E11L : Experiment 3 Report

Name: Jonathan Goh

UID : 404901382

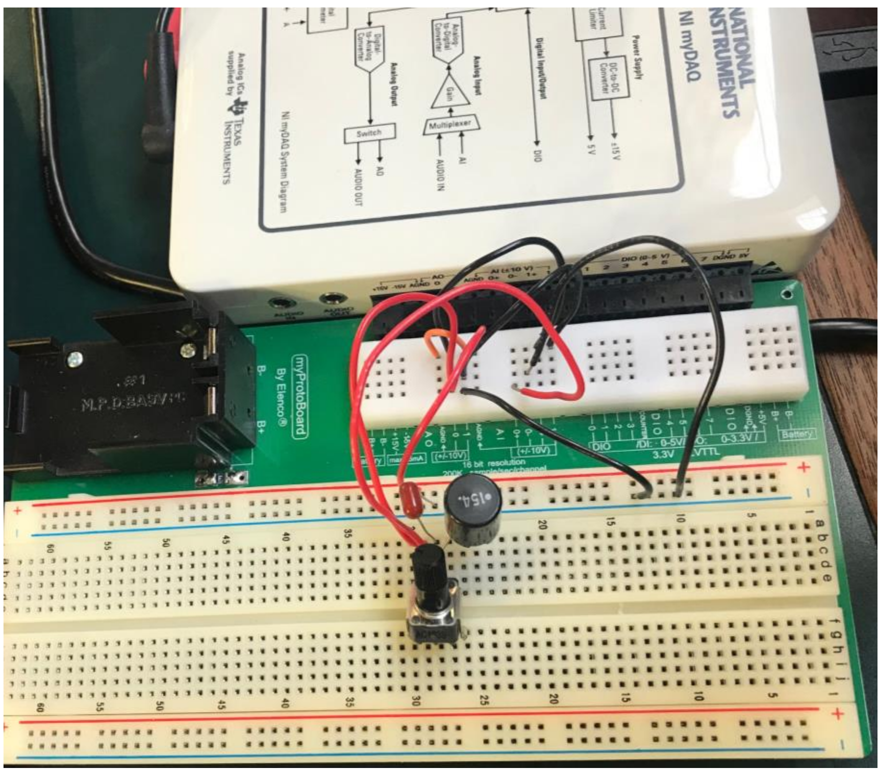
Lab 3. Underdamped RLC Circuit Design (Demonstration Experiment)

Lab 3 needs detail report

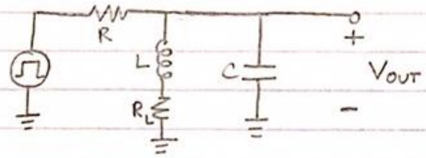
i. By changing the value of the resistor from Lab 1, acquire an underdamped response that overshoots the final voltage value by approximately 3 (take into account inductor resistance as well). Record the value of the resistor used, as well as the experimental value of the overshoot now considering the resistance of the inductor and the internal resistance of the generator.

ii. Design an underdamped system second-order circuit that has at least 4 visible peaks in oscillations before settling such that you can measure the period of oscillation. Compare the experimental value obtained by measuring the period of the oscillations with the theoretical value.

iii. Write the equation for $V_{OUT}(t)$ which describes the voltage across the capacitor as a function of time for your design. **(Show the necessary steps of your work)**



LAB 3: UNDERDAMPED RLC CIRCUIT DESIGN



$$iR + Li_L' + R_L i_L = V_s \quad , \quad i_L = \frac{c\,dv}{dt}$$

$$iR + \frac{Q}{C} = V_s \Rightarrow i = \frac{V_s - V_c}{R} \qquad i' = \frac{V_c'}{R}$$

$$i_L = i - i_c \Rightarrow \frac{V_s - V_c}{R} - CV_c'$$

$$\frac{di_L}{dt} = \frac{di}{dt} - \frac{di_c}{dt} = \frac{-V_c'}{R} - CV_c''$$

$$\left(\frac{V_s - V_c}{R}\right)R + L\left(\frac{-V_c}{R} - CV_c''\right) + \left(\frac{V_s - V_c}{R} - CV_c'\right)R_L = V_s$$

$$-V_c - \frac{L}{R}V_c' - LCV_c'' - \frac{R_L}{R}V_c - R_L C V_c' = \frac{-R_L}{R}V_s$$

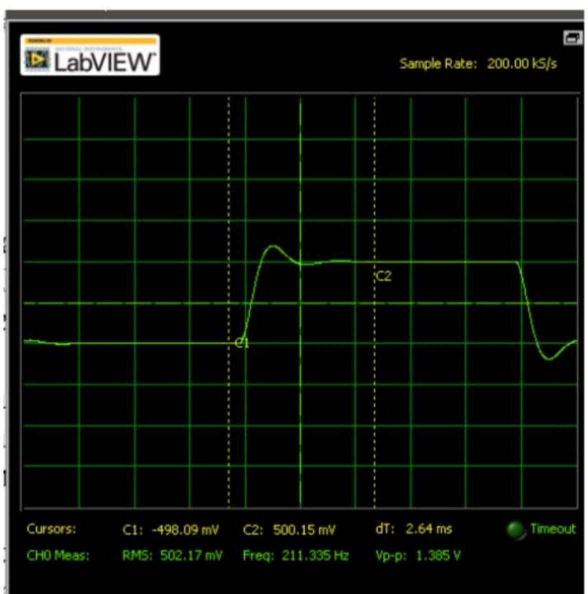
$$LCV_c'' + \left(\frac{L}{R} + R_L C\right)V_c' + \left(1 + \frac{R_L}{R}\right)V_c = \frac{R_L}{R}V_s$$

$$V_c'' + \left(\frac{1}{RC} + \frac{R_L}{L}\right)V_c' + \left(\frac{1}{LC} + \frac{R_L}{LCR}\right)V_c = \frac{R_L}{RLC}V_s$$

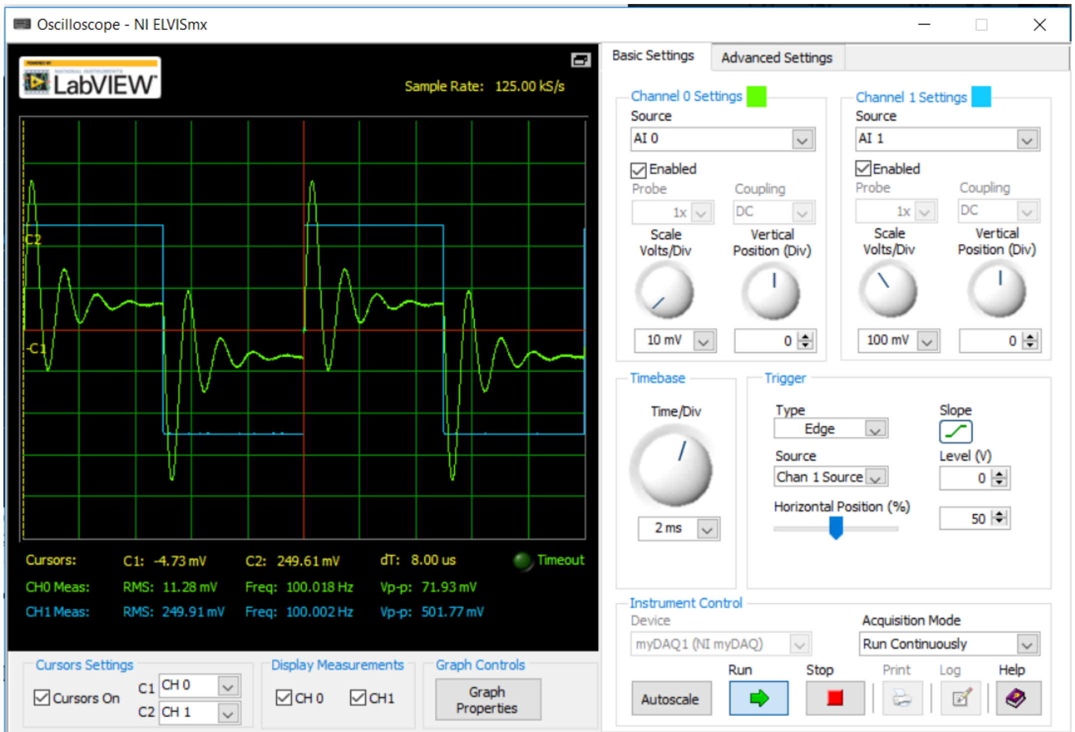
$$V_c = e^{-\alpha t}\left(A_1\cos(\omega t) + A_2\sin(\omega t)\right) + \frac{R_L}{R + R_L}V_s$$

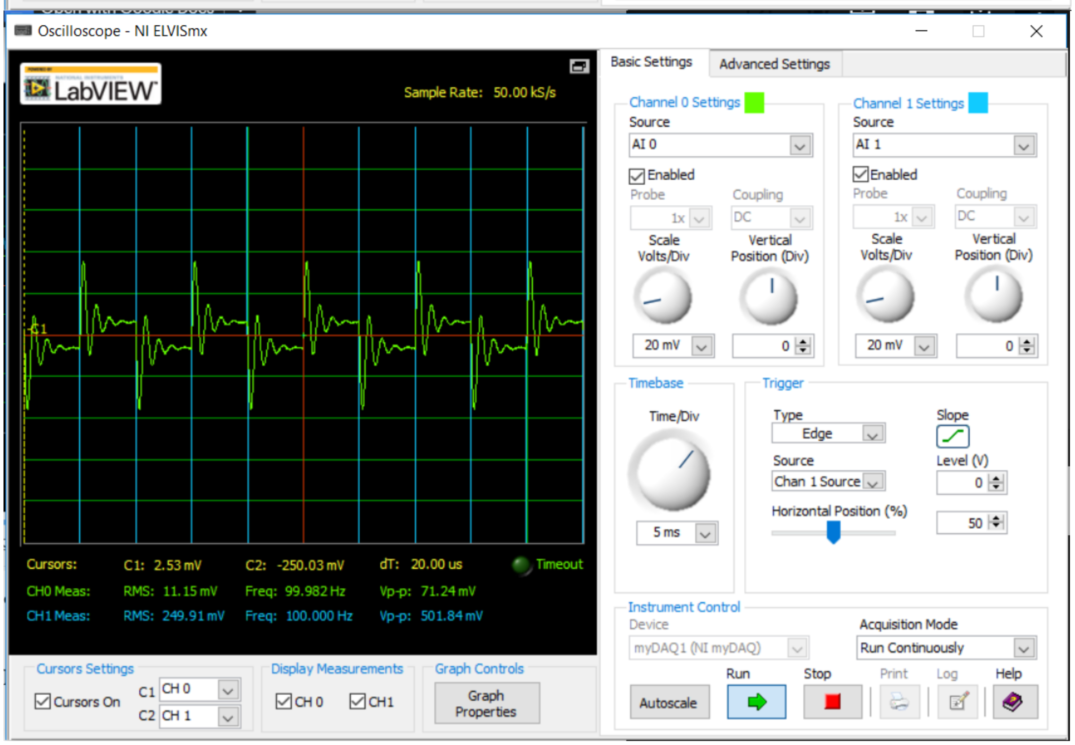
$$\alpha = \frac{1}{2}\left(\frac{R_L}{L} + \frac{1}{RC}\right) \quad , \quad \omega_0 = \sqrt{\left(\frac{1}{LC} + \frac{R_L}{LCR}\right)}$$

Work for V out, Alpha, and Omega



Part i). Resistances			
	R (Used) Ω	**R (Internal) Ω**	**Function Generator Ω**
Theoretical	580	0	0
Measured	540	245±10	430±10





Part ii). Time Between Peaks			
	Peak 1-2	**Peak 2-3**	**Peak 3-4**
Time Difference dT	1.23 ms	1.10 ms	1.18 ms

Discussion

3.1 How did the experimental damped frequency compare with that of the theoretical values?

The experimental values of the damped frequency were only very slightly different from that of the theoretical values, which shows that the derived differential equations and related formulas are accurate representations of the system.

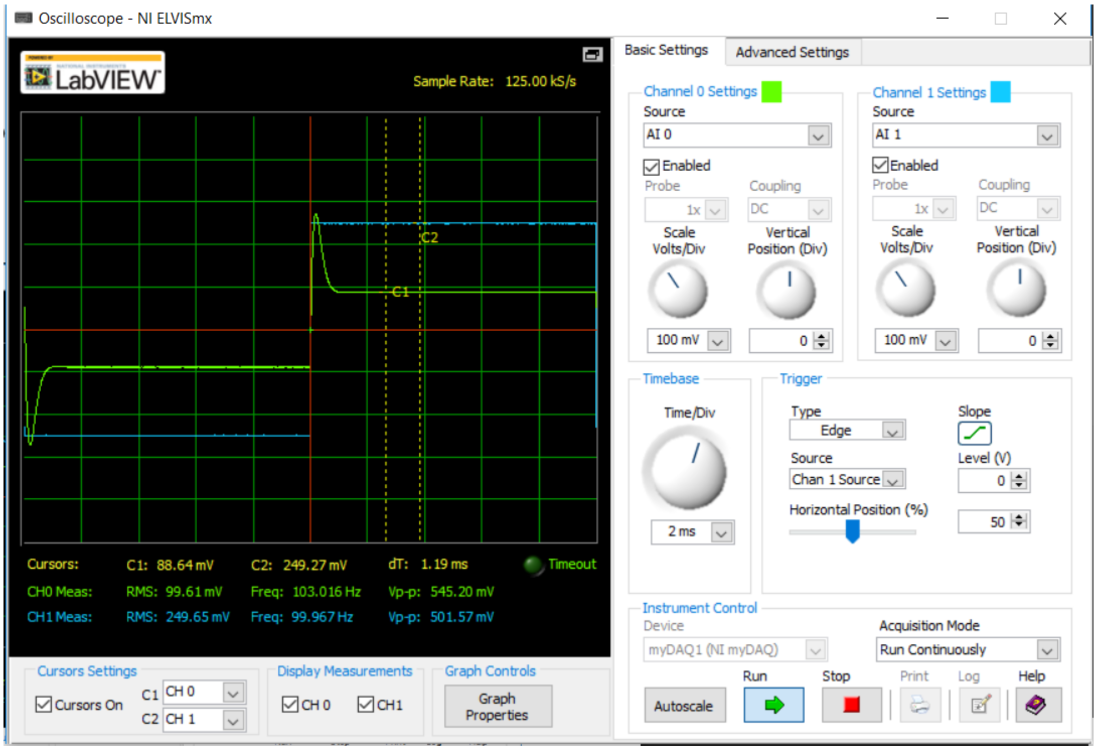
3.2 What happens if you try to make the overshoot smaller?

To make the overshoot smaller, the resistance of the potentiometer would need to decreased; to increase the overshoot, the resistance of the potentiometer would need to be increased. This can be observed through the voltage response of the circuit as the difference between the voltage spike and the final voltage of the response. Smaller overshoot lead to a smaller voltage spike.

Lab 4. Critically Damped RLC Circuit

i. Replace the resistor of the circuit in Lab 1 (Figure 3), again, but this time by a 10kΩ potentiometer. By using the oscilloscope output as a guide, adjust the resistance until you think the system is approximately critically damped. Record the output you obtain, remove the potentiometer, and measure the value of the resistance you used.

Response(Please attached necessary screenshots of oscilloscope):



The value of the resistance:

Experimental Value: 0.449k Ω or 449 Ω from the potentiometer

LAB 4: CRITICALLY DAMPED RLC CIRCUIT

CRITICAL DAMPING MEANS THAT

$\xi = 1 \quad , \quad \xi = \alpha / \omega_0$

$$\xi = \frac{1}{2}\left(\frac{R_L}{L} + \frac{1}{RC}\right) \Big/ \sqrt{\frac{1}{LC} + \frac{R_L}{RLC}}$$

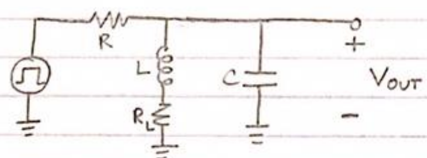
$R = ?\Omega$, $R_L = 245\Omega$, $L = 150\,mH$, $C = 0.22\,\mu F$

$$1 = \frac{1}{2}\left(\frac{245}{0.15} + \frac{1}{R(2.2\times10^{-7})}\right) \Big/ \sqrt{\frac{1}{0.15(2.2\times10^{-7})} + \frac{245}{245(0.15)(2.2\times10^{-7})}}$$

$\boxed{R = 326.151\,\Omega}$ for critical damping

THEORETICAL RESISTANCE

LAB 3: UNDERDAMPED RLC CIRCUIT DESIGN



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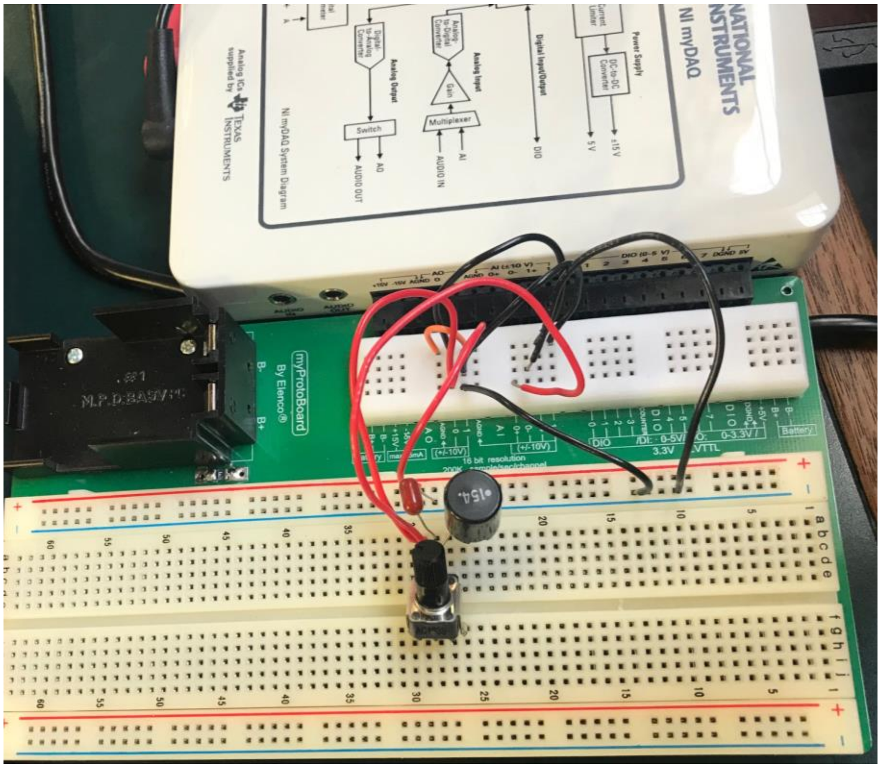

Discussion

4.1 How close was the value of resistance you ended up with when using the potentiometer to obtain a critically damped response, to the theoretical value you have derived? Consider the effects of inductor resistance as well.

My theoretical (critically damped) resistance calculated was 326.151 ohms, while the experimental resistance calculated was 449 ohms. The values obtained, though not the same, are definitely close enough to show that the calculations were accurate. This proves that the differential equation derived for a parallel RLC circuit with inductor resistance accurately models the circuit. A source of error, that could have caused the discrepancy with theoretical and calculated values is the internal resistance of the function generator which was not taken into account with the differential equation.

4.2 What did you observe in the output waveform as resistance varied?

Higher resistances of the potentiometer correspond to the voltage response being more underdamped and grew more peaks, while lower resistances of the potentiometer correspond to the voltage response becoming more overdamped and reducing the number of peaks.