ISE 4623/5023: Deterministic Systems Models / Systems Optimization

University of Oklahoma
School of Industrial and Systems Engineering
Fall 2020

Group Assignment 1 (100 points)

Group size: 3 or 4 people

For each problem, you need to present a detailed mathematical formulation (with sets and parameters if applicable, decision variables, objective function, and constraints clearly defined). You need to prepare a self-contained report for the assignment, to be submitted in Canvas (PDF format) by the beginning of the class on the due date, along with the Gurobi/Excel support files used to solve the problems.

Problem 1 (50 points): Supply chain transportation – Multicommodity Problem

Transportation plays a significant role in a supply chain and its efficiency. In particular, reducing the transportation costs between the manufacturers, warehouses, and retailers helps to provide more affordable product prices for customers.

A major manufacturing company, MC Inc., produces its star product, Product A, to be sold at five retailers across the state. First, the products are manufactured in one of two plants, then distributed to three warehouses for storage, and finally are distributed to each of the retailers. Each retailer sends a separate order to the manufacture's head office, which is then dispatched from the appropriate warehouse to the retailer. The company has collected data regarding weekly orders and production and would like to find a way to minimize the costs of the entire operation.

To expand their market share, the company decided to fabricate a second product, Product B. To fabricate Product B, not only Plants 1 and 2 were adapted accordingly, but also a third plant, Plant 3, was constructed.

Considering the degree of specialization and the difficulty associated with making each product, as well as the machines available at each plant, you have calculated the average number of units (of each product) that an average worker would be able to make in an hour. This information is given in Table 1. For example, Table 1 indicates that the average number of units of Product A that a worker can make in one hour in Plant 1 is 0.5. In other words, to produce one unit of Product A, you could have one worker to dedicate two hours of labor, or two workers to dedicate one hour.

Table 1. Average number of units produced per person per hour

Production Facility	Product A (units/hour-person)	Product B (units/hour-person)
Plant 1	0.5	0.1
Plant 2	0.5	0.1
Plant 3	0.2	0.4

The number of workers (devoted to production) in each plant is given in Table 2. Also, assume that each employee works 48 hours per week.

Table 2. Number of workers per plant

Production Facility	Workers
Plant 1	90
Plant 2	120
Plant 3	80

The production of the three plants is delivered to the warehouses on Monday morning before the orders form the retailers are received. The capacity of the warehouses is given in Table 3.

Table 3. Warehouse storage capacities

Warehouse Facility	Total storage capacity of the warehouse (in cubic feet)
Warehouse 1	650
Warehouse 2	600
Warehouse 3	750

Keep in mind that a unit of Product A occupies one cubic foot, while one unit of Product B occupies 2 cubic feet.

The weekly orders made by the retailers for the first week of October are given in Table 4.

Table 4. Weekly orders (in number of units) made by the retailers on the first week of October

Product Request	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5
Product A	175	120	140	100	160
Product B	100	150	110	300	230

The transportation costs (per pound) from the plants to warehouses and warehouses to retailers are given in Table 5 and Table 6, respectively. Keep in mind that one unit of Product 1 weighs one pound, while one unit of Product B weighs 3 pounds.

Table 5. Transportation costs, per pound, from production plants to warehouses

	Unit Shipping cost (per pound)		
From/To	Warehouse 1	Warehouse 2	Warehouse 3
Plant 1	$25	$85	$25
Plant 2	$50	$35	$95
Plant 3	$50	$40	$55

Table 6. Transportation costs, per pound, from warehouses to retailers

	Unit Shipping Cost (per pound)				
From/To	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5
Warehouse 1	$75	$50	$60	$75	$30
Warehouse 2	$85	$15	$85	$85	$90
Warehouse 3	$90	$85	$35	$35	$95

Additionally, due to diverse company regulations, there is a maximum weekly weight that can be sent from each plant to each warehouse (shown in Table 7) and from each warehouse to each retailer (shown in Table 8).

Table 7. Maximum weekly weight (of products) that can be sent from each plant to each warehouse

	Maximum weekly amount of product shipped (in pounds)		
From/To	Warehouse 1	Warehouse 2	Warehouse 3
Plant 1	120	150	170
Plant 2	150	160	180
Plant 3	150	170	180

Table 8. Maximum weekly weight (of products) that can be sent from each warehouse to each retailer

	Maximum weekly amount of product shipped (per pound)				
From/To	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5
Warehouse 1	160	190	110	180	150
Warehouse 2	170	190	150	140	120
Warehouse 3	140	160	180	120	100

In case you do not fully satisfy the demand of one or more retailers, you agreed to pay the retailers $500 per unit of Product A not supplied, and $1000 per unit of Product B not supplied.

Using the provided information, you are asked to:

1. (5 points) Draw the network that represents the company's distribution network.
2. (20 points) What is the transportation strategy (number of each product shipped from each plant to each warehouse, and from each warehouse to each retailer) that minimizes the total cost? Mathematically formulate this problem (indicating sets, parameters, decision variables, objective function, and constraints) and solve it using Excel or Gurobi. Show your results clearly and discuss.

3. (20 points) Now, assume that in addition to the transportation costs per pound (given in Tables 5 and 6) you have to pay a fixed toll fee (given in Table 9) when you ship anything from a given warehouse to a given retailer. What is the transportation strategy (number of each product shipped from each plant to each warehouse, and from each warehouse to each retailer) that minimizes the total cost? Is if different from the one found in the previous numeral?
   Mathematically formulate this problem (indicating sets, parameters, decision variables, objective function, and constraints) and solve it using Excel or Gurobi. Show your results clearly and discuss.

Table 9. Fixed toll fee from warehouses to retailers

	Fixed toll fee				
From/To	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5
Warehouse 1	$100	$600	$400	$800	$400
Warehouse 2	$600	$1000	$500	$900	$900
Warehouse 3	$700	$800	$600	$300	$850

4. (5 points) What is the maximum total profit that can be obtained from producing and selling Products A and B, if one unit of Product A is sold for $200 and one unit of Product B is sold for $250. Do not forget to consider the costs and transportation strategy from the previous numeral?

Problem 2 (25 points): Inventory Modeling

Thanks to all the analyses that you have performed on the transportation strategies of the MC Inc. supply chain, the company has been able to save thousands of dollars in operational costs for the past months. Given this, the management of MC Inc. has noticed your exceptional skills in optimization and operations research, and promotes you to be the head of the inventory and production planning department for its new plant ("Plant 4").

This new plant was built to produce both "Product A" and "Product B". However, this new plant will produce to supply only online orders (i.e., they will ship directly to the customers).

Currently, Plant 4 has 98 workers (that work 48 "regular work time" hours per week). If you decide to increase your workforce, the associated hiring and training costs are $500 per worker. On the other hand, if you reduce your workforce, the layoff costs are $700 per worker. Each worker in the plant is paid $15 per hour (during regular work time) and $25 per overtime hour. Keep in mind that, due to state regulations, a worker cannot be asked to work more than 10 overtime hours per week. Additionally, assume that you can hire or lay off workers only at the beginning of a month (for simplicity, assume that each month has exactly 4 weeks). Additional relevant inventory, production, and workforce costs for Plant 4 can be found in Table 10.

Table 10. Inventory, production, and workforce costs for Plant 4

Description	**Cost (Product A)**	**Cost (Product B)**
Material/production cost	$20/unit	$15/unit

Inventory holding cost	$4/unit/week	$5/unit/week
Marginal cost of stockout/backlog	$380/unit/week	$250/unit/week
Labor hours required	6/unit	4/unit
Cost of subcontracting	$40/unit	$50/unit

The expected demand for the next 12 weeks for each product is shown in Table 11.

Table 11. Expected demand (from online orders) of Product A and Product B for the next 12 weeks

Weeks	Product A	Product B
1	1100	700
2	1200	1300
3	1400	700
4	1500	200
5	1400	800
6	1000	2000
7	2000	1500
8	500	400
9	300	600
10	1700	800
11	2500	1000
12	1500	1100

Using the provided information, you are asked to:

1. (25 points) Find the inventory, production, and workforce strategy, for the next 12 weeks, that minimizes the total associated cost. Mathematically formulate this problem (indicating sets, parameters, decision variables, objective function, and constraints) and solve it using Excel or Gurobi. Show your results clearly and discuss.

Problem 3 (25 points): Traveling Salesperson Problem

Imagine that one of your friends, Ana, is a representative of MC Inc. in charge of promoting and selling large orders of "Product A" to wholesalers around the country. In the next two weeks, she would like to visit the major wholesalers in each of 14 nearby cities, so she decides to visit a different city each day. Figure 1 shows a depiction of the location of the 14 cities to be visited, whereas Table 12 shows the travel time (in hours) between each pair of cities.

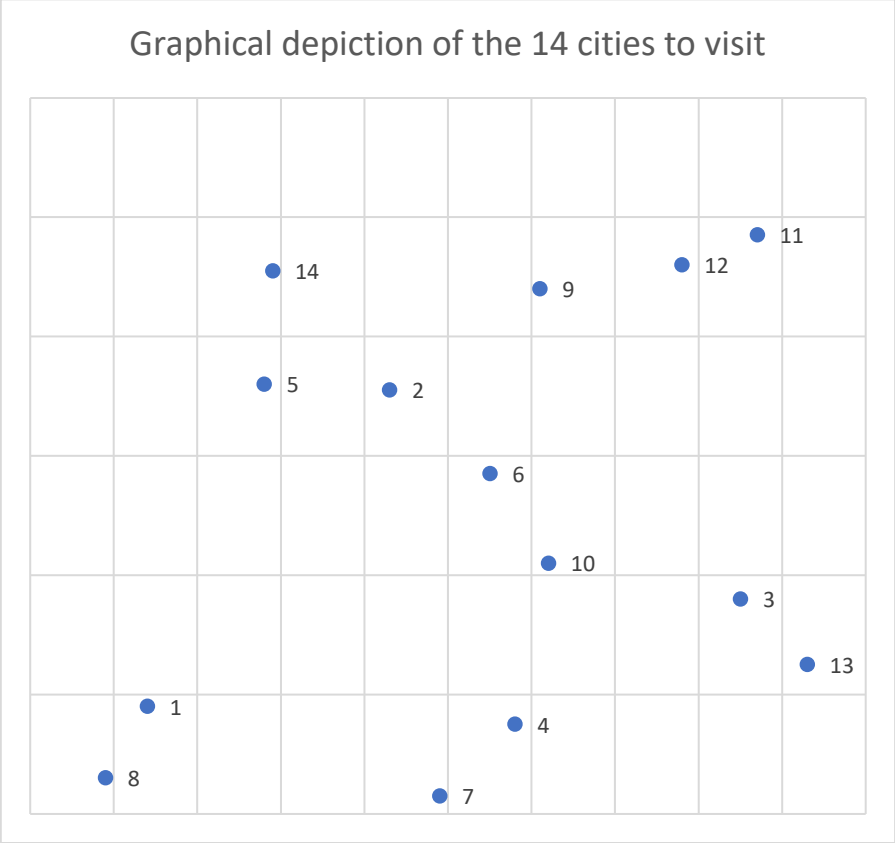

Figure 1. Representation of the 14 cities to visit

1. (25 points) Find the route that minimizes the total travel time associated with visiting all 14 cities (and coming back to Ana's city). Assume that Ana lives in city "1" (so she would have to start and end her route there). Given this, note that Ana could decide to either visit the wholesaler in city "1" in day 1, or should could decide to visit a wholesaler in a different city in day 1 (and visit the wholesaler in city "1" in her way back, in day 14). Mathematically formulate this problem (indicating sets, parameters, decision variables, objective function, and constraints) and solve it using Excel or Gurobi. Show your results, clearly indicating the obtained route (make your plot on top of Figure 1), the day in which Ana visits each wholesaler (use Table 13 to report this) and discuss.

Table 12. Travel time (in hours) between each pair of cities to be visited

		City													
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
City	**1**	0.0	6.0	7.3	4.4	5.6	5.7	3.8	1.3	8.4	5.4	10.8	9.8	7.9	7.5
	2	6.0	0.0	5.5	5.8	1.5	1.8	6.8	7.3	2.5	3.5	5.1	4.1	6.8	2.4
	3	7.3	5.5	0.0	3.4	6.7	3.7	4.9	8.2	5.7	2.4	6.1	5.6	1.4	7.8
	4	4.4	5.8	3.4	0.0	6.4	4.2	1.5	5.0	7.3	2.7	8.7	8.0	3.6	8.1
	5	5.6	1.5	6.7	6.4	0.0	3.1	7.2	6.9	3.7	4.5	6.4	5.4	8.0	1.9
	6	5.7	1.8	3.7	4.2	3.1	0.0	5.4	6.9	3.2	1.7	5.1	4.2	5.0	4.3
	7	3.8	6.8	4.9	1.5	7.2	5.4	0.0	4.0	8.6	4.1	10.1	9.4	4.9	9.0
	8	1.3	7.3	8.2	5.0	6.9	6.9	4.0	0.0	9.7	6.4	12.0	11.0	8.6	8.7
	9	8.4	2.5	5.7	7.3	3.7	3.2	8.6	9.7	0.0	4.6	2.8	1.7	7.1	3.2
	10	5.4	3.5	2.4	2.7	4.5	1.7	4.1	6.4	4.6	0.0	6.0	5.2	3.5	5.9
	11	10.8	5.1	6.1	8.7	6.4	5.1	10.1	12.0	2.8	6.0	0.0	1.0	7.2	5.8
	12	9.8	4.1	5.6	8.0	5.4	4.2	9.4	11.0	1.7	5.2	1.0	0.0	6.9	4.9
	13	7.9	6.8	1.4	3.6	8.0	5.0	4.9	8.6	7.1	3.5	7.2	6.9	0.0	9.2
	14	7.5	2.4	7.8	8.1	1.9	4.3	9.0	8.7	3.2	5.9	5.8	4.9	9.2	0.0

Table 13. Optimal scheduling of wholesaler visits according to Ana's travel plan

Wholesaler	**Day of Ana's visit**
Wholesaler City 1	
Wholesaler City 2	
Wholesaler City 3	
Wholesaler City 4	
Wholesaler City 5	
Wholesaler City 6	
Wholesaler City 7	
Wholesaler City 8	
Wholesaler City 9	
Wholesaler City 10	
Wholesaler City 11	
Wholesaler City 12	
Wholesaler City 13	
Wholesaler City 14	