

ST 705; Spring 2025

Linear Models and Variance Components

Lecture: Tuesdays/Thursdays, 11:45–13:00, 243 111 Lampe Drive

Lab: Mondays, 10:40–11:30, 5270 SAS Hall

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Course website: <https://jonathanpw.github.io/ST705>

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Office hours: 14:00–15:00 Mondays and 15:00–16:00 Tuesdays

Course Description: Theory of estimation and testing in full and non-full rank linear models. Normal theory distributional properties. Least squares principle and the Gauss-Markov theorem. Estimability and properties of best linear unbiased estimators. General linear hypothesis. Application of dummy variable methods to elementary classification models for balanced and unbalanced data. Analysis of covariance. Variance components estimation for balanced data.

Corequisite: ST 702.

Credit Hours: 3

Text(s): *A primer on linear models*

Author(s): John F Monahan

Grade Distribution:

Midterm 1 exam	30%
Midterm 2 exam	30%
Final exam	40%

Letter Grade Distribution:

≥ 93.00	A	73.00 - 76.99	C
90.00 - 92.99	A-	70.00 - 72.99	C-
87.00 - 89.99	B+	67.00 - 69.99	D+
83.00 - 86.99	B	63.00 - 66.99	D
80.00 - 82.99	B-	60.00 - 62.99	D-
77.00 - 79.99	C+	≤ 59.99	F

Midterm 1 exam period: Tuesday, 11 February 2025.

Midterm 2 exam period: Tuesday, 25 March 2025.

Final exam period: 12:00–14:30 on Tuesday, 29 April 2025 at 243 111 Lampe Drive.

Personal note to students: Please do not feel intimidated about interacting with me. Regardless of how busy or stressed I may appear to you, teaching your class is a part of my job, and I take that very seriously. I care deeply about the quality of your learning. Please always reach out to me if you have questions, concerns, or need help. I understand that it can be difficult and can even feel embarrassing to ask for help. However, I was once in your position, and I promise to always treat you with respect, empathy, and kindness. Nobody that ever did anything meaningful did so without first failing over and over again.

Course policies and commentary:

• Assignments

- **Practice homework problems will be posted each Tuesday evening. These problems will be relevant (if not identical) to the exam questions.**
- It is strongly encouraged for you to work with classmates on all homework assignments. You will learn a lot about what you do and do not understand when you discuss your ideas and solutions with others, and you will expand the way you think when you understand the ideas and solutions of others.
- If you are not confident that you have correctly solved all homework problems, then you are not making adequate use of available resources (i.e., discussions with classmates, lectures notes, textbook, open-source materials available online, office hours).
- Take responsibility for understanding solutions to all assignments. For example, if you find a solution on StackExchange, then convince yourself that the solution is actually correct.
- **Learn to distinguish between the things you *do* know and the things you *do not* know** (this is one of the most important results of all education). To understand, to *a* particular degree, that a given statement is true means that you can explain why the statement is true, to *the* particular degree.

• Exams

- All exams are closed-note, closed-book.
- Any communication with other individuals is strictly prohibited during exams.
- For each exam, each part of each problem has the following point distribution:
 - * 3 points – solution is correct, up to minor typos
 - * 2 points – solution is mostly correct, but has a small mistake
 - * 1 point – solution is sensible or on the right track
 - * 0 points – solution is not relevant to the question
- A study guide will be provided by the instructor prior to each exam.

• Attendance

- Use lecture time (or not) as you feel most productive, but do not use it in a way which is distracting to others.

- Lecture time is not for answering homework questions; please attend the lab or office hours for homework questions.

Tentative Course Outline:

Basic linear algebra: Vector spaces, Null space, and Column space, Idempotent matrix, the Projection matrix, Matrix decompositions (QR, Cholesky, eigenvalues and eigenvectors, SVD) (2 lectures)

Solving a system of linear equations: the existence of solutions, generalized inverse (2 lectures)

General linear models: different classes of linear models and related assumptions (1 lecture)

Least squares method: normal equations and their general solutions (2 lectures)

Reparameterization (1 lectures)

Identifiability and Estimability: linearly estimable functions of model parameters, least squares estimators of estimable functions (3 lectures)

The Gauss-Markov model: best linear unbiased estimators, the Gauss-Markov theorem, variance estimation, model misspecification, underfitting and overfitting (3 lectures)

The Aitken model: generalized least squares, estimability, Aitken's theorem (2 lectures)

Imposing constraints: estimation of model parameters with additional constraints, restricted normal equations and their solution, best estimation in a constrained model (2 lectures)

Distributional theory: Multivariate normal, Chi-squared, F and t distributions, distribution of quadratic forms, Cochran's theorem, maximum likelihood estimation in Gaussian linear models (2 lectures)

Hypothesis testing: General linear hypotheses, Testable hypotheses, General linear test, Likelihood ratio test (2 lectures)

Confidence intervals and multiple comparisons: confidence region, simultaneous confidence intervals (Bonferroni method, Scheffe's method, Tukey's method), Multiple testing (2 lectures)

Linear mixed effects model: definition of general mixed effects models, maximum likelihood and restricted maximum likelihood estimation, analysis of variance in mixed effects models, best linear unbiased prediction (3 lectures)

NCSU Policies, Regulations, and Rules: Students are responsible for reviewing the NC State University Policies, Rules, and Regulations (PRRs) which pertain to their course rights and responsibilities, including those referenced both below and above in this syllabus:

- Equal Opportunity and Non-Discrimination Policy Statement <https://policies.ncsu.edu/policy/pol-04-25-05> with additional references at <https://oied.ncsu.edu/divweb/policies/>
- Code of Student Conduct <https://policies.ncsu.edu/policy/pol-11-35-01>
- Grades and Grade Point Average <https://policies.ncsu.edu/regulation/reg-02-50-03>
- Credit-Only Courses <https://policies.ncsu.edu/regulation/reg-02-20-15>
- Audits <https://policies.ncsu.edu/regulation/reg-02-20-04>

Policy on Academic Integrity: Cheating, plagiarism and other forms of academic dishonesty will not be tolerated. Violations of academic integrity will be handled in accordance with the Student Discipline Procedures (NCSU REG 11.35.02).

Disability Services for Students: Reasonable accommodations will be made for students with verifiable disabilities. In order to take advantage of available accommodations, students must register with the Disability Resource Office at Holmes Hall, Suite 304, 2751 Cates Avenue, Campus Box 7509, 919-515-7653. For more information on NC State's policy on working with students with disabilities, please see the Academic Accommodations for Students with Disabilities Regulation (NCSU REG 02.20.01).

Privacy: Students may be required to disclose personally identifiable information to other students in the course, via digital tools, such as email or web-postings, where relevant to the course. Examples include online discussions of class topics, and posting of student coursework. All students are expected to respect the privacy of each other by not sharing or using such information outside the course.