DSA 595 Bayesian computations for machine learning
Problem set 8

March 19, 2025

1. Derive the multivariate posterior distribution kernel for (β, σ^2) based on a sample from the linear regression model
$$Y_i = x_i^\top \beta + U_i,$$
for $i \in \{1, \ldots, n\}$, where $U_1, \ldots, U_n \overset{\text{iid}}{\sim} \text{normal}(0, \sigma^2)$, and priors distributions
$$\begin{aligned}\beta &\sim \text{normal}_p(0, \tau^2 \cdot I_p)\\ \sigma^2 &\sim \text{inverse-gamma}(\alpha, \beta).\end{aligned}$$

2. Write an MH algorithm to draw samples from the posterior $\pi(\beta, \sigma^2 \mid y_1, \ldots, y_n)$. Generate synthetic data from the linear regression model with $p = 4$, and generate the x_i as:
$$\begin{aligned}x_1 &= (1, \ldots, 1)^\top\\ x_{i,2} &\sim \text{uniform}(0, 1)\\ x_{i,3} &\sim \text{uniform}(0, 30)\\ x_{i,4} &\sim \text{uniform}(0, 60),\end{aligned}$$
for $i \in \{1, \ldots, n\}$.

3. Modify your Metropolis-Hastings algorithm in problem 3 with the pre-burn-in adaptive proposal scaling and covariance strategy discussed in lecture this week. Demonstrate that you are able to target an acceptance rate close to the range of $(.4, .5)$, post-burn-in.