ST 705 Linear models and variance components
Homework problem set 10

April 4, 2023

1. Monahan exercise 5.2

2. Monahan exercise 5.3a-c

3. Monahan exercise 5.6

4. Monahan exercise 5.9

5. Let
$$Y \sim \mathrm{N}_2 \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \right\},$$
$$A = \frac{1}{8} \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix},$$
and $B = (1, -2)'$. Find the joint distribution of $Y'AY$ and $B'Y$.

6. Suppose that (X, Y) has a bivariate distribution (**not necessarily Gaussian**) with mean $(\mu_X, \mu_Y)'$ and covariance matrix
$$\begin{pmatrix} \sigma_X^2 & \sigma_{X,Y} \\ \sigma_{Y,X} & \sigma_Y^2 \end{pmatrix}.$$
   (a) Show that if $E(Y \mid X) = \beta_0 + \beta_1 X$, then $\beta_1 = \sigma_{Y,X}/\sigma_X^2$ and $\beta_0 = \mu_Y - \beta_1 \mu_X$.
   (b) Show that if $E(Y \mid X) = \beta_0 + \beta_1 X$ and $\mathrm{Var}(Y \mid X) = \tau^2$, then $\tau^2 = \sigma_Y^2 - \sigma_{Y,X}^2/\sigma_X^2$.