Problem Set 1

1. How many integers are there from 25 to 79 inclusive?

2. What is the 53rd integer in the sequence 86, 87, 88, $\cdots$?

3. The largest of 123 consecutive integers is 307. What is the smallest?

4. The smallest of r consecutive integers is n. What is the largest?

5. The largest of r consecutive integers is k. What is the smallest?

6. How many integers are there in the sequence $n,\ n+1,\ n+2,\ \cdots,\ n+h$?

7. How many integers x satisfy the inequalities $12 < \sqrt{x} < 15$, that is $\sqrt{x}$ exceeds 12, but $\sqrt{x}$ is less than 15?

8. How many integers are there in the sequences

   (a) 60, 70, 80, $\cdots$, 540; (b) 15, 18, 21, $\cdots$, 144;

   (c) 17, 23, 29, 35, $\cdots$, 221?

9. How many integers between 1 and 2000 (a) are multiples of 11; (b) are multiples of 11 but not multiples of 3; (c) are multiples of 6 but not multiples of 4?

10. What is the smallest number of coins needed to pay in exact change any charge less than one dollar? (Coins are in the denominations 1, 5, 10, 25 and 50 cents.)

11. A man has 47 cents in change coming. Assuming that the cash register contains an adequately large supply of 1, 5, 10 and 25 cent coins, with how many different combinations of coins can the clerk give the man his change?

12. A man has six pairs of cuff links scrambled in a box. No two pairs are alike. How many cuff links does he have to draw out all at once (in the dark) in order to be certain to get a pair that match?

13. A man has twelve blue socks and twelve black socks scrambled in a drawer. How many socks does he have to draw out all at once (in the dark) to be certain to get a matching pair? (Any two blue socks, or any two black socks, constitute a pair.)

14. The measure in degrees of an angle of a regular polygon is an integer. How many sides can such a polygon have?

15. A man has a large supply of wooden regular tetrahedra, all the same size. (A regular tetrahedron is a solid figure bounded by four congruent equilateral triangles; see Figure 1.3.) If he paints each triangular face in one of four colors, how many different painted tetrahedra can he make, allowing all possible combinations of colors? (Say that two blocks are different if they cannot be put into matching positions with identical colors on corresponding faces.)

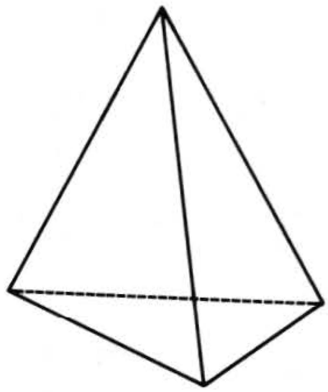

Figure 1.3

16. How many paths are there from one corner of a cube to the opposite corner, each possible path being along three of the twelve edges of the cube?

17. At formal conferences of the United States Supreme Court each of the nine justices shakes hands with each of the others at the beginning of the session. How many handshakes initiate such a session?

Problem 1.2, page 2. 10

There is 1 kind of block with six blue faces; 1 kind with five blue faces; 2 kinds with four blue faces, because the two red faces may be opposite or adjacent to each other; 2 kinds with three blue faces, because there may be, or may not be, two blue faces opposite one another. The number of different kinds of blocks with two blue faces is the same as the number with four blue faces; with one blue face, the same as with five blue faces; with no blue face, the same as six blue faces.

Problem 1.3, page 3, is solved on page 27.

Problem 1.4, page 4, is solved on page 59.

Problem 1.5, page 4, is solved on page 106.

Problem Set 1, page 5

1. 55 **2.** 138 **3.** 185 **4.** $n + r - 1$ **5.** $k - r + 1$ **6.** $h + 1$

7. 80 integers from $x = 145$ to $x = 224$.

8. (a) 49 (b) 44 (c) 35

Argument for part (c). Subtract 11 from each integer: 6, 12, 18, 24, $\cdots$, 210. Divide each by 6: 1, 2, 3, 4, $\cdots$, 35. These operations have not changed the number of elements.

9. (a) 181: the integers 11, 22, 33, $\cdots$, 1991;
(b) 121: delete from the integers in (a) the following: 33, 66, 99, $\cdots$, 1980; these are 60 in number; hence $181 - 60$;
(c) 167: delete from 6, 12, 18, 24, $\cdots$, 1998 the integers 12, 24, 36, $\cdots$, 1992; thus $333 - 166$.

10. 9: 4 cents, 2 nickels, 1 dime, 1 quarter, 1 fifty-cent piece (alternatively, replace 2 nickels, 1 dime by 1 nickel, 2 dimes).

11. 39

Let a, b, c denote a cents, b nickels, c dimes. Then with no 25 cent piece the solutions in triples a, b, c are

47, 0, 0	42, 1, 0	37, 2, 0	37, 0, 1	32, 3, 0
32, 1, 1	27, 4, 0	27, 2, 1	27, 0, 2	22, 5, 0
22, 3, 1	22, 1, 2	17, 6, 0	17, 4, 1	17, 2, 2
17, 0, 3	12, 7, 0	12, 5, 1	12, 3, 2	12, 1, 3
7, 8, 0	7, 6, 1	7, 4, 2	7, 2, 3	7, 0, 4
2, 9, 0	2, 7, 1	2, 5, 2	2, 3, 3	2, 1, 4

With one 25 cent piece the solutions are

22, 0, 0	17, 1, 0	12, 2, 0	12, 0, 1	7, 3, 0
7, 1, 1	2, 4, 0	2, 2, 1	2, 0, 2	

12. 7

13. 3

14. 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360

A regular polygon with n sides has exterior angle $360/n$ degrees, and so an interior angle has size $360 - (360/n)$ degrees. Hence we choose all positive integers n such that $360/n$ is an integer, except $n = 1$ and $n = 2$.

15. 36

Let the colors be denoted by R, G, B and W, say for red, green, blue and white. There are 4 cases of solids painted one color: all R, all G, all B, all W. Solids painted two colors yield 18 cases: if the colors are R and G there are 3 cases because the number of R-faces may be 1, 2 or 3; similarly there are 3 cases for each of the other color combinations RB, RW, GB, GW, BW. There are 12 different kinds of solids painted three colors: if the colors are R, G, B, there are 3 cases—for example, one case with two R-faces, one G-face and one B-face. Solids painted four colors can be of 2 kinds: orient the tetrahedron so that the bottom is R and there is a G-face towards you; then the other two faces can be BW or WB.

16. 6

17. 36