ST 705 Linear models and variance components
Lab practice problem set 11

April 4, 2022

1. Let $X \sim \mathrm{N}_p(\mu, \Sigma)$. Show that for any partition of components, i.e.,

$$X = \begin{pmatrix} X_1 \\ \vdots \\ X_m \end{pmatrix}, \qquad \mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_m \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \Sigma_{11} & \cdots & \Sigma_{1m} \\ \vdots & \ddots & \vdots \\ \Sigma_{m1} & \cdots & \Sigma_{mm} \end{pmatrix},$$

   $X_1, \ldots, X_m$ are mutually independent if and only if $\Sigma_{ij} = 0$ for every $i \neq j$.

2. Construct two random variables that have zero correlation, but are *not* independent.