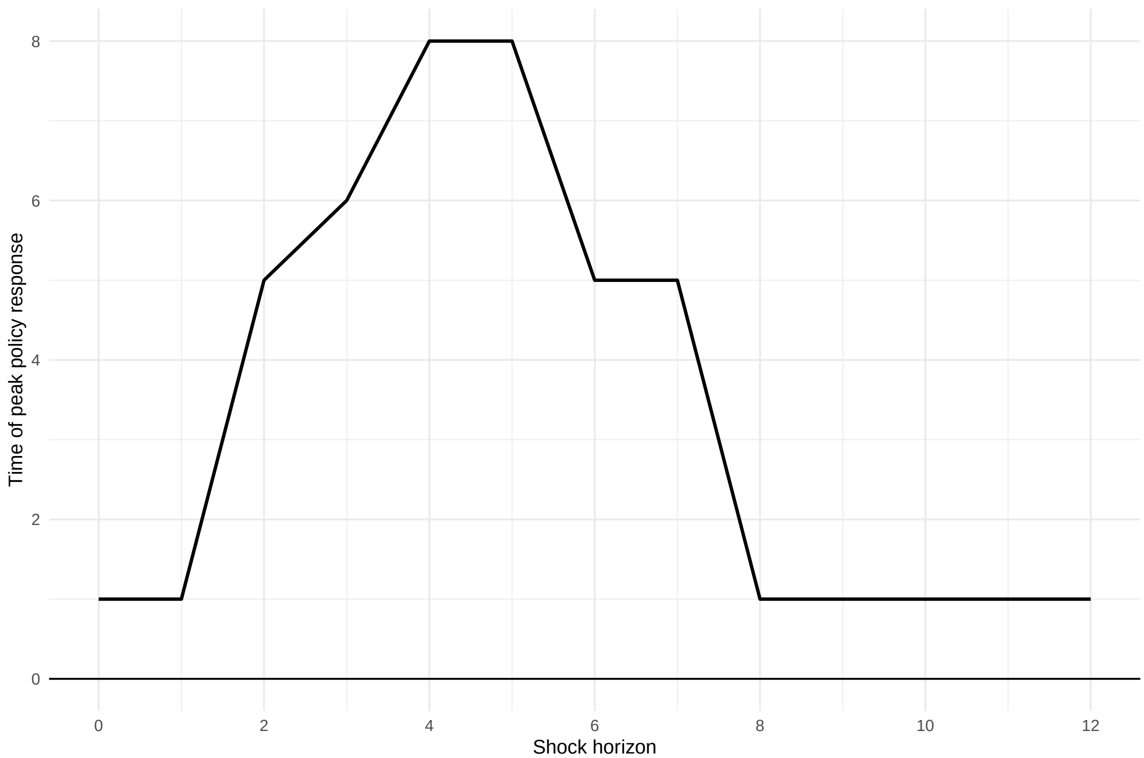
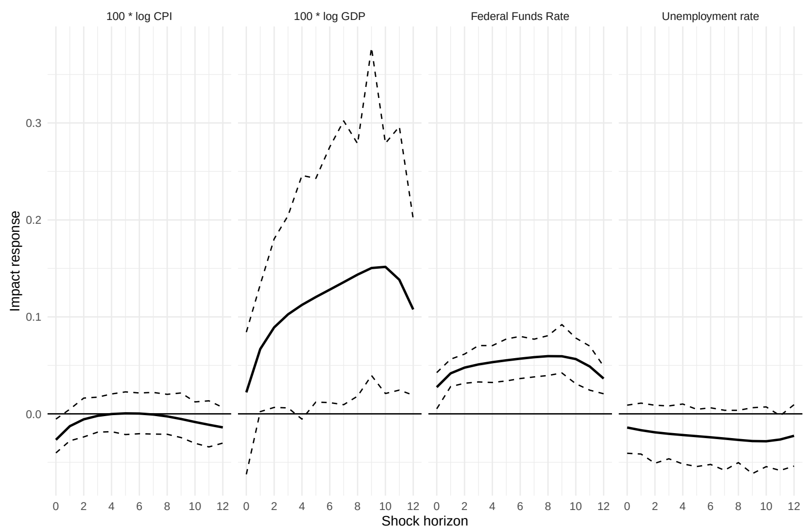
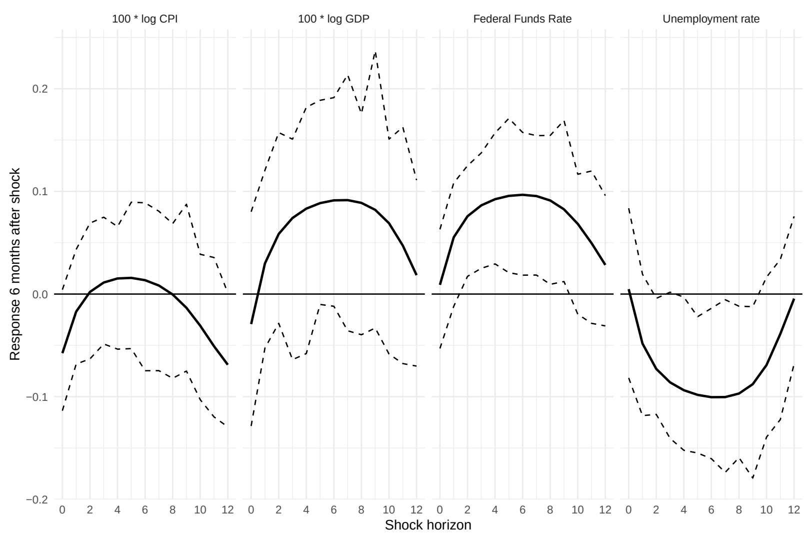


Figure 5: Horizon of Peak Fed Funds Response, by Shock Horizon



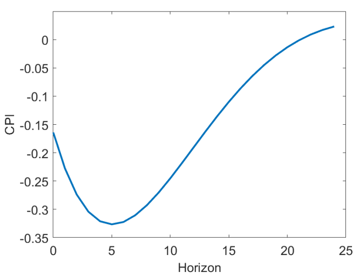
(a) Impact Effect



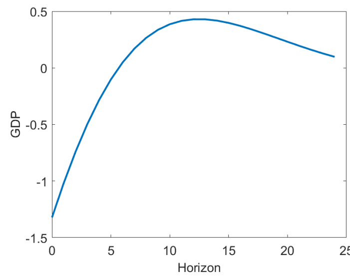
(b) 6-month Effect

Figure 6: Responses to Synthetic Shocks by Horizon

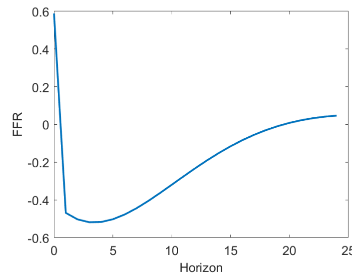
Figures show impact of different synthetic shocks on macroeconomic variables, on impact (top panel) and six months after the shock (bottom panel).



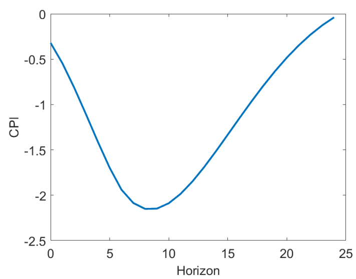
(a) Impact Effect: CPI



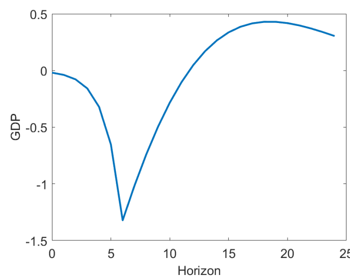
(b) Impact Effect: GDP



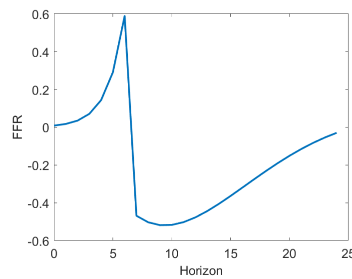
(c) Impact Effect: FFR



(d) 6-Month Effect: CPI



(e) 6-Month Effect: GDP



(f) 6-Month Effect: FFR

Figure 7: New Keynesian Responses to News Shocks by Horizon

Each panel plots how a news shock affects a macroeconomic variable on impact or after 6 months. In each case, the x-axis is the horizon of the news shock. These effects are calculated from a standard calibration (Galí, 2008) of the textbook New Keynesian model.

6 Conclusions

In this paper, we address three important questions about the identification of monetary policy shocks.

The first is: how should we compare the different estimated monetary policy shocks? The framework we develop in this paper is based on the idea that identified monetary policy shocks identify a common type of exogenous disturbance, but vary in its anticipated timing. By applying this method to identify the differences in nine well-known monetary shock series, we decompose each into its surprise and news components, the latter at multiple horizons. We find that most of these shocks have large news components.

Second, how can we map between empirical shocks and theory? By projecting fixed h-period ahead impulses onto imperfectly correlated empirical shocks, we can construct the responses to news shocks at multiple horizons as a linear combination of estimated impulse responses. In doing so, we are able to characterize the empirical responses of shocks which comport with the theory. We show that positive monetary surprises are contractionary and deflationary, but that news at longer horizons increases output, employment, and prices. At very long horizons the effect of monetary policy news is negligible.

Third, how can we make sense of these findings? As this is principally an empirical paper, we do not provide a full answer to this question. However, we do show that our results are not entirely at odds with standard macroeconomic theory. A standard New Keynesian framework can provide results which are at least qualitatively consistent with our findings.

These results suggest several directions for future research. Most obviously, they provide a framework for evaluating future monetary policy shocks, allowing them to be compared to those already in the literature. However, our specific findings also give some guidance on how empirical identification of MPSs might most valuably proceed. In particular, our results show that there is still much to be done to systematically capture monetary policy surprises distinct from news about the future. Beyond this, our findings on the effect of news shocks at multiple horizons set a target for future models to aim at.

References

Adams, Jonathan J. and Philip Barrett, "Shocks to inflation expectations," *Review of Economic Dynamics*, October 2024, *54*, 101234.

Aruoba, S. Borağan and Thomas Drechsel, "Identifying Monetary Policy Shocks: A Natural Language Approach," May 2024.

Barnichon, Regis and Geert Mesters, "Identifying Modern Macro Equations with Old Shocks*," *The Quarterly Journal of Economics*, November 2020, *135* (4), 2255–2298.

Bauer, Michael D. and Eric T. Swanson, "A Reassessment of Monetary Policy Surprises and High-Frequency Identification," *NBER Macroeconomics Annual*, May 2023, *37*, 87–155. Publisher: The University of Chicago Press.

Baumeister, Christiane and James D. Hamilton, "Structural Interpretation of Vector Autoregressions with Incomplete Identification: Revisiting the Role of Oil Supply and Demand Shocks," *American Economic Review*, May 2019, *109* (5), 1873–1910.

Campbell, Jeffrey R., Charles L. Evans, Jonas D. M. Fisher, Alejandro Justiniano, Charles W. Calomiris, and Michael Woodford, "Macroeconomic Effects of Federal Reserve Forward Guidance [with Comments and Discussion]," *Brookings Papers on Economic Activity*, 2012, pp. 1–80. Publisher: Brookings Institution Press.

Fieldhouse, Andrew J, Karel Mertens, and Morten O Ravn, "The Macroeconomic Effects of Government Asset Purchases: Evidence from Postwar U.S. Housing Credit Policy*," *The Quarterly Journal of Economics*, August 2018, *133* (3), 1503–1560.

Galí, Jordi, *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton University Press, 2008.

Gertler, Mark and Peter Karadi, "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*, January 2015, *7* (1), 44–76.

Gürkaynak, Refet S., Brian Sack, and Eric Swanson, "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models," *American Economic Review*, March 2005, *95* (1), 425–436.

Hansen, Stephen and Michael McMahon, "Shocking language: Understanding the macroeconomic effects of central bank communication," *Journal of International Economics*, March 2016, *99*, S114–S133.

Jarociński, Marek and Peter Karadi, "Deconstructing Monetary Policy Surprises—The Role of Information Shocks," *American Economic Journal: Macroeconomics*, April 2020, *12* (2), 1–43.

Kim, Hee Soo, Christian Matthes, and Toan Phan, "Severe weather and the macroeconomy," *Federal Reserve Bank of Richmond Working Papers, 21-14R*, 2022.

Känzig, Diego R., "The Macroeconomic Effects of Oil Supply News: Evidence from OPEC Announcements," *American Economic Review*, April 2021, *111* (4), 1092–1125.

Miranda-Agrippino, Silvia and Giovanni Ricco, "The Transmission of Monetary Policy Shocks," *American Economic Journal: Macroeconomics*, July 2021, *13* (3), 74–107.

Nakamura, Emi and Jón Steinsson, "High-Frequency Identification of Monetary Non-Neutrality: The Information Effect," *The Quarterly Journal of Economics*, August 2018, *133* (3), 1283–1330.

Romer, Christina D. and David H. Romer, "Federal Reserve Information and the Behavior of Interest Rates," *American Economic Review*, June 2000, *90* (3), 429–457.

_ **and** _ , "A New Measure of Monetary Shocks: Derivation and Implications," *American Economic Review*, September 2004, *94* (4), 1055–1084.

_ **and** _ , "Transfer Payments and the Macroeconomy: The Effects of Social Security Benefit Increases, 1952-1991," *American Economic Journal: Macroeconomics*, October 2016, *8* (4), 1–42.

Swanson, Eric T., "Measuring the effects of federal reserve forward guidance and asset purchases on financial markets," *Journal of Monetary Economics*, March 2021, *118*, 32–53.

_ , "The Macroeconomic Effects of the Federal Reserve's Conventional and Unconventional Monetary Policies," August 2023.

Wieland, Johannes F. and Mu-Jeung Yang, "Financial Dampening," *Journal of Money, Credit and Banking*, 2020, *52* (1), 79–113. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/jmcb.12681.

A Proofs

Proposition 1. The OLS estimator for the third stage regression (9) is

$$\hat{\varrho} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\hat{R} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'(y - X\hat{\phi})$$

so $\hat{\nu}$ is given by

$$\begin{aligned}\hat{\nu} &= \hat{R} - \mathbf{X}\hat{\varrho} \\ &= (y - X\hat{\phi}) - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'(y - X\hat{\phi}) = M_{\mathbf{X}}(y - X\hat{\phi})\end{aligned}$$

The OLS estimator for the fourth stage regression (**??**) is

$$\hat{\gamma} = (W'W)^{-1}W'\hat{\nu} = (W'W)^{-1}W'M_{\mathbf{X}}(y - X\hat{\phi})$$

Finally, the 2SLS estimator is $\hat{\beta} = (X'P_Z X)^{-1}X'P_Z y$, so $\hat{\gamma}$ can be written

$$\hat{\gamma} = (W'W)^{-1}W'M_{\mathbf{X}}(y - X(X'P_Z X)^{-1}X'P_Z y)$$

■

Proposition 2. The following expectations are conditional on the data:

$$\begin{aligned}\mathbb{E}[\hat{\gamma}] &= \mathbb{E}\left[(W'W)^{-1}W'M_{\mathbf{X}}(y - X\hat{\phi})\right] \\ &= \mathbb{E}\left[(W'W)^{-1}W'M_{\mathbf{X}}(X\phi + R - X\hat{\phi})\right] \\ &= \mathbb{E}\left[(W'W)^{-1}W'M_{\mathbf{X}}R\right] + \mathbb{E}\left[(W'W)^{-1}W'M_{\mathbf{X}}X(\phi - \hat{\phi})\right] \\ &= \mathbb{E}\left[(W'W)^{-1}W'\nu\right] + \mathbb{E}\left[(W'W)^{-1}W'M_{\mathbf{X}}X(\phi - \hat{\phi})\right] \\ &= \mathbb{E}\left[(W'W)^{-1}W'(W\gamma + u)\right] + \mathbb{E}\left[(W'W)^{-1}W'M_{\mathbf{X}}X(\phi - \hat{\phi})\right] \\ &= \gamma + \mathbb{E}\left[(W'W)^{-1}W'M_{\mathbf{X}}X(\phi - \hat{\phi})\right]\end{aligned}$$

which uses that W and u are orthogonal.

The 2SLS error $\phi - \hat{\phi}$, is given by

$$\phi - \hat{\phi} = \phi - (X'P_Z X)^{-1}X'P_Z y$$

$$= \phi - (X'P_ZX)^{-1}X'P_Z(X\phi + W\gamma + u)$$

$$= -(X'P_ZX)^{-1}X'P_Z(W\gamma + u) \tag{11}$$

Substitute this back in:

$$\mathbb{E}\left[\hat{\gamma}\right] = \gamma - \mathbb{E}\left[(W'W)^{-1}W'M_{\mathbf{X}}X(X'P_ZX)^{-1}X'P_Z(W\gamma + u)\right]$$

By assumption, Z is orthogonal to both W and u, so the equation becomes

$$\mathbb{E}\left[\hat{\gamma}\right] = \gamma$$

■

Proposition 3. The conditional variance of the estimator is

$$Var\left(\hat{\gamma}|W, X, Z\right) = Var\left(\hat{\gamma} - \gamma|W, X, Z\right)$$
$$= Var\left((W'W)^{-1}W'M_{\mathbf{X}}(y - X\hat{\beta}) - \gamma|W, X, Z\right)$$
$$= Var\left((W'W)^{-1}W'M_{\mathbf{X}}(X\beta + \gamma W + u - X\hat{\beta}) - \gamma|W, X, Z\right)$$
$$= Var\left((W'W)^{-1}W'M_{\mathbf{X}}(X(\beta - \hat{\beta}) + u)|W, X, Z\right)$$

u is not orthogonal to the IV error $\beta - \hat{\beta}$, which is given by equation (11). Substitute it in:

$$Var\left(\hat{\gamma}|W, X, Z\right) = Var\left((W'W)^{-1}W'M_{\mathbf{X}}(-X(X'P_ZX)^{-1}X'P_Z(W\gamma + u) + u)|W, X, Z\right)$$

$$= (W'W)^{-1}W'M_{\mathbf{X}}Var\left(\left(I - X(X'P_ZX)^{-1}X'P_Z\right)u - X(X'P_ZX)^{-1}X'P_ZW\gamma|W, X, Z\right)M_{\mathbf{X}}W(W'W)^{-1}$$

We can separate the interior term because u and W are orthogonal, i.e. $\mathbb{E}[W\gamma u'] = 0$:

$$Var\left(\left(I - X(X'P_ZX)^{-1}X'P_Z\right)u - X(X'P_ZX)^{-1}X'P_ZW\gamma|W, X, Z\right)$$
$$= Var\left(\left(I - X(X'P_ZX)^{-1}X'P_Z\right)u|W, X, Z\right) + Var\left(X(X'P_ZX)^{-1}X'P_ZW\gamma|W, X, Z\right)$$

The first term is given by

$$Var\left(\left(I - X(X'P_ZX)^{-1}X'P_Z\right)u|W, X, Z\right) = \left(I - X(X'P_ZX)^{-1}X'P_Z\right)\Omega\left(I - X(X'P_ZX)^{-1}X'P_Z\right)'$$

using $\Omega = \mathbb{E}[uu']$. And the second term is simply

$$Var\left(X(X'P_ZX)^{-1}X'P_ZW\gamma|W,X,Z\right) = 0$$

Accordingly, we can construct the entire variance matrix by

$$\begin{aligned}&Var\left(\hat{\gamma}|W,X,Z\right) = \\ &(W'W)^{-1}W'M_{\mathbf{X}}\left(I - X(X'P_ZX)^{-1}X'P_Z\right)\Omega\left(I - X(X'P_ZX)^{-1}X'P_Z\right)' M_{\mathbf{X}}W(W'W)^{-1}\end{aligned}$$

■

Proof of Proposition 5. By equation (4), the MPS w_t^c can be written as

$$\begin{aligned}w_t^c &= \lambda_a w_t^a + \lambda_b w_t^b \\ &= \lambda_a \sum_{h=0}^{H_{w^a}} \beta_h^a \nu_{h,t} + \lambda_a \xi_t^a + \lambda_b \sum_{h=0}^{H_{w^b}} \beta_h^b \nu_{h,t} + \lambda_b \xi_t^b \\ &= \sum_{h=0}^{H_{w^c}} \beta_h^c \nu_{h,t} + \xi_t^c\end{aligned}$$

where $\beta_h^c = \lambda_a\beta_h^a + \lambda_b\beta_h^b$, $H_{w^c} = \max\{H_{w^a}, H_{w^b}\}$ and $\xi_t^c = \lambda_a\xi_t^a + \lambda_b\xi_t^b$ is orthogonal to $\nu_{h,t}$ for all h. By equation (6), the term structure coefficients are given by

$$\begin{aligned}\gamma_h^c &= \left(\lambda_a\beta_h^a + \lambda_b\beta_h^b\right)\frac{Var(\nu_{h,t})}{Var(w_t^c)} \\ &= \lambda_a\gamma_h^a\frac{Var(w_t^a)}{Var(w_t^c)} + \lambda_b\gamma_h^b\frac{Var(w_t^b)}{Var(w_t^c)}\end{aligned}$$

When $Var(w_t^a)$ and $Var(w_t^b)$ are normalized to 1, the vector form of this equation is

$$\begin{aligned}\vec{\gamma}^c = \lambda_a\vec{\gamma}^a\frac{1}{Var(w_t^c)} + \lambda_b\vec{\gamma}^b\frac{1}{Var(w_t^c)} \\ \propto \lambda_a\vec{\gamma}^a + \lambda_b\vec{\gamma}^b\end{aligned}$$

■

Proof of Proposition 7. The synthetic MPS $\vec{\gamma}^s$ must be in the span of the observed MPS term structures, i.e. the columns of $\Gamma_{\mathcal{J}}$. The vector in this span minimizing $\||\vec{\gamma}^i - \vec{\gamma}\|_2$ is the projection of $\vec{\gamma}^i$ onto the span of the columns of $\Gamma_{\mathcal{J}}$. This is given by the familiar expression

$$\vec{\gamma}^s = \Gamma_{\mathcal{J}}(\Gamma'_{\mathcal{J}}\Gamma_{\mathcal{J}})^{-1}\Gamma'_{\mathcal{J}}\vec{\gamma}^i$$

■

Proof of Proposition 4. By equation (5), the MPI variance condition on w^j_{t-k} is

$$Var(\nu_t|w^j_{t-k}) = Var(\sum_{h=0}^{H_w}\gamma^j_h w^j_{t-h} + u_t|w^j_{t-k})$$

$$= Var(\gamma^j_h w^j_{t-k}|w^j_{t-k}) = (\gamma^j_k)^2 Var(w^j_t)$$

because the MPS is homoskedastic white noise, and orthogonal to u_t. Similarly, the total variance conditional on the history of MPS is

$$Var(\nu_t|\{w^j_{t-h}\}^{H_w}_{h=0}) = Var(\sum_{h=0}^{H_w}\gamma^j_h w^j_{t-h} + u_t|\{w^j_{t-k}\}^{H_w}_{k=0})$$

$$= \sum_{h=0}^{H_w} Var(\gamma^j_h w^j_{t-h}) = \left(\sum_{h=0}^{H_w}(\gamma^j_h)^2\right) Var(w^j_t)$$

Combining these two equations gives the ratio

$$\frac{Var(\nu_t|w^j_{t-k})}{Var(\nu_t|\{w^j_{t-h}\}^{H_w}_{h=0})} = \frac{(\gamma^j_k)^2}{\sum_{h=0}^{H_w}(\gamma^j_h)^2}$$

■

B Additional Plots

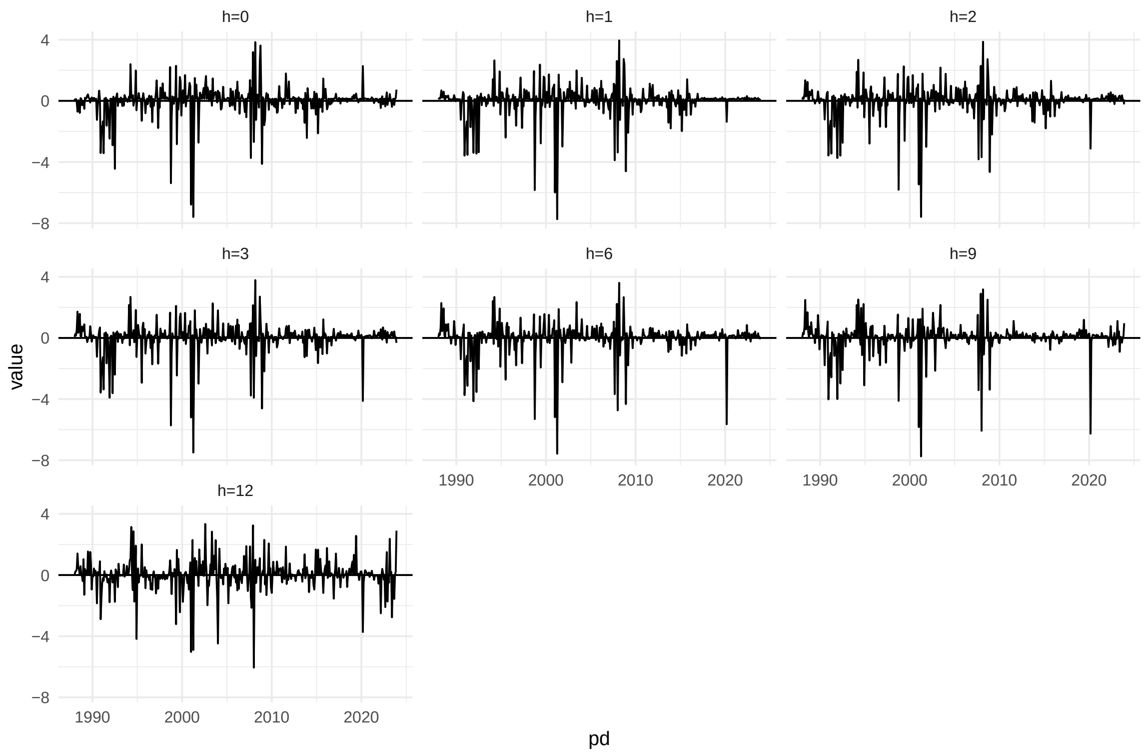


Figure 8: Synthetic Shocks: Time Series