

# DSA 595 Bayesian computations for machine learning

## Problem set 2

January 20, 2026

1. Derive the posterior distribution for  $\lambda$  based on a sample  $X_1, \dots, X_n \stackrel{\text{iid}}{\sim} \text{exponential}(\lambda)$ , and the prior distribution  $\lambda \sim \text{Pareto}(m, \alpha)$ . Assume  $n, m, \alpha$  are fixed and known.
2. Derive the posterior distribution for  $\sigma^2$  based on a sample  $X_1, \dots, X_n \stackrel{\text{iid}}{\sim} \text{normal}(\mu, \sigma^2)$ , and the improper prior density  $\pi(\sigma^2) \propto 1/\sigma^2$ . Assume  $n, \mu$  are fixed and known.
3. Derive the posterior distribution for  $\beta$  based on a sample from the linear regression model  $Y_i = x_i^\top \beta + U_i$ , independently, for  $i \in \{1, \dots, n\}$ , where  $U_1, \dots, U_n \stackrel{\text{iid}}{\sim} \text{normal}(0, \sigma^2)$ , and the prior distribution  $\beta \sim \text{normal}_p(0, \tau^2 \cdot I_p)$ , where  $I_p$  is the  $p \times p$  identity matrix. Assume  $n, p, \sigma^2, \tau^2, x_1, \dots, x_n$  are fixed and known.
4. Same as the previous problem, except assume the improper prior density  $\pi(\beta) \propto 1$ .
5. Suppose that  $X \sim \text{Poisson}(\theta)$  with prior density  $\pi(\theta) = 1/\theta$  for  $\theta > 0$ . Show that the posterior distribution  $\pi(\theta | x)$  is not defined for  $x = 0$ .