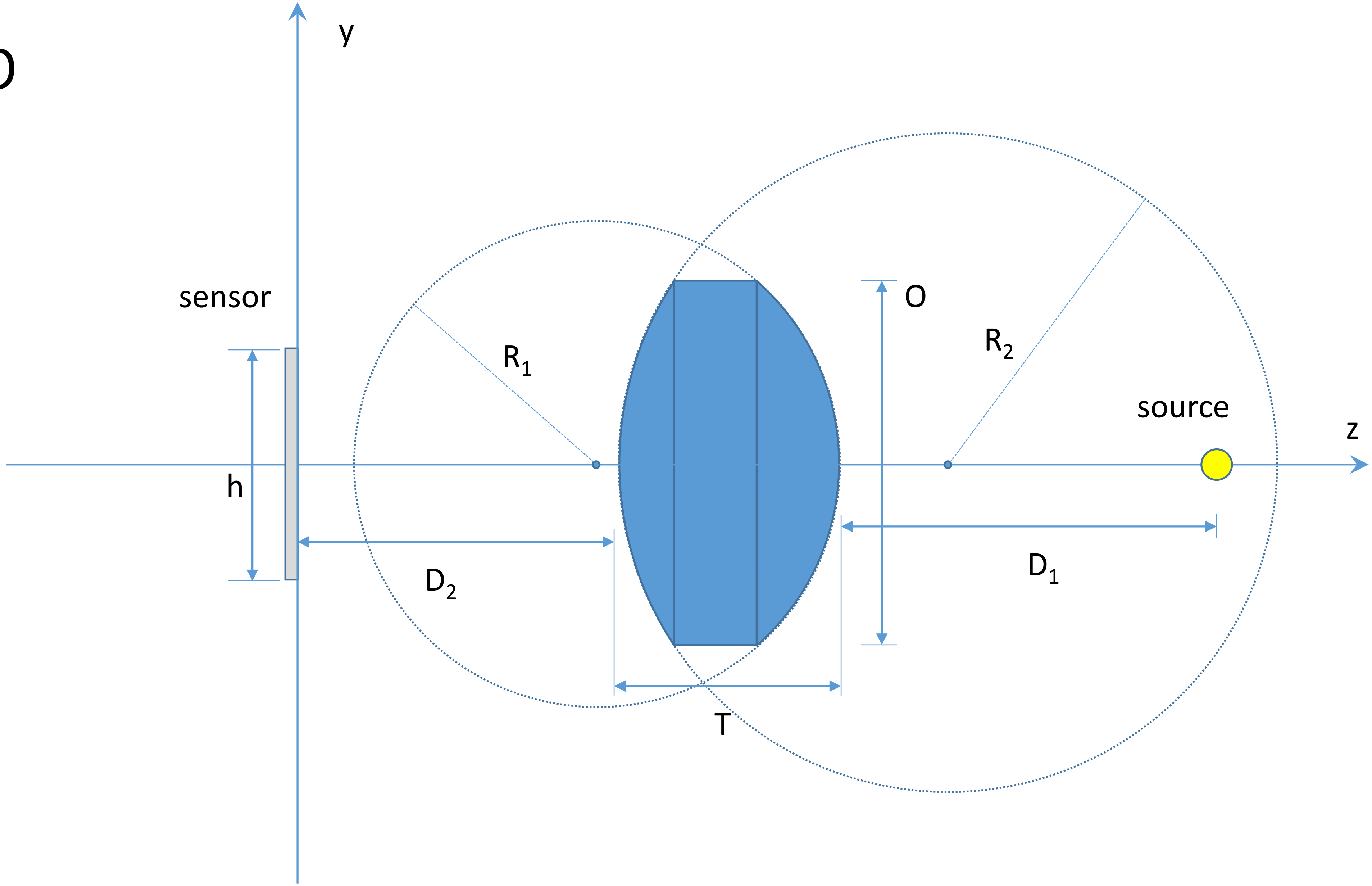
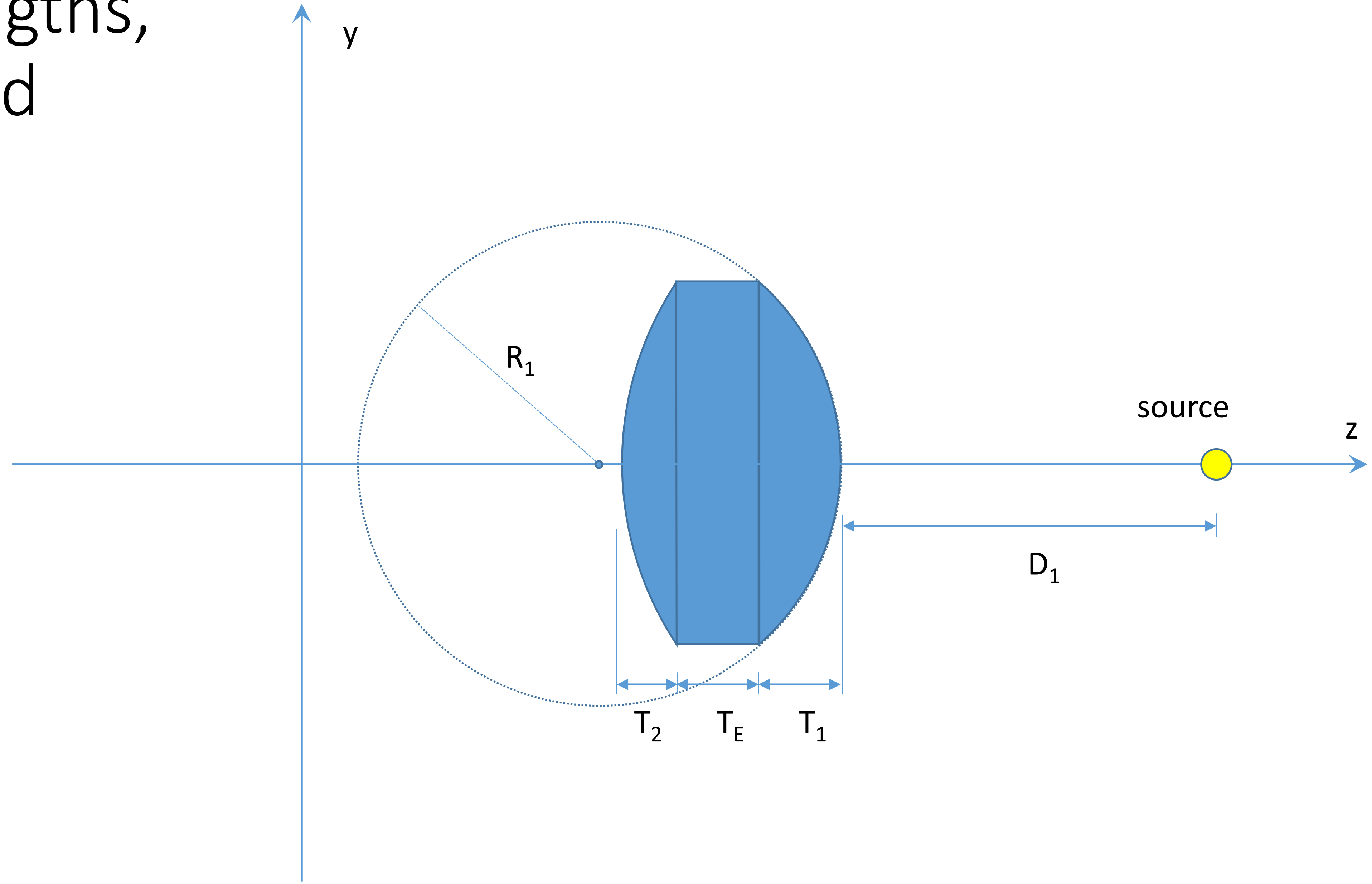
2D Set up



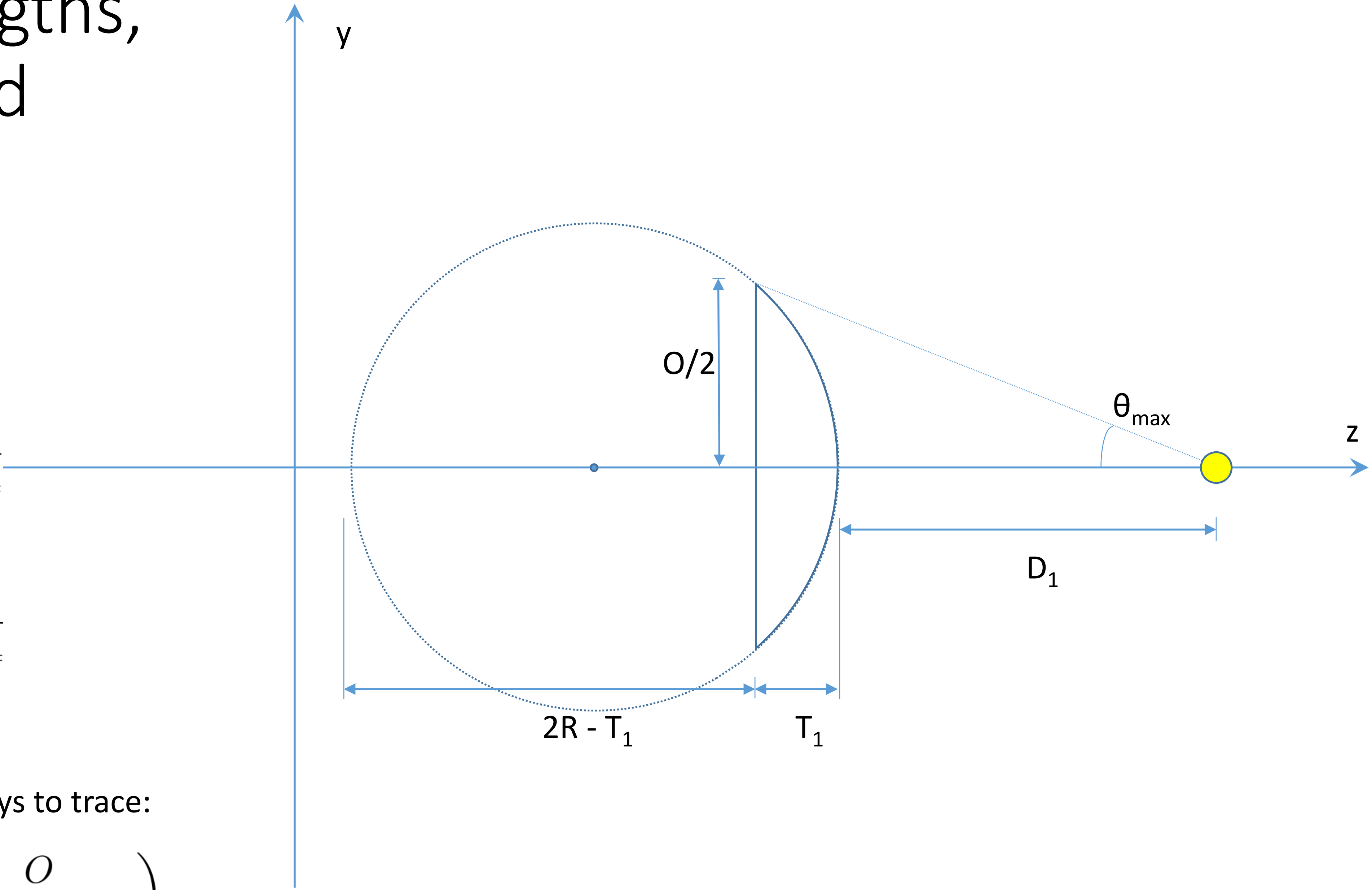
Useful lengths, points, and bounds



Useful lengths, points, and bounds

Power of a point:

$$(2R_1 - T_1)T_1 = O^2/4$$
$$\Rightarrow T_1^2 - 2R_1 + O^2/4 = 0$$
$$\Rightarrow T_1 = R_1 - \sqrt{R_1^2 - O^2/4}$$



Similarly for sphere 2:

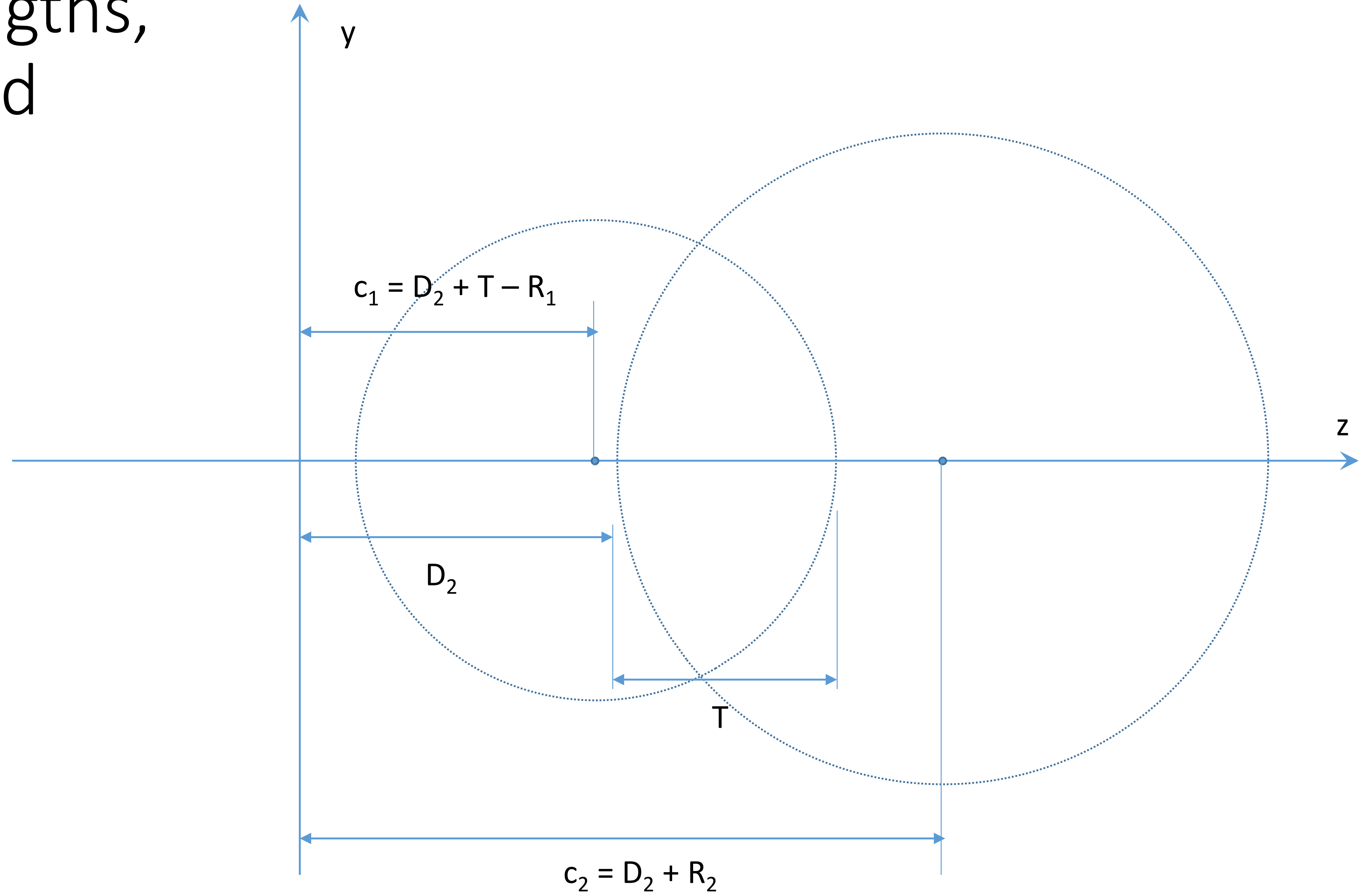
$$T_2 = R_2 - \sqrt{R_2^2 - O^2/4}$$

Bound on angles for rays to trace:

$$\theta_{\max} = \tan^{-1}\left(\frac{O}{2(D_1 + T_1)}\right)$$

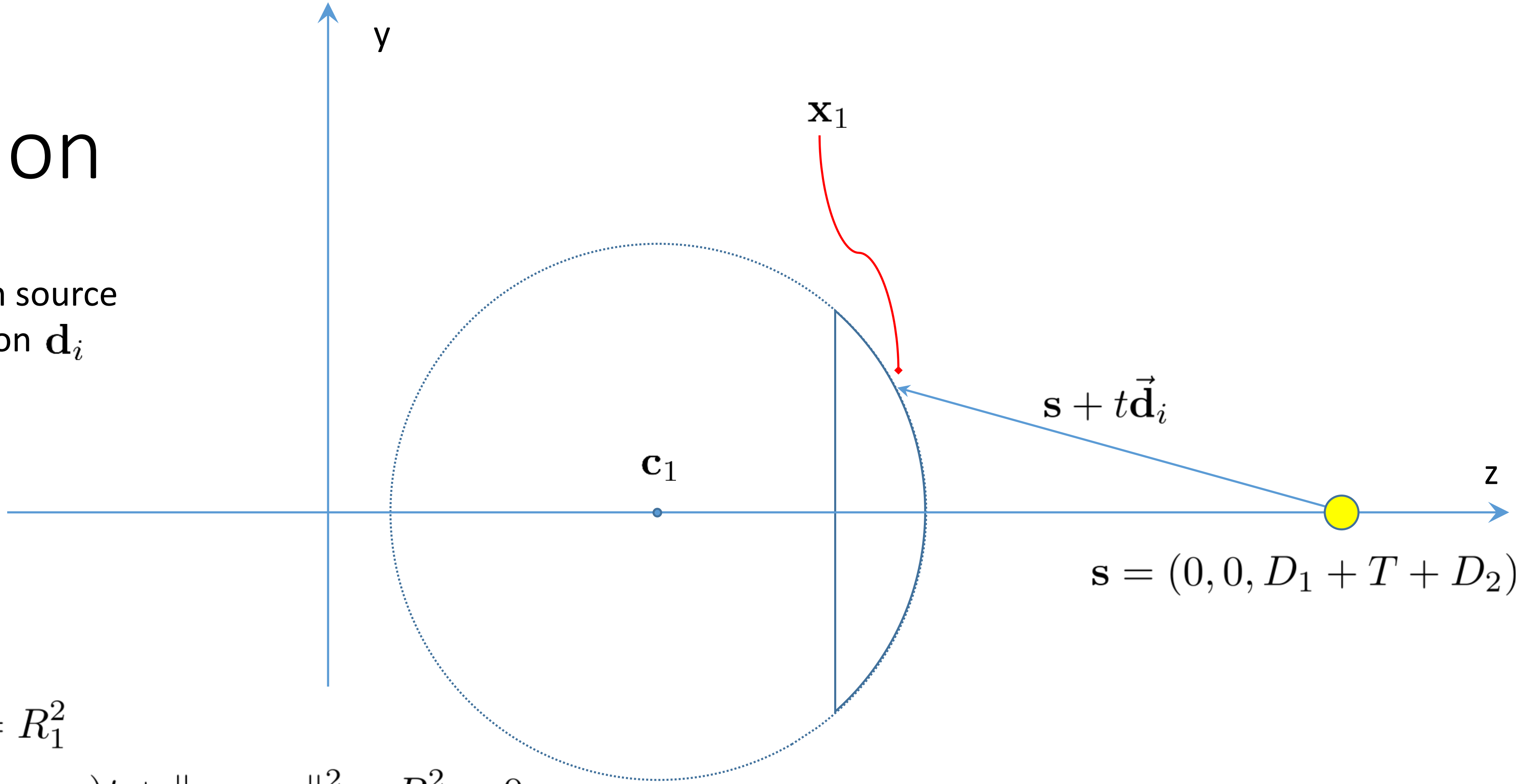
Useful lengths, points, and bounds

Centers of spheres



First intersection

Trace ray index i from source in normalized direction $\mathbf{d}_i$



Ray intersect sphere:
Solve for t:

$$\|\mathbf{s} + t\mathbf{d}_i - \mathbf{c}_1\|_2^2 = R_1^2$$

$$\Rightarrow \|\mathbf{d}_i\|_2^2 t^2 + 2\mathbf{d}_i \cdot (\mathbf{s} - \mathbf{c}_1)t + \|\mathbf{s} - \mathbf{c}_1\|_2^2 - R_1^2 = 0$$

$$\Rightarrow t = -\mathbf{d}_i \cdot (\mathbf{s} - \mathbf{c}_1) \pm \sqrt{(\mathbf{d}_i \cdot (\mathbf{s} - \mathbf{c}_1))^2 - \|\mathbf{s} - \mathbf{c}_1\|_2^2 + R_1^2} \qquad (1)$$

$$\Rightarrow t = -\mathbf{d}_i \cdot (\mathbf{s} - \mathbf{c}_1) - \sqrt{(\mathbf{d}_i \cdot (\mathbf{s} - \mathbf{c}_1))^2 - \|\mathbf{s} - \mathbf{c}_1\|_2^2 + R_1^2} \qquad (2)$$

(Choose minus because we want first intersection)

First refraction

Find the normal:

$$\mathbf{n}_1 = \frac{\mathbf{x}_1 - \mathbf{c}_1}{\|\mathbf{x}_1 - \mathbf{c}_1\|}$$

Let

$$\vec{\mathbf{d}}_{\mathbf{n}} = (\vec{\mathbf{d}} \cdot \vec{\mathbf{n}})\vec{\mathbf{n}}$$

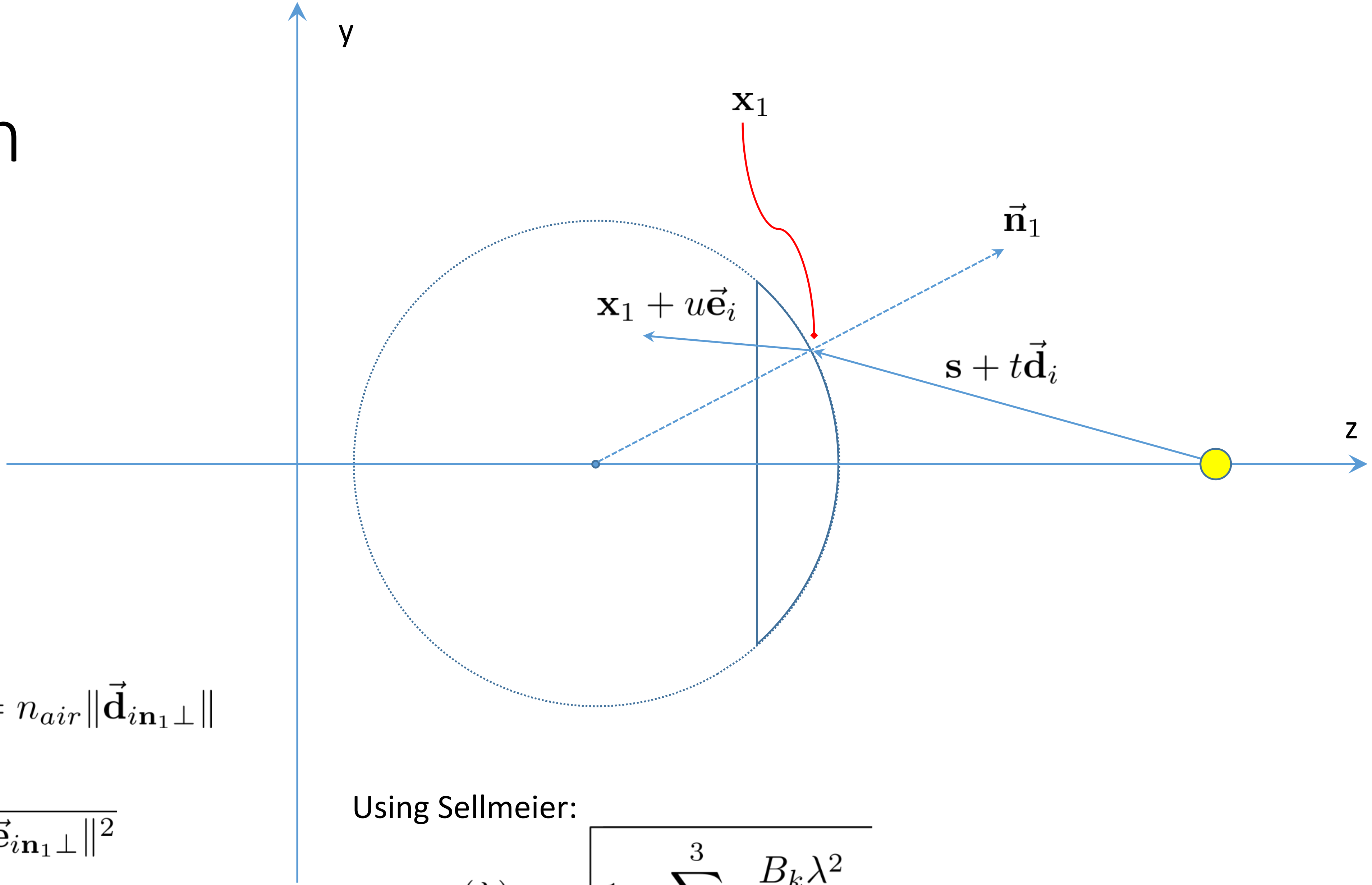
$$\vec{\mathbf{d}}_{\mathbf{n}\perp} = \vec{\mathbf{d}} - \vec{\mathbf{d}}_{\mathbf{n}}$$

Snell's Law:

$$n_{BK7}(\lambda)\|\vec{\mathbf{e}}_{i\mathbf{n}_1\perp}\| = n_{air}\|\vec{\mathbf{d}}_{i\mathbf{n}_1\perp}\|$$

Pythagorize!

$$\vec{\mathbf{e}}_{i\mathbf{n}} = -\vec{\mathbf{n}}_1\sqrt{1 - \|\vec{\mathbf{e}}_{i\mathbf{n}_1\perp}\|^2}$$



Using Sellmeier:

$$n_{BK7}(\lambda) = \sqrt{1 + \sum_{k=1}^{3} \frac{B_k\lambda^2}{\lambda^2 - C_k}}$$

Second intersection

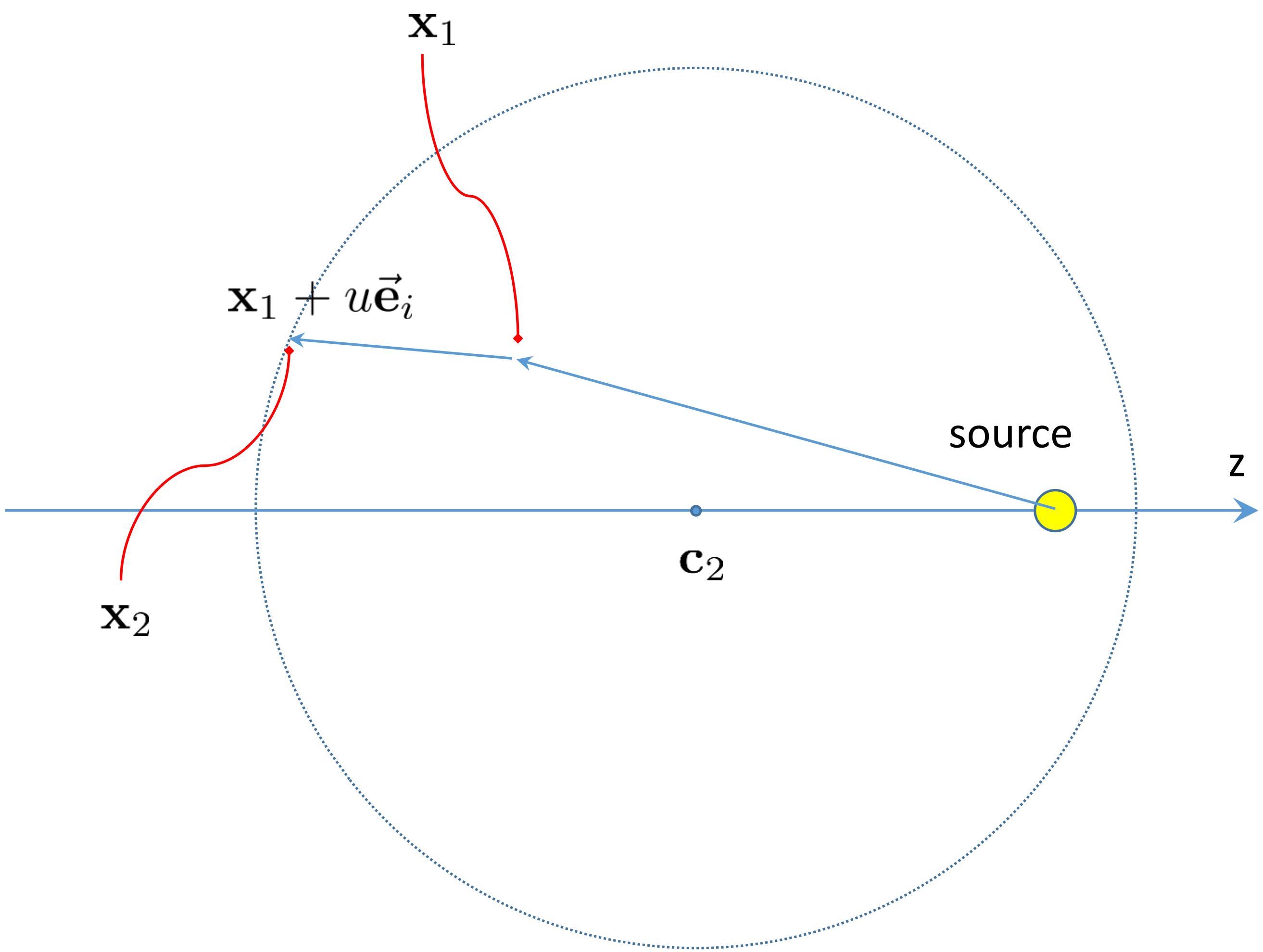
Ray intersect sphere:
Solve for u, reusing soln
from (1) of first intersection:

$$u = -\mathbf{e}_i \cdot (\mathbf{x}_1 - \mathbf{c}_2) + \sqrt{(\mathbf{e}_i \cdot (\mathbf{x}_1 - \mathbf{c}_2))^2 - \|\mathbf{x}_1 - \mathbf{c}_2\|_2^2 + R_2^2}$$

(Choose plus this time for non-negative u)

Check validity:

$$\mathbf{x}_{2z} \leq D_2 + T_2$$
$$\mathbf{x}_{2y}^2 + \mathbf{x}_{2x}^2 \leq O^2/4$$



Second refraction

Using similar notation as before:

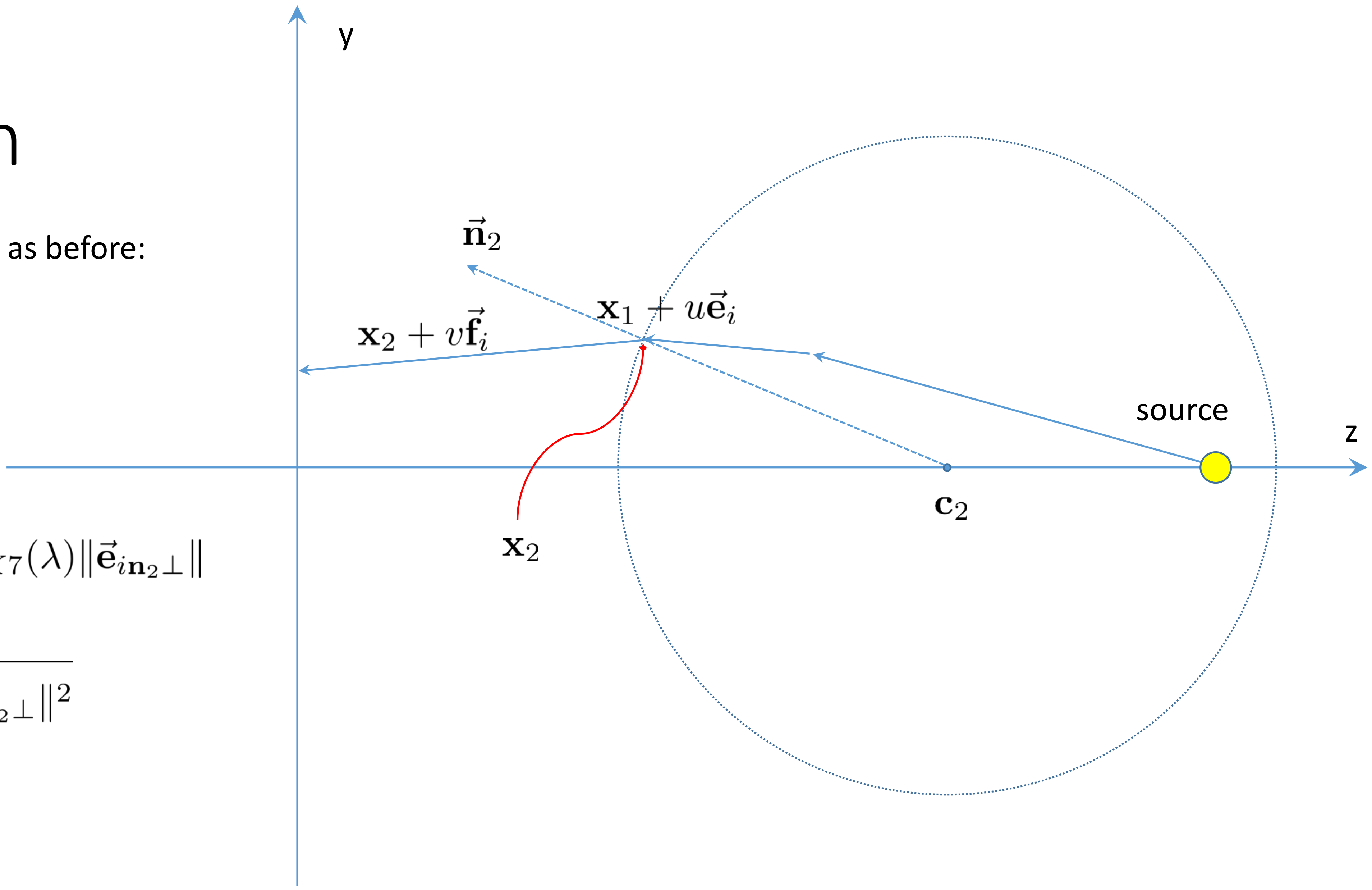
$$\mathbf{n}_2 = \frac{\mathbf{x}_2 - \mathbf{c}_2}{\|\mathbf{x}_2 - \mathbf{c}_2\|}$$

Snell's Law:

$$n_{air}\|\vec{\mathbf{f}}_{i\mathbf{n}_2\perp}\| = n_{BK7}(\lambda)\|\vec{\mathbf{e}}_{i\mathbf{n}_2\perp}\|$$

Pythagorize!

$$\vec{\mathbf{f}}_{i\mathbf{n}} = \vec{\mathbf{n}}_2\sqrt{1 - \|\vec{\mathbf{f}}_{i\mathbf{n}_2\perp}\|^2}$$



Hitting home

Solve for v:

$$x_{2z} + v\vec{f}_{iz} = 0$$