ST 495 Advanced computing for statistical methods
Homework problem set 12

April 21, 2022

No `R` packages are permitted for use in this assignment.

1. Let $Y_1, \ldots, Y_n \overset{\text{iid}}{\sim} \exp(\lambda)$. If the value of y_n is missing, then construct and implement an EM algorithm to find the MLE of λ. Implement a simulation study to repeat this algorithm for a large number of data sets, and plot a histogram of the MLE of λ for each data set.

2. Derive an EM algorithm for estimating the unknown parameters of data from an exponential mixture model

$$X_1, \ldots, X_n \overset{\text{iid}}{\sim} f(x) = \sum_{j=1}^{m} p_j \lambda_j e^{-\lambda_j x}$$

by introducing latent variables $Y_1, \ldots, Y_n \in \{1, \ldots, m\}$ with $P(Y_i = j) = p_j$, for $j \in \{1, \ldots, m\}$. Implement a simulation study to repeat this algorithm for a large number of data sets, and plot a histogram of the MLE of λ for each data set.

3. Let $X \sim \text{N}(0, 1)$. Implement a Monte Carlo integration strategy to approximate the mean and variance of the random variable $Y := X^2$.

4. Using the probability integral transform, generate data from an $\exp(\lambda)$ distribution from a sample of observations from the uniform$(0, 1)$ distribution.