THE UNIVERSITY OF TEXAS AT AUSTIN
McCombs School of Business

STA 372.5 **Spring 2018**

HOMEWORK #8 – Due Wednesday, April 11

1. Consider the following ARIMA(1, 1, 1) model for Y_t:

$$W_t = Y_t - Y_{t-1}$$

where

$$W_t = \mu + \beta(W_{t-1} - \mu) + \varepsilon_t + \theta\varepsilon_{t-1} \qquad \varepsilon_t \text{ iid } N(0, \sigma^2).$$

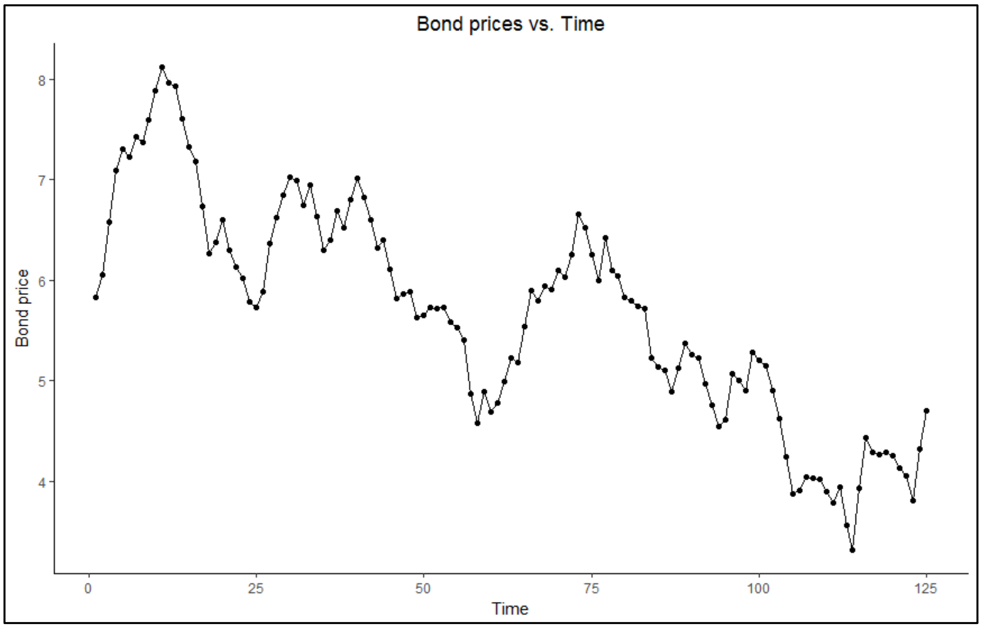
Suppose $\mu = 0$, $\beta = 0.5$, $\theta = -0.2$, $\sigma = 0.1$, $Y_{99} = 8.4$, $Y_{100} = 8.5$, and $\varepsilon_{100} = 0.05$.

(a) What is the distribution of Y_{101} given information through time period $t = 100$? What is the forecast of Y_{101} given information through time period $t = 100$? What is the probability the actual value of Y_{101} that occurs is between 8.4 and 8.6?

(b) What is the distribution of Y_{102} given information through time period $t = 100$? What is the forecast of Y_{102} given information through time period $t = 100$? What is the probability the actual value of Y_{102} that occurs is between 8.4 and 8.6?

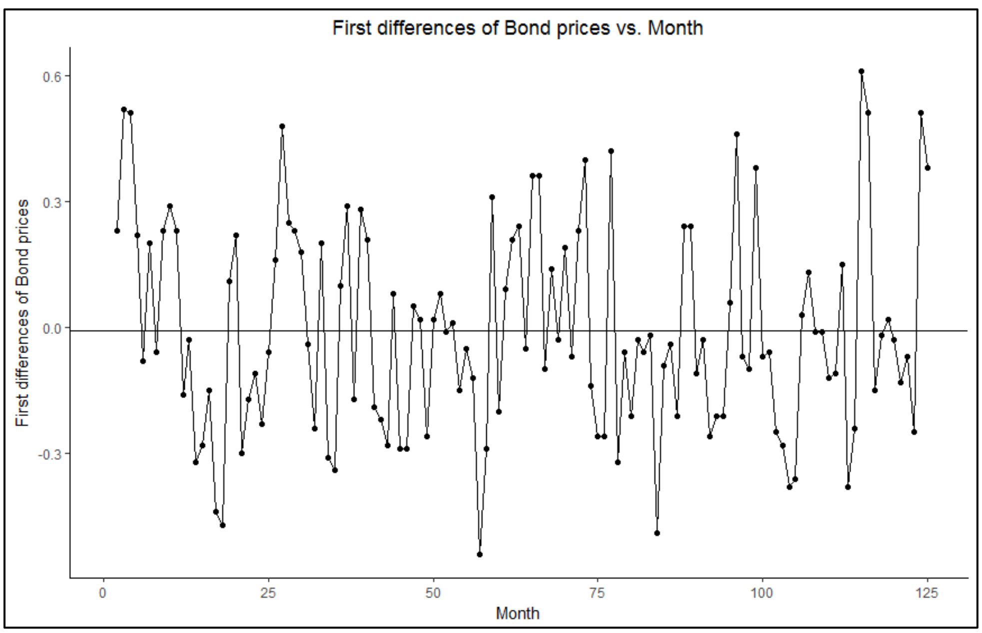
2. The file STA372_Homework8_Question2.dat on the *Data sets* page of the Canvas class website contains monthly U.S. 10-year bond yields for the period January, 1994 - May, 2004 (i.e. 125 months of data). You are asked to compute forecasts and 80% confidence intervals for 10-year bond yields in June through October of 2004 (i.e. in months 126-130). Note that there is no seasonality in the data so the original data will be analyzed rather than seasonally adjusted data.

(a) Plot the monthly U.S. 10-year bond yields for the period January, 1994 - May, 2004. Based on a visual inspection, do the 10-year bond yields appear to be stationary?



(b) Compute the autocorrelation function and augmented Dickey-Fuller test statistic for the monthly U.S. 10-year bond yields. Do they provide evidence that the time series of bond yields is nonstationary? Why or why not?

(c) Plot the first differences of the monthly U.S. 10-year bond yields for the period January, 1994 - May, 2004. Based on a visual inspection, do the first differences appear to be stationary?



(d) Compute the autocorrelation function and augmented Dickey-Fuller test statistic for the first differences of the monthly U.S. 10-year bond yields. Do they provide evidence that the first differences are stationary? Why or why not?

(e) Let Y_t represent the monthly U.S. 10-year bond yield in time period t and let $W_t = Y_t - Y_{t-1}$ represent their first differences.

What evidence is there in the autocorrelation function obtained in part (d) that the MA(1) model

$$W_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1} \qquad \varepsilon_t \text{ iid } N(0, \sigma^2)$$

is the appropriate model to use for the first differences W_t?

(f) Estimate the parameters μ, θ, and σ using the Arima command in R with an ARIMA(0, 1, 1) model. What are the estimates?

(g) Are the errors from this model independent?

(h) Using the appropriate commands in R, compute and plot the forecasts and 80% confidence intervals for the U.S. 10-year bond yields in June through October of 2004 (i.e. in time periods t = 126-130).

(i) What will happen to the 80% confidence intervals for the U.S. 10-year bond yields the further out in time the predictions are made? You do not need to compute any additional confidence intervals to answer this question. You only need to think about what the properties are for the confidence intervals of a nonstationary process as the forecasting horizon increases.