ST 705 Linear models and variance components
Lab practice problem set 6

February 14, 2023

1. Consider the model $Y_{ij} = \mu + \alpha_i + \beta_i x_{ij} + U_{ij}$, for $i \in \{1, \ldots, n\}$ and $j \in \{1 \ldots, m\}$. Further, assume that $\sum_{j=1}^{m}(x_{ij} - \bar{x}_{i\cdot})^2 > 0$ for all $i \in \{1, \ldots, n\}$. Derive the necessary and sufficient conditions for an estimable function $\lambda'\gamma$, where $\gamma := (\mu, \alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_n)'$.

2. Consider the model $Y_{ijk} = \mu + \alpha_i + \beta_j + \theta_{ij} + U_{ijk}$, with $k \in \{1, \ldots, m_{ij}\}$, $i \in \{1, \ldots, n\}$, $j \in \{1, \ldots, m\}$, and $E(U_{ijk}) = 0$. Find necessary and sufficient conditions for which $\lambda'\gamma$ is estimable for

$$\gamma = \begin{pmatrix} \mu & \alpha_1 & \cdots & \alpha_n & \beta_1 & \cdots & \beta_m & \theta_{11} & \cdots & \theta_{nm} \end{pmatrix}'.$$