ST 705 Linear models and variance components
Homework problem set 9

March 23, 2022

1. Exercise 4.2 from Monahan.
2. Exercise 4.3 from Monahan.
3. Exercise 4.9 from Monahan.
4. Exercise 4.12 from Monahan.
5. The problem of least squares regression can be understood as a special case of the more general problem of ridge regression. For an n-dimensional column vector y and an $n \times p$ design matrix X, the problem of ridge regression is to solve for the parameter vector b that minimizes
$$a\|b\|_2^2 + \|y - Xb\|_2^2,$$
where $a \geq 0$ is fixed.
   (a) Derive a closed-form expression of the ridge regression solution.
   (b) Assume that X has full column rank, and suppose that y is an observed instance of the random vector $Y = X\beta + U$, where $\beta \in \mathbb{R}^p$ is fixed and U satisfies the Gauss-Markov assumptions. Under what condition(s) is the ridge regression solution the best linear unbiased estimator (BLUE) for any β?
6. Show that if X is a p-dimensional random vector with mean μ and covariance Σ, A is a $p \times p$ matrix, and $Y = X'AX$, then $E(Y) = \mathrm{tr}(A\Sigma) + \mu' A\mu$.
7. For a random vector Y, with finite second moment, verify the following properties.
   (a) $\mathrm{E}(a'Y) = a'\mathrm{E}(Y)$, for a fixed vector a.
   (b) $\mathrm{Var}(a'Y) = a'\mathrm{Var}(Y)a$, for a fixed vector a.
   (c) $\mathrm{Cov}(a'Y, c'Y) = a'\mathrm{Var}(Y)c$, for fixed vectors a and c.
   (d) $\mathrm{Var}(A'Y) = A'\mathrm{Var}(Y)A$, for a fixed matrix A.