

# ST 495 Advanced computing for statistical methods

## Homework problem set 9

March 28, 2022

**No R packages are permitted for use in this assignment.**

1. Derive a confidence interval for  $\lambda$  based on a random sample  $Y_1, \dots, Y_n \stackrel{\text{iid}}{\sim} \exp(\lambda)$ . Design and implement a simulation study to demonstrate that the confidence interval achieves its nominal  $1 - \alpha$  level coverage for  $\alpha \in \{.01, .02, \dots, .99\}$ . Display the nominal versus empirical coverage as a scatter plot.
2. Derive a confidence interval for the intercept  $\beta_0$  based on the simple linear regression model  $Y_i = \beta_0 + \beta_1 x_i + U_i$ , for  $i \in \{1, \dots, n\}$ , where  $U_1, \dots, U_n \stackrel{\text{iid}}{\sim} N(0, \sigma^2)$ . Assume  $\sigma^2$  is known. Design and implement a simulation study to demonstrate that the confidence interval achieves its nominal  $1 - \alpha$  level coverage for  $\alpha \in \{.01, .02, \dots, .99\}$ . Display the nominal versus empirical coverage as a scatter plot.
3. Repeat problem 2, but with  $\sigma^2$  unknown. You will need to first construct an unbiased estimator for  $\sigma^2$ .
4. Derive a confidence interval for  $\beta_1$  in problem 2, but with  $\sigma^2$  unknown (recall that we did this in lecture for  $\sigma^2$  known). Design and implement a simulation study to demonstrate that the confidence interval achieves its nominal  $1 - \alpha$  level coverage for  $\alpha \in \{.01, .02, \dots, .99\}$ . Display the nominal versus empirical coverage as a scatter plot.
5. Derive a prediction interval for  $Y_{n+1} \sim N(x'_{n+1}\beta, \sigma^2)$ , assuming  $\sigma^2$  is unknown, based on the linear regression model  $Y \sim N_n(X\beta, \sigma^2 I_n)$  with  $X \in \mathbb{R}^{n \times p}$  and  $\text{rank}(X) = p$ . Recall from lecture that such an interval was derived in the case of  $\sigma^2$  known. Design and implement a simulation study to demonstrate that the prediction interval achieves its nominal  $1 - \alpha$  level coverage for  $\alpha \in \{.01, .02, \dots, .99\}$ . Display the nominal versus empirical coverage as a scatter plot.