Day 1 scribbles

July 20, 2020 10:40 AM

Welcome to

Geometry of Lattices!

While we wait to start, discuss:

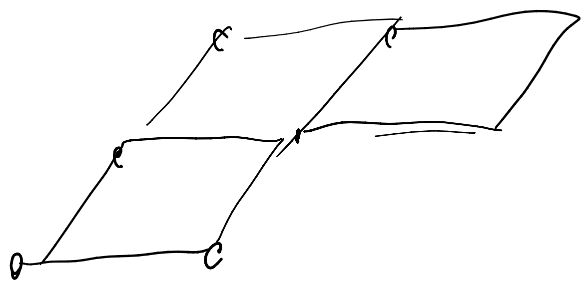
What is $\left\{\begin{array}{l}\text{the best} \\ \text{your favorite} \\ \text{an ideal}\end{array}\right\}$ number of dimensions?

And why?

Today

Explore 2-D lattices!

(check chat / Slack for notes.)



lattice: $\{mu+nv \mid m, n \in \mathbb{Z}\}$

fund parr: $\{ru+sv \mid r, s \in [0,1)\}$

Given any $au+bv$, divide the coeffs into $\underbrace{\text{integer part}}_{\text{lattice}}$ and $\underbrace{\text{fractional part}}_{\text{fund parr.}}$

into $\underbrace{\quad\quad\quad}_{\text{lattice}}$ $\underbrace{\quad\quad\quad}_{\text{fund parr.}}$

$$\left.\begin{aligned} a\begin{pmatrix}u_1\\u_2\end{pmatrix}+b\begin{pmatrix}v_1\\v_2\end{pmatrix}&=\begin{pmatrix}1\\0\end{pmatrix}\\ c\begin{pmatrix}u_1\\u_2\end{pmatrix}+d\begin{pmatrix}v_1\\v_2\end{pmatrix}&=\begin{pmatrix}0\\1\end{pmatrix}\end{aligned}\right\}\quad \begin{pmatrix}u_1 & v_1\\ u_2 & v_2\end{pmatrix}\begin{pmatrix}a & c\\ b & d\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

integer matrices that multiply to give $\boxed{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}}$ $\boxed{\text{identity}}$

Try Exp 7,8

(optional exps are optional)