E11L : Experiment 3 Report

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Lab1. Parallel RLC Circuit Response

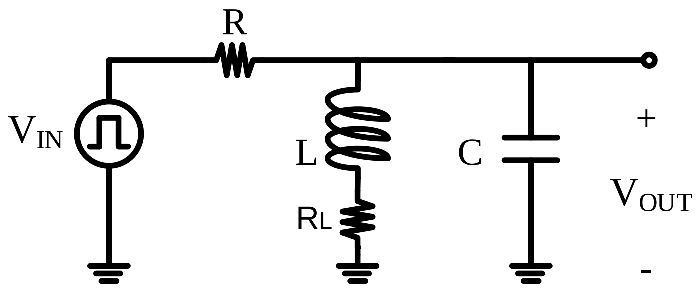
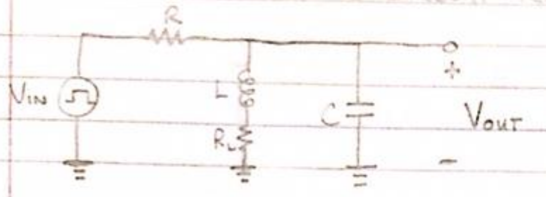


Fig.3. Parallel RLC Circuit (series inductor resistance included)

i. Using a 100 Ω resistor, 150 mH inductor, and 0.22 uF capacitor, build the parallel RLC circuit illustrated in Figure 3 (note that we consider the resistance of the inductor R_L, which is non-negligible). Considering the finite resistance of the inductor, calculate the theoretical damping factor of the circuit.

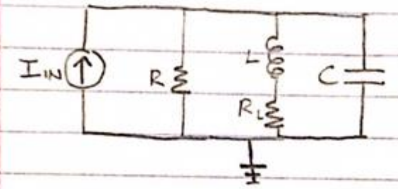
LAB 1: PARALLEL RLC CIRCUIT RESPONSE



MEASURED VALUES:
R: 99 Ω
L: 150 mH
R_L: 245 Ω

100 Ω RESISTOR, 150 mH INDUCTOR, 0.22 μF CAPACITOR

THEORETICAL DAMPING FACTOR



$$V_o = V_L + V_{R_L}$$
$$= L\frac{di}{dt} + R_L i$$
$$V_o' = L\frac{d^2i}{dt^2} + R_L\frac{di}{dt}$$

$$I_s = i_R + i + C\frac{dV_C}{dt}$$

$$\frac{V_o}{R} + i + \frac{C\,dv}{dt} \Rightarrow \frac{1}{R}\left(\frac{L\,di}{dt} + R_L i\right) + i + C\left(L\frac{d^2i}{dt^2} + R_L\frac{di}{dt}\right)$$

$$I_s = CL\frac{d^2i}{dt} + \left(\frac{L}{R} + CR_L\right)\frac{di}{dt} + \left(\frac{R_L}{R} + 1\right)i$$

$$\frac{I_s}{CL} = \frac{d^2i}{dt} + \frac{\left(\frac{L}{R} + CR_L\right)\frac{di}{dt}}{CL} + \frac{\left(\frac{R_L}{R} + 1\right)i}{CL}$$

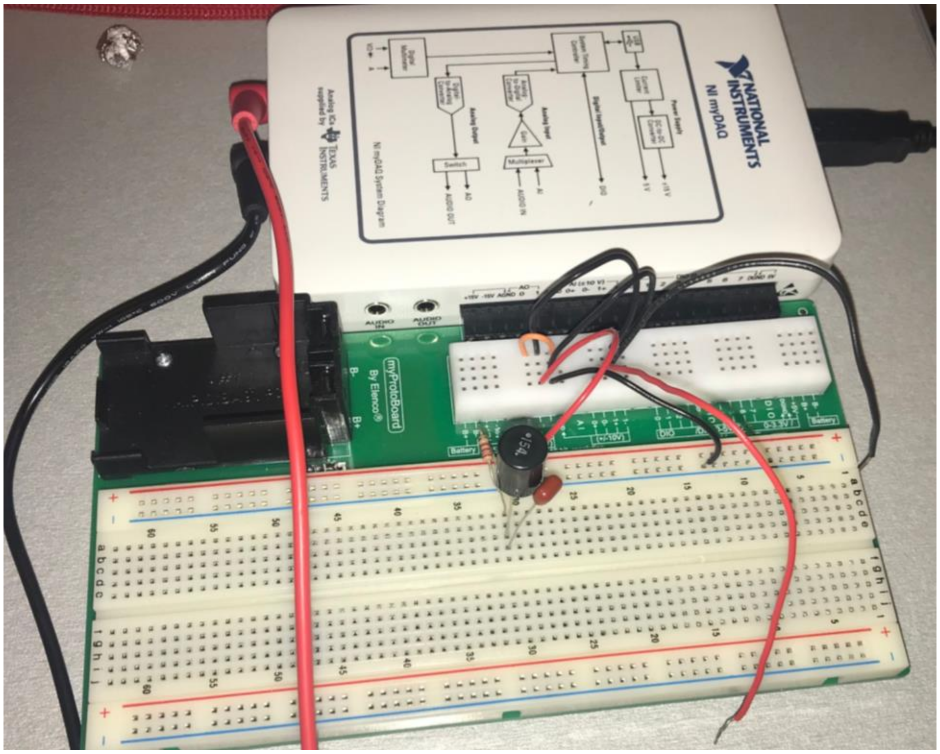
$$\omega^2 = \frac{\left(\frac{R_L}{R} + 1\right)}{CL} = 1.056 \times 10^8$$

$$\omega = 1.028 \times 10^4$$

$$2\alpha = \frac{\left(\frac{L}{R} + CR_L\right)}{CL} = 48009$$

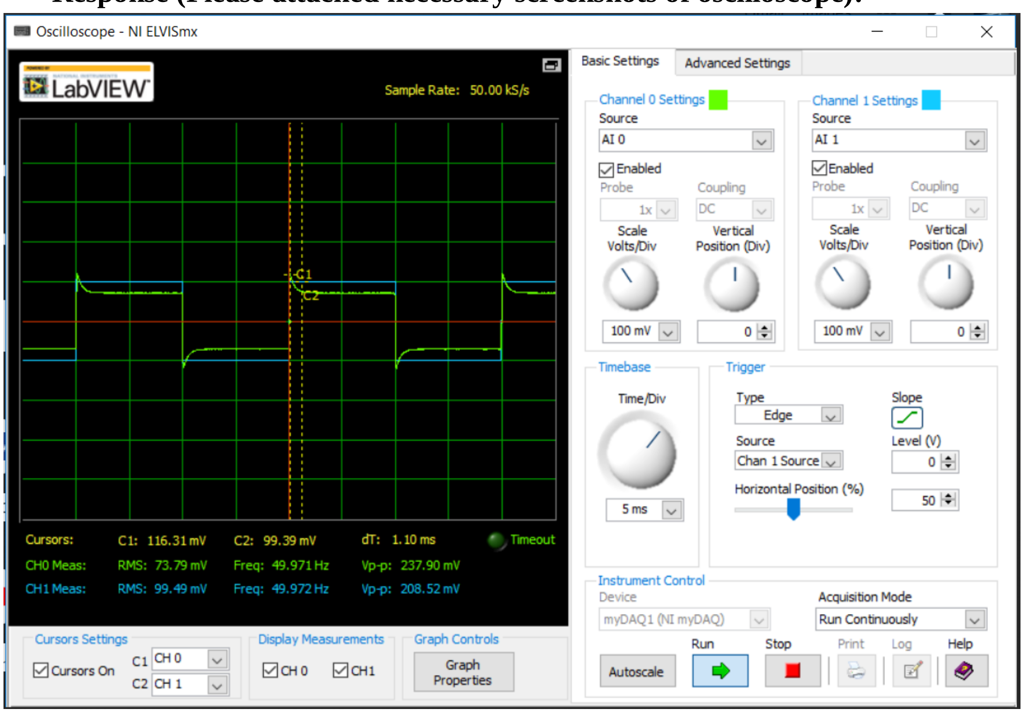
$$\alpha = 2.33$$

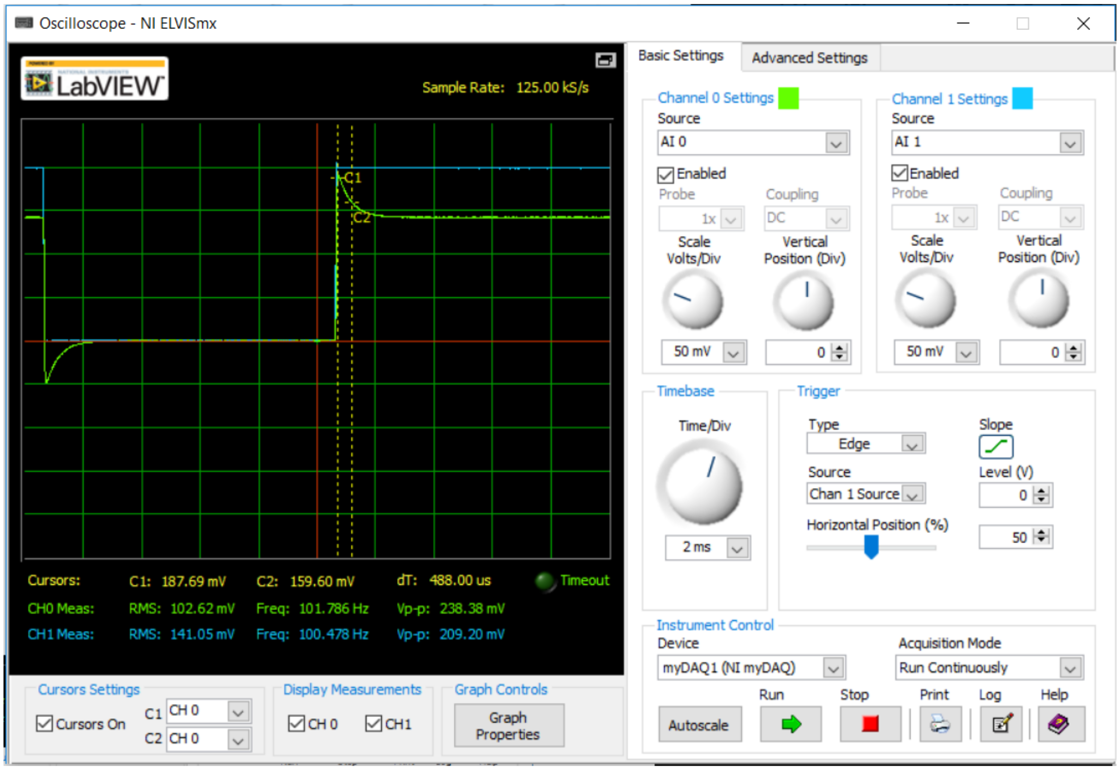
$$\boxed{\xi = \frac{\alpha}{\omega_o} = 2.33 \quad (>1) \text{ SO OVERDAMPED}}$$



ii. Construct the circuit and verify the above theoretical results by generating a square signal using the function generator of the myDAQ which steps from 0V to 1Vand measuring V_{OUT} using Oscilloscope. Take a screenshot of the output voltage response ($V_{OUT}(t)$). For the input frequency, set the function generator to 50Hz (adjustable if necessary).

Response (Please attached necessary screenshots of oscilloscope):





Discussion

1.1 What kind of damping is observed? Compare the theoretical damping factor with the measured one.

We can see that overdamping was observed based on the oscilloscope. Based on the theoretical value that was seen, we know that overdamping occurred since the value of 2.33 was greater than 1. Based on the experimentally measured value, we also observed overdamping; noted by the single oscillation. Overdamped response equation not given in terms of Vfinal and Vpeak. Experimentally calculated dT of 488 microseconds as seen from the second oscilloscope photo.

Lab2. Series RLC Circuit Analysis

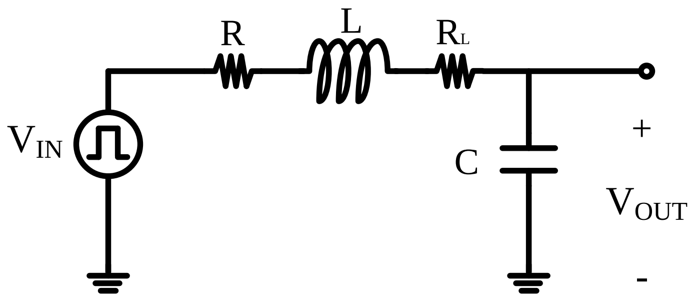
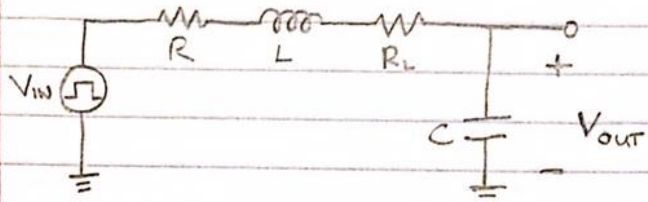


Fig.4 Series RLC Circuit (series inductor resistance included)

i. Using a 2.2 kΩ resistor, 150mH inductor, and 0.1uF capacitor, build the series RLC circuit illustrated in Figure 4 (note that we consider the resistance of the inductor R_L, which is non-negligible).

ii. Calculate the theocratical damping factor of the second order circuit. **(Show the necessary steps of your work)**

LAB 2: SERIES RLC CIRCUIT ANALYSIS



MEASURED VALUES
R: 2.15Ω
L: 150 mH
R_L: 245Ω

2.2 kΩ RESISTOR, 150 mH INDUCTOR, 0.1 µF CAPACITOR

THEORETICAL DAMPING FACTOR:

$$V_s = iR + L\frac{di}{dt} + iR_L + \frac{\int i\,dt}{C}$$

$$\Rightarrow (R+R_L)i + L\frac{di}{dt} + \frac{\int i\,dt}{C}$$

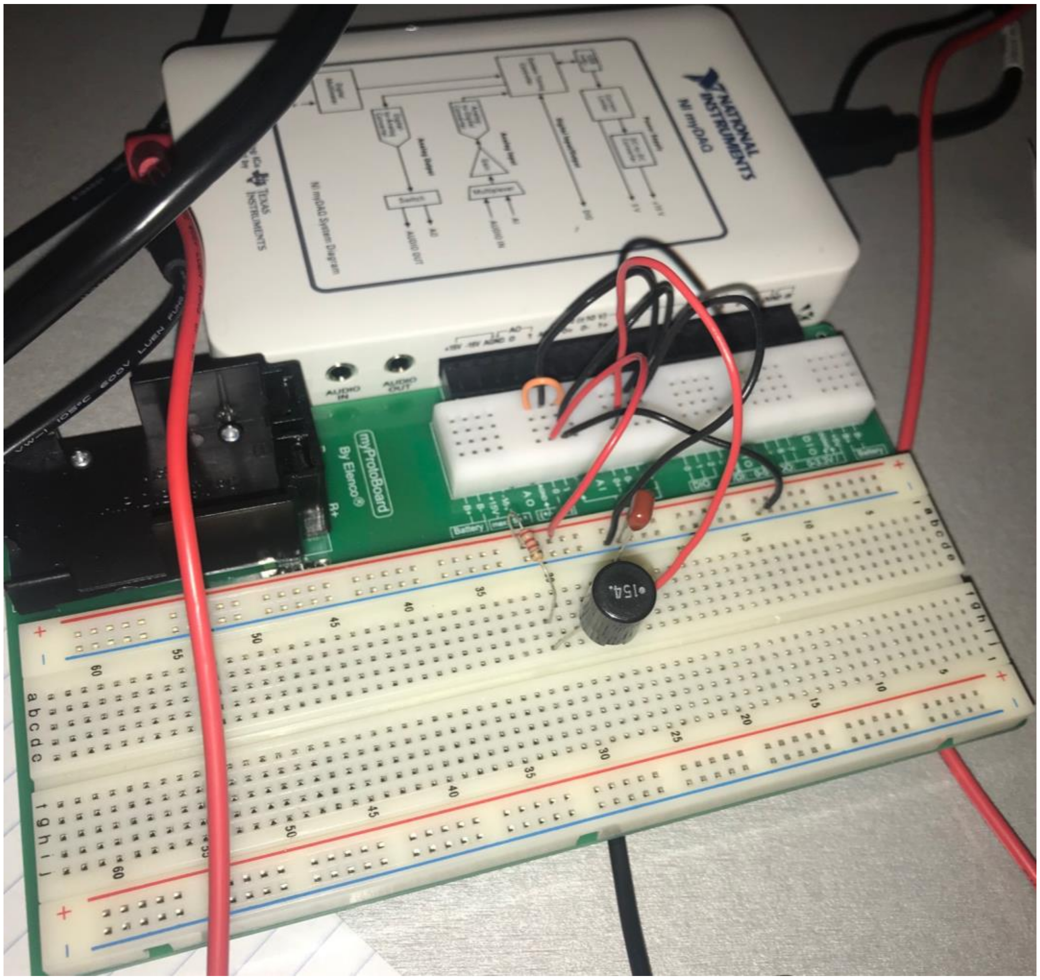
$$\Rightarrow L\frac{d^2i}{dt^2} + (R+R_L)\frac{di}{dt} + \frac{1}{C}i = 0$$

$$\Rightarrow 0 = \frac{d^2i}{dt^2} + \frac{(R+R_L)}{L}\frac{di}{dt} + \frac{1}{CL}i = 0$$

$$\omega^2 = \frac{1}{LC} = 6.67 \times 10^7$$

$$2\xi \times \omega = 15960$$

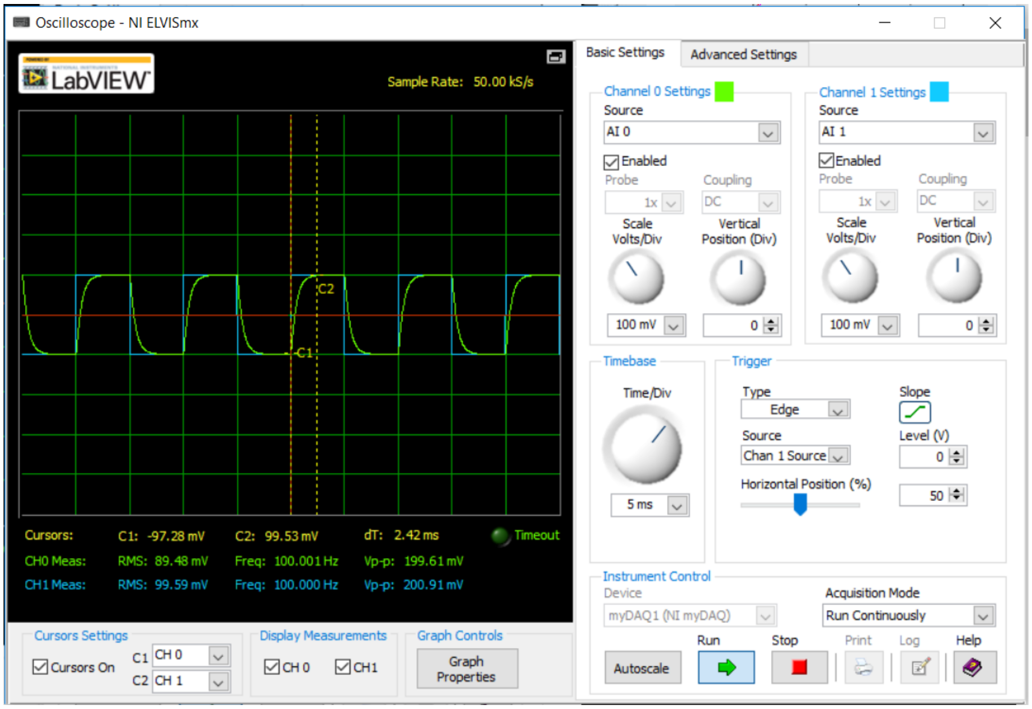
$$\xi = 0.977$$

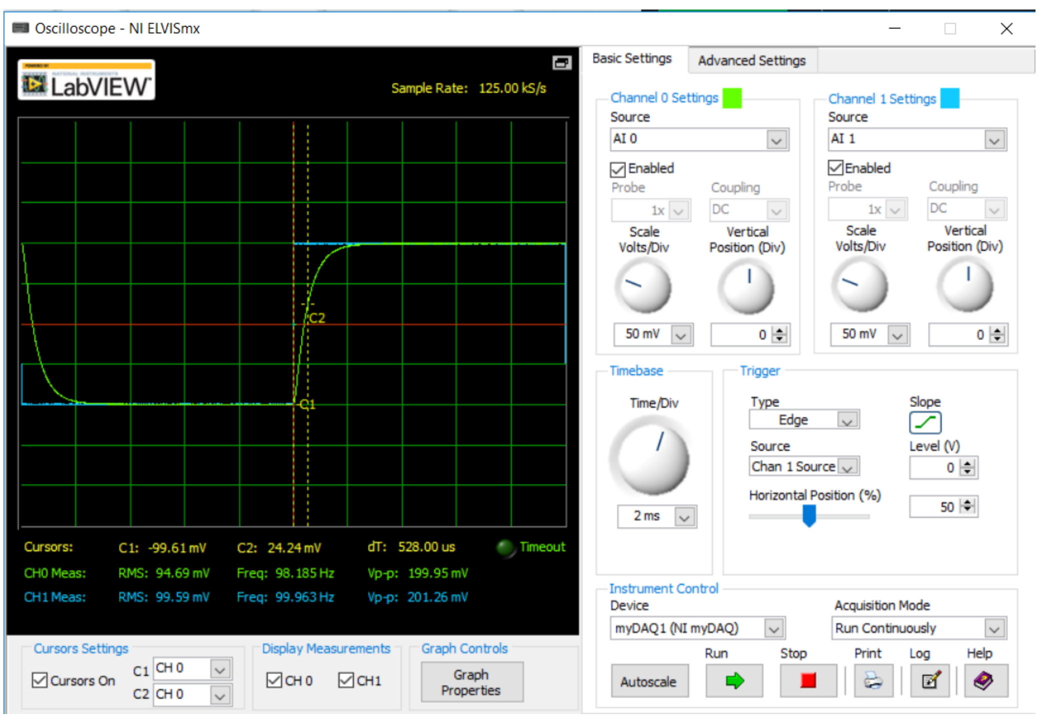


iii. Set the function generator to 100Hz (adjustable if desired) and output a square wave with 1V (p-p) as in Lab 1 of the laboratory.

iv. Connect one channel of the oscilloscope to view the input waveform, and another to measure the output voltage across the capacitor. Record your observations.

Response (Please attached necessary screenshots of oscilloscope):





Discussion

2.1 What kind of damping is observed? Compare the theoretical damping factor with the measured one.

We can see that critical damping was observed based on the oscilloscope. Based on the theoretical value that was seen, we know that critical damping occurred since the value of 0.977 was essentially equal to 1. Based on the experimentally measured value, we also observed critical damping. Using the given equation that relates voltage with the damping factor we get.

Experimentally calculated dT of 528 microseconds as seen from the second oscilloscope photo.

~~Lab 3. Underdamped RLC Circuit Design (Demonstration Experiment)~~

~~Lab 3 needs detail report~~

i. ~~By changing the value of the resistor from Lab 1, acquire an underdamped response that overshoots the final voltage value by approximately 3 (take into account inductor resistance as well). Record the value of the resistor used, as well as the experimental value of the overshoot now considering the resistance of the inductor and the internal resistance of the generator.~~

ii. ~~Design an underdamped system second-order circuit that has at least 4 visible peaks in oscillations before settling such that you can measure the period of oscillation. Compare the experimental value obtained by measuring the period of the oscillations with the theoretical value.~~

iii. ~~Write the equation for $V_{OUT}(t)$ which describes the voltage across the capacitor as a function of time for your design.~~ **~~(Show the necessary steps of your work)~~**

~~Resistances~~			
	~~R (Used) Ω~~	~~R (Internal) Ω~~	~~Function Generator Ω~~
~~Theoretical~~		~~0~~	~~0~~
~~Measured~~		~~245±10~~	~~430±10~~

~~Time Between Peaks~~			
	~~Peak 1-2~~	~~Peak 2-3~~	~~Peak 3-4~~
~~Time Difference dT~~	~~1.23 ms~~	~~1.10 ms~~	~~1.18 ms~~

~~Discussion~~

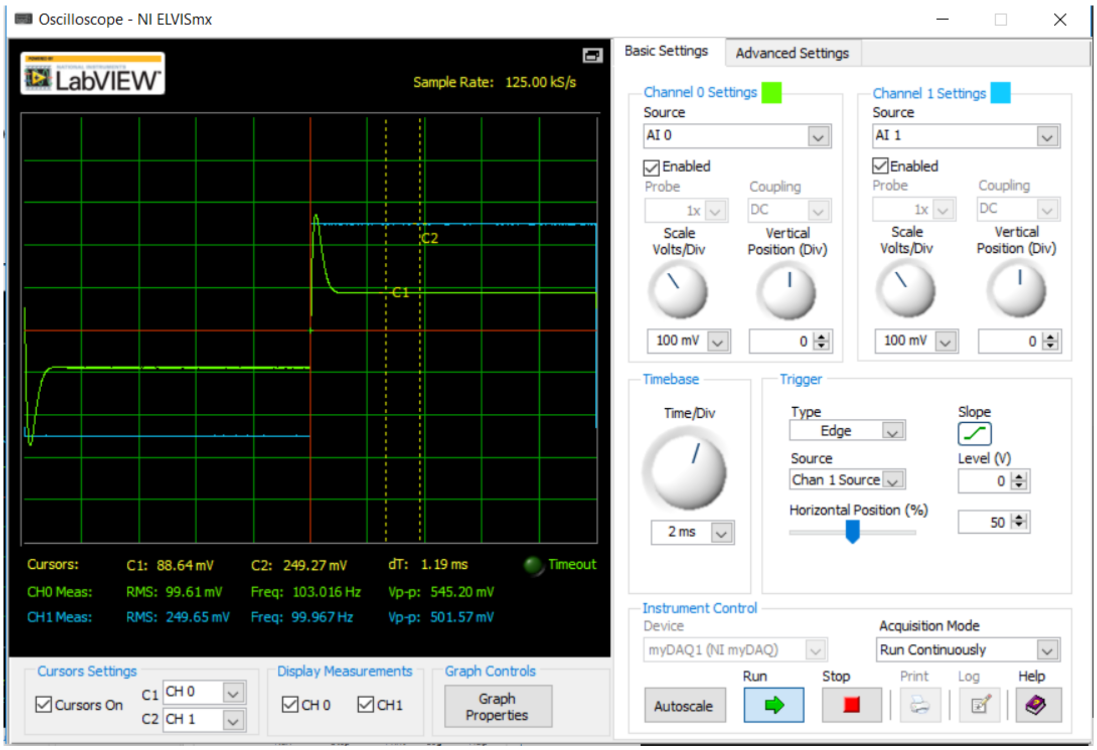
~~3.1 How did the experimental damped frequency compare with that of the theoretical values?~~

~~3.2 What happens if you try to make the overshoot smaller?~~

Lab 4. Critically Damped RLC Circuit

i. Replace the resistor of the circuit in Lab 1 (Figure 3), again, but this time by a 10kΩ potentiometer. By using the oscilloscope output as a guide, adjust the resistance until you think the system is approximately critically damped. Record the output you obtain, remove the potentiometer, and measure the value of the resistance you used.

Response(Please attached necessary screenshots of oscilloscope):



The value of the resistance:

Experimental Value: 0.449k Ω or 449 Ω from the potentiometer

LAB 4: CRITICALLY DAMPED RLC CIRCUIT

CRITICAL DAMPING MEANS THAT

$\xi = 1$, $\xi = \alpha / \omega_0$

$$\xi = \frac{1}{2}\left(\frac{R_L}{L} + \frac{1}{RC}\right) \Big/ \sqrt{\frac{1}{LC} + \frac{R_L}{RLC}}$$

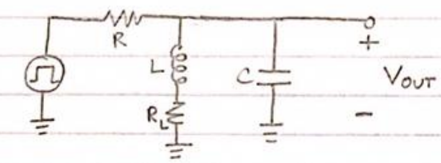
$R = ?\,\Omega$, $R_L = 245\,\Omega$, $L = 150$ mH , $C = 0.22\ \mu F$

$$1 = \frac{1}{2}\left(\frac{245}{0.15} + \frac{1}{R(2.2\times10^{-7})}\right) \Big/ \sqrt{\frac{1}{0.15(2.2\times10^{-7})} + \frac{245}{245(0.15)(2.2\times10^{-7})}}$$

$\boxed{R = 326.151\,\Omega}$ for critical damping

THEORETICAL RESISTANCE

LAB 3: UNDERDAMPED RLC CIRCUIT DESIGN



$$iR + L i_L' + R_L i_L = V_s \quad , \quad i_L = \frac{C\,dv}{dt}$$

$$iR + \frac{Q}{C} = V_s \Rightarrow i = \frac{V_s - V_c}{R} \qquad i' = \frac{V_c'}{R}$$

$$i_L = i - i_c \Longrightarrow \frac{V_s - V_c}{R} - C V_c'$$

$$\frac{di_L}{dt} = \frac{di}{dt} - \frac{di_c}{dt} = \frac{-V_c'}{R} - C V_c''$$

$$\left(\frac{V_s - V_c}{R}\right)R + L\left(\frac{-V_c}{R} - C V_c''\right) + \left(\frac{V_s - V_c}{R} - C V_c'\right)R_L = V_s$$

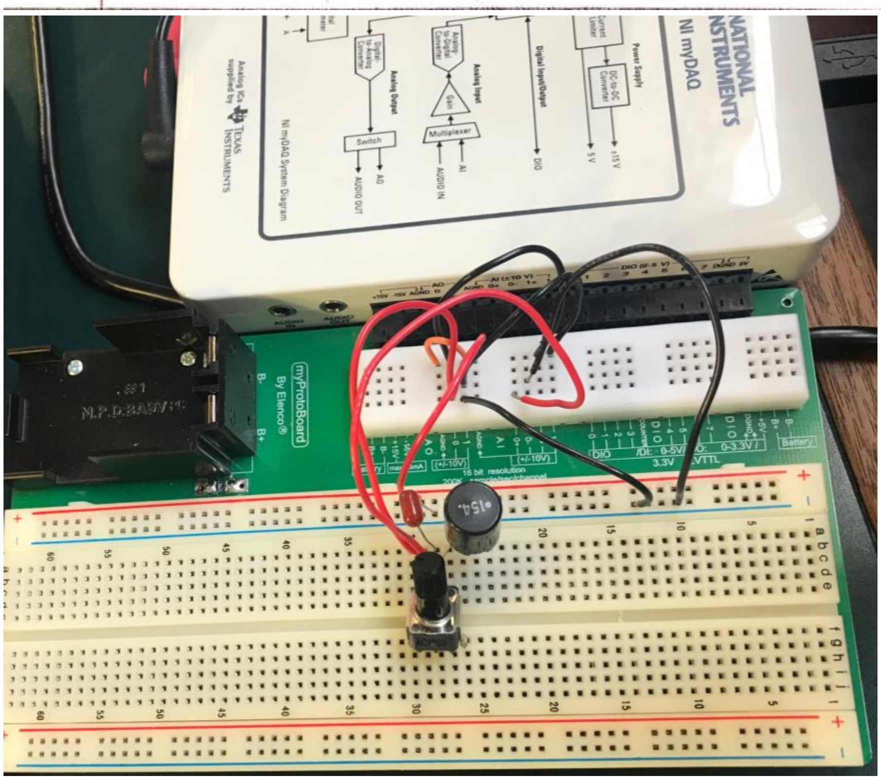
$$-V_c - \frac{L}{R}V_c' - LCV_c'' - \frac{R_L}{R}V_c - R_L C V_c' = \frac{-R_L}{R}V_s$$

$$LCV_c'' + \left(\frac{L}{R} + R_L C\right)V_c' + \left(1 + \frac{R_L}{R}\right)V_c = \frac{R_L}{R}V_s$$

$$V_c'' + \left(\frac{1}{RC} + \frac{R_L}{L}\right)V_c' + \left(\frac{1}{LC} + \frac{R_L}{LCR}\right)V_c = \frac{R_L}{RLC}V_s$$

$$V_c = e^{-\alpha t}\left(A_1\cos(\omega t) + A_2\sin(\omega t)\right) + \frac{R_L}{R+R_L}V_s$$

$$\alpha = \frac{1}{2}\left(\frac{R_L}{L} + \frac{1}{RC}\right) \quad , \quad \omega_0 = \sqrt{\left(\frac{1}{LC} + \frac{R_L}{LCR}\right)}$$



Discussion

4.1 How close was the value of resistance you ended up with when using the potentiometer to obtain a critically damped response, to the theoretical value you have derived? Consider the effects of inductor resistance as well.

My theoretical (critically damped) resistance calculated was 326.151 ohms, while the experimental resistance calculated was 449 ohms. The values obtained, though not the same, are definitely close enough to show that the calculations were accurate. This proves that the differential equation derived for a parallel RLC circuit with inductor resistance accurately models the circuit. A source of error, that could have caused the discrepancy with theoretical and calculated values is the internal resistance of the function generator which was not taken into account with the differential equation.

4.2 What did you observe in the output waveform as resistance varied?

Higher resistances of the potentiometer correspond to the voltage response being more underdamped and grew more peaks, while lower resistances of the potentiometer correspond to the voltage response becoming more overdamped and reducing the number of peaks.