ST 705 Linear models and variance components
Lab practice problem set 11

April 4, 2023

1. Recall the definition of the multivariate normal distribution from class:

   Definition 1 *The p-dimensional random vector Y is said to follow the multivariate normal distribution with mean μ and covariance matrix Σ if for every p-dimensional vector v such that $v'\Sigma v \neq 0$,*

   $$v'Y \sim N(v'\mu, v'\Sigma v).$$

   Denote $Y \sim N_p(\mu, \Sigma)$. ■

   Prove that if Σ is nonsingular, then $Y \sim \mathrm{N}_p(\mu, \Sigma)$ if and only if Y has density,

   $$f(y) = \det(2\pi\Sigma)^{-\frac{1}{2}} e^{-\frac{1}{2}(y-\mu)'\Sigma^{-1}(y-\mu)}.$$

2. Let $X \sim \mathrm{N}_p(\mu, \Sigma)$. Show that for any partition of components, i.e.,

   $$X = \begin{pmatrix} X_1 \\ \vdots \\ X_m \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_m \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \cdots & \Sigma_{1m} \\ \vdots & \ddots & \vdots \\ \Sigma_{m1} & \cdots & \Sigma_{mm} \end{pmatrix},$$

   $X_1, \ldots, X_m$ are mutually independent if and only if $\Sigma_{ij} = 0$ for every $i \neq j$.