

# DSA 595 Bayesian computations for machine learning

## Problem set 8

March 19, 2025

1. Derive the multivariate posterior distribution kernel for  $(\beta, \sigma^2)$  based on a sample from the linear regression model

$$Y_i = x_i^\top \beta + U_i,$$

for  $i \in \{1, \dots, n\}$ , where  $U_1, \dots, U_n \stackrel{\text{iid}}{\sim} \text{normal}(0, \sigma^2)$ , and priors distributions

$$\beta \sim \text{normal}_p(0, \tau^2 \cdot I_p)$$

$$\sigma^2 \sim \text{inverse-gamma}(\alpha, \beta).$$

2. Write an MH algorithm to draw samples from the posterior  $\pi(\beta, \sigma^2 \mid y_1, \dots, y_n)$ . Generate synthetic data from the linear regression model with  $p = 4$ , and generate the  $x_i$  as:

$$x_1 = (1, \dots, 1)^\top$$

$$x_2 \sim \text{uniform}(0, 1)$$

$$x_3 \sim \text{uniform}(0, 30)$$

$$x_4 \sim \text{uniform}(0, 60).$$

3. Modify your Metropolis-Hastings algorithm in problem 3 with the pre-burn-in adaptive proposal scaling and covariance strategy discussed in lecture this week. Demonstrate that you are able to target an acceptance rate close to the range of  $(.4, .5)$ , post-burn-in.