ST 705 Linear models and variance components
Lab practice problem set 12

April 14, 2022

1. Suppose that (X, Y) has a bivariate distribution (**not necessarily Gaussian**) with mean $(\mu_X, \mu_Y)'$ and covariance matrix

$$
\begin{pmatrix}
\sigma_X^2 & \sigma_{X,Y} \\
\sigma_{Y,X} & \sigma_Y^2
\end{pmatrix}.
$$

   (a) Show that if $E(Y \mid X) = \beta_0 + \beta_1 X$, then $\beta_1 = \sigma_{Y,X}/\sigma_X^2$ and $\beta_0 = \mu_Y - \beta_1 \mu_X$.

   (b) Show that if $E(Y \mid X) = \beta_0 + \beta_1 X$ and $\text{Var}(Y \mid X) = \tau^2$, then $\tau^2 = \sigma_Y^2 - \sigma_{Y,X}^2/\sigma_X^2$.

2. Let

$$
Y \sim N_2 \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \right\},
$$

$$
A = \frac{1}{8} \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix},
$$

   and $B = (1, -2)'$. Find the joint distribution of $Y'AY$ and $B'Y$.