DAY 1: WHICH NOTES WORK WELL TOGETHER?

MUSIC: THE NUMBER THEORY OF SOUND J-LO MC2023

This class will be driven almost entirely by **exploration**. The goal is not just to discover the answers to a given set of questions, but to discover the questions as well! The worksheets provide some questions to get you started, but you are not required to go through them all; try to come up with your own questions and conjectures as a group and let that guide your exploration.

Explorations

Using the "Frequency Pair Tester" (available either using the QR code above or at rb.gy/t2lbz), develop a sense for the relationship between the *frequency* of a sound and your *experience* of the sound.

Exploration 1. Play various pairs of frequencies, and rank them based on "chili level;" how *spicy* is the resulting sound? Do not rank based on how much you *like* the sound, and (if you have music theory background) try to avoid comparing the sounds to intervals you already know. Focus on your gut reaction, the way it makes you feel at a visceral level.

- **1:** mild, simple, peaceful; the notes feel like they belong together; makes you feel calm/relaxed
- **4:** medium, intriguing, uncertain; the notes have a bit of tension between them; makes you raise your eyebrows or feel very slightly uncomfortable
- **7:** spicy, complex, harsh; the notes are strongly clashing with each other; makes your muscles stiff and hair stand on end

Discuss your answer with your group; how similar are your responses?

Additional notes and questions:

By default, the frequency pair tester is producing *sine waves*. Specifically, if you choose frequencies f_1 and f_2, your device produces a wave where the air pressure after t seconds is given by

$$w(t) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t). \tag{1}$$

Here are some things to try:

(1) Start by playing one note at a time (you can leave a box blank to prevent it from playing). As you change the frequency, how does the sound change?
(2) To rate randomly generated pairs, use *Options / FAST MODE*.

(3) If you want to hear each note one at a time, use *Options / Timing: Consecutive*. (But you should only assign chili levels to the way they sound when played simultaneously.)
(4) If a frequency is very low, the pressure from the corresponding sine wave changes so slowly that your ear can't sense any difference. If you use *Options / Waveform: Clicks*, the site will play a given number of clicks every second; this can be helpful if you want to hear lower frequencies more easily. **Avoid using this option with high frequencies,[1] as the resulting sound will be quite harsh.** (Remember rule 1!)
(5) Try to develop a mathematical model for spiciness. When will two different pairs of frequencies have a similar quality? Is there a function that takes a pair of positive real numbers as input and returns an approximate chili level?
(6) If you think you have an idea for how to predict spiciness mathematically, try to give a physical explanation in terms of the shape of the pressure wave 1 (you may want to use *Options / Graphical investigation*).

Optional: Exploring extreme cases

(7) Turn on *Options / Waveform: Clicks* and leave one frequency blank (so only a single note plays). If the frequency is extremely low, you will hear individual clicks. What is the highest frequency at which you can hear individual clicks? What is the lowest frequency at which you can hear a musical note?
(8) Test your ability to tell frequencies apart!
   (a) Use *Options / Waveform: Sine* and *Options / Timing: Consecutive*.
   (b) One group member chooses a starting frequency (e.g. 500) and a second frequency that is slightly lower, equal, or slightly higher (e.g. 490, 500, or 510), but does not say which.
   (c) Everyone else says whether they think the note went up, down, or stayed the same.
   (d) Repeat several times (and take turns). If anyone is consistently answering correctly, gradually decrease the difference (e.g. use 493, 500, and 507).

   What is the smallest difference for which you can consistently answer correctly? Does this smallest difference change if you pick a different starting frequency?
(9) Repeat the experiment above, but now with *Options / Timing: Simultaneous* (instead of same/higher/lower, you just need to guess same/different). Can you do better than before? Why or why not? Can you explain what's going on mathematically using Equation 1?

[1] If your curiosity gets the better of you, I recommend (a) starting with a low frequency and increasing very slowly (no more than 50Hz at a time), and (b) ensuring only you can hear the sound (use headphones, or set the volume extremely low and hold your ear directly to the speaker).