ST 705 Linear models and variance components
Homework problem set 7

March 4, 2025

1. Assume that $Y = X\beta + U$, where X is an $n \times p$ matrix with $\text{rank}(X) = k < p$, and assume $\lambda'\beta$ is estimable.

   (a) Construct an argument to determine the rank of the matrix $\begin{pmatrix} X \\ \lambda' \end{pmatrix}$.

   (b) Construct an argument to determine the rank of the matrix $\begin{pmatrix} X \\ \lambda'(I - P_{X'}) \end{pmatrix}$.

2. Let X be an $n \times p$ matrix with $\text{rank}(X) = r$, and C be a $(p - r) \times p$ matrix with $\text{rank}(C) = p - r$, such that $\text{col}(X') \cap \text{col}(C') = \{0\}$. Show that $C(X'X + C'C)^{-1}C' = I_{p-r}$.

3. Let X be an $n \times p$ matrix with $\text{rank}(X) = r$, and C be a $(p - r) \times p$ matrix with $\text{rank}(C) = p - r$, such that $\text{col}(X') \cap \text{col}(C') = \{0\}$. Show that

$$\text{rank} \begin{pmatrix} X \\ C \end{pmatrix} = p.$$

4. Monahan problem 3.26.

5. Monahan problem 4.1.

6. Monahan problem 4.2.

7. Monahan problem 4.3.