

ST 705 Linear models and variance components

Homework problem set 6

February 14, 2023

1. Monahan exercise 2.11
2. Monahan exercise 2.12
3. Monahan exercise 2.14
4. Suppose that there exists a solution to the system of equations $Ax = c$. Then the general form of a solution is

$$x_z = Gc + (I - GA)z,$$

where z is an arbitrary vector of appropriate dimension and $G := (A'A)^g A'$ (do NOT need to show). Find the z that minimizes the Euclidean norm of x_z .

5. In the simple linear regression model $y_i = \beta_0 + x_i\beta_1 + u_i$ for $i \in \{1, \dots, n\}$, show that β_0 is estimable **by finding** a vector a and scalar c such that $E(c + a'y) = \beta_0$.
6. Prove that if $\lambda^{(1)'}\beta, \dots, \lambda^{(k)'}\beta$ are estimable, then so is

$$\sum_{j=1}^k d_j \lambda^{(j)'} \beta,$$

for any scalar constants d_1, \dots, d_k .

7. Show that if the least squares estimator $\lambda'\hat{\beta}$ is the same for all solutions $\hat{\beta}$ to the normal equations, then $\lambda'\beta$ is estimable.