ST 705 Linear models and variance components
Lab practice problem set 10

March 30, 2022

1. Recall the definition of the multivariate normal distribution from class:

   Definition 1 *The p-dimensional random vector Y is said to follow the multivariate normal distribution with mean μ and covariance matrix Σ if for every p-dimensional vector v such that $v'\Sigma v \neq 0$,*

   $$v'Y \sim N(v'\mu, v'\Sigma v).$$

   Denote $Y \sim N_p(\mu, \Sigma)$. ■

   Prove that if Σ is nonsingular, then $Y \sim \mathrm{N}_p(\mu, \Sigma)$ if and only if Y has density,

   $$f(y) = \det(2\pi\Sigma)^{-\frac{1}{2}} e^{-\frac{1}{2}(y-\mu)'\Sigma^{-1}(y-\mu)}.$$