ST 705 Linear models and variance components
Homework problem set 5

February 9, 2022

1. Show that $I_n - P_X$ is the unique symmetric projection matrix onto null(X').

2. Suppose that there exists a solution to the system of equations $Ax = c$. Then the general form of a solution is

$$x_z = Gc + (I - GA)z,$$

where z is an arbitrary vector of appropriate dimension and $G := (A'A)^g A'$ (do NOT need to show). Find the z that minimizes the Euclidean norm of x_z.

3. Suppose that the $m \times n$ matrix A has the form

$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

where A_1 is an $n \times n$ nonsingular matrix, and $m > n$. Define $A^+ := (A'A)^{-1}A'$, and prove that $\|A^+\|_2 \leq \|A_1^{-1}\|_2$.

4. Let $Q = X(X'V^{-1}X)^g X'V^{-1}$, with $V > 0$ and symmetric, and show that Q is a projection onto col(X).

5. Exercise A.34 from Monahan.

6. Exercise A.35 from Monahan.

7. Exercise A.50 from Monahan.