AI Pricing in the Macroeconomy: Predictions for the Phillips Curve and Monetary Policy

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Abstract

What are the macroeconomic implications of the increasing use of AI for price-setting? To address this question, I develop an AI pricing theory where firms choose how much data and computation to use when setting prices. However the firm's pricing model degrades, and they have to periodically reestimate the model at a cost. This structure is consistent with evidence that firms using more AI pricing are larger and earn higher markups, while at the aggregate level AI pricing becomes more prevalent and prices become more flexible as computing costs fall. Pricing dynamics resemble menu cost models, which I show leads to an AI Phillips Curve where the slope is an increasing function of AI usage. Embedding this relationship in a New Keynesian model shows that as AI pricing rises, optimal monetary policy responds more aggressively to inflation. Based on the model, I make four predictions about the macroeconomic future given increasing AI pricing.

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JEL-Codes: E31, E52, E58, O33

1 Introduction

Price-setting is changing. Over the past decade, firms are increasingly using extensive data and algorithms augmented by artificial intelligence (henceforth "AI pricing") to set prices (Adams et al., 2026). The expanding use of these technologies may have implications for macroeconomic price stickiness. There is already evidence in the data: concurrent with the rise of AI pricing, the frequency of price adjustment (FPA) has increased secularly (Cavallo, 2018). These trends raise several questions. Will AI pricing make prices more flexible in the future? How will it affect the economy's Phillips curve? How should monetary policy respond?

This paper makes several predictions. These questions are forward-looking, and relevant data are currently scarce. So I rely on economic theory to address them. My approach is to model AI pricing in a sufficiently rich way to be consistent with the limited existing macroeconomic evidence, but also in a sufficiently familiar way that it is possible to use existing theoretical results to draw conclusions. Properly balanced, this strategy allows for broad and relevant macroeconomic predictions to be drawn quickly, without first writing an entirely new theory of price setting from the ground up.

I model firms as setting prices as functions of consumer and product characteristics. Firms choose how much data to use, e.g. they can set prices the same for all consumers, or they might carefully set unique prices for each consumer, or anything in between. Higher complexity comes at a cost. Thus far, this structure resembles the model of AI adoption in Adams et al. (2026) and inherits its ability to match salient patterns: firms that do more AI pricing are likely to be larger and have higher markups.

Next, I add a dynamic element in order to address business cycle questions. Firms' pricing models degrade over time: the structure of the economy can change, expensive datasets effectively depreciate as they lose coverage of an evolving consumer base, etc. Therefore firms have to periodically re-estimate their pricing models, but doing so comes at a cost. With these assumptions, the firm's problem resembles a standard menu cost model. This allows the observed rise of AI pricing to explain at least a share of the observed rise in FPA.

Aggregation is where the menu cost structure becomes really useful. A large body of work has built micro-founded menu cost theory over the past decades, and therefore it is possible to apply existing results to the macroeconomics of AI pricing. First, I lean on Alvarez et al. (2023) in order to cleanly linearize the dynamic mean field game (MFG) describing how firms behave over the business cycle, and how their choices aggregate to determine inflation. Second, I apply new results from Adams (2026) that allows the infinite-dimensional MFG to be written tractably as a single dynamic discrete time equation: the *Primary Eigenvalue Discretization.* When trend inflation is zero, menu cost models are closely approximated by a traditional Calvo Phillips curve (Alvarez et al., 2017; Auclert et al., 2024) albeit with a non-traditional calibration. This is true of the AI pricing model as well. Specifically, it implies a Calvo-style Phillips curve, where the slope is an increasing function of the intensity of AI in price setting.

With the AI Phillips curve in hand, it is straightforward to draw conclusions for future monetary policy. The microeconomic model is portable: AI pricing can replace the pricing block of existing macroeconomic models. I demonstrate by applying it to a textbook framework, yielding an AI New Keynesian model. With a general equilibrium model, it is possible to predict how rising AI pricing will change the effects of macroeconomic shocks, and calculate the implications for optimal policy. For example, in a standard calibration, a future rise in AI pricing will imply that optimal monetary policy will respond more aggressively to inflation.

Altogether, this theory leads to the following predictions:

1. AI Pricing will continue to rise as costs decline.
2. As AI pricing rises, prices will become more flexible, as measured by the frequency of price adjustment.
3. The Phillips curve will become steeper, so that inflation is more elastic to real costs.
4. Monetary policy, when conducted optimally, will be more responsive to inflation, and thus interest rates will become more volatile.

However, they are mainly qualitative; precise quantitative predictions require data that do not exist. In particular, evidence is needed to discipline the AI pricing cost function. And it will become increasingly important; the microeconomics of pricing is changing permanently. Thus this need is a call to action for new empirical work: what is the relationship between firms' use of AI pricing and their frequency of price adjustment?

This paper joins a rapidly growing literature on the macroeconomic consequences of AI, which is already too voluminous to summarize here. Adams et al. (2026) is the most closely related, where firms' use of AI pricing is inferred from their demanded skills observed in the universe of online job postings. A more narrow literature has studied how algorithms and online shopping have affected pricing behavior. Gorodnichenko and Talavera (2017) and Gorodnichenko et al. (2018) find that online prices are more flexible and exhibit stronger passthrough than traditional retail. One reason is that brick-and-mortar retail engages in relatively little geographic price discrimination Cavallo et al. (2014); DellaVigna and Gentzkow (2019). At the micro level, the use of algorithmic pricing in a variety of industries has been associated with price flexibility (Brown and MacKay, 2023; Assad et al., 2024; Aparicio et al., 2024a,b).

The paper is organized as follows. Section 2 develops a model of AI-assisted pricing where firms choose how many demand factors to incorporate before setting prices. Section 3 aggregates the model, derives the mean-field game, and characterizes steady-state welfare. Section 4 uses the primary eigenvalue decomposition to derive the AI Phillips curve and analyze how AI adoption affects its slope. Section 5 embeds this Phillips curve in a New Keynesian model, calibrates the parameters, and studies how AI pricing affects shock transmission and optimal monetary policy. Section 6 concludes.

2 Model

This section develops a model of AI-assisted pricing. The key innovation is that firms choose how sophisticated a pricing model to build (that is, how many demand factors to incorporate) before setting prices. This creates a link between AI adoption and the slope of the Phillips curve.

2.1 The Price-Setting Problem

There are many monopolistic firms indexed by i. Each firm i sells a measure M of products, indexed by j. This product index may be finely defined, for example at the individual customer level. The firm's optimal price for product j depends on its demand curve. Unlike in standard price-setting models, the firm does not know this demand curve perfectly. Specifically, it does not know the quality or preference shifter Z_{ijt} that enters product j's demand. Firms must instead learn about Z_{ijt} by estimating a demand model that conditions on observable characteristics. This structure introduces a role for AI pricing: firms can invest in learning more about demand by estimating richer models with more factors.

2.1.1 The Demand Structure

Each product j has an associated quality or preference shifter Z_{ijt} that enters demand, but also represents the ij-specific component of marginal cost. This shifter depends on a high-dimensional set of observed factors $F_{ij}(k)$ for $k \in [0, 1]$. The log shifter z_{ijt} is given by the index

$$z_{ijt} \equiv \log Z_{ijt} = \int_0^1 \beta_{it}(k) F_{ij}(k)\, dk. \tag{1}$$

The factors represent the firm's data: observed characteristics of product j. They are orthogonal across k and distributed standard normal in the population (across both j and k). These factors can be thought of as principal components of high-dimensional product characteristics.

The loading coefficients $\beta_{it}(k)$ map factors to demand. They have two components:

$$\beta_{it}(k) = \omega(k) w_t + \hat{\beta}_{it}(k) \tag{2}$$

where $w_t \equiv \log W_t - \log \bar{W}$ is the deviation of aggregate nominal marginal cost from its long-run mean, and $\hat{\beta}_{it}(k)$ is the idiosyncratic component. The function $\omega(k)$ controls how the aggregate component w_t maps to k-indexed factor loadings. I assume that $\omega(k) = 0$ for $k > A_i$, so that the aggregate cost component enters only through modeled factors. This reflects the idea that firms estimate the most salient loadings first, and aggregate data on w_t are widely available.

The idiosyncratic coefficients follow a Brownian motion

$$d\hat{\beta}_{it}(k) = \sigma_\beta \, dW_{it}^{\beta}(k), \tag{3}$$

with independent shocks across k. Because the factors are orthonormal, the preference shifter inherits a Brownian motion plus an aggregate drift:

$$dz_{ijt} = \left(\int_0^{A_i} \omega(k) F_{ij}(k) \, dk \right) dw_t + \sigma_\beta \, dW_{ijt}^{Z} \tag{4}$$

where the integral runs only over the modeled factors because $\omega(k) = 0$ for $k > A_i$.

To keep the distribution of z_{ijt} non-degenerate, I also assume that each coefficient occasionally resets to zero. Specifically, the reset events occur according to independent Poisson processes with intensity ζ such that the stationary variance of z_{ijt} is

$$V_z \equiv Var(z_{ijt}) = \frac{\sigma_z^2}{2\zeta}$$

2.1.2 Firms' Information

Firms do not observe z_{ijt} directly. Instead, firm i estimates a demand model that uses a share $A_i \in [0, 1]$ of the available factor dimensions. Because the factors are independent and symmetric across k, A_i can be interpreted as the share of the idiosyncratic variance of z_{ijt} that is explained by the firm's model. Decompose the index into the modeled and unmodeled components,

$$z_{ijt}^{A_i} \equiv \int_0^{A_i} \beta_{it}(k) F_{ij}(k) \, dk \tag{5}$$

$$z_{ijt}^{\neg A_i} \equiv \int_{A_i}^{1} \beta_{it}(k) F_{ij}(k) \, dk \tag{6}$$

Because the factors are orthonormal, the variance of the idiosyncratic components is proportional to the measure of factors included. Define $V_{\hat{z}}$ as the stationary variance of $\int_0^1 \hat{\beta}_{it}(k)F_{ij}(k)\,dk$. Then:

$$\text{Var}(z_{ijt}^{A_i}|w_t) = \text{Var}\left(\int_0^{A_i} \hat{\beta}_{it}(k)F_{ij}(k)\,dk\right) = A_i V_{\hat{z}} \tag{7}$$

$$\text{Var}(z_{ijt}^{\neg A_i}|w_t) = \text{Var}\left(\int_{A_i}^{1} \hat{\beta}_{it}(k)F_{ij}(k)\,dk\right) = (1 - A_i)V_{\hat{z}} \tag{8}$$

2.1.3 Price Setting

Firms are monopolistic and choose prices for their products. But unlike a standard menu cost model, the firm does not directly choose a separate price for each product. Instead, it chooses a pricing rule that maps product characteristics into prices. Conditional on choosing share A_i of factor dimensions, I write the firm's log price as a sum of factor-specific price components,

$$\log P_{ijt} = \mu + \log \bar{W} + \int_0^{A_i} p_{it}(k)F_{ij}(k)\,dk \tag{9}$$

where $p_{it}(k)$ is the firm's price coefficient on factor dimension k, μ is the frictionless optimal log markup, and $\bar{W}$ is the economy-wide long-run average nominal marginal cost (e.g. the wage). This implicitly assumes that trend inflation is zero. Note that firms only set prices based on fixed observables; prices do not update continuously without changes to the pricing rule. This rules out any direct indexing to the economy-wide price level or marginal cost W_t. But because the aggregate component w_t is embedded in the modeled factor coefficients $\beta_{it}(k)$, firms can respond to aggregate shocks by adjusting their factor loadings.

With this setup, firms do not pay any menu cost to change the price of any particular product. They pay a *compute cost* to update their pricing rule. Specifically, if a firm wants to change the loading $p_{it}(k)$ for factor dimension k, it pays a compute cost ψ. Therefore total compute costs paid by firms will depend on how many factor dimensions A_i they choose to update, but not directly on the number of products M.

This structure will lead to an (s, S)-style inaction region for price loadings $p_{it}(k)$. When price loadings are close to optimal, firms are unwilling to pay the compute cost ψ to make a small improvement. But if price loadings are far from optimal, then firms will be willing to pay the fixed cost and make a discrete adjustment. Because the $\beta_{it}(k)$ factor loadings exhibit rare large Poisson jumps at rate ζ, these will typically induce endogenous adjustments. For tractability, I assume that whenever the kth factor receives a Poisson shock, the firm can also change $p_{it}(k)$ costlessly, so that the Poisson shock always causes an adjustment.

2.1.4 Gaps

This fixed cost structure leads to inaction regions for each factor dimension, analogous to price inaction regions in standard menu cost models, e.g. Golosov and Lucas Jr. (2007). There are two related gap concepts. The *gross gap* on factor k is the difference between the price loading and the full factor loading:

$$g_{it}(k) \equiv p_{it}(k) - \beta_{it}(k) = p_{it}(k) - \omega(k)w_t - \hat{\beta}_{it}(k) \tag{10}$$

The gross gap captures the firm's total mispricing on factor k, including any failure to respond to aggregate marginal cost w_t.

The *loading gap* is the difference between the price loading and the idiosyncratic component:

$$x_{it}(k) \equiv p_{it}(k) - \hat{\beta}_{it}(k) \tag{11}$$

In the recursive problem that follows, I will write it such that the loading gap is the firm's state variable for factor k: it evolves as a Brownian motion between resets, since $p_{it}(k)$ is fixed and $\hat{\beta}_{it}(k)$ follows (3). The two gaps are related by

$$g_{it}(k) = x_{it}(k) - \omega(k)w_t \tag{12}$$

When aggregate marginal cost rises ($w_t > 0$) and if $\omega(k) > 0$, then gross gaps fall for any fixed loading gaps. This creates pressure to raise prices.

The optimal frictionless price of product j satisfies

$$\log P^*_{ijt} = \log Z_{ijt} + \mu + \log W_t$$

As is common, I represent the firm's problem using a second order approximation around the frictionless optimal price. The profit lost in every period relative to the optimal $\log P^*_{ijt}$ is the quadratic $-\mathbf{B}\left(\log P_{ijt} - \log P^*_{ijt}\right)^2$ where $\mathbf{B} = \frac{\eta(\eta-1)}{2}$ and η is the price elasticity of demand. Therefore firm i's total loss $\mathcal{L}_{it}$ across products is

$$\mathcal{L}_{it} = \int_0^M -\mathbf{B}\left(\log P_{ijt} - \log Z_{ijt} - \mu - \log W_t\right)^2 \, dj \tag{13}$$

In menu cost models, this leads to a standard representation in terms of price gaps across products. Lemma 1 shows that in the AI pricing model, the loss can be represented in terms of the loading gaps across factors.

Lemma 1. *If $\omega(k) = 0$ for $k > A_i$, then the firm's loss satisfies*

$$\mathcal{L}_{it} = -\mathbf{B}M \int_0^{A_i} g_{it}(k)^2 \, dk - \mathbf{B}M(1 - A_i)V_{\hat{z}} \tag{14}$$

where $g_{it}(k) = p_{it}(k) - \beta_{it}(k) = x_{it}(k) - \omega(k)w_t$ is the gross gap on factor k and $V_{\hat{z}}$ is the variance of the idiosyncratic component of z_{ijt}.

Proof. Substitute the pricing rule (9) and the index (1) into the loss (13):

$$\mathcal{L}_{it} = -\mathbf{B} \int_0^M \left(\int_0^{A_i} p_{it}(k)F_{ij}(k) \, dk - \int_0^1 \beta_{it}(k)F_{ij}(k) \, dk \right)^2 dj$$

The $\omega(k) = 0$ for $k > A_i$ assumption implies that the w_t term enters only through the modeled factors. Split the β integral at A_i and use (2):

$$= -\mathbf{B} \int_0^M \left(\int_0^{A_i} (p_{it}(k) - \omega(k)w_t - \hat{\beta}_{it}(k))F_{ij}(k) \, dk - z_{ijt}^{\neg A_i} \right)^2 dj$$

where $z_{ijt}^{\neg A_i} = \int_{A_i}^1 \hat{\beta}_{it}(k)F_{ij}(k) \, dk$ is the unmodeled component. Using the definition $g_{it}(k) = p_{it}(k) - \omega(k)w_t - \hat{\beta}_{it}(k)$:

$$= -\mathbf{B} \int_0^M \left(\int_0^{A_i} g_{it}(k)F_{ij}(k) \, dk - z_{ijt}^{\neg A_i} \right)^2 dj$$

Expand the square. The cross term between the modeled factor integral and $z_{ijt}^{\neg A_i}$ vanishes when integrating over j, because factors are orthogonal across k. This leaves:

$$\mathcal{L}_{it} = -\mathbf{B} \int_0^M \left(\int_0^{A_i} g_{it}(k)F_{ij}(k) \, dk \right)^2 dj - \mathbf{B} \int_0^M \left(z_{ijt}^{\neg A_i} \right)^2 dj \tag{15}$$

For the first term, write the squared integral as a double integral and integrate over j. Because factors are orthonormal ($\int_0^M F_{ij}(k)F_{ij}(\kappa) \, dj = M\mathbf{1}_{k=\kappa}$), the double integral collapses:

$$\int_0^M \left(\int_0^{A_i} g_{it}(k)F_{ij}(k) \, dk \right)^2 dj = M \int_0^{A_i} g_{it}(k)^2 \, dk$$

For the second term, the same orthonormality gives:

$$\int_0^M \left(z_{ijt}^{\neg A_i} \right)^2 dj = M \int_{A_i}^1 \hat{\beta}_{it}(k)^2 \, dk = M(1 - A_i)V_{\hat{z}}$$

Substituting back into equation (15) yields (14). □

Lemma 1 establishes that the firm's problem separates across factor dimensions. Conditional on the choice of A_i, the firm solves a standard fixed cost problem for each factor's gross gap $g_{it}(k)$. The second term in (14) is the irreducible loss from unmodeled heterogeneity; it does not depend on the firm's pricing decisions and vanishes as $A_i \to 1$.

The assumption $\omega(k) = 0$ for $k > A_i$ implies that firms choose modeled factors $k \in [0, A_i]$ that entirely capture the effects of aggregate marginal costs. This reflects the idea that firms estimate the most important and salient loadings first, and aggregate data on w_t are widely available.[1]

2.2 The Inner Loop: A Fixed Cost Problem for Each Factor

Given Lemma 1, the firm's problem for each modeled factor dimension $k \leq A_i$ is a classic fixed cost problem. The firm's idiosyncratic state variable is the loading gap $x_{it}(k) = p_{it}(k) - \hat{\beta}_{it}(k)$, which measures mispricing on factor k. When the firm pays the compute cost ψ to update factor k, it reoptimizes $p_{it}(k)$.

For simplicity, I assume that the aggregate component w_t enters each modeled factor uniformly:

$$\omega(k) = \begin{cases} 1 & k \leq A_i \\ 0 & k > A_i \end{cases} \tag{16}$$

This satisfies the condition in Lemma 1. Under this assumption, the gross gap and loading gap are related by $g_{it}(k) = x_{it}(k) - w_t$ for each modeled factor $k \leq A_i$. The firm's problem can be written in terms of either state gap. I use the loading gap $x_{it}(k)$ as the state variable because it evolves as a pure Brownian motion between adjustments: since $p_{it}(k)$ is fixed and $\hat{\beta}_{it}(k)$ follows (3),

$$dx_{it}(k) = -\sigma_\beta \, dW_{it}^{\beta}(k) \tag{17}$$

with independent shocks across k. The gross gap inherits this diffusion plus the aggregate drift from w_t.

The firm's problem is to choose an inaction region $[\underline{x}_i(t), \bar{x}_i(t)]$ within which it will not adjust its price loading, and a reset value $x_i^*(t)$ that it reoptimizes too when the loading gap exits the inaction region.

[1] Relaxing this assumption would add an interesting new channel by which aggregate shocks fail to passthrough to pricing behavior, similar to inaction models with incomplete information. But I shut down this channel in order to simplify the analysis and conclusions.

2.2.1 The Hamilton-Jacobi-Bellman Equation

I now write the firm's problem recursively, and drop time t subscripts and factor k arguments for clarity when possible.

Let $v_i(x,t)$ denote the value function for the inner loop problem, where x is the loading gap on a representative factor dimension. Because factors are symmetric, the value function is the same for each $k \leq A_i$. I write the value function $v_i(x,t)$ relative to the counterfactual firm's value if they were able to reset price loadings frictionlessly, but still only utilized A_i factors. Thus when applying Lemma 1, the value function does not yet contain the $\mathbf{B}M(1-A_i)V_{\hat{z}}$ loss term due to unmodeled factors.

Inside the inaction region, the value function satisfies the Hamilton-Jacobi-Bellman (HJB) equation

$$\rho v_i(x,t) = -\mathbf{B}M(x-w)^2 + \partial_t v_i(x,t) + \frac{\sigma_\beta^2}{2}v''(x,t) + \zeta\left(v_i(x_i^*(t),t) - v_i(x,t)\right) \tag{18}$$

The flow payoff $-\mathbf{B}M(x-w)^2$ is the instantaneous loss from the gross gap $g = x - w$. The $\partial_t v_i(x,t)$ term captures how the evolution of w_t affects the firm's value. The $\frac{\sigma_\beta^2}{2}v''$ term is due to the diffusion of the loading gap. The final term reflects the Poisson resets: at rate ζ, the coefficient $\hat{\beta}_{it}(k)$ resets to zero. Because the firm can costlessly reoptimize at these times, this also resets the loading gap to its optimal value $x_i^*(t)$.

2.2.2 Boundary Conditions

The boundary conditions are standard value-matching, reset optimality, and smooth-pasting conditions:

$$v_i(\underline{x}_i(t),t) + \psi = v_i(x_i^*(t),t) \tag{19}$$

$$v_i(\bar{x}_i(t),t) + \psi = v_i(x_i^*(t),t) \tag{20}$$

$$v_i'(x_i^*(t),t) = 0 \tag{21}$$

$$v_i'(\underline{x}_i(t),t) = 0 \tag{22}$$

$$v_i'(\bar{x}_i(t),t) = 0 \tag{23}$$

Value matching says that at the boundary, the firm is indifferent between paying the compute cost ψ to reset and remaining at the boundary: the value at the boundary plus the cost equals the value at the reset point. The reset optimality and smooth-pasting conditions say that the firm chooses the critical points optimally. These five conditions pin down the three endogenous objects ($\underline{x}_i(t)$, $\bar{x}_i(t)$, and $x_i^*(t)$) and the shape of the value function (which has two degrees of freedom).

2.2.3 Properties of the Solution

Written this way, the firm's problem for any given factor is standard. And the symmetry of the problem (there is no trend inflation) implies several useful well-known properties for the steady state problem, i.e. when aggregate marginal cost is constant.

First, the steady-state reset point is $x_{i,SS}^* = 0$. Because $\hat{\beta}_{it}(k)$ follows a driftless Brownian motion, the loading gap $x_{it}(k)$ also has no drift when w_t is constant. The symmetry of the problem then implies that the optimal reset point is at the center of the inaction region. When the firm resets, it sets $p_{it}(k) = \hat{\beta}_{it}(k)$, eliminating the idiosyncratic component of the gross gap.

Second, the steady-state inaction region is symmetric: $\bar{x}_{i,SS} = -\underline{x}_{i,SS}$. This follows from the symmetric diffusion and the quadratic loss function. Let $\bar{x}_{i,SS}(t) \equiv \frac{\ell_i}{2} > 0$ denote the half-length of the steady-state inaction region. Then the firm tolerates loading gaps in the interval $[-\frac{\ell_i}{2}, \frac{\ell_i}{2}]$.

Third, the steady-state value function $v_{i,SS}(x)$ satisfies

$$(\rho + \zeta)v_{i,SS}(x) = -\mathbf{B}Mx^2 + \frac{\sigma_\beta^2}{2}v_{i,SS}''(x) + \zeta v_{i,SS}(0) \tag{24}$$

with boundary conditions $v_{i,SS}(-\frac{\ell_i}{2}) + \psi = v_{i,SS}(\frac{\ell_i}{2}) + \psi = v_{i,SS}(0)$ and $v_{i,SS}'(-\frac{\ell_i}{2}) = v_{i,SS}'(\frac{\ell_i}{2}) = 0$.

2.2.4 Steady-State Firm Value

The value function $v_i(x, t)$ measures the firm's value lost relative to a frictionless-adjusting firm that only utilizes the A_i factors. But mitigating the value lost from not observing factors $k \in [A_i, 1]$ is the main incentive to increase model complexity, so it must be accounted for.

Lemma 1 states that the flow loss from not utilizing all factors is $\mathbf{B}M(1-A_i)V_{\hat{z}}$. Discounting at rate ρ, this implies that the total firm value $Value_i(x, t)$ relative to the frictionless counterfactual is

$$Value_i(x, t) = A_i v_i(x, t) - \frac{\mathbf{B}MV_{\hat{z}}}{\rho}(1 - A_i) \tag{25}$$

The $A_i M$ coefficient on appears because the relative value $v_i(x, t)$ is per-factor, and there are A_i modeled factors. The HJB (18) implies both terms are scaled by the number of products M and the demand curvature $\mathbf{B}$.

This completes the inner loop: given a choice of A_i, the firm's pricing problem reduces to a continuum of menu cost problems, one for each factor dimension $k \leq A_i$. Greater A_i increases the value of the firm, but at some cost. The next section addresses the A_i decision.

2.3 The Outer Loop: Choosing Model Complexity (A_i)

The firm's fixed cost decision took the share of modeled factors A_i as given. This section characterizes the firm's choice of A_i.

2.3.1 Costs and Benefits of Model Complexity

When making their A_i choice, firms make a once-and-for-all decision based on the steady-state expected value of their firm after a price reset.[2] In principle the incentive to increase A_i can be dynamic, but the once-and-for-all decision making reflects the fixed costs of large-scale modeling, such as computational resources, datasets, and software systems.

Modeling more factor dimensions requires building a more complex pricing system. I represent this as a one-off setup cost $\Psi(A_i)$ that is increasing in A_i. The firm maximizes the expected steady state value after a reset, i.e. $Value_{i,SS}(0,t)$. Therefore the firm's decision is

$$\max_{A_i\in[0,1]} A_i M v_i(0,t) - \frac{\mathbf{B} M V_{\hat{z}}}{\rho}(1-A_i) - \Psi(A_i)$$

per equation (25).

The first order condition characterizing the firm's optimal A_i choice is thus

$$v_i(0,t) + \frac{\mathbf{B} M V_{\hat{z}}}{\rho} = \Psi'(A_i) \tag{26}$$

The left-hand side is the marginal benefit of additional complexity: if a given factor is modeled, then it contributes relative loss $v_i(0,t)$ (which is negative but less so than the unmodeled case) instead of the much more negative loss $-\frac{\mathbf{B} M V_{\hat{z}}}{\rho}$. This marginal benefit must equal the marginal cost $\Psi'(A_i)$.

2.3.2 Comparative Statics

The first-order condition (26) immediately yields comparative statics for the optimal model complexity A_i^*.

[2] This arrangement is similar to rational inattention models where agents choose an information structure once, and then the economy runs in perpetuity given that decision (Sims, 2003; Maćkowiak and Wiederholt, 2009).

Proposition 1 (Comparative Statics for AI Adoption)**.** *If $\Psi(A_i)$ is strictly convex, and there is a unique optimal share of modeled factors $A_i^* \in (0,1)$, then it satisfies:*

1. $\partial A_i^*/\partial \mathbf{B} > 0$*: higher demand elasticity increases AI adoption.*
2. $\partial A_i^*/\partial M > 0$*: more products increase AI adoption.*
3. $\partial A_i^*/\partial \psi < 0$*: higher compute costs decrease AI adoption and firm value.*
4. $\partial A_i^*/\partial \lambda_i < 0$ *a proportional increase λ_i of the fixed cost $\lambda_i \Psi(A_i)$ decreases AI adoption and firm value.*

and the signs are reversed if $\Psi(A_i)$ is strictly concave.

Proof: Appendix A

Results (1) and (2) describe the incentives to use AI pricing: if firms face more elastic demand, or face more markets, then the incentive to accurately model costs and demand are increased, and the firm will choose larger A_i. Results (3) and (4) describe the costs: if the variable cost ψ of reestimating the model is large, firms will benefit from the technology less often, reducing the incentive to adopt high model complexity. And if the fixed cost $\Psi(A_i)$ uniformly increases, so does the marginal cost, and firms will choose lower A_i. Changes to costs also change the value of the firm in an unambiguous way: if they cause A_i to rise, then the value of the firm rises as well, because costs are lower and firms price more accurately.

3 Aggregation

Section 2 laid out the environment faced by firms, their optimal pricing behavior, and their optimal choice of model complexity. This section describes aggregation and the evolution of the distribution of loading gaps.

3.1 The Distribution of Loading Gaps

In this section I suppose that there is a unit measure of symmetric firms, and drop the i subscripts for clarity.

For each factor k, a firm's loading gap x follows an (s, S) policy with inaction region $[\underline{x}(t), \bar{x}(t)]$ and reset point $x^*(t)$. The boundaries depend on time t through the aggregate state. Because factors are symmetric, the cross-sectional distribution of loading gaps $h(x, t)$ for any particular factor evolves according to the Kolmogorov forward equation (KFE):

$$\partial_t h(x,t) = \frac{\sigma_\beta^2}{2}\partial_x^2 h(x,t) - \zeta h(x,t) + F(t)\delta(x - x^*(t)) \tag{27}$$

where $F(t)$ is the frequency of price adjustment (FPA), the flow of factor dimensions being reset at time t, and $\delta(\cdot)$ is the Dirac delta. The $\frac{\sigma_\beta^2}{2}\partial_x^2 h(x,t)$ term captures the diffusion of loading gaps due to coefficient drift, the $-\zeta h(x,t)$ term captures resets from the Poisson resets, and $F(t)\delta(x-x^*(t))$ captures the flow of firms re-entering at the reset point.

The boundaries are absorbing, implying the Dirichlet boundary conditions

$$h(\underline{x}(t),t) = h(\bar{x}(t),t) = 0 \tag{28}$$

When firms hit either boudnary, they immediately reset to $x^*(t)$. The flow of resetting firms is the FPA, which is determined by conservation:

$$F(t) = \frac{\sigma_\beta^2}{2}\left(\partial_x h(\underline{x}(t),t) - \partial_x h(\bar{x}(t),t)\right) + \zeta \tag{29}$$

The first term is the diffusive flux across the boundaries, and the ζ term is the flow of Poisson resets. Adams (2025) proves that the FPA $F(t)$ enters the KFE as in equation (27), and satisfies equation (29).

3.2 The Mean-Field Game (MFG)

In the symmetric economy, the MFG is a set of time-varying functions for each factor k: a value function $v_k(x,t)$, a reset point $x_k^*(t)$, inaction boundaries $\underline{x}_k(t)$, $\bar{x}_k(t)$ if $k \le A_i$, and a distribution $h_k(x,t)$ of loading gaps.

For each k, These functions satisfy a HJB equation (18) with boundary conditions (19)–(23) if $k \le A_i$ (and only (21) if $k > A_i$), the KFE (27), and absorbing boundaries (28) if $k \le A_i$, given an distribution $h_k(x,0)$.

3.3 Aggregate Loading Gap

The aggregate loading gap $X_k(t)$ for any particular factor k is the cross-sectional average $\int_{\underline{x}(t)}^{\bar{x}(t)} x\, h_k(x,t)\, dx$, where h is now indexed by k. Given the assumption that aggregate shocks affect these loading gaps symmetrically, the distributions $h_k(x,t)$ must also be obey certain symmetries: for factors $k \le A_i$ all distributions $h_k(x,t)$ are symmetric, while for unmodeled factors $k > A_i$ all distributions $h_k(x,t)$ are also symmetric.

For any factor k, the average loading gap loading gap $X_k(t)$ is

$$X_k(t) = \int_{\underline{x}(t)}^{\bar{x}(t)} x\, h_k(x,t)\, dx \tag{30}$$

and because the steady state distributions are symmetric around zero, the steady state average loading gap is $X = 0$. Therefore by symmetry, the economy-wide loading gap is

$$X(t) \equiv \int_0^1 X_k(t)\, dk = A_i X_{A_i}(t) + (1 - A_i) X_{\neg A_i}(t) \tag{31}$$

where $X_{A_i}(t)$ denotes the average loading gap for $k \leq A_i$ modeled factors, and $X_{\neg A_i}(t)$ denotes the average loading gap for $k > A_i$ unmodeled factors.

As usual, the aggregate loading gap is useful because it determines the price level in the economy. To a first order approximation, the economy-wide price index is

$$\log P(t) = \int_0^1 \int_0^1 p_{it}(k) dk di = \int_0^1 \int_0^1 \left(x_{it}(k) + \hat{\beta}_{it}(k) \right) dk di = X(t)$$

because $\hat{\beta}_{it}(k)$ is entirely idiosyncratic.

3.4 Steady-State Welfare

AI pricing affects steady-state welfare through two channels: mispricing costs and compute costs. Denote the welfare losses (relative to perfectly flexible prices) associated with these channels as $\mathcal{W}_\chi$ and $\mathcal{W}_\psi$ respectively. Costs due to the initial fixed cost $\Psi(A_i)$ are also relevant losses, but I will consider only the ongoing welfare costs after the initial cost is paid.[3] The total welfare cost is

$$\mathcal{C} = \mathcal{C}_\chi + \mathcal{C}_\psi$$

The mispricing cost $\mathcal{C}_\chi$ induces misallocation: consumers end up with too few of the goods that are overpriced, and too many of those that are underpriced. Per Cavallo et al. (2023), this

[3] One reason to do so is that the model is relatively uninformative about the level of the fixed cost $\Psi(A_i)$. Only the marginal cost $\Psi'(A_i)$ appears in the first order conditions, so conclusions about the level will be sensitive to the assumed functional form. Moreover, the choice of A_i is not easily measured in the data, further complicating the issue.

loss is measured up to second order by

$$C_\chi = \frac{\eta}{2} M Var \left(\log P_{ijt} - \log Z_{ijt} - \mu - \log W_t\right)^2 \tag{32}$$

as each (i, j) reflects a unique product. The loss due to ongoing compute costs is

$$C_\psi = (F - \zeta)\psi \tag{33}$$

where F is the frequency of price adjustment. ζ is the frequency of free adjustments, so $F - \psi$ is the frequency of costly adjustments.

Lemma 2 (Steady-State Welfare Cost)**.** *If all firms are symmetric, then the total steady-state welfare cost is*

$$C(A_i) = \frac{\eta M \sigma_\beta^2}{2\zeta}\left[1 - A_i \operatorname{sech}(s\bar{x})\right] + \frac{A_i \zeta \psi}{\cosh(s\bar{x}) - 1} \tag{34}$$

where $s \equiv \sqrt{2\zeta/\sigma_\beta^2}$*.*

Proof: Appendix A

Lemma 2 expresses the total welfare cost as a function of AI adoption A_i. The first term is the mispricing cost C_χ, which is decreasing in A_i because greater AI adoption reduces the variance of loading gaps, and thus the misallocation of products. The second term is the compute cost C_ψ, which is increasing in A_i because greater AI adoption increases the frequency of costly price adjustments. Proposition 2 gives the net effect.

Proposition 2 (Steady-State Welfare)**.** *The marginal effect of AI adoption on steady-state welfare is*

$$\frac{\partial C}{\partial A_i} = -\frac{\eta M \sigma_\beta^2}{2\zeta}\operatorname{sech}(s\bar{x}) + \frac{\zeta\psi}{\cosh(s\bar{x}) - 1}$$

The first term is the marginal benefit from reduced mispricing; the second is the marginal cost from increased compute expenditure.

Proof. The proposition follows from Lemma 2. Differentiating (34) with respect to A_i:

$$\frac{\partial C_\chi}{\partial A_i} = -\frac{\eta M \sigma_\beta^2}{2\zeta}\operatorname{sech}(s\bar{x})$$

$$\frac{\partial C_\psi}{\partial A_i} = \frac{\zeta\psi}{\cosh(s\bar{x}) - 1}$$

□

4 The AI Phillips Curve

This section uses the primary eigenvalue decomposition (PED) to characterize how AI adoption shapes the Phillips curve. The PED, developed in Adams (2026), approximates the impulse response of the distribution to aggregate shocks using a single dominant eigenvalue. This yields a tractable representation of price dynamics.

4.1 The Primary Eigenvalue Decomposition

The MFG (Section 3.2) is highly non-linear. Following insights from Alvarez et al. (2023), it is possible to linearize the entire MFG around the stationary distribution. This gives a set of PDEs that are linear, and thus easily solved and characterized. This linearization is an accurate approximation for small marginal cost shocks.

The Alvarez-Lippi-Souganidis linearization is valuable for solving the distributional dynamics. However while the MFG becomes linear, it remains infinite-dimensional. Fortunately, further tractability gains are possible; Adams (2026) develops a low-dimensional approximation that accurately captures aggregate behavior with only a few linear equations: the *Primary Eigenvalue Decomposition.*

With zero trend inflation ($\bar{\pi} = 0$), the PED reduces the infinite-dimensional MFG of a Calvo-plus menu cost model to two pricing equations. The first governs optimal price setting; the second governs the evolution of the price level.

Property 1 (Primary Eigenvalue Decomposition)**.** *In a Calvo-plus menu cost model with zero trend inflation, diffusion variance $\sigma^2/2$, inaction region width ℓ, and random reset rate ζ, the linearized price dynamics are accurately approximated by*

$$p_t^* = (1 - \beta\theta)w_t + \beta\theta\,\mathbb{E}_t p_{t+1}^* \tag{35}$$

$$p_t = (1 - \theta)p_t^* + \theta\, p_{t-1} \tag{36}$$

where p_t^* is the optimal reset price, p_t is the aggregate price level, w_t is nominal marginal cost, $\beta = e^{-\rho}$ is the discretized discount factor, and θ is given by

$$\theta = \exp\left(-\zeta - \frac{\sigma^2 2\pi^2}{\ell^2}\right)$$

Adams (2026) derives property 1 allowing for trend inflation more generally, which requires accounting for the dynamics of the FPA. But the FPA term disappears when trend inflation is zero, so the representation becomes much simpler. In fact, the linear system is isomorphic to that of a pure Calvo model where θ captures only the random reset rate. What changes in the AI pricing model? AI affects the value of the θ coefficient.

4.2 The AI Phillips Curve

The main result of this section maps the AI pricing model to the menu cost framework, yielding the AI Phillips curve.

Theorem 1 (The AI Phillips Curve)**.** *The PED linear discrete time approximation of the AI pricing model implies that the inflation rate π_t satisfies*

$$\pi_t = \lambda(A_i)\, mc_t + \beta\, \mathbb{E}_t \pi_{t+1} \tag{37}$$

where mc_t is the real marginal cost, and the slope is given by

$$\lambda(A_i) \equiv A_i \frac{(1-\theta_{A_i})(1-\beta\theta_{A_i})}{\theta_{A_i}} + (1-A_i)\frac{(1-\theta_{\neg A_i})(1-\beta\theta_{\neg A_i})}{\theta_{\neg A_i}} \tag{38}$$

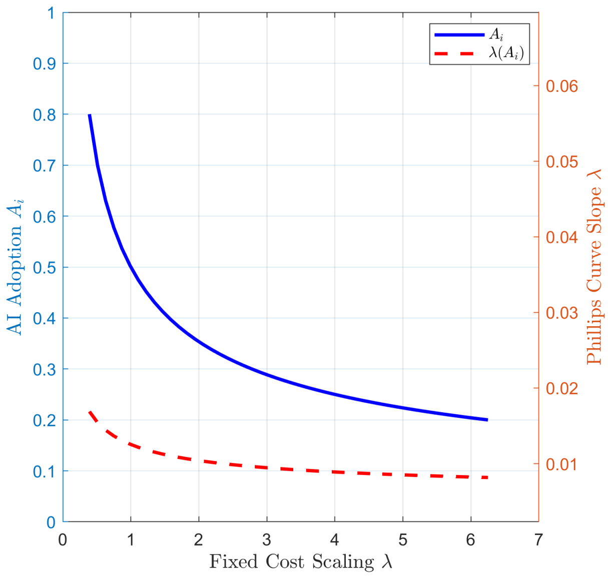
$$\theta_{A_i} \equiv \exp\left(-\zeta - \frac{\sigma_\beta^2 2\pi^2}{\ell^2}\right) \qquad\qquad \theta_{\neg A_i} \equiv \exp\left(-\zeta\right)$$

Proof: Appendix A

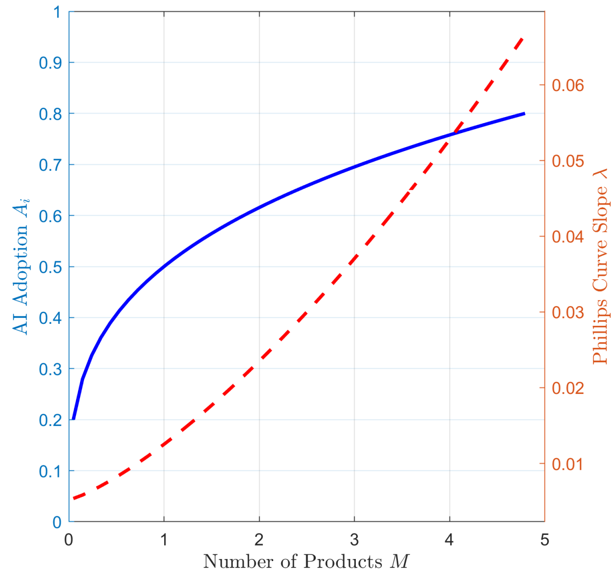
The theorem holds because the MFG for each factor k is isomorphic to the MFG of a Calvo-plus menu cost model. The PED equations themselves do not neatly aggregate across factors, but when combined to yield a New Keynesian Phillips Curve for each k, they do. As a result the A_i choice only affects the slope of the Phillips curve $\lambda(A_i)$, and does so linearly, because no elements of θ_{A_i} depend on the A_i choice itself.

4.3 How AI Adoption Affects the Phillips Curve Slope

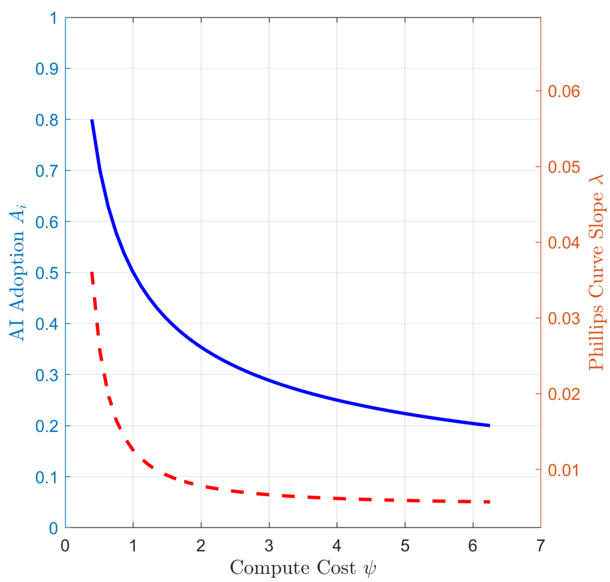
The AI Phillips curve (37) is a weighted average of a traditional New Keynesian Phillips curve with Calvo parameter $\theta_{\neg A_i}$, and a menu cost Phillips curve with effective Calvo parameter θ_{A_i}. Because $\theta_{A_i} < \theta_{\neg A_i}$, it implies that the slope $\frac{(1-\theta_{A_i})(1-\beta\theta_{A_i})}{\theta_{A_i}}$ associated with the A_i modeled factors is greater than the slope $\frac{(1-\theta_{\neg A_i})(1-\beta\theta_{\neg A_i})}{\theta_{\neg A_i}}$ from the $1 - A_i$ unmodeled factors. This is to be expected; Auclert et al. (2024) show that menu cost models are closely approximated by a Calvo Phillips curve with a steeper slope. When firms choose a higher model complexity by raising A_i, they therefore increase the slope of the AI Phillips curve $\lambda(A_i)$.



(a) Fixed cost scaling λ



(b) Number of products M



(c) Compute cost ψ

Figure 1: Effect of model parameters on the Phillips curve slope $\lambda(A_i)$. Panel (a): higher fixed costs reduce A_i^*, lowering λ through a pure composition effect. Panels (b) and (c): product scope and compute costs affect both A_i^* and the stickiness θ_{A_i}.

To demonstrate this effect of A_i on the slope, Figure 1a plots how $\lambda(A_i)$ is affected by a proportional increase in the fixed cost $\Psi(A_i)$. Specifically, I let $\Psi(A_i) = \lambda_0 A_i^\epsilon$ with $\epsilon > 1$, and plot how the optimal A_i choice (solid blue line) and slope $\lambda(A_i)$ (dashed red line) are affected. The remaining model parameters are calibrated per Table 1. As the marginal cost rises, firms choose lower model complexity, and the Phillips curve slope declines.

The other parameters affecting A_i operate through multiple channels. When the reset cost ψ increases, A_i decreases per Proposition 1. However, a larger cost ψ also makes firms less willing to reprice, increasing the length of the inaction region ℓ. Figure 1c shows that this reduces θ_{A_i} and further increases the slope.

Similarly, when the number of products M increases, the scale effect incentivizes firms to increase their A_i choice, as shown in Figure 1b. This also causes firms to reduce their inaction region length ℓ, raising θ_{A_i}. Both the direct and indirect effects reduce the slope of the Phillips curve $\lambda(A_i)$.

Therefore, to understand how A_i affects the slope of the Phillips curve, it is critical to understand the fundamental force that causes A_i adoption. Whether it is driven by costs or incentives implies different outcomes. In Adams et al. (2026) we argued that the rise of AI pricing was driven mainly by falling costs, which allowed a simple pricing model to fit a variety of stylized facts in the time series and cross section. In the quantitative analysis that follows, I will assume that A_i trends are driven by a decline in the fixed cost $\Psi(A_i)$ (Figure 1a), but improving predictions of inflation behavior will require accurately measuring how AI pricing affects the micro pricing moments that determine θ_{A_i}.

5 An AI pricing New Keynesian Model

This section embeds the model in a stationary economy and calibrates it to match the distribution of price changes. I then explore how the economy's response to shocks depends on AI adoption.

5.1 The Three Equation Model

The AI Phillips curve can be embedded in an otherwise standard DSGE model by replacing the pricing block. As an example, I will introduce it by replacing the usual Phillips curve in the textbook New Keynesian framework (Galí, 2008) with a fixed natural real rate. The resulting AI pricing New Keynesian Model is given by three dynamic equations:

$$\begin{aligned}
&\text{AI Phillips Curve} && \pi_t = \lambda(A_i)\,\omega\, y_t + \beta\,\mathbb{E}_t[\pi_{t+1}] + u_t^c \\
&\text{Euler Equation} && \gamma y_t = \mathbb{E}_t[\gamma y_{t+1} - (i_t - \mathbb{E}_t[\pi_{t+1}])] + u_t^d \\
&\text{Taylor Rule} && i_t = \phi_\pi \pi_t + \phi_y y_t + u_t^m
\end{aligned}$$

where y_t denotes the output gap which is related to marginal cost by $mc_t = \omega y_t$, and i_t is the nominal interest rate.[4] u_t^c, u_t^d, and u_t^m denote exogenous cost-push, demand, and monetary wedges respectively.

5.2 Calibration

The model has three structural parameters to calibrate: the diffusion variance σ_β^2, the Poisson reset rate ζ, and the compute cost ψ. It also has a cost function $\Psi(A_i)$ which determines the optimal AI adoption level, a fourth object to calibrate. I calibrate these values to match the recent distribution of price changes measured in France by Alvarez et al. (2024).

The economy-wide distribution of price changes is a mixture of a Calvo distribution for the unmodeled factors, and a Calvo-plus distribution for the modeled factors. The A_i choice determines the weights. Observed moments are therefore linear combinations, e.g. the economy-wide frequency and variance of price changes are

$$F = A_i F^{\mathrm{CP}} + (1 - A_i) F^{\mathrm{C}} \tag{39}$$

$$\sigma_{\Delta p}^2 = \frac{A_i F^{\mathrm{CP}} (\sigma_{\Delta p}^{\mathrm{CP}})^2 + (1 - A_i) F^{\mathrm{C}} (\sigma_{\Delta p}^{\mathrm{C}})^2}{F} \tag{40}$$

where the superscripts CP and C denote Calvo-plus (modeled) and Calvo (unmodeled) components, respectively. The Calvo component has $F^{\mathrm{C}} = \zeta$ and $(\sigma_{\Delta p}^{\mathrm{C}})^2 = \sigma_\beta^2 / \zeta$, while the Calvo-plus expressions are given below.

The following proposition formalizes how the model parameters map to observable moments.

Proposition 3 (Price Change Moments)**.** *Consider the zero-drift steady state. Define the composite parameter $s \equiv \sqrt{2\zeta / \sigma_\beta^2}$ and let $\bar{x} = \ell/2$ denote the steady state upper bound of the inaction region for modeled factors.*

(i) Calvo-plus moments (modeled factors). *The frequency, variance, and kurtosis of price changes*

[4]Following Galí (2008), the composite parameter $\omega \equiv \gamma + \frac{\varphi + \alpha}{1 - \alpha}$ combines the inverse elasticity of intertemporal substitution γ, the inverse Frisch elasticity of labor supply φ, and the capital share α. I use $\gamma = 1$, $\varphi = 1$, and $\alpha = 0.33$, which implies $\omega \approx 3$.

for the share A_i of modeled factors are:

$$F^{CP} = \frac{\zeta \cosh(s\bar{x})}{\cosh(s\bar{x}) - 1} \tag{41}$$

$$(\sigma_{\Delta p}^{CP})^2 = \frac{\sigma_\beta^2}{\zeta}\big(1 - \operatorname{sech}(s\bar{x})\big) \tag{42}$$

$$\kappa_{\Delta p}^{CP} = \frac{\mu_4^{CP}}{\big((\sigma_{\Delta p}^{CP})^2\big)^2} \tag{43}$$

where the fourth moment is

$$\mu_4^{CP} = \frac{6\sigma_\beta^4}{\zeta^2}\big(1 - \operatorname{sech}(s\bar{x})\big) - \frac{6\bar{x}^2\sigma_\beta^2}{\zeta}\operatorname{sech}(s\bar{x}) \tag{44}$$

(ii) Calvo moments (unmodeled factors). *The share $1 - A_i$ of unmodeled factors follow pure Calvo dynamics with exogenous reset rate ζ. The frequency, variance, and kurtosis are:*

$$F^C = \zeta \tag{45}$$

$$(\sigma_{\Delta p}^C)^2 = \frac{\sigma_\beta^2}{\zeta} \tag{46}$$

$$\kappa_{\Delta p}^C = 6 \tag{47}$$

The kurtosis equals 6 because the ergodic distribution is Laplace (double-exponential), not Gaussian; see Lemma 4.

(iii) Economy-wide moments. *The observed price change moments are weighted averages:*

$$F = A_i F^{CP} + (1 - A_i)F^C \tag{48}$$

$$\sigma_{\Delta p}^2 = \frac{A_i F^{CP}(\sigma_{\Delta p}^{CP})^2 + (1 - A_i)F^C(\sigma_{\Delta p}^C)^2}{F} \tag{49}$$

$$\kappa_{\Delta p} = \frac{\mu_4}{\sigma_{\Delta p}^4} \tag{50}$$

where the economy-wide fourth central moment is

$$\mu_4 = \frac{A_i F^{CP}\mu_4^{CP} + (1 - A_i)F^C\mu_4^C}{F}$$

and $\mu_4^C = 6\big((\sigma_{\Delta p}^C)^2\big)^2 = 6\sigma_\beta^4/\zeta^2$.

Proof. See Appendix A.1. □

Proposition 3 shows that the economy-wide kurtosis $\kappa_{\Delta p}$ depends on A_i even though the component kurtoses are pinned down by the other parameters. The Calvo component has kurtosis 6 (Laplace distribution), while the Calvo-plus component has higher kurtosis due to boundary adjustments. The mixture kurtosis interpolates between them, providing the fourth identifying restriction needed to pin down A_i.

5.2.1 Baseline Parameters

The data provide discipline on the pricing parameters $(\sigma_\beta^2, \zeta, \bar{x})$ but not on the AI adoption share A_i. The share of firms using algorithmic pricing is difficult to measure and varies across sectors. Chen et al. (2016) estimate that a large plurality of sellers of popular products on Amazon use algorithms to dynamically set prices. Calder-Wang and Kim (2024) estimate that roughly a quarter of apartment buildings in 2019 were setting rents algorithmically. These figures suggest substantial but incomplete adoption. Given the uncertainty, I set $A_i = 0.5$ as a baseline normalization, and explore robustness to alternative values in the quantitative exercises.

With A_i fixed, the calibration procedure solves for three parameters $(\sigma_\beta^2, \zeta, \bar{x})$ to match three moments: the frequency of price adjustment F, the standard deviation of price changes $\sigma_{\Delta p}$, and the economy-wide kurtosis $\kappa_{\Delta p}$. The parameters enter the Calvo-plus formulas in Proposition 3(i), and aggregate to economy-wide moments via part (iii). I solve the system numerically using a standard nonlinear least-squares routine.

Table 1 reports the baseline calibration. Following the macroeconomics literature, I set the discount rate $\rho = 0.04$ (annual) and the elasticity of substitution $\eta = 6$, which implies $\mathbf{B} = \eta(\eta - 1)/2 = 15$.

The calibration targets three moments from French CPI microdata (Alvarez et al., 2024): frequency of price adjustment $F = 1.26$ (annualized from 10.5% monthly), standard deviation of price changes $\sigma_{\Delta p} = 0.076$, and kurtosis $\kappa_{\Delta p} = 3.41$ (adjusted for heterogeneity) and economy-wide kurtosis 5.05.

The Calvo-plus structure (combining random Poisson resets with endogenous boundary adjustments) is essential for matching both the frequency and the shape of price changes. A pure menu cost model ($\zeta = 0$) can match frequency and standard deviation but generates too much kurtosis. A pure Calvo model can match frequency but has no notion of price change size.

Parameter	Symbol	Value	Source
Fixed parameters			
Discount rate	ρ	0.04	Standard (annual)
Household discount factor	β	0.997	$e^{-\rho/12}$ (monthly)
Elasticity of substitution	η	6	Standard
Profit loss curvature	$\mathbf{B}$	15	$\eta(\eta-1)/2$
Inverse intertemporal elasticity	γ	1	Standard
Inverse Frisch elasticity	φ	1	Standard
Labor share	$1-\alpha$	0.67	Standard
Output gap–MC elasticity	ω	3	$\gamma+(\varphi+\alpha)/(1-\alpha)$
Calibration targets (French CPI)			
Frequency of price adjustment	F	1.26	Alvarez et al. (2024)
Std dev of price changes	$\sigma_{\Delta p}$	0.10	Alvarez et al. (2024)
Kurtosis of price changes	$\kappa_{\Delta p}$	5.05	Alvarez et al. (2024)
Calibrated parameters			
Diffusion variance	σ_β^2	0.0073	Match moments
Poisson reset rate	ζ	0.85	Match moments
Inaction boundary	$\bar{x}$	0.88	Match moments
AI-specific parameters			
Baseline AI adoption	$\bar{A}$	0.5	Normalization

Table 1: Model calibration. Fixed parameters are standard and resemble the textbook New Keynesian calibration (Galí, 2008). Price adjustment parameters are calibrated to match frequency, standard deviation, and kurtosis of price changes from French CPI microdata.

5.3 Effects of AI on Shock Transmission

How does AI adoption affect the economy's response to shocks? This section explroes the impulse response functiond (IRFs) to structural shocks, and evaluates how they depend on the chosen A_i level. Figure 2 plots the impulse responses to each shock type for three values of A_i. The solid, dashed, and dotted lines correspond to low, medium, and high AI adoption.

The first column of Figure 2 plots the effects of a positive demand shock (increase in u_t^d) which raises output (first row), inflation (second row), and the interest rate (third row). When A_i increase, inflation is more responsive to the shock, because it affects price setting through the Phillips curve. The demand shock affects the output gap, and any change in the path of output gaps causes a greater change in inflation when $\lambda(A_i)$ is large. The output gap becomes somewhat less responsive because it is dampened by monetary policy: inflation responds the most when A_i is high, so interest rates respond the most via the Taylor rule.

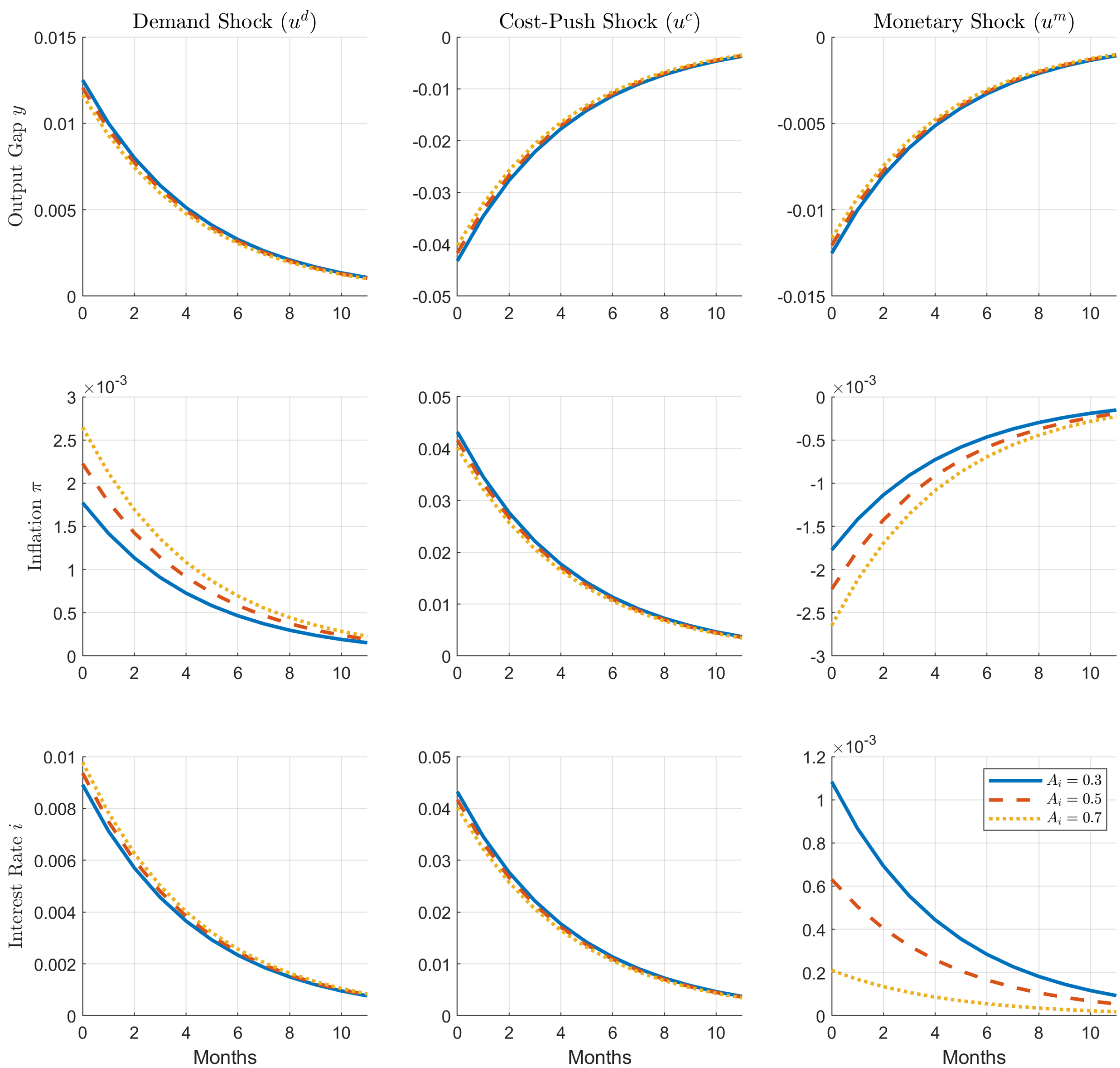


Figure 2: Impulse responses to demand, cost-push, and monetary shocks for different AI adoption levels. Each column shows the response of output gap, inflation, and interest rate. Solid: $A_i = 0.3$; dashed: $A_i = 0.5$; dotted: $A_i = 0.7$. All shocks are normalized to 0.01 magnitude.

The second column of Figure 2 plots the effects of a positive cost-push shock (increase in u_t^c). This has textbook effects, raising inflation, lowering output, and raising interest rates when the Taylor rule has a typical calibration. The dynamic effect of a cost-push shock is barely changed by the choice of A_i. This is because cost-push shocks drive inflation *directly*: the shock appears in the Phillips curve, so its effect on inflation is not mediated by the slope coefficient $\lambda(A_i)$.

The third column of Figure 2 plots the effects of a positive monetary policy shock (increase in u_t^m). As usual, a monetary policy tightening raises the nominal interest rate, reducing output and inflation. However, the effects are determined by the A_i choice. Similar to the demand shock, the channel by which a monetary policy shock affecting the Euler equation directly transmits to inflation is via the output gap in the Phillips curve. And therefore the Phillips curve slope $\lambda(A_i)$ determines the size of the inflation response. When A_i is higher, the same monetary policy shock will cause a deeper deflation.

5.4 Optimal Monetary Policy

How should monetary policy adapt to rising AI adoption? This section analyzes optimal policy in the AI-NK model.

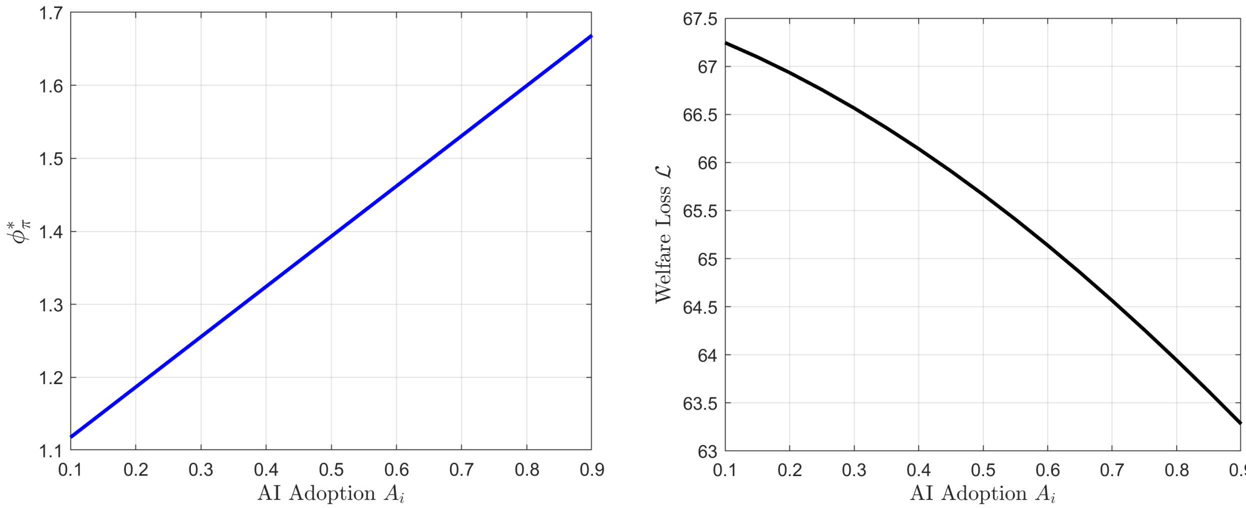
In the canonical New Keynsian model, the optimal Taylor rule inflation coefficient ϕ_π is increasing in the slope of the Phillips curve (Clarida et al., 1999; Woodford, 2003). Given that the AI-NK model is isomorphic to a traditional Calvo model, and the slope $\lambda(A_i)$ is increasing in the AI share, it comes as no surprise that the optimal inflation response is also increasing in A_i.

To demonstrate, I consider a standard objective function for the central bank:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \xi y_t^2\right] \tag{51}$$

thus the central bank makes a once-and-for-all commitment to a linear policy rule that minimizes the quadratic loss function. When the loss function represents a second-order approximation to consumer welfare losses, the $\xi > 0$ weight on the output gap is typically small. However I will choose $\xi = 1$ which more closely approximates the public's preferences elicited from surveys (Pfajfar and Winkler, 2024).

I calculate the optimal inflation coefficient ϕ_π in the model while varying the A_i share in the economy. All other parameters remain fixed, calibrated as in Table 1, so this should be interpreted as a change in the cost $\Psi(A_i)$. Figure 3a plots the optimal inflation response coefficient. As expected, when A_i increases, optimal ϕ_π increases as well. This is because $\lambda(A_i)$ determines how sensitive inflation is to the output gap, so when A_i is larger, small changes to real activity cause large changes to inflation. Therefore the central bank is incentivized to respond more aggressively. Figure 3b shows the loss (51) achieved by the central bank, which is decreasing in A_i. Prices are relatively more flexible, so when monetary policy is conducted optimally, the dynamic loss is reduced. Thus the dynamic welfare consequences of increased A_i mirror the steady-state consequences from Section 3.4.



(a) Optimal inflation response ϕ_π^*

(b) Minimized welfare loss $\mathcal{W}$

Figure 3: Optimal monetary policy as a function of AI adoption. Panel (a): optimal inflation coefficient in the Taylor rule. Panel (b): minimized welfare loss under optimal policy.

6 Conclusion

This paper developed a tractable model of AI-driven price setting, in which firms choose the complexity of their pricing models. The model implied an AI Phillips curve in which the slope depends on the degree of AI adoption. Embedded in a traditional New Keynesian framework, the AI-NK model shows that higher AI adoption steepens the Phillips curve, amplifying inflation responses to demand and monetary shocks. Optimal monetary policy responds more aggressively to inflation when AI adoption is higher.

In the introduction I made four predictions about the effects of AI on inflation dynamics. Here I summarize how the analysis supports each prediction. First, AI pricing will continue to rise as costs decline: Proposition 1 shows that when the compute cost ψ or fixed cost Ψ declines, firms optimally choose higher model complexity A_i. Second, as AI pricing rises, prices will become more flexible, as measured by the frequency of price adjustment: the Calvo-plus structure implies that modeled factors adjust more frequently than unmodeled factors, so as A_i increases the overall frequency of price adjustment F increases as well (equation (39)). Third, the Phillips curve will become steeper, so that inflation is more elastic to real costs: the AI Phillips curve slope $\lambda(A_i)$ is increasing in A_i (equation (38)), so as firms adopt more AI pricing the Phillips curve steepens. Fourth, monetary policy, when conducted optimally, will be more responsive to inflation, and thus interest rates will become more volatile: Figure 3a shows that the optimal inflation response coefficient ϕ_π increases in A_i, so optimal monetary policy responds more aggressively to inflation when AI adoption is higher.

Economists need more and better data on AI pricing. The predictions are mainly qualitative, because there is limited empirical evidence to discipline the observed A_i level or its cost structure. Future work should focus on measuring AI pricing adoption more accurately, and in particular on estimating how AI pricing affects micro pricing moments such as the FPA or moments of price changes. With better measurement, this theory and future refinements will be able to give more precise quantitative predictions about how AI pricing will affect inflation dynamics and monetary policy.

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Appendix A Proofs

Proof of Proposition 1. All results follow from implicit differentiation of the first-order condition (26). The left-hand side is $Mv_i(0) + \frac{\mathrm{B}MV_{\hat{z}}}{\rho}$, and the right-hand side is $\Psi'(A_i)$, which is increasing in A_i if Ψ is strictly convex.

Increasing $\mathbf{B}$ or M raises both terms on the left-hand side. Thus A_i^* rises to maintain equality, proving the first and second properties.

Next, if the compute cost of adjustment ψ falls, $v_i(0)$ must increase (become less negative) because the firm must become more valuable when adjustments are cheaper. This increases the left-hand side, so A_i^* must rise to restore equality. Additionally, modeled factors A_i must have higher value functions than unmodeled factors, so increasing A_i must unambigously increase the firm's value, proving the third property.

A uniform increase in the model complexity cost $\Psi(A_i)$ increases the right-hand side, so A_i must fall, and modeled factors have higher value functions, proving the fourth property.

If instead Ψ is strictly concave, then $\Psi'(A_i)$ is decreasing in A_i, and the logic is reversed. If strictly concave, there may not be a unique solution in general, but uniqueness remains under marginal parameter changes.

□

Proof of Lemma 2. From (32), the mispricing cost is proportional to the cross-sectional variance of price gaps $\tilde{p}_{ijt} \equiv \log P_{ijt} - \log Z_{ijt} - \mu - \log W_t$ in the steady state. Per equation (13) this relates to the distribution of (negative) firm losses $\tilde{h}^{\mathcal{L}}(i)$:

$$\mathcal{C}_\chi = -\frac{\eta}{2\mathcal{B}} \int_0^1 \mathcal{L}_i \tilde{h}^{\mathcal{L}}(i) di$$

where I have dropped time subscripts to reflect the steady state. Note that the unit measure of firms are ex ante symmetric and have a continuous distribution of products, so all firms make the same loss and $\tilde{h}^{\mathcal{L}}(i) = 1$.

Lemma 1 implies that firm losses are quadratic in gross gaps:

$$\mathcal{C}_\chi = \frac{\eta}{2} \int_0^1 M \left(\int_0^{A_i} g_{it}(k)^2 \, dk + (1 - A_i) V_{\hat{z}} \right) di = \frac{\eta}{2} M \left(\int_0^{A_i} \int_0^1 g_{it}(k)^2 \, di dk + (1 - A_i) V_{\hat{z}} \right)$$

$$\implies \mathcal{C}_\chi = \frac{\eta}{2} M \left(A_i \int_{-\bar{x}}^{\bar{x}} x^2 h(x) dx + (1 - A_i) V_{\hat{z}} \right) \tag{52}$$

using that $g_{it}(k) = x_{it}(k)$ in the steady state, and that loading gaps are distributed symmetrically across k and i with steady state distribution $h(x)$.

For $k \leq A_i$ modeled factors, Lemma 3 gives the ergodic density $f^{\mathrm{CP}}(x) \propto \sinh(s(\bar{x} - |x|))$ on $[-\bar{x}, \bar{x}]$. The variance is

$$\int_{-\bar{x}}^{\bar{x}} x^2 f^{\mathrm{CP}}(x)\, dx = \frac{\sigma_\beta^2}{\zeta}\big(1 - \operatorname{sech}(s\bar{x})\big)$$

For $k > A_i$ unmodeled factors, Lemma 4 gives the Laplace ergodic density with variance

$$\int_{-\infty}^{\infty} x^2 f^{\mathrm{C}}(x)\, dx = \frac{\sigma_\beta^2}{\zeta}$$

Substituting into (52) yields $\mathcal{C}_\chi = \frac{\eta M \sigma_\beta^2}{2\zeta}[1 - A_i \operatorname{sech}(s\bar{x})]$.

The compute cost from (33) is

$$\mathcal{C}_\psi = (F(A_i) - \zeta)\, \psi = \big(A_i F^{\mathrm{CP}} + (1 - A_i)F^{\mathrm{C}} - \zeta\big)\, \psi$$

with $F^{\mathrm{CP}} = \zeta \cosh(s\bar{x})/(\cosh(s\bar{x}) - 1)$ and $F^{\mathrm{C}} = \zeta$ per Proposition 3. Substituting,

$$= (A_i\zeta \cosh(s\bar{x})/(\cosh(s\bar{x}) - 1) - A_i\zeta)\, \psi = (1/(\cosh(s\bar{x}) - 1))\, A_i\zeta\, \psi$$

Summing the $\mathcal{C}_\chi$ and $\mathcal{C}_\psi$ components completes the proof. □

Proof of Theorem 1. The factor k MFG is isomorphic to a Calvo-plus menu cost model. Property 1 implies that the discrete time linear approximation is

$$x_{k,t}^* = (1 - \beta\theta_k)w_t + \beta\theta_k\, \mathbb{E}_t x_{k,t+1}^* \tag{53}$$

$$x_{k,t} = (1 - \theta_k)x_{k,t}^* + \theta_k\, x_{k,t-1} \tag{54}$$

where $x_{k,t}$ is the discretizaiton of the average gap $X_k(t)$, $x_{k,t}^*$ is the discretization of the reset point $x_k^*(t)$, and θ_k is the factor k coefficient.

Combine equations (53) and (54) to get the factor k Phillips curve:

$$x_{k,t} - x_{k,t-1} = \frac{(1 - \theta_k)(1 - \beta\theta_k)}{\theta_k} mc_t + \beta\mathbb{E}_t\left[x_{k,t+1} - x_{k,t}\right] \tag{55}$$

Aggregate across factors using $p_t = \int_0^1 x_{k,t}\, dk$:

$$p_t - p_{t-1} = \left(\int_0^1 \frac{(1 - \theta_k)(1 - \beta\theta_k)}{\theta_k}\, dk\right) mc_t + \beta\mathbb{E}_t\left[p_{t+1} - p_t\right] \tag{56}$$

Then substitute with the inflation rate $\pi_t = p_t - p_{t-1}$ and AI slope $\lambda(A_i)$ to complete the proof. $\square$

A.1 Price Change Moments

Proof of Proposition 3. The Calvo-plus model with zero drift has a symmetric inaction region $(-\bar{x}, \bar{x})$ in steady state. Define $s \equiv \sqrt{2\zeta/\sigma_\beta^2}$ and let $h(x)$ denote the steady-state density from Lemma 3.

Frequency. Price changes occur either at the boundaries $\pm\bar{x}$ (endogenous adjustment) or at Poisson resets (exogenous adjustment). Let $F_{\bar{x}}$ denote the boundary-adjustment frequency. The diffusive probability flux at the boundaries is

$$F_{\bar{x}} = \frac{\sigma_\beta^2}{2}\big(h'(-\bar{x}) - h'(\bar{x})\big)$$

Using Lemma 3,

$$h'(-\bar{x}) = -h'(\bar{x}) = \frac{s^2}{2(\cosh(s\bar{x}) - 1)}$$

so $F_{\bar{x}} = \frac{\sigma_\beta^2 s^2}{2(\cosh(s\bar{x})-1)} = \frac{\zeta}{\cosh(s\bar{x})-1}$. Adding Poisson resets yields

$$F^{\mathrm{CP}} = F_{\bar{x}} + \zeta = \frac{\zeta\cosh(s\bar{x})}{\cosh(s\bar{x}) - 1}$$

which is equation (41).

Second moment. Conditional on an adjustment, the price change equals $\pm\bar{x}$ at boundary adjustments and equals $-x$ at a Poisson reset from interior state x. Therefore

$$\mathbb{E}[(\Delta p)^2] = \frac{F_{\bar{x}}}{F^{\mathrm{CP}}}\bar{x}^2 + \frac{\zeta}{F^{\mathrm{CP}}}\int_{-\bar{x}}^{\bar{x}} x^2 h(x)\,dx$$

The integral evaluates to

$$\int_{-\bar{x}}^{\bar{x}} x^2 h(x)\,dx = \frac{2}{s^2} - \frac{\bar{x}^2}{\cosh(s\bar{x}) - 1}$$

Substituting, using $F_{\bar{x}}/F^{\mathrm{CP}} = \operatorname{sech}(s\bar{x})$ and $\zeta/F^{\mathrm{CP}} = 1 - \operatorname{sech}(s\bar{x})$, and simplifying yields equation (42).

Fourth moment. The fourth central moment is

$$\mathbb{E}[(\Delta p)^4] = \frac{F_{\bar{x}}}{F^{\mathrm{CP}}}\bar{x}^4 + \frac{\zeta}{F^{\mathrm{CP}}}\int_{-\bar{x}}^{\bar{x}} x^4 h(x)\,dx$$

The integral evaluates to

$$\int_{-\bar{x}}^{\bar{x}} x^4 h(x)\,dx = \frac{24}{s^4} - \frac{\bar{x}^4}{\cosh(s\bar{x}) - 1} - \frac{6\bar{x}^2}{s^2(\cosh(s\bar{x}) - 1)}$$

Using $F_{\bar{x}}/F^{\text{CP}} = \operatorname{sech}(s\bar{x})$ and $\zeta/F^{\text{CP}} = 1 - \operatorname{sech}(s\bar{x})$, the $\bar{x}^4$ terms cancel and

$$\mu_4^{\text{CP}} = \mathbb{E}[(\Delta p)^4] = \frac{24}{s^4}\big(1 - \operatorname{sech}(s\bar{x})\big) - \frac{6\bar{x}^2}{s^2}\operatorname{sech}(s\bar{x})$$

Substituting $s^2 = 2\zeta/\sigma_\beta^2$ gives equation (44). The kurtosis is $\kappa_{\Delta p}^{\text{CP}} = \mu_4^{\text{CP}}/\big((\sigma_{\Delta p}^{\text{CP}})^2\big)^2$.

Unmodeled factors follow a traditional Calvo model, so there is no inaction region: prices only reset at Poisson rate ζ. Between resets, the price gap x follows a Brownian motion with variance σ_β^2. The frequency of adjustment is simply the Poisson rate: $F^{\text{C}} = \zeta$.

At each reset, the price change equals the current price gap, drawn from the ergodic Laplace distribution (Lemma 4). The variance is:

$$(\sigma_{\Delta p}^{\text{C}})^2 = \mathbb{E}[x^2] = \frac{\sigma_\beta^2}{\zeta}$$

By Lemma 4, the ergodic distribution is Laplace with kurtosis 6. Since price changes equal the price gap at reset, $\kappa_{\Delta p}^{\text{C}} = 6$.

The economy-wide moments aggregate the Calvo-plus and Calvo components. The total adjustment intensity is $F = A_i F^{\text{CP}} + (1 - A_i)F^{\text{C}}$. An observed price change is drawn from the Calvo-plus component with probability $A_i F^{\text{CP}}/F$ and from the Calvo component with probability $(1 - A_i)F^{\text{C}}/F$. Therefore the economy-wide second and fourth central moments are intensity-weighted averages, which gives equations (49) and (50).

□

Appendix B Steady-State Distributions

This appendix derives the ergodic distributions for both Calvo-plus (modeled) and pure Calvo (unmodeled) factors.

Lemma 3 (Calvo-Plus Ergodic Distribution)**.** *Consider a price gap x that follows a Brownian motion with variance σ_β^2, is reset to zero when $|x|$ hits the boundary $\bar{x}$, and is also reset to zero at Poisson* *rate ζ. In steady state, the distribution $h(x)$ on $[-\bar{x}, \bar{x}]$ satisfies*

$$\frac{\sigma_\beta^2}{2}\partial_{xx}h(x) = \zeta h(x) - F\delta(x) \tag{57}$$

with absorbing boundary conditions $h(\pm\bar{x}) = 0$. Define $s \equiv \sqrt{2\zeta/\sigma_\beta^2}$. The solution is

$$h(x) = \frac{s}{2(\cosh(s\bar{x}) - 1)}\sinh\big(s(\bar{x} - |x|)\big) \tag{58}$$

for $x \in [-\bar{x}, \bar{x}]$, and $h(x) = 0$ otherwise.

Proof. Away from the reset point, the KFE reduces to $\frac{\sigma_\beta^2}{2}h''(x) = \zeta h(x)$, which has characteristic roots $\pm s$. On $[0, \bar{x}]$, the general solution is $h(x) = Ae^{sx} + Be^{-sx}$. The boundary condition $h(\bar{x}) = 0$ gives $A = -Be^{-2s\bar{x}}$. Symmetry requires $h'(0) = 0$ for the left and right pieces to match smoothly. Normalization $\int_{-\bar{x}}^{\bar{x}} h(x)\,dx = 1$ pins down the constant. The result follows. □

Lemma 4 (Calvo Ergodic Distribution)**.** *Consider a price gap x that follows a Brownian motion with variance σ_β^2 and is reset to zero at Poisson rate ζ (with no boundaries). In steady state, the distribution $h(x)$ on $\mathbb{R}$ satisfies*

$$\frac{\sigma_\beta^2}{2}\partial_{xx}h(x) = \zeta h(x) - \zeta\delta(x) \tag{59}$$

Define $s \equiv \sqrt{2\zeta/\sigma_\beta^2}$. The solution is the Laplace (double-exponential) distribution:

$$h(x) = \frac{s}{2}e^{-s|x|} \tag{60}$$

This distribution has mean zero, variance σ_β^2/ζ, and kurtosis 6.

Proof. Away from the origin, the KFE reduces to $\frac{\sigma_\beta^2}{2}h''(x) = \zeta h(x)$, with solutions $h(x) = Ce^{-s|x|}$ that decay as $|x| \to \infty$. At $x = 0$, the delta function source requires a jump in the derivative: $h'(0^+) - h'(0^-) = -2\zeta/\sigma_\beta^2 \cdot 1 = -s^2$. With $h(x) = Ce^{-s|x|}$, we have $h'(0^+) = -Cs$ and $h'(0^-) = Cs$, so the jump is $-2Cs = -s^2$, giving $C = s/2$. The variance is $\mathbb{E}[x^2] = 2/s^2 = \sigma_\beta^2/\zeta$. The fourth moment is $\mathbb{E}[x^4] = 24/s^4$, so the kurtosis is $\kappa = (24/s^4)/(2/s^2)^2 = 6$. □