Portfolio Risk Analysis

FI6012: Technical Assignment

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Course: MSc. in Computational Finance

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Var Indroduction

According to Danielsson, (2011) after volatility, Value at Risk (VaR) is the most common measurement of risk. The modelling of this measurement, enables financial institutions to quantify the risk that their capital could be potentially exposed to, in terms of losses, over a specified risk horizon (Alexander, 2008b). To obtain the level of capital reserves which were need in order to survive a crisis in the financial market, the VaR approach was the standard risk measurement tool under the Basel II Accord, stating “regulatory capital is based on .. the 1% 10-day VaR if the value-at-risk approach is used” (Alexander, 2008b, p.401). However it has come under vigorous review since the financial crisis as one of the most important believed causes of the crisis was the mismanagement of risk. As a result it has been found that a problem exists in regards to the accuracy of the measurement, with assumptions causing notable model risk (Dorić & Nikolić-Dorić, 2011). One other criticism of the VaR being implemented as risk measurement, is its inability to estimate the magnitude of notably extraordinary losses. As a result, to overcome the dilemma of shortages of subadditivity in VaR, numerous other risk measurements were proposed. One such measurement is Expected Shortfall (ES), which is the most common alternative to VaR. Proposed by Artzner et al., (1996), it provides the worst conditional expectation measure of risk and contributes an answer for the following question: *“What is expected loss when losses exceed VaR?”* (Danielsson, 2011, p.85). According to Alexander, (2008b, p.128), the statistical modelling of ES calculates an estimation of the expected loss, given that VaR is exceeded by the loss.

Reasearch Aims and Hypotheses

The rationale behind this paper lies two-fold. Firstly within the scope, is to determine the most accurate type of historical simulation (HS) approach for calculating VaR and ES values. Secondly, to answer the question, what assumptions and characteristics of each model enhances or hinders their suitability? In this paper, four different approaches to HS are simulated and examined, with the objective to obtain estimations to 1% VaR of the S&P 500 Index. The approaches are as follows:

Approach 1	HS and applying scaling factors through the square root of time
Approach 2	HS with non-overlapping 10 day periods,
Approach 3	HS with overlapping adjusted daily returns
Approach 4	Filtered Historical simulation (FHS) using overlapping

Table 1: Historical Simulation Approaches

Backtesting will be incorporated in to each of the approaches to ensure validation of the VaR results which are obtained. Backtesting will come in the form of the violation ratio (VR) and normalised shortfall (NS), for estimations of VaR and ES. VR and NS will act as simple performance indicators of the approaches. A risk model deemed to be reliable, should be capable of generating the number of violations as expecting, whilst not observing a clustering of violations (Boucher, Danίelsson, Kouontchou, Maillet, & Bertrand B. Maillet a, 2014) . Independence tests and Bernoulli coverage test will also be incorporated in to this work. The data collected for testing and estimation scope of the S&P 500 index was collected during the following time period: 1st January 1990 to 3rd March 2017.

The single index portfolio, the S&P 500 index will be the subject of the papers analysis and testing. It was chosen due to its ability to be seen from the perspective of a portfolio of 500 companies which are all equally weighted. Therefore it acts as an excellent surrogate for a multi-asset portfolio. If the paper were to be adopted to simulate the VaR and ES of a multi-stock portfolio, the appropriate weights would have to be applied to the historical returns, which in turn will measure the total VaR. This measurement will be broken down in to two components, systematic VaR and specific VaR. The systematic VaR component is due to the movements of the index as a whole, whilst the specific VaR component is due to *"idiosyncratic movements of each stock price".* (Alexander, 2008a, p.179).

The form of the portfolio, whether is it daily rebalanced or fixed weights, is not relevant due to the total historical VaR being "*calculated from a distribution of reconstructed portfolio returns"*(Alexander, 2008a p.179). Nonetheless, in order for an accurate estimation to be obtained, the portfolio weights must be adjusted. This results in additional costs for rebalancing, which must be accounted for.

The computing programme package Matlab is used to compute all estimations and backtesting discussed in this paper, with the code located in the Appendix. The remainder of this paper will be formatted as follows: The methodology for each approach will be discussed and analysed; the results obtained from the simulated approaches will be examined; Lastly, the paper will conclude by discussing the observations of its calculations.

Historical Simulation

Historical Simulation (HS) is a nonparametric method for risk forecasting. It collects raw market values of risk in a specified period of time, and calculates the variance over this period to be implemented in the VaR methodology. According to Danielsson, (2011, p.95), it is reliant on the assumption of the possibility *"that history repeats itself".* This assumption means that correlations and volatilities which occur in the returns of the historical sample, may also be replicated in the future. HS is also dependent on the assumption that the chosen sample period describes the properties of assets accurately. This methodology in an instance, is taking the estimated VaR to be equal to an alpha quantile of the historical returns, where the skewness and excessive kurtosis have been captured (Boudoukh, Richardson, & Whitelaw, 1998). However, when applied implemented as a risk measure for computing, VaR is more significant and a different risk horizon may be necessary in its estimation. Linear parametric VaR models have the ability to capture skewness, however the following assumptions must be applied, the distribution of the risk factor returns being normal mixture or student-t (Alexander, 2008a, p.129). Under the Basel regulatory framework, estimations of 10-day risk reported by banks are to be formatted to VaR-based models (Committee on Banking Supervision, 2013). It is also noteworthy that the calculation of VaR, does not include the assumption of normality (Danielsson, 2011, p.79)

VaR models which contain a specific distribution of returns, are known to be static. The risk horizon, h, for these models can be obtained, through the scaling of 1-day VaR up to h-day Var, when h>1. For normal linear VaR, the scaling factor to use is, which is accepted by the Basel II regulatory framework, is the square root of time.

$$VaR_{h=5} = (VaR_{h=1}).(\sqrt{5})$$

If the returns have a stable distribution, it is not guaranteed they are normally distributed (Alexander, 2008a, p.146). An advantage of HS is that it does not assume that daily returns are independently and identically distributed (i.i.d), as the distributions of returns can display fat tails and skewness. Another advantage is from empirical distributions, HS can obtain how dependent risk factor returns structure is (Alexander, 2008a, p.141). As mentioned previously, 4 different approaches to HS will be included in this paper.

Breakdown of testing periods

A estimation period of approximately 3 to 5 years of data is recommended by the Basel regulations, for HS models, however in order to examine the significance of the length of historical data, as well as the vitality of VaR and ES estimation models, a large sample size of data was implemented in to this project. In turn, setting the testing window to a sufficiently large period, would improve the performance of Bernoulli coverage tests and also the independence test. This will prevent rare violations being obtained, which are can occur when a small sample of data is used (Danielsson, 2011, p.160). The estimation window was set in the range of 500 to 2500 days with intervals of 500 days. The testing window on the other hand, includes the time period of the 2007-2008 financial crisis.

	Estimation window	Testing window	Total Period Size
Size (Trading days)	500	2500	3009
	1000	2500	3509
	1500	2500	4009
	2000	2500	4509
	2500	2500	5009

Table 2: Historical Data for Historical Simulation

Preliminary Testing of Returns

The distribution of S&P 500 returns over the chosen time period were examined over an array of standardised tests for normality and autocorrelation. From viewing Figure 1, high levels of excess kurtosis is evident in the daily returns distributions, as well as indications of non-normality and slight skewness. There is a high probability of the S&P 500 realising sizeable negative outlier returns than those that are forecasted by the normal distribution, due to the leptokurtic distributions. Upon testing, the returns are deemed to be autocorrelated which differs from the assumption that i.i.d is necessary for stable distribution, and hence the scaling rule is portrayed as unreliable. Evidence of volatility clustering is displayed in Figure 3, whilst also volatility regimes in Figure 4.

From viewing the following figure 4, it is evident that there were periods which experienced high levels of volatility but also periods of low levels. This observation would raise the belief that inaccuracies could exist if a simple HS approach was incorporated as periods with abrupt increases in volatility, such as crisis event, can at times result in an overestimation or underestimation of VaR.

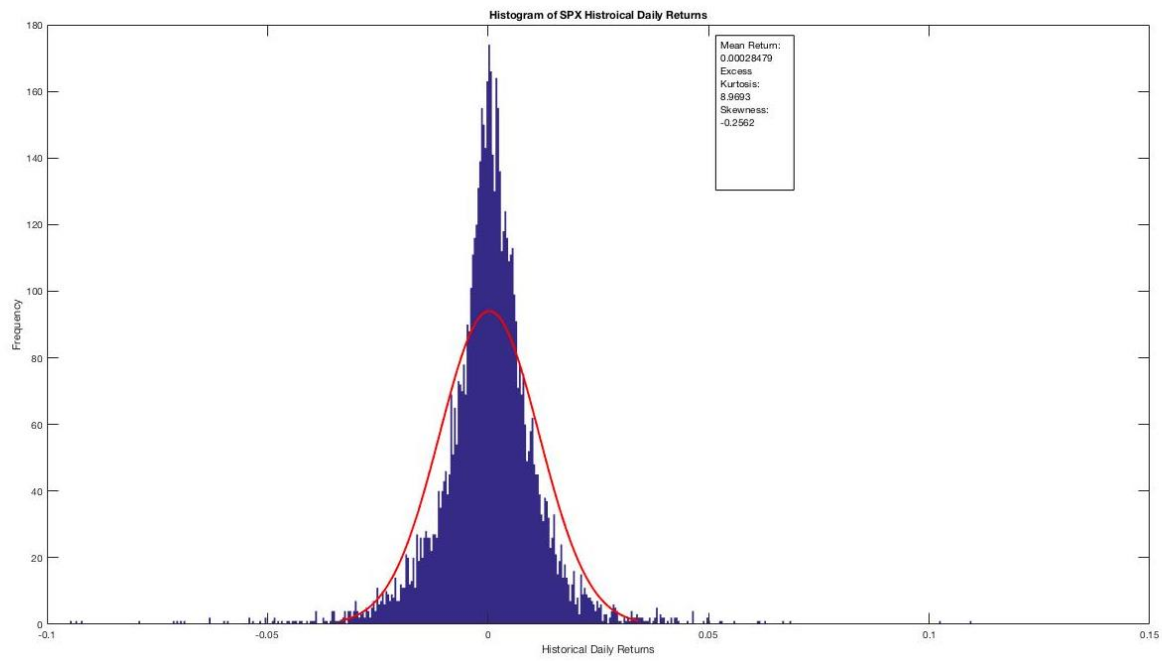

Figure 1: Histogram with whole sample of data

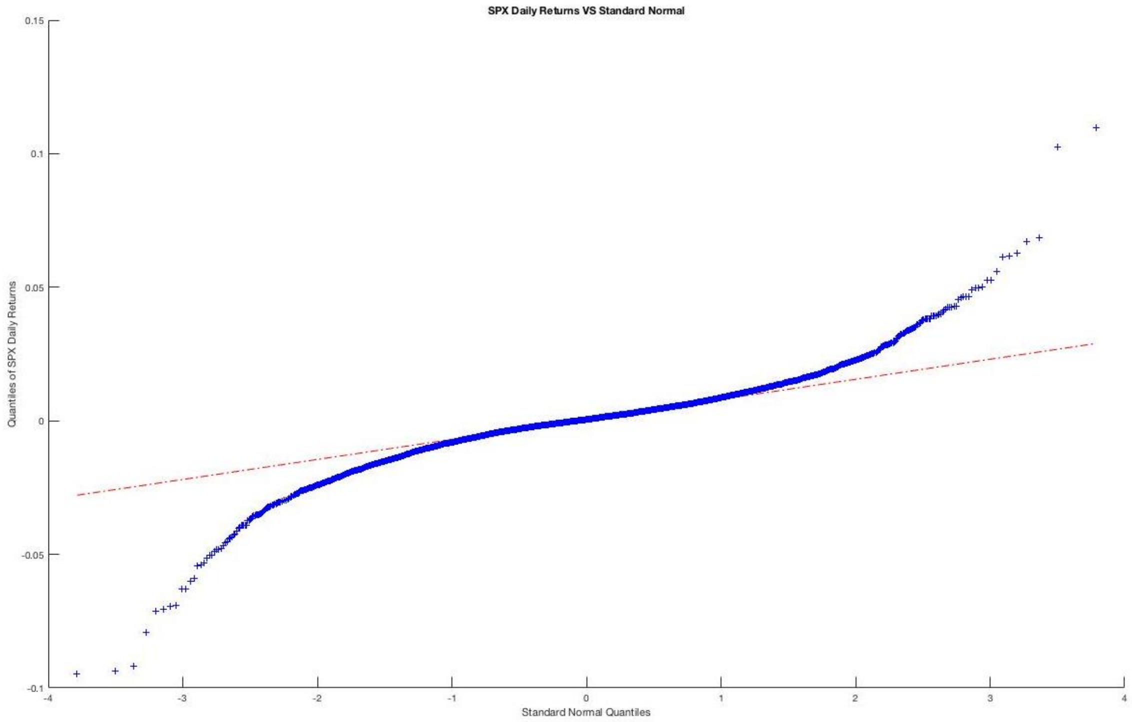

Figure 2: QQPLOT

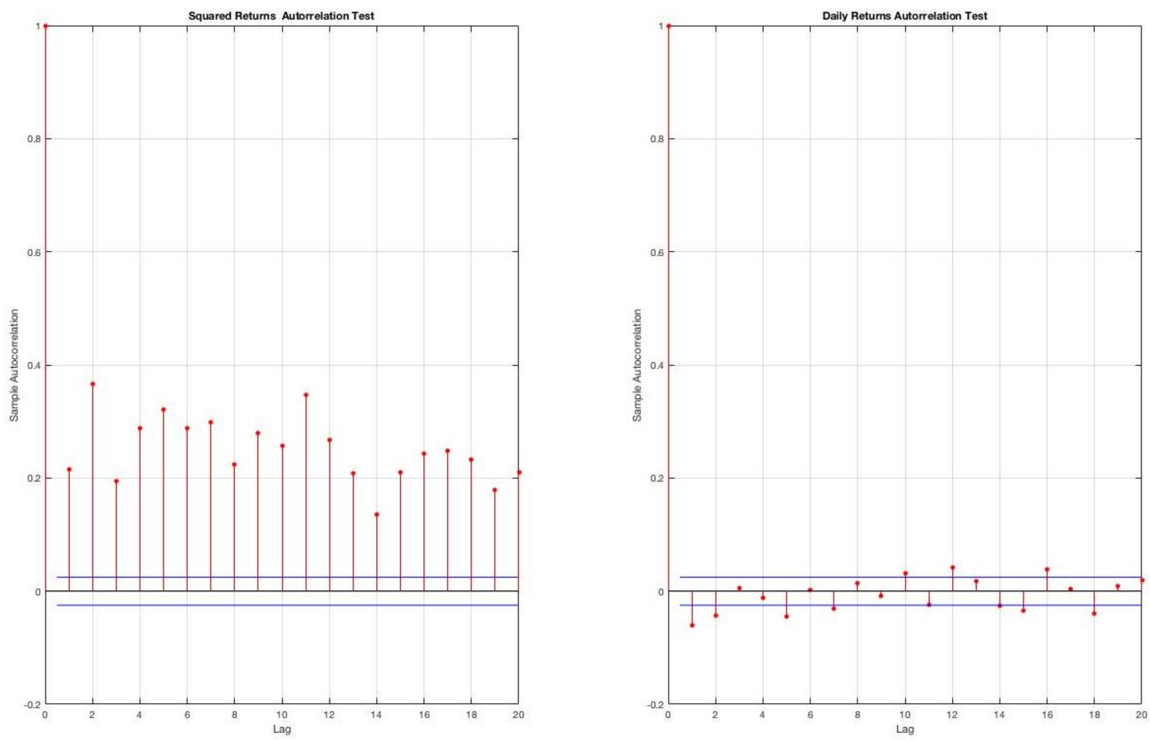

Figure 3: Autocorrelation Test for Squared Returns and Returns

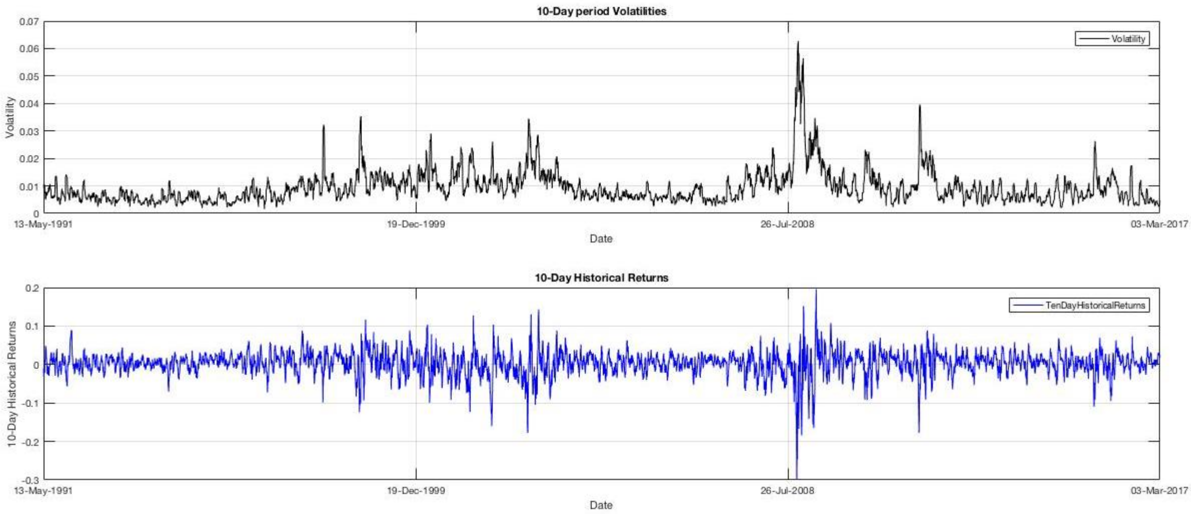

Figure 4: Comparison of Historical Returns and 10-day period volatility

Approach 1: HS - Square root of time as scaling factor

The first approach will have HS being used to obtain an estimation of 1-day VaR which will then be scaled to 10 days, by the square root of time. In order for an accurate estimation of VaR to be obtained, there should be a complete absence of structural gaps in volatility. When this absence is in place, no estimation error should be found in the approach and therefore under these circumstances, HS performs better than parametric alternatives (Danielsson, 2011). To ensure that no structural breaks exist, a visual check of the daily volatility was undertaking over the specified period of time. Another notable issue which exists within this approach is using the square root of the time horizon as a scaling factor. The issue arises as this assumption assumes that the returns are i.i.d (Alexander, 2008a), i.e. the returns will not contain any autocorrelation. The scaling rule is severely inaccurate, when returns are autocorrelated, or when there is a clustering of volatility.(Alexander, 2008a). Practitioners are already aware that there are limitations to implementing HS for risk forecasting and explains why the scaling rule is still being applied, even though it possibly be wrong (Alexander, 2008a). Instead of scaling, the approach could apply 10-day historical returns, however using a long history of data would lower the value of more recent information, which would reflect the present market more accurately. (Boudoukh et al., 1998). Model Risk can also possibly be introduced to the VaR estimation, if the returns are not i.i.d. (Alexander, 2008a). However this issue should not be significant in this approach as the approach is essentially correct for 1% VaR (Danielsson, 2011). Alternative parametric methods other than the HS approach, tend to underperform in comparison where there is an absence of structural breaks, due to the HS approach not containing an estimation error (Danielsson, 2011).

Approach 2: HS – non-overlapping 10-day returns

The goal of the subsequent approach is the same as the first, to estimate 10-day VaR, however it differs by incorporating non-overlapping 10 day returns in its estimation. It also meets the same issues of the first in the form of structural breaks existing in volatility. To implement this approach and compute a valid estimation, it is necessary to gather a large amount of data. In order to calculate an estimation of 1% Var, a minimum time period of 1000 days is required. However, the estimation of ES will be equal to VaR, if the time period in which data is collected in, is only 1000 days. *"The minimum recommended sample size for HS is* $3/p$*"* is taken as the general rule when choosing the estimation window size, where p significance level (Danielsson, 2011, p.98) . It is also noteworthy that old data is unlikely to be representative of the trends which exist in the market presently.

Approach 3: HS – overlapping adjusted returns

A volatility weighting approach was introduced by Hull & White, (1998), which allowed volatility adjustment to be included into HS for obtain VaR. Through the ratio of present daily volatility to that of a historical sample period, the real historical returns are adjusted (Hull & White, 1998). Consider a historical sample with 4 years of returns. Denoting the unadjusted historical portfolio returns as $\{r_{t,T}\}_{t=1}^{T}$, and the returns volatility as $\{\hat{\sigma}_t\}_{t=1}^{T}$, where T = time at end of the sample (1000 days). Regarding σ_{1000} as the present level of volatility, then adjusted returns of the first day, which reflect the present volatility regime can be calculated as:

$$\hat{r}_{t,T} = \left(\frac{\sigma_{1250}}{\sigma_1}\right) r_t$$

This approach will estimate 10-day VaR using overlapping periods, with adjust returns. A further discussion of the application of GARCH to approach 3 is discussed in the next section.

Approach 4: FHS – overlapping daily data

The fourth approach implements Filtered historical simulation (FHS) which will estimate VaR and ES that will be conditional on more recent market conditions (Gurrola-Perez & Murphy, 2015). This is approach was developed by Barone-Adesi, Giannopoulos, Barone-Adesi, & Giannopoulos, (2000) where the idea of adjusting volatility to HS was expanded to incorporate overlapping data in such a way "*that does not create blunt tails for the h-day portfolio return distribution"* (Alexander, 2008b). This model is able to adjust sample returns to echo present day market conditions, whilst also releasing them from data limitations. An attractive trait of FHS is its un-reliance on assumptions about the returns distribution. By incorporating asymmetric GARCH in to Approach 3 and 4, the daily log returns were simulated throughout the risk horizon, to model the volatility. Standardised residuals, which are i.i.d, are then calculated by dividing the volatility estimated by the GARCH, into the simulated/observed returns. The GARCH forecast for the second day is updated by the return, and from the sample, is subsequently multiplied by a new random residual. This procedure is consequently repeated for each day until the end of the risk horizon has been reached. The complete return for the VaR horizon is calculated by the sum of the estimated returns. Simulating this approach thousands of time, obtains 10-day returns distribution, whilst the alpha quantile produces the VaR estimation. In theory, not only can the regime-switching behavior of volatility, be captured through implementing this approach and approach three to estimate VaR, but so can returns and volatility clustering. The dependence on a parametric model such as GARCH, by these HS approaches is evident, however GARCH performs a different role in each. It operates as a means to obtain the statistical empirical volatilities, when applied in Approach 3. However, by using manually estimated volatility, adjusted returns methods can still be applied in the absence of a GARCH model. In approach 4, FHS, a GARCH model is necessary for modelling how volatility will involve in the future.Thus, a comprehensive assessment should be completed for the selection of which GARCH model to apply, as different models can produce contrasting results. In this paper, the simple GARCH(1,1) model will be implemented. According to Alexander, (2008b), using a sample which contains overlapping data in the FHS model, will alter the shape of the tail of the distributions. The GARCH model does not solve this issue which in turn could lead to an inaccurate VaR estimation (Alexander, 2008b). The 10-day returns for Approach 4 will be calculated by simulating 10 daily returns with random standardized residuals, and finding the sum of all these values. Due to the FHS being fused with Monte Carlo simulation, there is no repetitive computation needed. This paper implemented 5,000 simulations into Approach 4. The number of simulations have a significant impact upon the outcomes.

The following assumptions can be made from the brief deliberation of the four approaches: When incorporating the entire data sample, the FHS model should return the most precise VaR and ES estimations. If only given a section of the data sample, the simple HS model should outperform the FHS approach, given that there is an absence in structural breaks in volatility. As a large estimation window is necessary in order to obtain accurate estimations from the second approach, it can be forecasted the first and third approaches will provide more precise estimations in comparison.

Estimation and Backtesting

Backtesting will be incorporated in to each of the approaches to ensure validation of the VaR results obtained. Backtesting will come in the form of applying the violation ratio (VR) and normalised shortfall (NS), for estimations of VaR and ES, which will act as simple performance indicators. In order for VaR to be estimated and backtested, estimation and testing windows are specified to break down the collected data sample. A comparison of the estimation results is then made in contrast to the following 10 days return. The estimations and backtesting are then repeated, after the start of the window is moved to the following day. The number of exceedances divided by the expected number of exceedance is how VR is obtained. NS on the other hand, is the mean of the ratios between the returns which are observed on VaR violation dates, to the returns which returns on ES violation dates (Danielsson, 2011). Results can be seen in Table 3 below:

	Estimation Window Size	Violation Ratio	Normalised Shortfall
Approach 1	500	1.12	-1.2085
	1000	1.36	-1.1315
	1500	1.08	-1.1895
	2000	0.92	-1.2174
	2500	1.16	-1.1278
Approach 2	500	2.48	-1.6067
	1000	1.8	-1.6115
	1500	0.76	-1.9256
	2000	0.8	-1.383
	2500	0.72	-1.4318
Approach 3	500	1.44	-1.3292
	1000	1.44	-1.2858
	1500	1.32	-1.1766
	2000	1.04	-1.2482
	2500	1.16	-1.1534
Approach 4	500	1.04	-1.3706
	1000	1.04	-1.3363
	1500	0.92	-1.3917
	2000	0.88	-1.3835
	2500	1	-1.3385

Table 3: Violation Ratio and Normalized Shortfall results

In regards to using different data samples, how each approach performed was inconsistent. However, their performance improved, once a larger estimation window was implemented, especially the results of Approach 2. Approach 3 and 4 ability to predict future risks was poor when a small period of data was used.

The indication that ES is a more competent measurement of risk than VaR is evident in the NS comparison graph. Their performance is more superior in contrast with VR. However, the backtesting of ES is not as credible when compared with the backtesting to VaR. (Danielsson, 2011). It is also worth noting that the VaR model produced the historical observations that were implemented in to the comparison with ES, as opposed to actual returns. If VaR is exceeded by the return initially, there will be exceedances for ES, however if VaR is not, there will be none for ES as it is dependent on the results of VaR. Therefore, if there are errors in the estimation of VaR, they will have a direct effect on the performance of ES backtesting. The conclusion after completing the various HS approaches and backtesting, is that over a lengthy time horizon, each approach produces approximately the correct number of violations, which are expected. Secondly, the amount of days used in the estimation window will have a significant impact upon the results. From examining the results, it can be found that when backtesting is implemented over a risk horizon of a long period, a simple HS approach with a scaling factor, will not be significantly impacted by the estimation window size. In regards to HS with non-overlapping days, meaningful estimations can only be produced when the estimation window is a large amount of days.

Appendix A: Graphs

Figure 5: Comparison of 4 Approaches at Estimation Window = 500

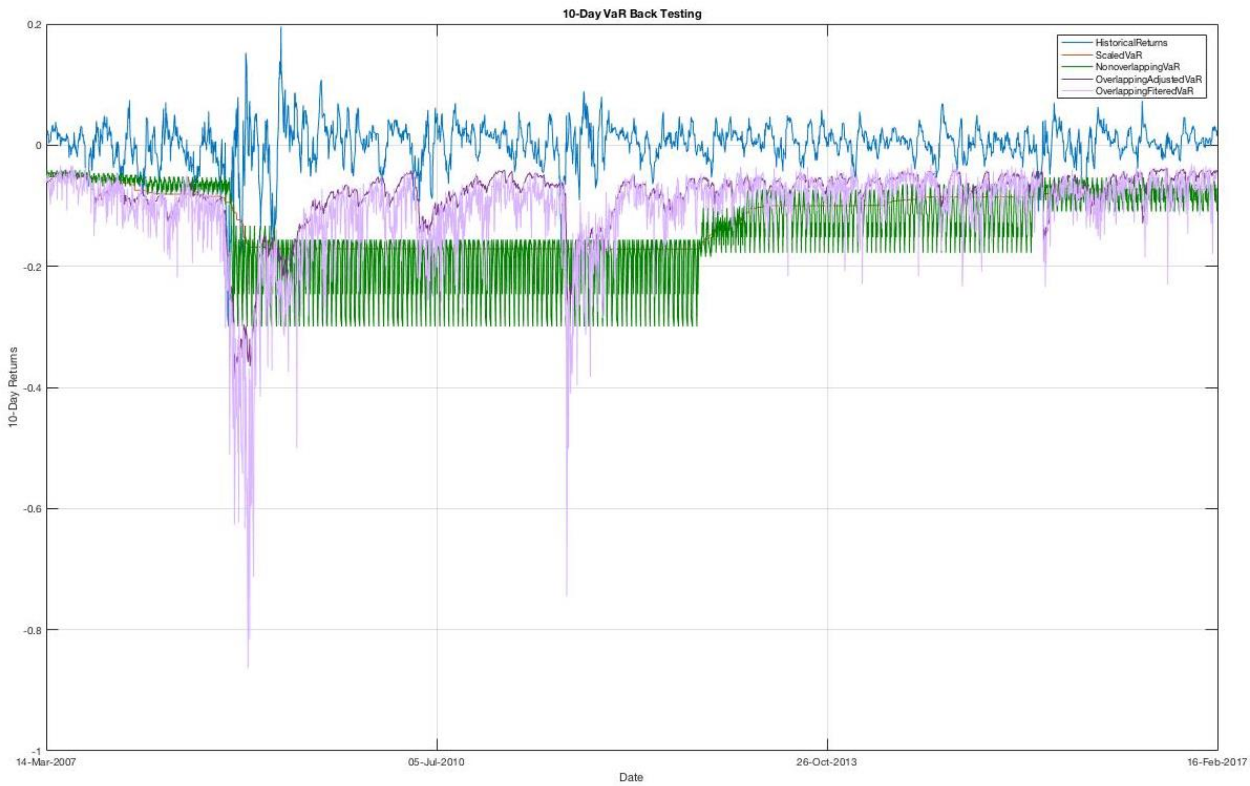

Figure 6: Comparison of 4 Approaches at Estimation Window = 1000

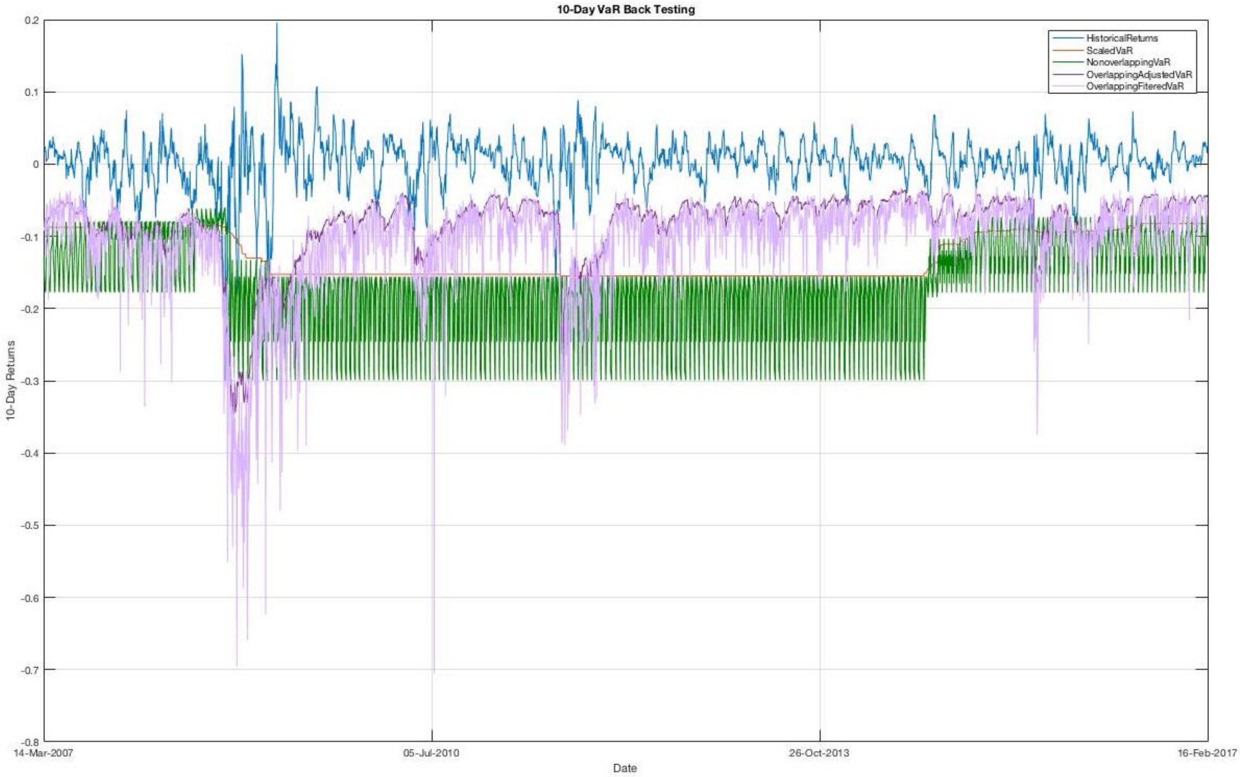

Figure 7: Comparison of 4 Approaches at Estimation Window = 1500

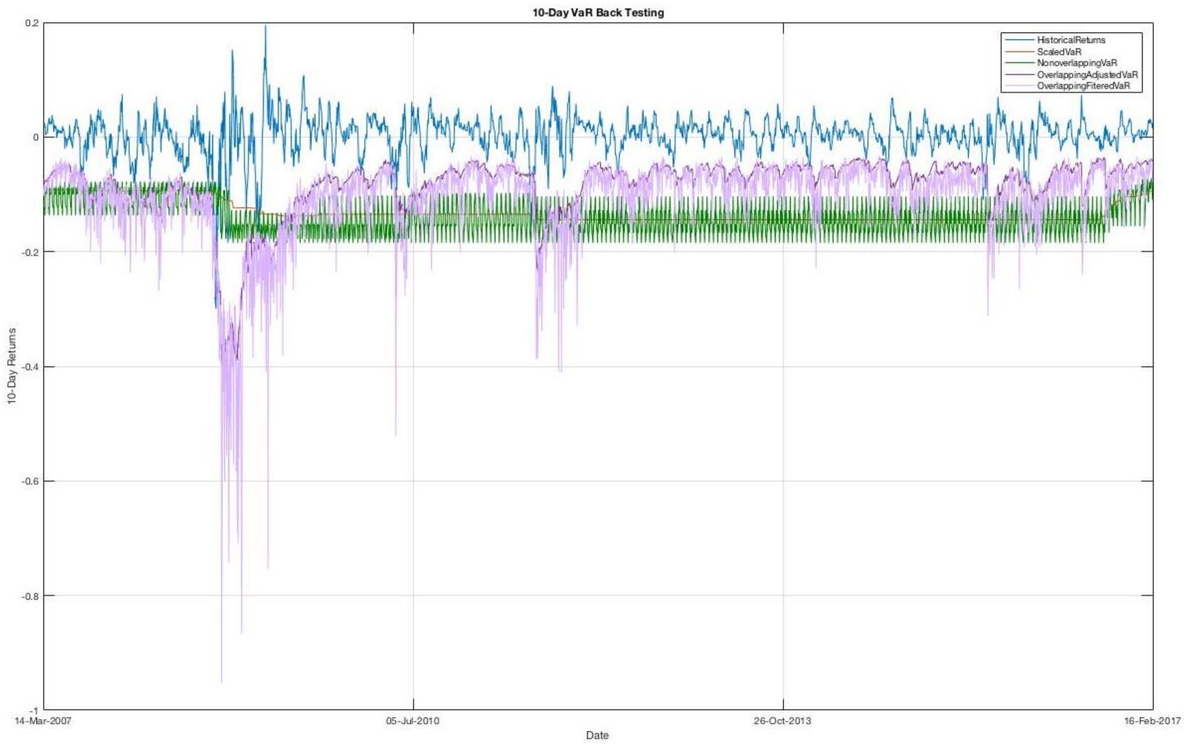

Figure 8: Comparison of 4 Approaches at Estimation Window = 2000

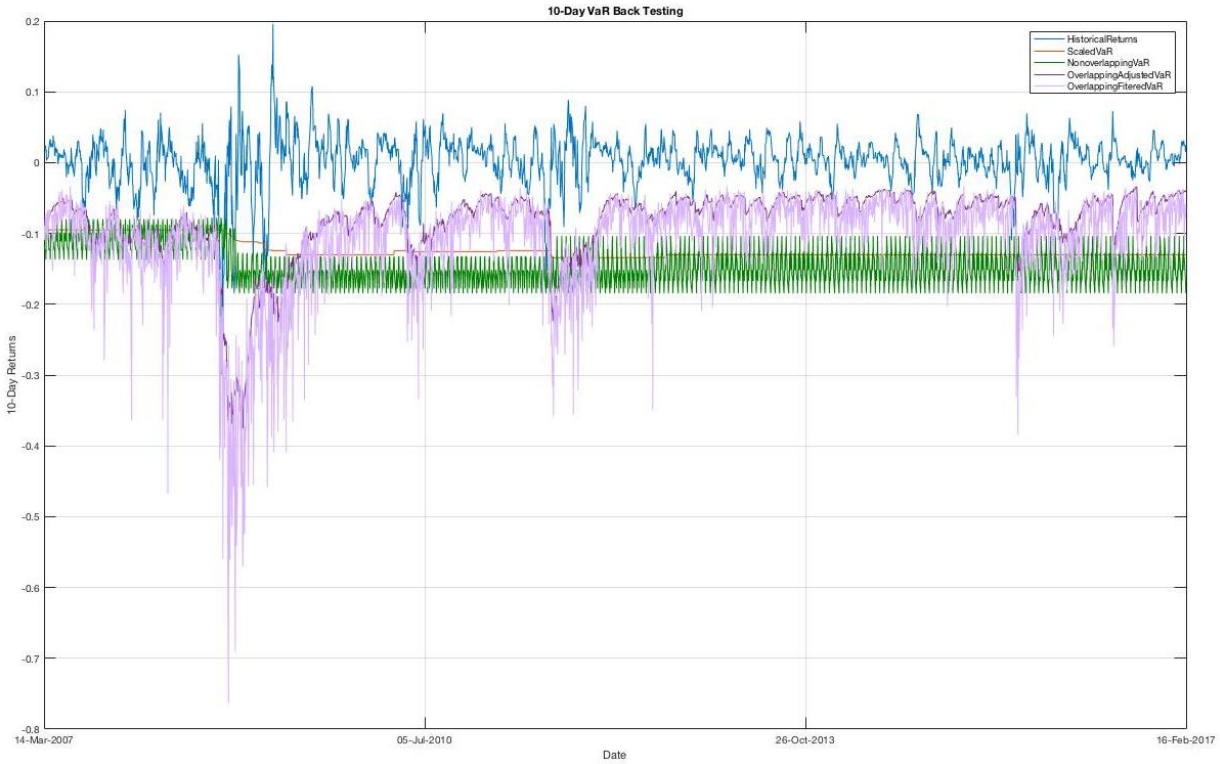

Figure 9: Comparison of 4 Approaches at Estimation Window = 2500

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Appendix B: Matlab Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Name: Jonathan Nolan                                                  %
% Student Number: 16071514                                              %
% Module: Portfolio Risk Analysis (FI6012)                              %
% Assignment: Technical Assignment                                      %
%                                                                       %
% This code uses historical SPX_VIX_Ret to calculate the 10-day VaR     %
% and ES of an equity portfolio using 4 different approaches:           %
%   1st approach: 1-day Var/ES scaled to 10-day using sqrt(time)        %
%   2nd approach: Non-overlapping 10-day periods                        %
%   3rd approach: Overlapping adjusted returns                          %
%   4th approach: Overlapping 10 day periods and filtered historical    %
%                 simulation                                            %
%                                                                       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Clear workspace of past values before running code
clear


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Import equity portfolio data, calculate returns and dimensions needed
% for initlising arrays.

%Initialise Parameters
alpha = 0.01;                                 % significance level
H_1 = 1;                                      % 1-day risk horizon
H_2 = 10;                                     % 10-day risk horizon
Scale_Factor = sqrt(H_2);                     % square root of time
value = 1;                                    % portfolio value

% Initialise Share Price Data:
% All data taken from Bloomberg on 01/03/2017
% Daily SPX VIX Index values from 03/01/1990 - 01/03/2017
% Import equity portfolio data, calculate returns and dimensions needed
% for initlising arrays
SPX = xlsread('SPX_Historical_Last_Prices.xlsx',1, 'A:B','basic');

%calculates the daily returns from the prices
Returns = diff(log(SPX(:,2)));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Mod_Returns = mod(length(Returns), 500);              %Modulo returns
length_sample = length(Returns) - Mod_Returns + 9;    %Sample Length
Returns = Returns(end-length_sample+1 : end);         %Returns
Dates = SPX(end-length_sample+1:end, 1);              %Dates
Number_of_Dates = x2mdate(Dates, 0);                  %Converts date
number to matlab serial date
Sample_SPX = SPX(end-length_sample+1:end,2);          %sample size

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% Data Analysis and Pretesting
FirstTimeseries = fints(Number_of_Dates, Sample_SPX, {'Historical_SPX'}, 0,
'SPX_index' );
SecondTimeseries = fints(Number_of_Dates, Returns, {'SPX_returns'}, 0,
'SPX_returns' );


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Graphs
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Comparison of Historical S&P 500 Returns with Index
figure(1)
subplot(2, 1, 2);
x = plot(FirstTimeseries);
set(x(1), 'color', 'blue');
xlabel('Date');ylabel('Historical S&P500 Index');
title('S&P 500 Index')
subplot(2,1,1);
plot(SecondTimeseries);
xlabel('Date');ylabel('Historical S&P500 Returns');
title('S&P 500 Returns');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Create Histograms for Returns Data
% Plot histogram of daily Optimized Portfolio Daily Returns
figure (2)
histfit(Returns, 500)
title('Histogram of SPX Histroical Daily Returns')
xlabel('Historical Daily Returns')
ylabel('Frequency')
string_1 = {'Mean Return:' ,num2str(mean(Returns)),...
 'Excess Kurtosis:' ,num2str(kurtosis(Returns)-3),...
 'Skewness: ' ,num2str(skewness(Returns))};
annotation('textbox', [0.6,0.7,0.055,0.21],'String', string_1);
Histogram_Kurtosis = kurtosis(Returns);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%QQplot Comparison of Daily Returns versus Standard Normal
figure(3)
qqplot(Returns);
xlabel('Standard Normal Quantiles');
ylabel('Quantiles of SPX Daily Returns');
title('SPX Daily Returns VS Standard Normal')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Autocorrelation Test
figure(4)
subplot(1, 2, 2)
autocorr(Returns);
title('Daily Returns Autorrelation Test');
subplot(1, 2, 1)
autocorr(Returns.^2);
title('Squared Returns  Autorrelation Test');
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Calculate the following:
% 10-day historical returns
% 10-day period daily volatilities

Ten_day_returns = zeros(length_sample-9, 1);
Volatility = zeros(length_sample-9, 1);

for i = 1: length_sample-9
        Ten_day_returns(i,1) = sum(Returns(i:i+9));
        Volatility(i,1) = std(Returns(i:i+9));
end


Ten_day_date = Number_of_Dates(10:length_sample);
ThirdTimeseries = fints(Ten_day_date,
Ten_day_returns,{'TenDayHistoricalReturns'},0,...
'TenDayReturns' );
FourthTimeseries = fints(Ten_day_date, Volatility, {'Volatility'},0,
'TenDayVolatilities' );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Graph of 10-Day Historical Returns Versus 10-day period Volatilities
figure(5)
subplot(2, 1, 2);
y = plot(ThirdTimeseries);
set(y(1), 'color', 'blue');
xlabel('Date');ylabel('10-Day Historical Returns');
title('10-Day Historical Returns')
subplot(2,1,1);
plot(FourthTimeseries);
xlabel('Date');ylabel('Volatility');
title('10-Day period Volatilities');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Graph comparing the returns versus the 10-day returns
figure(6)
subplot(2, 1, 2);
y = plot(ThirdTimeseries);
set(y(1), 'color', 'blue');
xlabel('Date');ylabel('10-Day Historical Returns');
title('10-Day Historical Returns')
subplot(2,1,1);
plot(SecondTimeseries);
xlabel('Date');ylabel('Historical S&P500 Returns');
title('S&P 500 Returns');
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Set Testing Window length
Test_Window = 2500;

% Adjust Date Length
Dates = Number_of_Dates(length_sample-Test_Window-8: length_sample-9);

%Historical Observations
Historical_OB = Ten_day_returns(end-Test_Window+1: end);

%creates Daily VaR matrix
VaR_Daily = zeros(Test_Window, 1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Approaches
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%1st Approach: 1-day Var/ES scaled to 10-day using sqrt(time)

%Creates matrix for values
Approach_1_VaR = zeros(Test_Window, 5);
Approach_1_ES = zeros(Test_Window, 5);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for l_length = 1:5

    %Estimation Window
    Approach1_Est_Window = 500*l_length;

    % Set Estimation Window length
    T = Approach1_Est_Window+Test_Window;

    % Length adjustment
    Sample_Returns = Returns(length_sample-T-9: length_sample);

    for j_n = 1 : Test_Window
        j_n_1 = Approach1_Est_Window+j_n;
        j_n_2 = Approach1_Est_Window+j_n+9;
        Approach1_Ret = Sample_Returns(j_n: j_n+Approach1_Est_Window-1);
        length1 = length(Approach1_Ret);
        Sort_1 = sort(Approach1_Ret);
        Optimised_1 = floor(length1*alpha);
        Daily_VaR(j_n, l_length) = -Sort_1(Optimised_1)*value;
        Approach_1_VaR(j_n, l_length) = Scale_Factor*Daily_VaR(j_n,
l_length);
        Approach_1_ES(j_n, l_length) =
Scale_Factor*mean(Sort_1(1:Optimised_1))*value;
    end
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%2nd Approach: Non-overlapping 10-day periods

%Creates matrix for values
Approach_2_VaR = zeros(Test_Window, 5);
Approach_2_ES = zeros(Test_Window, 5);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for l_length = 1:5
    %Estimation Window
    Approach2_Est_Window = 500*l_length;
    % Set Estimation Window length
    T2 = Approach2_Est_Window + Test_Window;
    Sample_Returns2 = Returns(length_sample-T2-9: length_sample);
    for j_n = 1 : Test_Window
        j_n_1 = Approach2_Est_Window+j_n;
        j_n_2 = Approach2_Est_Window+j_n+9;

        for Approach2_Ret = zeros(Approach2_Est_Window/10,1);
        c = 0;
        for u = j_n+9 : 10 : j_n+Approach2_Est_Window;
            Approach2_Ret(c+1) = sum(Sample_Returns2(u-9:u));
            c = c + 1;
        end

        length2 = length(Approach2_Ret);
        Sort_2 = sort(Approach2_Ret);
        Optimised_2 = floor(length2*alpha);

        if Optimised_2 == 0
            Optimised_2 = 1;
        end
        Approach_2_VaR(j_n, l_length) = -Sort_2(Optimised_2)*value;
        Approach_2_ES(j_n, l_length) = mean(Sort_2(1:Optimised_2))*value;
        end
    end;
end;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%3rd Approach: Overlapping 10-day VaR&ES with Garch(1, 1) adjusted returns

Approach_3_VaR = zeros(Test_Window, 5);
Approach_3_ES = zeros(Test_Window, 5);        %

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for l_length = 1:5
    Approach3_Est_Window = 500*l_length;%Estimation Window
    T = Approach3_Est_Window+Test_Window;% Set Estimation Window length
    Sample_Returns3 = Returns(length_sample-T-9: length_sample);

    for j_n = 1 : Test_Window
        j_n_1 = Approach3_Est_Window+j_n;
        j_n_2 = Approach3_Est_Window+j_n+9;

        % Garch(1,1) fitting and calibration
        % Estimate conditional volatilities using MATLAB infer function
        Garchsample = Sample_Returns3(j_n: j_n+Approach3_Est_Window-1);
        Mdl = garch(1,1);
        EstMdl = estimate(Mdl, Garchsample, 'display', 'off');
        Garch_vol = sqrt(infer(EstMdl,Garchsample,'E0', Garchsample(1)));
        Constant = EstMdl.Constant;
        if isempty(EstMdl.ARCH)==1
            Parameter1 = 0;
        else
            Parameter1 = cell2mat(EstMdl.ARCH);
        end
        if isempty(EstMdl.GARCH)==1
           Parameter2 = 0;
        else
           Parameter2 = cell2mat(EstMdl.GARCH);
        end;

        %Approach 3
        Adjust_returns = zeros(Approach3_Est_Window, 1);
            for p = 1 : Approach3_Est_Window
                Adjust_returns(p, 1) =
(Garch_vol(end)/Garch_vol(p,1))*Garchsample(p, 1);
            end
        len = (Approach3_Est_Window/10)+9*(Approach3_Est_Window/10-1);
        Approach3_Ret = zeros(len, 1);
        g = 0;
        for n = 1 : 10
            for m = n+9 : 10 : Approach3_Est_Window
            Approach3_Ret(g+1, 1) = sum(Adjust_returns(m-9:m));
            g = g + 1;
            end
        end
        length3 = length(Approach3_Ret);
        length3 = length3 - mod(length3, 10);
        Sort_3 = sort(Approach3_Ret);
        Optimised_3 = floor(length3*alpha);
```

```
        if Optimised_3 == 0
           Optimised_3 = 1;
        end
        Approach_3_VaR(j_n, l_length) = -Sort_3(Optimised_3)*value;
        Approach_3_ES(j_n,l_length) = mean(Sort_3(1:Optimised_3))*value;
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%4th Approach: Overlapping 10-day VaR&ES with Garch(1, 1) filtered returns

Approach_4_VaR = zeros(Test_Window, 5);
Approach_4_ES = zeros(Test_Window, 5);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for l_length = 1:5
    Approach4_Est_Window = 500*l_length;
    T = Approach4_Est_Window+Test_Window;
    Sample_Returns4 = Returns(length_sample-T-9: length_sample);

    for j_n = 1 : Test_Window
        j_n_1 = Approach4_Est_Window+j_n;
        j_n_2 = Approach4_Est_Window+j_n+9;

        % Garch(1,1) fitting and calibration
        % Estimate conditional volatilities using MATLAB infer function
        Garchsample4 = Sample_Returns4(j_n: j_n+Approach4_Est_Window-1);
        Mdl = garch(1,1);
        EstMdl = estimate(Mdl, Garchsample4, 'display', 'off');
        Garch_vol4 = sqrt(infer(EstMdl,Garchsample4,'E0',
Garchsample4(1)));
        Constant = EstMdl.Constant;
        if isempty(EstMdl.ARCH)==1
            Parameter1 = 0;
        else
            Parameter1 = cell2mat(EstMdl.ARCH);
        end
        if isempty(EstMdl.GARCH)==1
           Parameter2 = 0;
        else
           Parameter2 = cell2mat(EstMdl.GARCH);
        end;

        %Approach 4
        Adjust_returns = zeros(Approach4_Est_Window, 1);
        for p = 1 : Approach4_Est_Window
            Adjust_returns(p, 1) =
(Garch_vol4(end)/Garch_vol4(p,1))*Garchsample4(p, 1);
        end
        len = Approach4_Est_Window/10+9*(Approach4_Est_Window/10-1);
        returns3 = NaN(len, 1);
        g = 0;
        for n = 1 : 10
            for m = n+9 : 10 : Approach4_Est_Window
            returns3(g+1, 1) = sum(Adjust_returns(m-9:m));
            g = g + 1;
            end
        end
```

```
        Standard_Residuals = Garchsample4./Garch_vol4;
        sigma0 = Garch_vol4(end);
        r0 = Garchsample4(end);
        conditionalvariance = NaN(11, 1);
        Forecast_returns = NaN(10, 1);
        Simulations = 5000;
        Approach_Ret4 = NaN(Simulations, 1);
        conditionalvariance(1, 1) = Constant + Parameter1*(r0^2) +
Parameter2*(sigma0^2);

        b = 1;
        while b < Simulations+1
            for c = 2: 11
                z = randperm(Approach4_Est_Window,1);
                Forecast_returns(c-1, 1) = sqrt(conditionalvariance(c-1,
1))*Standard_Residuals(z, 1);
                conditionalvariance(c, 1) = Constant +
Parameter1*(Forecast_returns(c-1, 1)^2)...
                    + Parameter2*(conditionalvariance(c-1, 1));
            end
            Approach_Ret4(b, 1) = sum(Forecast_returns);
            b = b + 1;
        end

        length4 = length(Approach_Ret4);
        Sort_4 = sort(Approach_Ret4);
        Optimised_4 = ceil(length4*alpha);
        Approach_4_VaR(j_n,l_length) = -Sort_4(Optimised_4)*value;
        Approach_4_ES(j_n,l_length) = mean(Sort_4(1:Optimised_4))*value;
    end
end


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Create Matrices for Backtesting VaR and ES
Violation_Ratio = zeros(4,5);                         % Violation Ratio
Norm_Shortfall = zeros(4,5);                          % Normalised Shortfall
bertest = zeros(4, 5);                                % Bernolii Test
indtest = zeros(4, 5);                                % Independence Test

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Bernolii coverage test - Independence test
First_Sample_Test = zeros(Test_Window, 5);
Second_Sample_Test = zeros(Test_Window, 5);
Third_Sample_Test = zeros(Test_Window, 5);
Forth_Sample_Test = zeros(Test_Window, 5);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


for O_period = 1:5


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %Exceedances
    %Violation Ratio
```

```
    %Normalised Shortfall

    %Approach 1
    Approach1_exceedances = find(Historical_OB < -
Approach_1_VaR(:,O_period));
    Violation_Ratio(1, O_period) =
length(Approach1_exceedances)/(alpha*(Test_Window));
    Norm_Shortfall(1, O_period) =
mean(Historical_OB(Approach1_exceedances)./-
Approach_1_ES(Approach1_exceedances, O_period));

    %Approach 2
    Approach2_exceedances = find(Historical_OB < -
Approach_2_VaR(:,O_period));
    Violation_Ratio(2, O_period) =
length(Approach2_exceedances)/(alpha*(Test_Window));
    Norm_Shortfall(2, O_period) =
mean(Historical_OB(Approach2_exceedances)./-
Approach_2_ES(Approach2_exceedances, O_period));

    %Approach 3
    Approach3_exceedances = find(Historical_OB < -
Approach_3_VaR(:,O_period));
    Violation_Ratio(3, O_period) =
length(Approach3_exceedances)/(alpha*(Test_Window));
    Norm_Shortfall(3, O_period) =
mean(Historical_OB(Approach3_exceedances)./-
Approach_3_ES(Approach3_exceedances, O_period));

    %Approach 4
    Approach4_exceedances = find(Historical_OB < -
Approach_4_VaR(:,O_period));
    Violation_Ratio(4, O_period) =
length(Approach4_exceedances)/(alpha*(Test_Window));
    Norm_Shortfall(4, O_period) =
mean(Historical_OB(Approach4_exceedances)./-
Approach_4_ES(Approach4_exceedances, O_period));


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    % Bernoulli coverage test and independance test
    First_Sample_Test(Approach1_exceedances, O_period) = 1;
    bertest(1, O_period) = bern_test(alpha,First_Sample_Test(:, O_period));
    indtest(1, O_period) = ind_test(First_Sample_Test(:, O_period));

    Second_Sample_Test(Approach2_exceedances, O_period) = 1;
    bertest(2, O_period) = bern_test(alpha,Second_Sample_Test(:,
O_period));
    indtest(2, O_period) = ind_test(Second_Sample_Test(:, O_period));

    Third_Sample_Test(Approach3_exceedances, O_period) = 1;
    bertest(3, O_period) = bern_test(alpha,Third_Sample_Test(:, O_period));
    indtest(3, O_period) = ind_test(Third_Sample_Test(:, O_period));

    Forth_Sample_Test(Approach4_exceedances, O_period) = 1;
    bertest(4, O_period) = bern_test(alpha,Forth_Sample_Test(:, O_period));
    indtest(4, O_period) = ind_test(Forth_Sample_Test(:, O_period));
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %displays matrices containing the answers
    disp([O_period, Violation_Ratio(1, O_period), Norm_Shortfall(1,
O_period), bertest(1, O_period), 1-chi2cdf(bertest(1, O_period),1),...
        indtest(1, O_period),1-chi2cdf(indtest(1, O_period),1);...
        O_period, Violation_Ratio(2, O_period), Norm_Shortfall(2, O_period)
bertest(2, O_period),1-chi2cdf(bertest(2, O_period),1),...
        indtest(2, O_period), 1-chi2cdf(indtest(2, O_period),1);...
        O_period, Violation_Ratio(3, O_period), Norm_Shortfall(3,
O_period), bertest(3, O_period), 1-chi2cdf(bertest(3, O_period),1),...
        indtest(3, O_period),1-chi2cdf(indtest(3, O_period),1);...
        O_period, Violation_Ratio(4, O_period), Norm_Shortfall(4, O_period)
, bertest(4, O_period),1-chi2cdf(bertest(4, O_period),1),...
        indtest(4, O_period), 1-chi2cdf(indtest(4, O_period),1)] );


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    Data = [Historical_OB -Approach_1_VaR(:,O_period) -
Approach_2_VaR(:,O_period)  -Approach_3_VaR(:,O_period) -
Approach_4_VaR(:,O_period)];
    FifthTimeseries = fints(Dates, Data, {'HistoricalReturns',...
    'ScaledVaR', 'NonoverlappingVaR','OverlappingAdjustedVaR',
'OverlappingFiteredVaR'}, 0, 'Back testing 10-day VaR' );
    figure(O_period+6)
    plot(FifthTimeseries);
    xlabel('Date');
    ylabel('10-Day Returns');
    title('10-Day VaR Back Testing');
    colormap(jet);

end
```

```
%%Bernoulli Coverage test
function res=bern_test(p,v)
    a = p^(sum(v))*(1-p)^(length(v)-sum(v));
    b = (sum(v)/length(v))^(sum(v))*(1-(sum(v)/length(v)))^(length(v)-sum(v));
    res = -2*log(a/b);
end
```

```
%%Independence Test
function res = ind_test(V)
T = length(V);
J = zeros(T,4);
for i = 2:T
J(i,1) = V(i - 1) == 0 & V(i) == 0;
J(i,2) = V(i - 1) == 0 & V(i) == 1;
J(i,3) = V(i - 1) == 1 & V(i) == 0;
J(i,4) = V(i - 1) == 1 & V(i) == 1;
end
V_00 = sum(J(:,1));
V_01 = sum(J(:,2));
V_10 = sum(J(:,3));
V_11 = sum(J(:,4));
p_00 = V_00/(V_00 + V_01);
p_01 = V_01/(V_00 + V_01);
p_10 = V_10/(V_10 + V_11);
p_11 = V_11/(V_10 +V_11);
hat_p = (V_01 + V_11)/(V_00 + V_01 + V_10 + V_11);
a = (1 - hat_p)^(V_00 + V_10)*(hat_p)^(V_01 +V_11);
b = (p_00)^(V_00)*(p_01)^(V_01)*(p_10)^(V_10)* p_11^(V_11);
res = -2 * log(a/b);
end
```