ST 705 Linear models and variance components
Homework problem set 5

February 6, 2024

1. Monahan exercise 2.8.

2. Monahan exercise 2.9.

3. In lecture, we proved a lemma that $(X'X)^g X'$ is a generalized inverse of X.

    (a) Verify that $X(X'X)^g$ is a generalized inverse of X'.

    (b) We proved that $P_X := X(X'X)^g X'$ is the unique projection onto column(X). Is $(X'X)^g X'$ the unique generalized inverse of X?

4. Show that $I_n - P_X$ is the unique symmetric projection matrix onto null(X').

5. Let $X = QR$ where Q has orthonormal columns. Prove that if rank$(X) = $ rank(Q), then $P_X = QQ'$.

6. Let $Q = X(X'V^{-1}X)^g X'V^{-1}$, with $V > 0$ and symmetric, and show that Q is a projection onto col(X).

7. Prove that if a (symmetric) matrix is positive definite, then all of its eigenvalues are greater than zero.